

## **The Impact of Disability Insurance on a Life Insurer's Risk Situation**

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### **ABSTRACT**

Because of the rising demand of disability insurance, the management of the added disability risk within an insurance portfolio is becoming increasingly important. The aim of this paper is therefore to study the impact of disability insurance on the risk situation of a portfolio also consisting of annuity and term life insurance. Particularly, the overall risk is examined and potential hedging strategies are identified. Our results show that the addition of disability insurance to the portfolio lowers the overall risk, but is a less efficient tool to hedge shocks to mortality. However, the added disability risk is very sensitive to shocks and is not hedged by life insurances.

*Keywords:* Disability insurance, life, insurance, longevity risk, mortality risk, natural hedging, risk management

*JEL classification:* G22, G23, G32, J11

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## 1. INTRODUCTION

Due to the reform of the German statutory pension program in 2001, the coverage of disabilities by the German social security system has been considerably reduced and hence, individual disability insurance is considered to be existential (see Graf von der Schulenburg, 2005; Schwark, 2011). The importance of disability insurance has been additionally pointed out by Chandra and Samwick (2005), who concluded that precautionary saving is a less useful hedge against disability risk. As a result, the demand of disability insurance has been increasing. During the last decade, the total number of disability insurance policies sold, the sum insured and also the corresponding shares in the German life insurance market have been increasing, according to reports about the business development in the German life insurance market issued by the German Insurance Association (GDV) from 2003 to 2011. In order to thoroughly analyze the liability side of a life insurance company, disability insurance policies must therefore be added to the portfolio containing annuity and term life insurances. As other contributions to research usually regard a portfolio with annuity and life insurance only, the aim of this paper is to study the risk situation of a life insurance company that additionally offers disability insurance policies and that is subject to mortality and disability risk.

Mortality in general is influenced by various factors including socioeconomic factors, biological variables, government policies, environmental influences, health conditions and health behaviors (see Rogers, 2002). Stochastic models to model mortality and to capture longevity risk were introduced by Cairns, Blake, and Dowd (2006a) and Dahl (2004) for example and a review on discrete and continuous time models for mortality rates is given by Cairns, Blake, and Dowd (2008). Alternatively, the model proposed by Lee and Carter (1992) may be fitted to the mortality data. This model has been extended, for instance, by Cox, Lin, and Pedersen (2010) to capture deviations in mortality rates due to permanent longevity jumps and temporary mortality jumps. In contrast, Brouhns, Denuit, and Vermunt (2002a) have improved it by embedding it in a Poisson regression model. In this paper, we will utilize the model by Brouhns, Denuit, and Vermunt (2002a) to model the mortality rates. We thereby neglect the impact of macroeconomic fluctuations and causes of death on mortality trends in the Lee-Carter model, as studied by Hanewald (2011), and acknowledge that ignoring the link between macroeconomic fluctuations and the mortality index may lead to an underestimation of the true probability of default of the assumed insurance company, as concluded by Hanewald, Post, and Gründl (2011).

In the literature concerning mortality risk, one objective is to hedge this risk which comprises all forms of uncertainty that are related to future mortality rates (see, e.g., Cairns, Blake, and Dowd, 2006a). The mortality risk consists of systematic and unsystematic risk as well as basis risk (see Gatzert and Wesker, 2012a). The systematic mortality risk refers to changes in the underlying mortality intensity and affects all individuals (see, e.g., Cox and Lin, 2007; Dahl and Møller, 2006). Hence, this component of the mortality risk is not diversifiable (see, e.g., Cairns, Blake, and Dowd, 2008), but may be reduced either by including a safety loading in the premium or by transferring parts of it to the insured, i.e. by offering mortality-linked contracts (see Dahl, 2004; Dahl, Melchior, and Møller, 2008; Richter and Weber, 2011). The systematic risk itself includes the longevity risk which follows from higher than expected survival probabilities because of unforeseen improvements in medical science for instance (see, e.g., Blake et al., 2006; Cox, Lin, and Pedersen, 2010; Wetzel and Zwiesler, 2008). The impact of longevity risk on the solvency of pension funds has been studied by Hári et al. (2008). In contrast to systematic risk, the unsystematic mortality risk refers to the randomness of deaths in a life insurance portfolio, given a fixed mortality intensity, and therefore, it is diversifiable (see, e.g., Dahl and Møller, 2006). This risk may be decreased by expanding the insured population, i.e. risk pooling (see, e.g., Biffis, Denuit, and Devolder, 2010). The third component of the mortality risk, the basis risk is induced by adverse selection which results in the difference between mortality of policyholders and the mortality observed in the population as a whole (see Gatzert and Wesker, 2011; Gatzert and Wesker, 2012a).

The management of mortality risk is required by modern risk management practice and therefore, it is a part of enterprise risk management (see Cairns, Blake, and Dowd, 2008). The mortality risk inherent to life insurance or annuity contracts may be hedged using financial instruments such as longevity bonds which are also known as mortality bonds (see Dowd et al., 2006). In general, longevity bonds are financial instruments whose payments depend on the outcome of a survivor index at a certain time  $t$  (see, e.g., Blake et al., 2006). Here, the survivor index describes the proportion of survivors at time  $t$  in relation to an initial reference population (see, e.g., Cairns, Blake, and Dowd, 2006b). In theory, longevity bonds are regarded as an opportunity to perfectly match the liabilities and to reduce the systematic risk to zero. In practice though, the existence of basis risk prevents a perfect hedge by longevity bonds because, for instance, either the maturities of liabilities and bonds do not match or the age group underlying the survivor index does not correspond to the age group of policyholders (see Barbarin, 2008; Hári et al., 2008). Depending on the type of bond used, the longevity bond may be subdivided into longevity zeros and survivor bonds for instance.

Longevity zeros are a financial security with a single payout at maturity, whose value at maturity is equal to the survivor index, whereas the coupons of survivor bonds are linked to the proportion of a reference population which is still alive when the coupon is due (see, e.g., Blake and Burrows, 2001; Blake et al., 2006). For further information on pricing and the effectiveness of longevity bonds as a hedging tool, we additionally refer to Cairns, Blake, and Dowd (2006a), Ngai and Sherris (2011), Wills and Sherris (2010) and references therein.

Similar to survivor bonds, survivor swaps are defined as swaps whose future outcome is linked to at least one survivor index. Nevertheless, survivor swaps have several advantages over survivor bonds. These swaps may exploit the natural hedging opportunities between two lines of life insurance business or two life insurers and, in addition, they are introduced at lower transaction costs than a bond issue. Also, the cancellation is easier and the flexibility is higher (see Dowd et al., 2006). Based on the availability of survivor swaps, risk-minimizing strategies for a portfolio consisting of general life insurance can therefore be derived, as performed by Dahl, Melchior, and Møller (2008). An alternative design of swaps as a risk management tool is proposed by Cox and Lin (2007), who construct a so-called mortality swap by means of strike levels for the mortality.

Instead of applying bonds and swaps, an insurance company also has the opportunity to transfer the mortality risk to the financial market via mortality forwards (see Ribeiro and di Pietro, 2009). Mortality forwards, also referred to as q-forwards, pay an actual mortality rate in exchange for a fixed mortality rate (see Ngai and Sherris, 2011). From an investor's point of view, these mortality derivatives are also of interest as they may serve as a tool to diversify the investment portfolio (see Dahl, Melchior, and Møller, 2008). The effectiveness of hedging strategies that incorporate mortality forwards were analyzed by Cairns (2011) and Ngai and Sherris (2011).

As an alternative method to mortality-linked financial instruments, the risk management may utilize natural hedging effects to absorb the mortality risk. Natural hedging is a dynamic process that aims to stabilize the aggregate cash outflows resulting from an insurance product portfolio (see Cox and Lin, 2007) and that is based on the adverse development of the value of liabilities due to changes in mortality (see Milevsky and Promislow, 2002). Cox and Lin (2007) empirically study the existing natural hedging effect in the insurance industry and conclude that insurers, who utilize natural hedging, charge a lower premium than otherwise similar insurers and, thus, have a competitive advantage. For this reason, previous literature has proposed several models to analyze the natural hedging. Bayraktar and Young (2007), for

instance, price life insurance and pure endowment individually and in combination. They reveal that combined pricing leads to lower premiums than a separate pricing. In contrast, Gründl, Post, and Schulze (2006) study the impact of natural hedging based on a shareholder value maximizing strategy and detect that natural hedging is preferred if equity is scarce, but may lower the shareholder value otherwise. Wetzel and Zwiesler (2008) decompose the variance of the cash flows and derive a product mix which minimizes the variance of the prediction risk, whereas Tsai, Wang, and Tzeng (2010) propose an optimization strategy based on the Conditional Value-at-Risk of the total loss proportion. They additionally compare their results with the immunization strategy by the use of mortality durations and mortality convexities, which is implemented by Wang et al. (2010). A Delta-Gamma hedging framework for mortality risk is developed by Luciano, Regis, and Vigna (2012) and extended by Luciano, Regis, and Vigna (2011) by the integration of natural hedging. In contrast, Gatzert and Wesker (2012b) construct a multi-period framework taking into account the assets and liabilities of a life insurance company in order to quantify the effectiveness of natural hedging with and without continued underwriting activities. A similar model was used by Gatzert and Wesker (2011) to evaluate natural hedging under adverse selection and by Gatzert and Wesker (2012a) to assess natural hedging in combination with mortality contingent bonds. None of these papers though has integrated disability insurances in the portfolio to examine possible hedging effects between annuity, disability and term life insurance and to study the impact of disability risk.

In this paper, we aim to fill this gap and, thus, we incorporate disability insurances in the multi-period framework by Gatzert and Wesker (2012b) to analyze and to quantify existing hedging effects between annuity, disability and term life insurance. We therefore consider the insurance company as a whole and provide a more holistic view of an insurer's long term risk situation. However, possible basis risk, financial instruments or optional reinsurance will not be regarded in this paper. In order to ensure the comparability between the modeling of these insurance types, we implement a Markov renewal model for all three of these. Our results show that, although disability insurance is less efficient to hedge mortality risk and life insurances in general are not suitable to limit the impact of disability risk, an optimal product mix may nevertheless decrease the solvency risk inherent to the insurance portfolio.

The remainder of this article is structured as follows. In Section 2, the mortality model, the pricing model and the framework are presented. Section 3 contains results of the numerical analyses and Section 4 concludes.

## 2. MODEL FRAMEWORK

In this chapter, we begin with the modeling of mortality rates based on the model proposed by Lee and Carter (1992) and continue with the computation of survival probabilities and the associated waiting time distributions. Then, we introduce general Markov renewal models as a model framework for life and disability insurance and state the specific set of states used in this paper, i.e. active and dead as well as disabled in the case of disability insurance. Afterwards, we present the model framework for the insurance company as stated in Gatzert and Wesker (2012a) and name the risk measures utilized in this paper.

### 2.1 The modeling of mortality rates

Lee and Carter (1992) proposed a method which has been frequently applied to model and predict different mortality rates (Lee, 2000). In their model, Lee and Carter (1992) split the natural logarithm of the age-specific mortality rate  $\mu_x(t)$  at age  $x$  during period  $t$  into age-specific components  $\alpha_x$  and  $\beta_x$  as well as a time-varying parameter  $\kappa_t$ :

$$\ln(\mu_x(t)) = \alpha_x + \beta_x \cdot \kappa_t + \varepsilon_x(t) \Leftrightarrow \mu_x(t) = e^{\alpha_x + \beta_x \cdot \kappa_t + \varepsilon_x(t)}.$$

While  $e^{\alpha_x}$  is the time-independent general shape of the mortality rate,  $\kappa_t$  describes the time trend of the mortality rate and  $\beta_x$  indicates the sensitivity of the mortality rate regarding changes in the time trend. The homoskedastic centered error terms of the model are denoted by  $\varepsilon_x(t)$  and two components are subject to the following constraints:

$$\sum_t \kappa_t = 0 \text{ and } \sum_x \beta_x = 1.$$

Based on the approach by Lee and Carter (1992), Brouhns, Denuit, and Vermunt (2002a) integrate this approach in a Poisson regression model:

$$D_{xt} \sim \text{Poisson}(E_{xt} \cdot \mu_x(t)).$$

This modified model assumes that the number of deaths recorded at age  $x$  during period  $t$  ( $D_{xt}$ ) is Poisson distributed with an expected value equal to the product of the mortality rate  $\mu_x(t)$  and the exposure-to-risk  $E_{xt}$ . The exposure-to-risk thereby denotes the number of person years from which  $D_{xt}$  occurred and, according to Brouhns, Denuit, and Vermunt (2002b), may be computed with

$$E_{xt} = \frac{-n(t-1) \cdot q_x(t)}{\ln(p_x(t))},$$

where  $p_x(t)$  denotes the one-period survival probability,  $q_x(t)$  the one-period death probability and  $n(t-1)$  the observed population size by age  $x$  and year  $t-1$ . The resulting log-likelihood, which is given by

$$L((\alpha_x), (\beta_x), (\kappa_t)) = \sum_{x,t} \{ D_{xt} (\alpha_x + \beta_x \cdot \kappa_t) - E_{xt} \cdot e^{\alpha_x + \beta_x \cdot \kappa_t} \} + const,$$

can be maximized by utilizing the iterative, uni-dimensional Newton method as generally proposed by Goodman (1979). Brouhns, Denuit, and Vermunt (2002a) state the corresponding updating scheme for the Poisson regression.

Using the methodology in Box and Jenkins (1976) or the modeling procedure in Pandit and Wu (1990), an appropriate ARIMA time series model is fitted to the estimated time trend of the mortality rate  $\kappa_t$ . The obtained ARIMA model is then used to forecast the time trend and, by incorporating the time-constant components  $\alpha_x$  and  $\beta_x$ , the mortality rate. Based on the projected mortality rate  $\mu_x(t)$ , the one-period survival probability for a person aged  $x$  in period  $t$  can be computed by  $p_x(t) = \exp(-\mu_x(t))$  (see Brouhns, Denuit, and Vermunt, 2002a). Furthermore, the associated waiting time distribution  ${}_x F_{ij}(s, t)$ , which describes the probability that a transition from an alive state  $i$  to the dead state  $j$  occurs up to time  $t$  on condition that an  $x$ -year old individual entered state  $i$  at time  $s$  and that  $j$  is the next state, can be calculated by

$${}_x F_{ij}(s, t) = \sum_{\vartheta=0}^{t-s} {}_x f_{ij}(s, s + \vartheta) \text{ with}$$

$${}_x f_{ij}(s, s + \vartheta) = \begin{cases} (1 - p_{x+\vartheta-1}(s + \vartheta - 1)) \cdot \prod_{\vartheta'=1}^{\vartheta-1} p_{x+\vartheta'-1}(s + \vartheta' - 1), & \text{if } s \leq T - 1, \vartheta \in [1, T - s] \\ 0 & s = T \text{ or } \vartheta = 0 \end{cases},$$

where

$T$  denotes the time horizon and  ${}_x f_{ij}(s, s + \vartheta)$  depicts the probability that a transition from an alive state  $i$  to the dead state  $j$  takes place exactly at time  $s + \vartheta$ , given that  $j$  is the next state and that an individual aged  $x$  entered state  $i$  at time  $s$ . Both, active and disabled states are regarded as alive states. Since homogeneous age groups will be regarded in the subsequent analysis, the index  $x$  is omitted in the following.

## 2.2 The discrete time non-homogeneous Markov renewal process

Based on the notations and definitions given in D'Amico, Guillen, and Manca (2009), the model framework applied to disability insurance, term life insurance and annuity insurance is presented in this subchapter. This general framework consists of a discrete time non-homogeneous Markov renewal process and the associated probabilities will be marked with additional superscripts in the next subchapter to distinguish between the different policy types.

Let  $J_n$  and  $T_n$  be two random variables that denote the state occupied at the  $n$ -th transition and the time of the  $n$ -th transition, respectively. The state space for  $T_n$  is described by  $\mathbb{N}$ , whereas the state space for  $J_n$  is referred to as  $I = \{1, \dots, m\}$ , with  $m$  being the final and absorbing state 'death'. Then, for the associated Markov renewal process  $(J_n, T_n)$ , the following transition probabilities are defined  $\forall i, j \in I, \forall s, t \in \mathbb{N}, s \leq t$ :

$$Q_{ij}(s, t) = P[J_{n+1} = j, T_{n+1} \leq t | J_n = i, T_n = s] \text{ and } p_{ij}(s) = \lim_{t \rightarrow \infty} Q_{ij}(s, t).$$

The first transition probability  $Q$  thereby describes the probability that the successive state  $j$  is entered up to time  $t$  and the second transition probability  $p$  denotes that state  $j$  is the next state occupied, regardless of the time of the associated transition. The matrix  $P(s) = [p_{ij}(s)]$  is introduced as the transition matrix, which corresponds to the embedded non-homogeneous Markov chain in the process. Both transition probabilities  $Q$  and  $p$  are conditional upon the state  $i$  being entered at time  $s$  and are constrained by the following assumptions:

1.  $Q_{ij}(s, s) = 0, \forall i, j \in I, \forall s \in \mathbb{N}$  and
2.  $Q_{ii}(s, t) = 0, t - s > 0, s = 1, 2, \dots$

The first assumption forbids multiple transitions at any time  $s$  and the second restriction excludes artificial transitions from a state to itself. Based on the previously defined transition probabilities and on condition that the successive state  $j$  is known, the distribution function of the waiting time in the current state  $i$  is defined as

$$F_{ij}(s, t) = P[T_{n+1} \leq t | J_n = i, J_{n+1} = j, T_n = s] = \begin{cases} Q_{ij}(s, t) / p_{ij}(s) & \text{if } p_{ij}(s) \neq 0 \\ 1 & \text{if } p_{ij}(s) = 0. \\ 0 & \text{if } i = j = m \end{cases}$$

This waiting time distribution is crucial in the context of disability insurance as well as term life and annuity insurance as it determines the time of disability and of death. The same notation as compared to the waiting time distribution in the previous subchapter was used on purpose because the waiting time distributions for transitions to the state ‘death’ will be based on the Poisson regression model by Brouhns, Denuit, and Vermunt (2002a). Furthermore, with  $m$  being the number of states in the regarded model, the following probabilities can be defined:

$$b_{ij}(s,t) = P[J_{n+1} = j, T_{n+1} = t | J_n = i, T_n = s] = \begin{cases} Q_{ij}(s,t) - Q_{ij}(s,t-1) & \text{if } t > s \\ 0 & \text{if } t = s \end{cases} \text{ and}$$

$$d_{ij}(s,t) = P[T_{n+1} > t | J_n = i, T_n = s] = \begin{cases} 1 - \sum_{j=1}^m Q_{ij}(s,t) & \text{if } i = j \\ 0 & \text{if } i \neq j. \end{cases}$$

Probability  $b_{ij}(s,t)$  is almost equivalent to probability  $Q$  except that the transition to state  $j$  takes place exactly at time  $t$ . The distribution  $d_{ij}(s,t)$  represents the probability that the current state  $i$ , which was entered at time  $s$ , will not be left up to time  $t$ . For notational reasons, this definition makes sense iff  $i=j$ . The previously defined probabilities  $b_{ij}(s,t)$  and  $d_{ij}(s,t)$  are extended by conditioning them on time already spent the present state, i.e. the initial backward recurrence time (see, e.g., D’Amico, Guillen, and Manca, 2009; Stenberg, Manca, and Silvestrov, 2007):

$$b_{ij}(l,s;t) = P[J_{n+1} = j, T_{n+1} = t | J_n = i, T_n = l, T_{n+1} > s] = \begin{cases} 0 & \text{if } d_{ii}(l,s) = 0 \text{ or } t = s \\ \frac{b_{ij}(l,t)}{d_{ii}(l,s)} & \text{otherwise,} \end{cases}$$

$$d_{ij}(l,s;t) = P[T_{n+1} > t | J_n = i, T_n = l, T_{n+1} > s] = \begin{cases} \frac{d_{ii}(l,t)}{d_{ii}(l,s)} & \text{if } i = j \\ 0 & \text{if } i \neq j \text{ or if } d_{ii}(l,s) = 0. \end{cases}$$

The difference between the original and extended probabilities is that  $b_{ij}(s,t)$  and  $d_{ij}(s,t)$  are conditional upon state  $i$  being entered at time  $s$ , while  $b_{ij}(l,s;t)$  and  $d_{ij}(l,s;t)$  assume that the health state  $i$  did not change after time  $l$  up to time  $s$ . These probabilities are of importance for the computation of the book values as defined below.

The Markov renewal model can be simulated using the probabilities  $b_{ij}(s,t)$  and  $d_{ij}(s,t)$ . After each transition at time  $s$  or at the beginning when  $s=0$ , the current state  $i$  is recorded and a random number  $\xi$  is drawn. If  $\xi > 1 - d_{ii}(s,T)$ , the last transition during the time horizon has occurred at time  $s$  and the policyholder stays in state  $i$ . Otherwise, the next state  $k$  is entered at time  $u$  and the pair  $(k,u)$  satisfies the following inequalities:

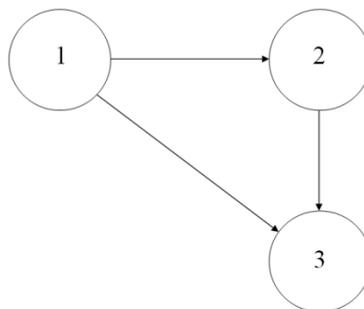
$$\sum_{j=1}^k \sum_{u'=s+1}^{u-1} b_{ij}(s,u') < \xi \leq \sum_{j=1}^k \sum_{u'=s+1}^u b_{ij}(s,u').$$

The inequalities essentially express the inversion method because the left side is equal to zero in the case of  $u=0$  and the right side is equal to  $1 - d_{ii}(s,T)$  as soon as  $u=T$  and  $k=m$ .

### 2.3 Multiple state models in disability and life insurance

In the context of disability insurances, several state models have been suggested and implemented. Generally, a three-state-model is utilized to display the health states of the policyholder: active (1), disabled (2) and dead (3). In the case of permanent disability, recoveries, i.e. transitions from the disabled state to the active state, are not allowed (see, e.g., Pitacco, 2004), while in the case of potentially temporary disability, recoveries are considered (see, e.g., Christiansen, 2012). In addition, the disabled state can be further split according to the duration of the disability and further states can be added to account for lapses and pensioners (see, e.g., D'Amico, Guillen, and Manca, 2009; Haberman and Pitacco, 1999). In this paper, we will employ a three-state-model without recoveries to model the disability insurance contract. The corresponding set of states and set of transitions are reflected in Figure 1.

**Figure 1:** Set of states and set of transitions for the disability insurance model



Mortality, and hence term life as well as annuity insurances, can be modeled using a two-state-model with transitions only from the active state (1) to the dead state (2) (see

Macdonald, 1996). To ensure the comparability between the model used for the disability insurance on one hand and the term life as well as annuity insurance on the other hand, we will apply a Markov renewal model to all insurance contracts. In the case of term life and annuity insurances, the transition probability from the active state to the dead state is equal to one and the waiting time distribution is only affected by the rate of mortality. The associated set of states and set of transitions are depicted in Figure 2.

**Figure 2:** Set of states and set of transitions for the term life and annuity insurance model



## 2.4 Model of an insurance company

The regarded fictitious insurance company offers disability, term life and annuity insurances and, analogous to Gatzert and Wesker (2012b), its risk situation is analyzed by applying a simplified balance sheet as exhibited in Table 1.

**Table 1:** Balance sheet of the insurance company at time  $t$

Assets	Liabilities
$A(t)$	$B^A(t)$
	$B^D(t)$
	$B^L(t)$
	$E(t)$
$A(t)$	$A(t)$

The market value of the assets of the insurance company at time  $t$  is denoted by  $A(t)$ . Furthermore, the book value of the liabilities for the offered insurance contracts is referred to by  $B^k(t)$  with superscript  $k$  specifying the regarded type of insurance: annuity insurance ( $A$ ), disability insurance ( $D$ ) and term life insurance ( $L$ ).  $E(t)$  is the insurer's equity at time  $t$ .

The shareholder of the insurance company make an initial investment  $E(0)$  and future values of the equity are determined as the difference between assets  $A(t)$  and liabilities  $L(t) = B^A(t) + B^D(t) + B^L(t)$ . After the payment of benefits and in return for the initial investment, the

insurance company is obliged to pay a dividend to the shareholders at time  $t$  if the earnings during period  $[t-1, t]$  are positive. In this case, the shareholders receive a constant fraction  $r_e$  of the earnings:

$$div(t) = r_e \cdot \max(E^*(t) - E(t-1); 0)$$

with  $E^*(t)$  being the value of equity at time  $t$  before the dividend is paid (see Gatzert and Wesker, 2012a).

The assets of the insurer are invested in the capital market and yield a rate of return  $\varepsilon_t$  at each time  $t$ . This rate of return  $\varepsilon_t$  is a normal distributed random variable with an expected value  $\mu_\varepsilon$  and a standard deviation of  $\sigma_\varepsilon$ . Hence,

$$A(t) = A(t-1) \cdot \exp(\varepsilon_t) \text{ with } \varepsilon_t \sim N(\mu_\varepsilon, \sigma_\varepsilon^2).$$

In addition to the yield, the asset base is also influenced by premiums, benefits and the previously described dividends. At the beginning of the time horizon, i.e. time  $t = 0^+$ , the asset base of the insurance company consists of the initial equity by the shareholders and the premiums received in  $t = 0$ . Thus, shocks to input parameters do not affect the initial balance sheet. From the insurer's perspective and for all insurance contracts, the premiums are received in advance, while the benefits and dividends are paid in arrears. The resulting development of the cash flows is depicted in Figure 3.

**Figure 3:** Development of the cash flows in a discrete time environment

$t = 0^+$	$t = 1^-$	$t = 1^+$	$t = 2^-$	$t = 2^+$	...
$+E(0)$	$- div$		$- div$		...
$+ premiums$		$+ premiums$		$+ premiums$	...
	$- benefits$		$- benefits$		...

The book values of liabilities of each insurance contract at each time  $t$ , i.e. the prospective reserve for all cash flows after time  $t$ , are computed based on the respective Markov renewal model which is extended by rewards. In this paper, we follow the approach in Stenberg, Manca, and Silvestrov (2007) to incorporate rewards in the Markov renewal model and to derive a closed-form solution. The descriptions of rewards used in the reward model are summarized in Table 2.

**Table 2:** Description of rewards

Parameter	Description
$\psi_1^A$	benefit (annuity insurance), also: annuity
$\psi_1^D$	premium (disability insurance)
$\psi_2^D$	benefit (disability insurance)
$\psi_1^L$	premium (term life insurance)
$\psi_2^L$	benefit (term life insurance)

The risk-free interest rate is denoted by  $r$ . Let  $B_i^A(t)$  be the expected book value of liabilities resulting from the annuity insurance at time  $t$  for being in state  $i$  and let reward  $\psi_1^A$  be the annuity paid in arrears. Then, with  $n_1^A(t)$  denoting the number of alive policyholders at time  $t$ ,  $B_i^A(t)$  is computed as

$$\begin{aligned} B_1^A(t) &= n_1^A(t) \cdot \left( d_{11}^A(0, t; T) \cdot \sum_{\vartheta=1}^{T-t} \psi_1^A \cdot e^{-r \cdot \vartheta} + \sum_{\vartheta=1}^{T-t} b_{12}^A(0, t; t + \vartheta) \cdot \left[ \sum_{\vartheta'=1}^{\vartheta-1} \psi_1^A \cdot e^{-r \cdot \vartheta'} \right] \right) \\ &= n_1^A(t) \cdot \left( \sum_{\vartheta=1}^{T-t} \left( \frac{1 - F_{12}^A(0, t + \vartheta)}{1 - F_{12}^A(0, t)} \right) \cdot \psi_1^A \cdot e^{-r \cdot \vartheta} \right). \end{aligned}$$

The first summand considers the case that the policyholder stays in the initial active state which implies the reception of annuities throughout the considered time horizon. The second summand accounts for death occurring during the contract duration inferring a stop of annuities at and after the time of death. As dead policyholders do not receive further annuities, being in state 2 implies zero further liabilities, i.e.  $B_2^A(t) = 0, \forall t$ .

The expected book value of the liabilities of the disability insurance at time  $t$ , given that the individual has been in health state  $i$  since time  $s$ , is denoted by  $B_i^D(s, t)$ . In this case,  $\psi_1^D$  describes the premium and  $\psi_2^D$  the disability benefit, while no death benefits are considered and, thus,  $B_3^D(s, t) = 0, \forall s, t$ . The number of active policyholders at time  $t$  is described with  $n_1^D(t)$  and the number of disabled policyholders at time  $t$  is specified with  $n_2^D(t)$ . The expected book values are calculated as

$$\begin{aligned} B_1^D(\bullet, t) &= \\ n_1^D(t) \cdot &\left( d_{11}^D(0, t; T) \cdot \sum_{\vartheta=0}^{T-t-1} -\psi_1^D \cdot e^{-r \cdot \vartheta} + \sum_{\vartheta=1}^{T-t} b_{12}^D(0, t; t + \vartheta) \cdot \left[ \sum_{\vartheta'=0}^{\vartheta-1} -\psi_1^D \cdot e^{-r \cdot \vartheta'} + B_2^D(0, t + \vartheta) \cdot e^{-r \cdot \vartheta} \right] \right) \\ &\left( + \sum_{\vartheta=1}^{T-t} b_{13}^D(0, t; t + \vartheta) \cdot \left[ \sum_{\vartheta'=0}^{\vartheta-1} -\psi_1^D \cdot e^{-r \cdot \vartheta'} \right] \right) \end{aligned}$$

and

$$B_2^D(s, t) = n_2^D(t) \cdot \left( d_{22}^D(t-s, t; T) \cdot \sum_{\vartheta=1}^{T-t} \psi_2^D \cdot e^{-r \cdot \vartheta} + \sum_{\vartheta=1}^{T-t} b_{23}^D(t-s, t; t+\vartheta) \cdot \left[ \sum_{\vartheta'=1}^{\vartheta} \psi_2^D \cdot e^{-r \cdot \vartheta'} \right] \right).$$

In the first closed-form formula, the expected book value for being in the active state is determined. The first summand describes that the initial active state is not left and hence, only premiums are received. On the contrary, the second summand considers a transition to the disabled state during the contract duration. Here, premiums are received up to the inception of disability and the expected book value of the upcoming disability benefits is assessed by  $B_2^D(s, t)$ . At last, direct transitions from the active state to death are included in the third summand. In this case, premiums are received up to the time of death. The second closed-form solution computes the expected book value of liabilities, given a disabled policyholder. The disability benefits are either paid for the remaining time horizon, as described in the first summand, or paid up to the time of death, as depicted in the second summand.

The expected book value for the term life insurance at time  $t$ ,  $B_i^L(t)$ , is computed as follows:

$$\begin{aligned} B_1^L(t) &= n_1^L(t) \cdot \left( d_{11}^L(0, t; T) \cdot \sum_{\vartheta=0}^{T-t-1} -\psi_1^L \cdot e^{-r \cdot \vartheta} + \sum_{\vartheta=1}^{T-t} b_{12}^L(0, t; t+\vartheta) \cdot \left[ \sum_{\vartheta'=0}^{\vartheta-1} -\psi_1^L \cdot e^{-r \cdot \vartheta'} + \psi_2^L \cdot e^{-r \cdot \vartheta} \right] \right) \\ &= n_1^L(t) \cdot \left( \sum_{\vartheta=0}^{T-t-1} \left( \psi_2^L \cdot \frac{F_{12}^L(0, t+\vartheta+1) - F_{12}^L(0, t+\vartheta)}{1 - F_{12}^L(0, t)} \cdot e^{-r \cdot (\vartheta+1)} - \psi_1^L \cdot \frac{1 - F_{12}^L(0, t+\vartheta)}{1 - F_{12}^L(0, t)} \cdot e^{-r \cdot \vartheta} \right) \right) \end{aligned}$$

$$B_2^L(t) = 0.$$

Here,  $n_1^L(t)$  stands for the number of alive policyholders at time  $t$ . The first summand in the first closed-form solution notes that the policyholder stays in the initial active state and hence, premiums are received for each point in time during the contract duration. The second summand considers the case that the death of the policyholder occurs before the end of the duration and therefore, premiums are paid up to the time of death and then, the death benefit is paid. As described by  $B_2^L(t)$ , the book value of the liabilities for dead policyholders is equal to zero.

In the subsequent numerical analysis, the three insurance types are supposed to have the same individual contract volume<sup>1</sup>  $V$ . Thus, based on the previous formulas for the expected book values, fair premiums and benefits must be computed according to the equivalence principle. The potential default of the insurance company is not considered in the premium calculation because external institutions are supposed to fulfill remaining contractual obligations in the case of insolvency (see, e.g., Gatzert and Wesker, 2012b). For the annuity insurance, the annuity  $\psi_1^A$  must solve the following equation:

$$V = \frac{B_1^A(0)}{n_1^A(0)} = d_{11}^A(0; T) \cdot \sum_{\vartheta=1}^T \psi_1^A \cdot e^{-r \cdot \vartheta} + \sum_{\vartheta=1}^T b_{12}^A(0; \vartheta) \cdot \left[ \sum_{\vartheta'=1}^{\vartheta-1} \psi_1^A \cdot e^{-r \cdot \vartheta'} \right].$$

The premium  $\psi_1^D$  and the disability benefit  $\psi_2^D$  of the disability insurance policy must fulfill the following equation

$$\begin{aligned} V^D &= d_{11}^D(0, T) \cdot \sum_{\vartheta=0}^{T-1} \psi_1^D \cdot e^{-r \cdot \vartheta} + \sum_{\vartheta=1}^T \left( b_{12}^D(0, \vartheta) + b_{13}^D(0, \vartheta) \right) \cdot \sum_{\vartheta'=0}^{\vartheta-1} \psi_1^D \cdot e^{-r \cdot \vartheta'} \\ &= \sum_{\vartheta=1}^T b_{12}^D(0, \vartheta) \cdot \frac{B_2^D(0, \vartheta)}{n_2^D(\vartheta)} \cdot e^{-r \cdot \vartheta} \\ &= \sum_{\vartheta=1}^T b_{12}^D(0, \vartheta) \cdot \left\{ d_{22}^D(\vartheta; T) \cdot \sum_{\vartheta'=1}^{T-\vartheta} \psi_2^D \cdot e^{-r \cdot \vartheta'} + \sum_{\vartheta'=1}^{T-\vartheta} b_{23}^D(\vartheta; \vartheta + \vartheta') \cdot \left[ \sum_{\vartheta''=1}^{\vartheta'} \psi_2^D \cdot e^{-r \cdot \vartheta''} \right] \right\} \cdot e^{-r \cdot \vartheta} \\ &\stackrel{!}{=} V, \end{aligned}$$

while for the term life insurance, the premium  $\psi_1^L$  and the death benefit  $\psi_2^L$  must satisfy this equation:

$$V^L = d_{11}^L(0, T) \cdot \sum_{\vartheta=0}^{T-1} \psi_1^L \cdot e^{-r \cdot \vartheta} + \sum_{\vartheta=1}^T b_{12}^L(0, \vartheta) \cdot \sum_{\vartheta'=0}^{\vartheta-1} \psi_1^L \cdot e^{-r \cdot \vartheta'} = \sum_{\vartheta=1}^T b_{12}^L(0, \vartheta) \cdot \psi_2^L \cdot e^{-r \cdot \vartheta} \stackrel{!}{=} V.$$

These equations may be solved by a bisection method, as described in Dadkhah (2011), because the expressions are continuous on the interval  $[0, \infty)$  and monotonically increasing in the respective premium or benefit.

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<sup>1</sup> In this paper, at time 0, the contract volume corresponds to the present value of expected benefit payments.

## 2.5 Risk measurement

In our numerical analysis, we will utilize several risk measures to evaluate the risk situation of the insurance company at each point in time, namely the probability of default, the mean loss and the expected shortfall as downside risk measures as well as the standard deviation of the liabilities as a deviation risk measure.

The probability of default ( $PD$ ) measures the frequency of default and is defined as the first point in time when the liabilities  $L(t)$  exceed the assets  $A(t)$ :

$$PD = P(T_d \leq T) \text{ with } T_d = \begin{cases} \inf \{t \in [0, T] : A(t) < L(t)\} & \text{if } \exists t \in [0, T] : A(t) < L(t) \\ T+1 & \text{otherwise.} \end{cases}$$

The second downside risk measure is the mean loss which measures the discounted expected loss in the case of a default of the insurer. Thus, unlike the probability of default, the mean loss ( $ML$ ) also considers the difference between the assets and liabilities at the time of default and it is defined as:

$$ML = E \left[ \left( L(T_d) - A(T_d) \right) \cdot e^{-rT_d} \cdot \mathbf{I}\{T_d \leq T\} \right].$$

Here,  $\mathbf{I}\{\cdot\}$  stands for an indicator function. Unlike the mean loss, the third applied risk measure, the expected shortfall ( $ES$ ), quantifies the expected loss conditional upon default (see Hull, 2011). It may be computed based on the probability of default and the mean loss:

$$ES = \frac{ML}{PD}.$$

Therefore, the expected shortfall extends the mean loss, as it also assesses the likelihood of default, and as a result, it may serve as an indicator for the magnitude of default (see Gatzert and Wesker, 2012b). The last measure that we will apply is the standard deviation of liabilities at each time  $t$ , i.e.

$$\sigma(L(t)) = \sigma(B^A(t) + B^D(t) + B^L(t)).$$

### 3. NUMERICAL ANALYSIS

In this section, the needed mortality rates are estimated and forecasted based on demographic data from Spain. After introducing the various input parameters, the risk situation of an insurance company offering annuity, disability and term life insurances is analyzed. Then, shocks to general mortality rates and mortality rates of policyholders are studied. Afterwards, the impact of disability risk and its sensitivity are examined.

#### 3.1 Mortality estimation and projections

The mortality rates for all regarded groups of insured are based on the number of deaths and the exposure-to-risk in the Spanish male population from 1950 to 2009 which are retrieved from the Human Mortality Database<sup>2</sup>. The demographic parameters, as defined in Lee and Carter (1992), were estimated with the regression model in Brouhns, Denuit, and Vermunt (2002a) and the results of  $\alpha_x$  and  $\beta_x$  are displayed in Appendix A.1. Because of its non-stationarity, the first differenced series of the estimated mortality trend  $\kappa_t$  was modeled with an ARIMA model and the order of the autoregressive as well as of the moving average dependence were determined by the modeling procedure given in Pandit and Wu (1990). The resulting ARIMA(2,1,1) model was tested by the Box-Pierce test and, using a significance level of 5%, by ACF as well as PACF and this analysis showed no significant residual autocorrelation. The indicated ARIMA model has the following parameters with their respective standard error in parentheses: an autoregressive part with parameters  $\phi_1 = 0.4978$  (0.1157),  $\phi_2 = 0.5020$  (0.1157) and a moving average part with parameter  $\theta_1 = -0.9842$  (0.0502). The estimated as well as forecasted mortality trend  $\kappa_t$  is shown in Appendix A.2.

#### 3.2 Input parameters

In this numerical analysis, we will solely regard male policyholders either aged 35 or aged 75. While the 35-year old males are buying either disability or term life insurances, the 75-year old males are just buying the annuity insurance. For both age groups, the contract duration is set to  $T=25$  years in year 2010. The three Markov renewal models, which are used to model these insurance contracts, access the same transition matrix with the entries being either equal to zero or to one, depending whether the respective transition is element of the set of transitions. But there are three exceptions: The empirical analyses in D'Amico, Guillen, and Manca (2009) and Hagen et al. (2011) suggest that the transition probability from the active

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<sup>2</sup> For further information, we are referring to the official website of the database [www.mortality.org](http://www.mortality.org).

state to the disabled state ( $p_{12}^D(s)$ ) is roundabout 20% across the ages 35 to 60 years, whereas the transition probability for an active policyholder to state death ( $p_{13}^D(s)$ ) is consequently equal to 80%. Therefore, we assume a homogeneous Markov renewal model with a transition probability  $p_{12}^D(s) = 20\%$  in our setup. In addition,  $p_{33}(s)$  is set to one to ensure a stochastic embedded Markov chain. The mortality rate for disabled insured is derived from the mortality of the whole population. Segerer (1993) points out that for a policyholder aged 35 at disablement, the mortality amounts to roughly 1100% of the active life mortality. Also, the mortality for disabled individuals is decreasing in age and is about 150% of the active life mortality at age 60. Hence, as a simplification, we multiply the general mortality  $\mu_x(t)$  with an assumed linear function  $c_x$  at age  $x$  where  $c_{35}=1100\%$  and  $c_{60}=150\%$  to obtain the mortality for disabled insured. The transition probabilities as well as the mortality rates are subject to variation. Based on the empirical results in D'Amico, Guillen, and Manca (2009), the waiting time from the active state to the disabled state is assumed to be logistically distributed with a mean of 30 years and a standard deviation of 10.88 years.

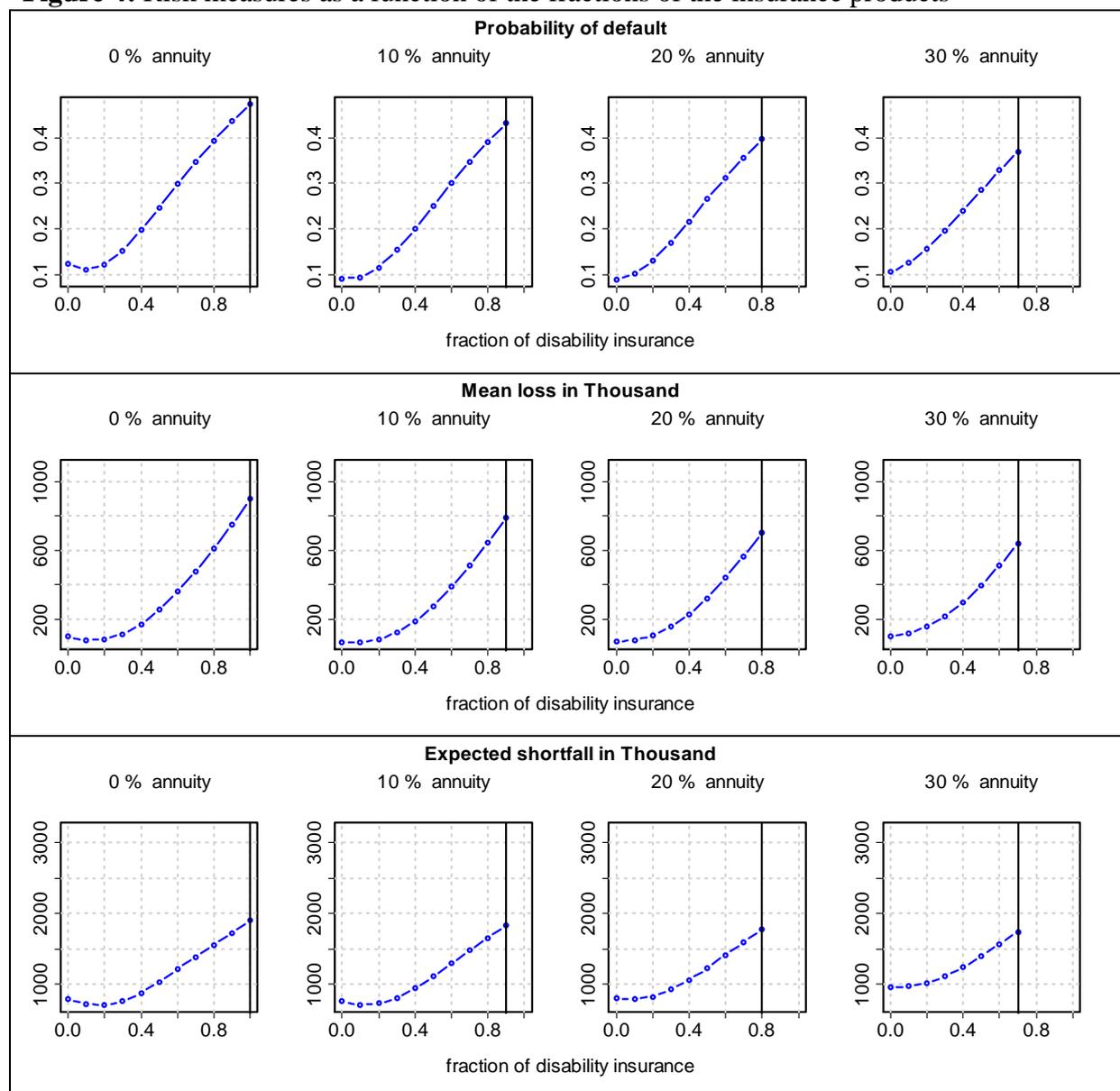
Independent of the resulting portfolio composition, the total amount of contracts sold is set to 10,000. In addition, the risk-free interest rate is fixed to  $r=3\%$  and the individual contract volume  $V$  is equal to 10,000 to ensure the comparability between the three regarded insurance types. Based on this risk-free interest rate and contract volume, the fair annuity  $\psi_1^A$  is equal to 808.64 and the single premium for the annuity insurance is  $V$ , whereas the fair premium for the disability insurance  $\psi_1^D$  is 572.44 and the fair disability benefit  $\psi_2^D$  is 46,309.59. Moreover, the resulting fair premium for the term life insurance  $\psi_1^L$  is equal to 566.25 and the death benefit  $\psi_2^L$  is 586,988.44.

In order to analyze the risk situation of the insurance company, a Monte Carlo simulation with 100,000 simulation paths is used and, for each simulation run and for each insurance type, 10,000 contracts are simulated based on the underlying Markov renewal model. The actual number of each insurance contract in the regarded portfolio is then considered by multiplying the number of policyholder in each state at any time with the share of the contract in the portfolio. The shareholders make an initial investment of 10 million at the beginning and receive a constant fraction of 25% of the earnings as a dividend in result. At each point in time, the book values for each type of contract is computed with the actuarial discount rate of  $r=3\%$ . In addition, the assets are calculated with an expected rate of return of  $\mu_\epsilon=5\%$  and a volatility of  $\sigma_\epsilon=8\%$ . The risk measures are then computed based on the determined liabilities and assets at each time. These input parameters are chosen for illustration purposes only.

### **3.3 The risk situation of an insurance company without shocks to input parameters**

In order to examine the risk situation of the fictive insurance company without any shocks to input parameters, the parameters used for the calculation of fair premiums and fair benefits coincide with the realized parameters, especially the mortality rate and the transition probability from the active to the disabled state. In addition, the total number of contracts in the portfolio and the contract volume for each insurance type remain constant to isolate the potential hedging effects. This first analysis is regarded as the base case and serves as a benchmark for the subsequent analysis.

Generally, the payouts resulting from the annuity insurance contracts decline over time, whereas the payouts for term life insurance increase. In contrast, the payout structure of disability insurance potentially involves the payment of several premiums and benefits. From the insurer's perspective, the cash inflow based on premium gradually declines as active policyholders either die or become disabled. The evolution of benefits being paid comprises policyholders, who become disabled and, thus, start to receive benefits, and disabled policyholder, who die during the contract duration implying a stop of benefits. Hence, depending on the chosen disability rates and mortality rates for disabled policyholders, the payment of benefits may theoretically fluctuate over time. In our analysis, the number of policyholders, who become disabled, exceeds the number of dying disabled policyholders at any time during the time horizon and, therefore, the cash outflow is steadily rising. Overall, the different payment structures provide opportunities for counterbalancing the payments, and, as a result, for risk reduction. The outcome of the different risk measures depending on the portfolio structure are shown in Figure 4.

**Figure 4:** Risk measures as a function of the fractions of the insurance products

The analysis of this base case revealed that portfolios solely consisting of disability insurance exhibit the highest risk. The risk can be reduced by either adding term life or annuity insurances to the portfolio. Within a portfolio consisting of term life and disability insurance, the optimal proportion of disability insurance is between 10% (according to the PD and ML) and 20% (according to the ES). In contrast, the optimal fraction of disability insurance in a portfolio also containing annuity insurances is between 30% (according to PD) and 70% (according to ES). Thus, increasing the proportion of annuity insurance lowers the optimal proportion of disability insurance. The addition of term life insurance to this portfolio leads to an even lower overall risk. In the case of three different insurance types in the portfolio, the PD is lowest for 20% annuity insurance and 80% term life insurance, while the ML and the

ES are smallest for 80% term life insurance and 10% of the other two insurance types. Regardless of the risk measure, the optimal portfolio features a high percentage of term life insurance and, depending on the initial portfolio composition and the regarded risk measure, the overall risk can be reduced up to 94%. Because the differences between the insurance types cannot arise from different contract values, the timing of the payment matters and a balanced portfolio composition may smooth the cash flows and also the liabilities from the various insurance types (see Figure 5).

**Figure 5:** Variance of liabilities

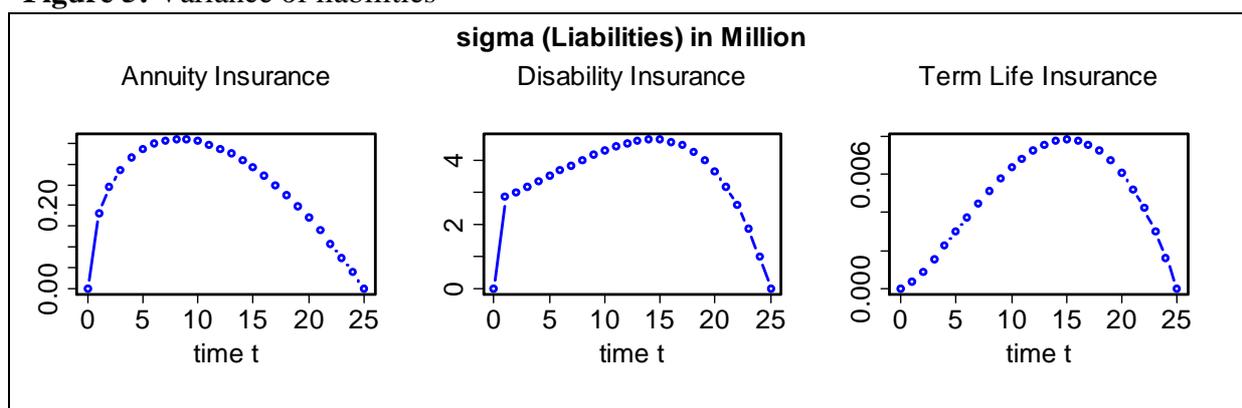


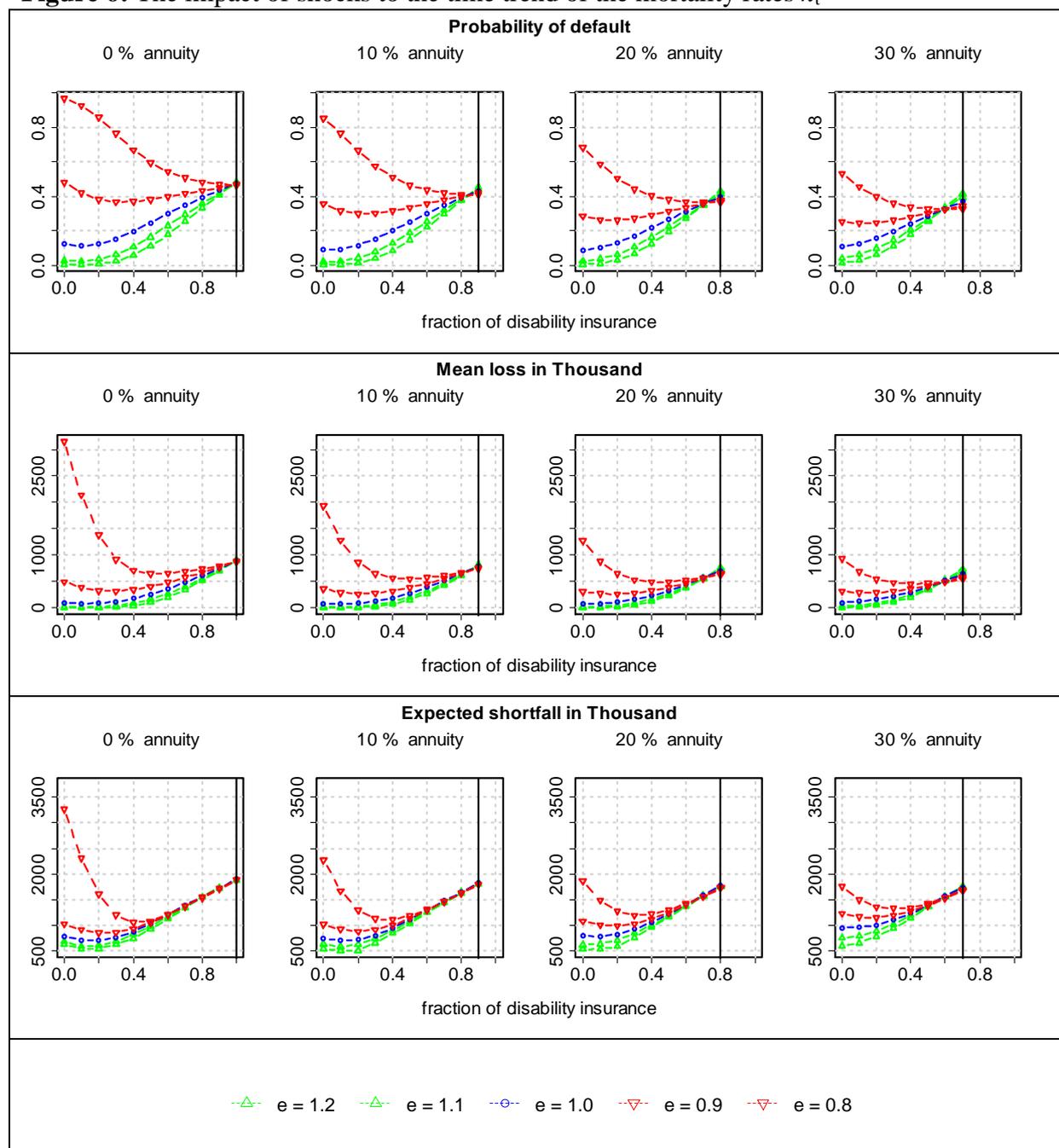
Figure 5 depicts the evolution of the standard deviation of the liabilities over the time horizon. The standard deviation of the liabilities resulting from annuity insurances reaches its maximum in the first half of the time horizon, whereas the variation of the liabilities arising from disability insurances and term life insurances have their maximum in the second half on the time horizon. As shown in this figure, disability insurances have the highest variation, followed by annuity insurances and, hence, the addition of term life insurances to the insurance portfolio may considerably lower the overall standard deviation over the time horizon. This figure also reveals that a disadvantage of disability insurances as a potential hedging tool may be the relatively high variation of liabilities.

Overall, these results imply that pure life insurers may decrease the risk inherent to their portfolio by adjoining a relatively small number of either disability or annuity insurance contracts. In contrast, pure annuity providers and especially pure disability insurers can reach the optimal composition only by a severe change of the portfolio. In all cases, changing the mixture in an existing portfolio through price adjustments for example can be difficult because providers of only one insurance type may lack the experience and the expertise with other insurance types. Also, altering the business composition of an insurance company can introduce additional expenses and operational risk (see Wang et al., 2010). So, in general,

instead of altering the portfolio and besides the usage of reinsurance, derivatives such as swaps may be of interest to balance the exposure to risk and utilize existing natural hedging opportunities (see, e.g., Dowd et al., 2006). This way, insurance companies may keep focusing on their specialized field.

### **3.4 The effect of natural hedging on the liability side**

The impact of shocks to mortality is modeled by multiplying the time trend of the mortality rates trend  $\kappa_t$  with the factor  $e$  after the calculation of the fair premium and fair benefits. Since the forecasted time trend is negative during the time horizon (see Figure A.2), factor  $e$  being smaller than one describes an increase in mortality (see red graphs in Figure 6) and, consequently, factor  $e$  being greater than one relates to a decrease in mortality (see green graphs in Figure 6). This constant shock for all  $\kappa_t$  implies a non-identical change of the mortality rates across all ages because of the age-specific sensitivity  $b_x$ . The base case, where the assumed and the realized parameters coincide, is depicted in Figure 6 and all following figures with the color blue.

**Figure 6:** The impact of shocks to the time trend of the mortality rates  $\kappa_t$ 

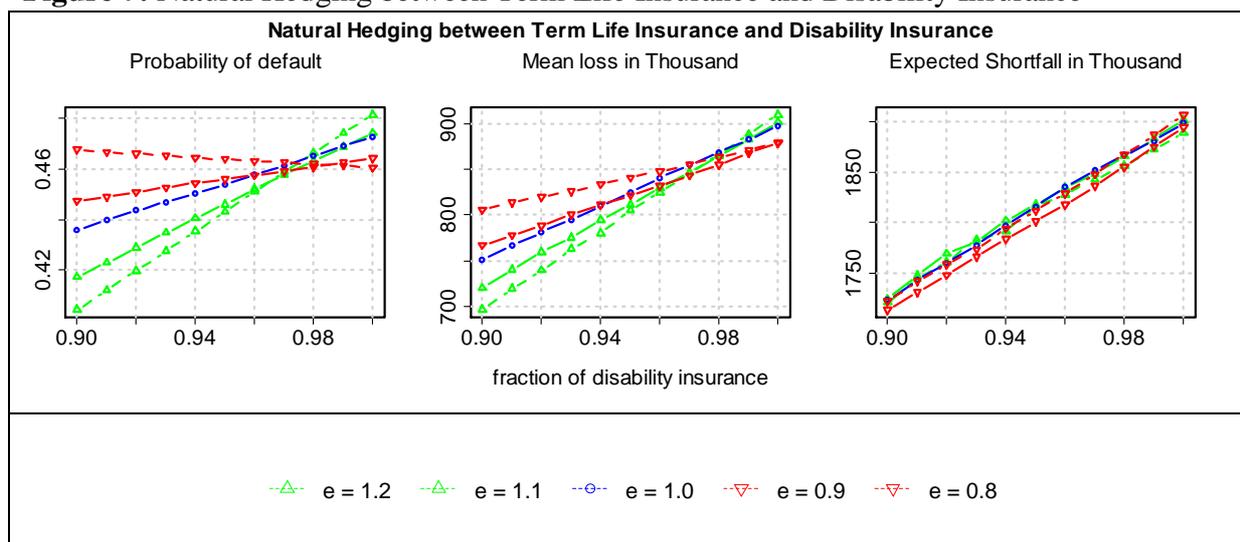
In the case of term life insurance, an increase in mortality has negative effects on the liabilities which is displayed in Figure 6, e.g. in the case with 0% annuity and 0% disability insurance. This insurance type is identified to have the highest sensitivity with regard to mortality risk. On the contrary, negative effects on the liabilities of annuity insurances arise from a reduced mortality because fewer annuities will be paid. This is illustrated in the plot with 30% annuity for example, where a decrease of mortality causes a higher PD in the case with 70% disability insurance. As concluded in Gatzert and Wesker (2012b) for instance, natural hedging opportunities exist between annuity and term life insurances. Thus, increasing

the proportion of annuity insurance contracts in the portfolio results in an augmented fraction of term life insurances to immunize against shocks to mortality (see Figure 6).

Shifts in mortality have a minor influence on the risk inherent to disability insurance contracts but an increase of mortality is slightly positive with respect to the insurer's risk situation. Here, the ML is the most sensitive and the ES the least sensitive to changes in mortality. In comparison with the other two insurance types, disability insurances are the least sensitive to changes in mortality risk because shifts in mortality are counterbalanced among the disability insurance contracts. For instance, a higher mortality implies that fewer premiums are paid by active policyholders, but at the same time fewer benefits are paid to disabled policyholders. Thus, in reality, it is less important to hedge the mortality risk inherent to disability insurance.

No natural hedging effect between annuity insurance and disability insurance was detected, because, analogous to annuity insurance, an increase in mortality resulted in lessened liabilities of disability insurance. However, a minor natural hedging effect between term life and disability insurance was discovered (see plots in Figure 6 with 0% annuity). To isolate the interactions between these two insurance types, the proportion of annuity insurances is set to zero. Since the plots for the different shocks to the time trend of the mortality intersect as soon as the fraction of disability insurance is within 90% and 100%, this interval is magnified in the subsequent figure to better evaluate the natural hedging effect (see Figure 7)

**Figure 7: Natural Hedging between Term Life Insurance and Disability Insurance**

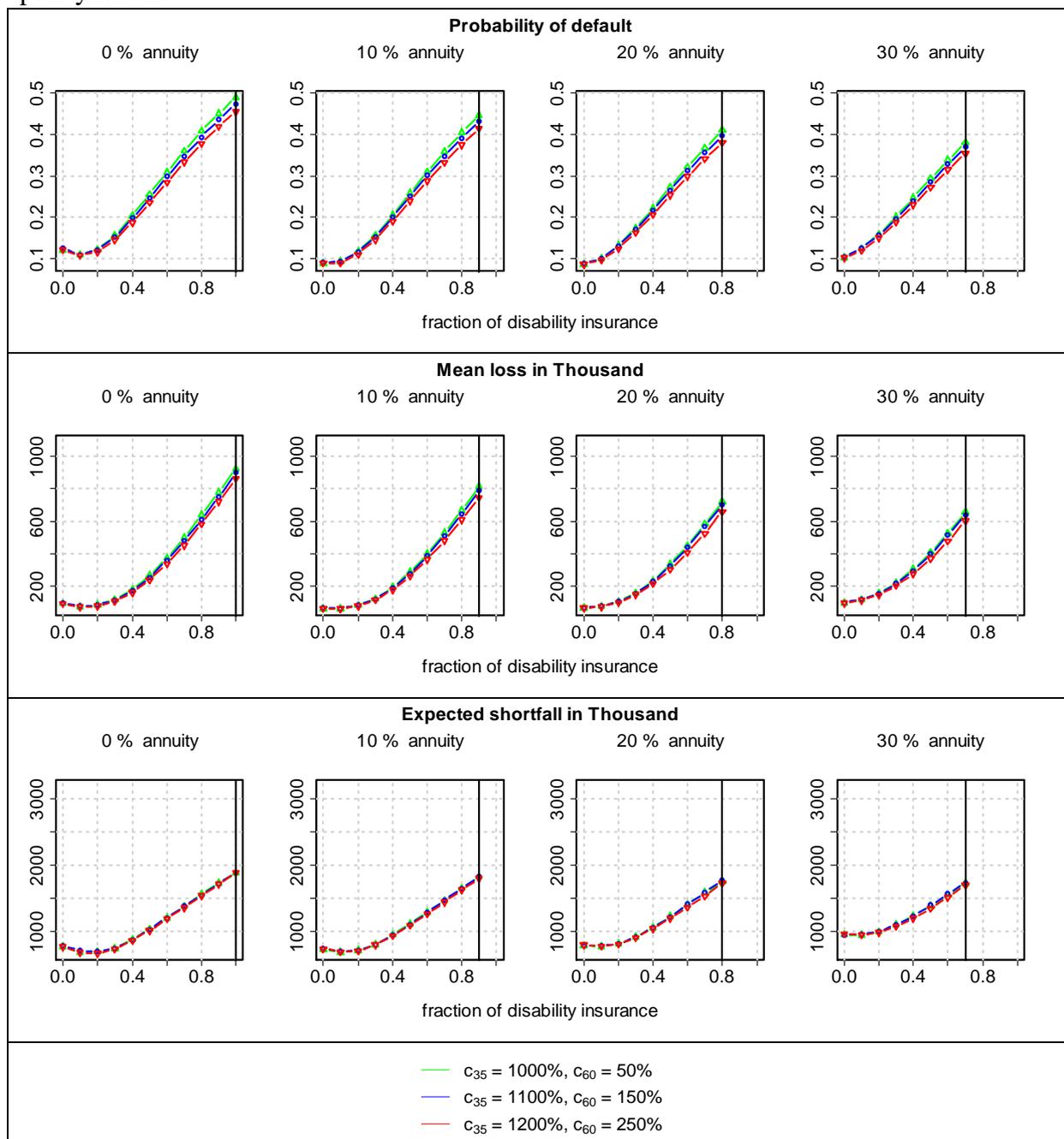


The optimal fraction of disability insurance, which is needed to immunize the portfolio against shocks to mortality, is marked by the intersection of the graphs for different mortality rates. Figure 7 reveals that the insurance company must sell between 94% to 99% disability

insurance contracts to become immune to changes of the risk measures PD and ML caused by shifts in mortality, i.e. the difference between realized and assumed mortality. In comparison with annuity insurances, a much higher amount of disability insurances is required to hedge the same amount of term life insurances, which implies that disability insurances are a far less efficient hedging tool for mortality risk because of their low sensitivity to this risk. This inefficiency is also reflected by the plot for the ES where no clear hedging strategy can be identified. In addition, the optimal hedge ratio between disability and term life insurance is much higher than one (between 94% to 6% and 99% to 1%), i.e. in order to hedge the mortality risk inherent to one term life insurance, the portfolio must contain between 16 and 99 disability insurance contracts. As a result, term life insurances may be an attractive tool for pure disability insurers to hedge the mortality risk on condition that they possess enough expertise to introduce term life insurance despite their specialization on disability insurance. However, it is not recommendable that pure life insurers utilize disability insurance as a hedging tool. Overall, the portfolio can be arranged either to minimize the overall risk inherent in the portfolio or to immunize the portfolio against shocks to mortality. Hence, there is a trade-off between the risk level and the immunization, as both cannot be achieved at the same time.

In the previous analysis, we have focused on shifts in general mortality. In the case of disability insurance, the mortality rate of disabled policyholders can be decomposed into general mortality and a specific factor that describes the relationship between active life mortality and the mortality of disabled insured (see Segerer, 1993). In this paper, the mortality rate was multiplied by a linear function that was defined by two function values  $c_{35}$  and  $c_{60}$ . In Figure 8, a lower mortality of disabled policyholders is plotted with a green graph, whereas a higher mortality was marked red.

**Figure 8:** Impact on the risk measures due to shifts of the mortality rate of disabled policyholders



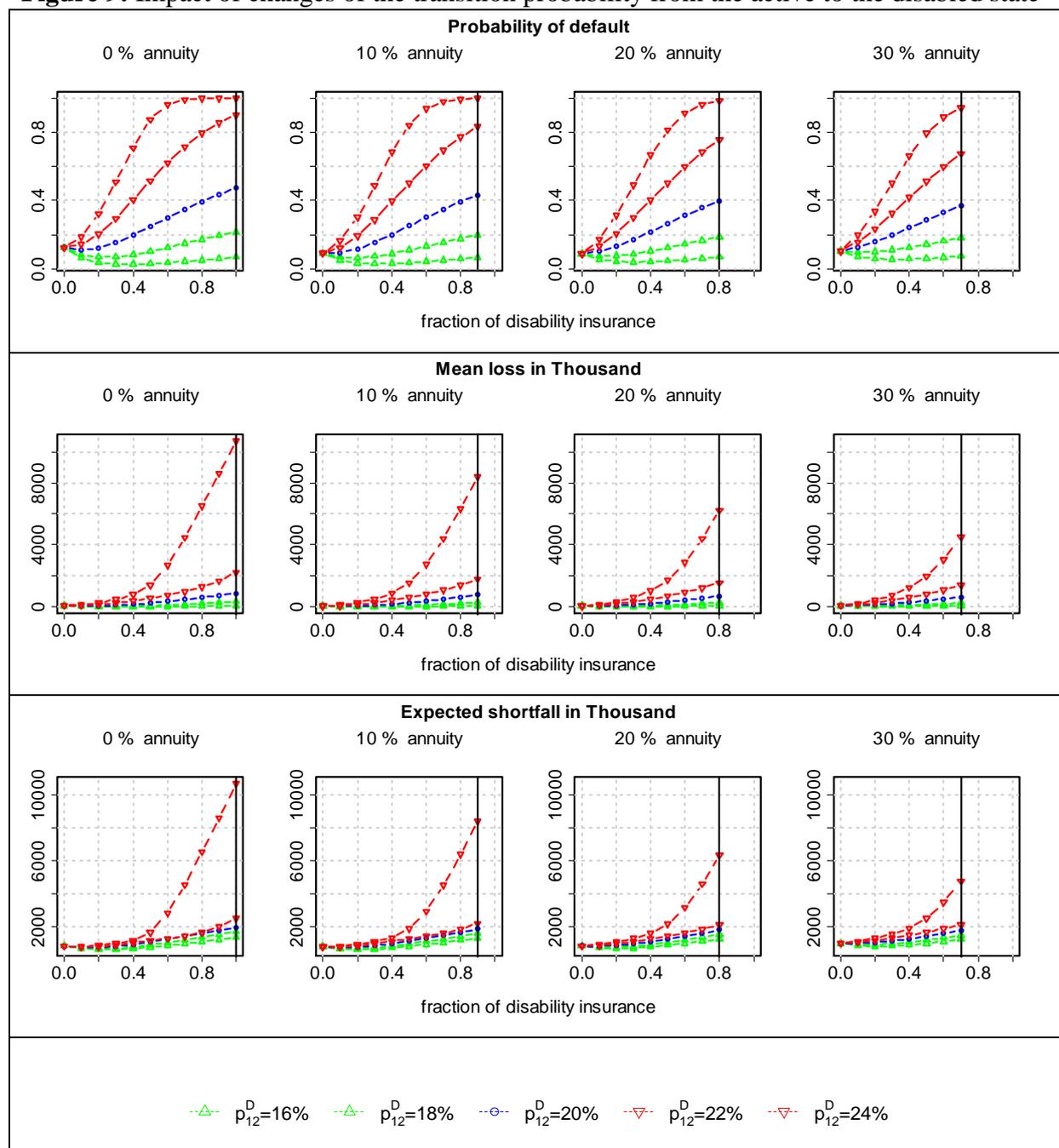
A higher mortality of disabled policyholders has a positive effect on the risk situation of the disability insurance provider, since benefit recipients die earlier than assumed, and, as a result, liabilities resulting from this contract are decreased. The opposite holds for a lower mortality. Shocks to this specific factor  $c_x$  have no effect on the risk inherent to annuity and term life insurance and, therefore, these insurance types cannot immunize the portfolio against this shift. Nevertheless, the regarded risk measures are less sensitive to changes of this specific factor, particularly with regard to shifts in the transition probability and the waiting

time from the active to the disabled state, which will be analyzed in the following subchapter. Hence, the immunization of this shock appears to be of minor importance for insurance companies.

### **3.5 The impact of shocks to disability risk**

The disability risk is influenced by the transition probability  $p_{12}(s)$  from the active to the disabled state as well as by the waiting time  $F_{12}(s,t)$ , which describes when the transition to the disabled states occurs. Therefore, the sensitivity of these factors will be studied separately.

A decrease in the transition probability implies that fewer policyholders become disabled during the time horizon (see green graphs in Figure 9). This alteration is favorable for the insurer because fewer benefits will be paid than was assumed in the calculation of the fair benefit. As a consequence, the risk situation is improved, which is shown in Figure 9. The opposite case being pictured by red graphs results in a worsened overall risk in the portfolio.

**Figure 9:** Impact of changes of the transition probability from the active to the disabled state

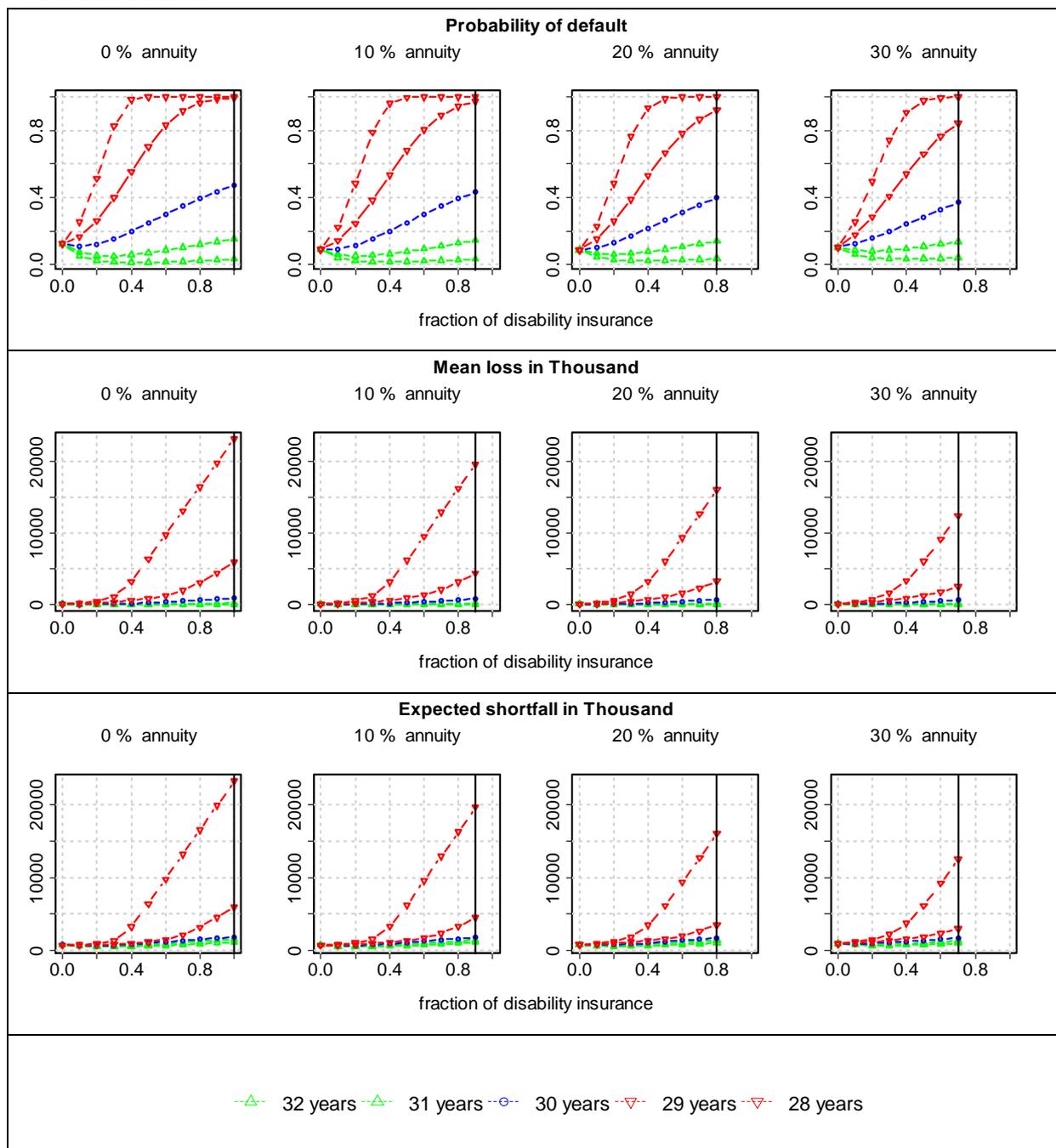
Both term life and annuity insurance are not affected by changes of the transition probability from the active to the disabled state and, thus, there are no hedging strategies that involve any of these insurances. Nevertheless, disability insurance is highly sensitive to changes of the underlying disability rate. In a portfolio only consisting of disability insurance, in the case of a decrease by 10% (20%) of the underlying transition probability, the probability of default decreases by 55% (85%), whereas an increase by 10% (20%) results in a rise of the probability of default by 90% (111%). The mean loss was found to be much more sensitive to changes than the probability of default and, in contrast, the expected shortfall was less

sensitive than the PD in case of smaller changes but more sensitive in the case of higher changes. Simply using a single risk measure may therefore underestimate the impact of disability risk on the risk situation.

Consequently, estimating and forecasting the transition probability based on empirical data is a critical step in the evaluation of disability insurance contracts. But due to continuously changing working conditions for instance, future trends of disability are difficult to predict and, thus, derivatives specifically designed to hedge these changes of the transition probability may be of future interest. Alternatively, the exposure to this risk may be reduced by increasing the amount of other insurance types, e.g. annuity insurance. With 30% annuity insurance for instance, the sensitivity of the probability of default is slightly reduced, whereas the impact on the ML and the ES is considerably lowered.

These risk measures are also observed to be highly sensitive to changes of the expected waiting time from the active to the disabled state. In general, the waiting time distribution describes when a policyholder transfers to another state, and, hence, a lower expected waiting time implies that policyholders become disabled earlier than assumed which has a negative impact on the risk situation of the insurer because benefits are paid earlier and less premiums are received. In Figure 10, lower expected waiting times are depicted in red and the contrary is shown in green.

**Figure 10:** Impact of changes of the expected waiting time for transitions from the active to the disabled state



Just as in the case of shocks to the transition probability from the active to the disabled state, changes of the expected waiting time for the same transition do not impact the annuity and term life insurance. Figure 10 shows that a shift towards faster transitions to the disabled state negatively affects the risk situation of the insurance company. A decrease of the expected waiting time by 3.3% results in a 100% to 560% higher risk depending on the regarded risk measure and an increase by the same percentage yields a risk reduction by 17% (ES) to 73% (ML), whereupon the PD is less sensitive than ML and ES. In comparison with changes to the

transition probability, shocks to the expected waiting time from the active to the disabled state have an even higher impact. Thus, with the expected waiting time being a representative for the waiting time distribution, this distribution must also be selected and calibrated with care. Analogous to the risk resulting from changes of the underlying transition probability, the exposure to risk due to alteration of the expected waiting time can be either lessened by transferring parts of it to the capital market via derivatives or by adding other insurance types to the portfolio.

#### **4. CONCLUSION**

In this paper, an insurance portfolio consisting of annuity, disability and term life insurances was studied. Specifically, we considered assets and liabilities of an insurance company in order to quantify the effect of mortality and disability risk on the insurer's overall risk situation and, ultimately, to identify risk-minimizing strategies. Within this framework, we aimed to examine the diversification effects in the whole insurance portfolio and to determine the optimal portfolio composition, which reduces the overall risk. In addition, we focused on strategies to hedge shocks to mortality and studied the effect of the disability risk.

The numerical analysis revealed that disability risk cannot be hedged by life insurances. Hence, future research may study portfolios with comparable insurance products, e.g. personal accident insurance or dread disease covers with total and permanent disability coverage, in order to identify potential strategies that may reduce the impact of disability risk. In addition, the proposal of new financial instruments may partially transfer the disability risk to the capital market. Moreover, further research may discuss disability insurance contracts that link disability benefits to actual disability experience by redistributing possible surplus to the insured and additional studies may also analyze different designs of disability insurance, e.g. with and without recoveries or with different disability levels. Finally, safety margins or premiums loadings for the disability risk may be included to lessen the impact of shocks to the transition probability and the waiting time distribution.

The mortality risk inherent to term life insurances can be hedged by disability insurances, which turned out to be a far less efficient hedging instrument than annuity insurances. Further studies may need to inspect whether the positive influence of increasing mortality on the risk inherent to disability insurances is generally observed in reality or depending on the input parameters. In addition, the age of policyholders when signing the contract, the contract duration and adverse selection affect the effectiveness of natural hedging (see Gatzert and

Wesker, 2011) and, therefore, future research may study these influences when disability insurances are added to an insurance portfolio. Overall, this paper has provided first insight with regard to the risk assessment of disability insurance within an insurance portfolio and has identified future research issues in the field of disability insurance.

## REFERENCES

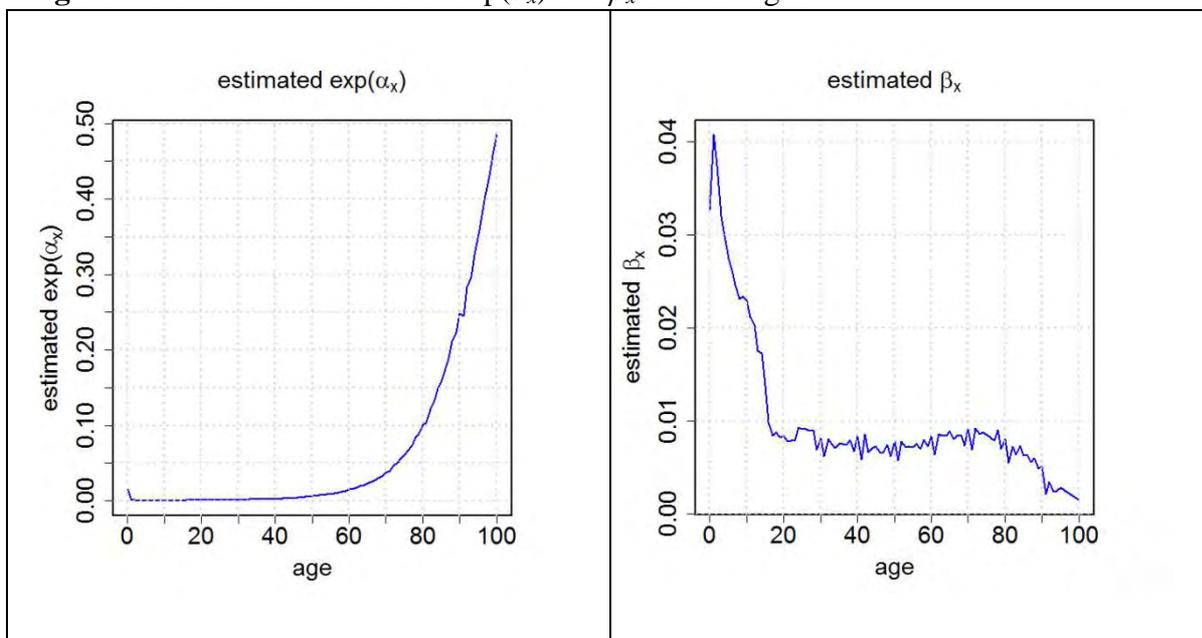
- Barbarin, J. (2008): Heath-Jarrow-Morton Modeling of Longevity Bonds and the Risk Minimization of Life Insurance Portfolios. *Insurance: Mathematics and Economics*, 43, 41-55.
- Bayraktar, E., Young, V. R. (2007): Hedging Life Insurance with Pure Endowments. *Insurance: Mathematics and Economics*, 40, 435-444.
- Biffis, E., Denuit, M., Devolder, P. (2010): Stochastic Mortality under Measure Changes. *Scandinavian Actuarial Journal*, 4, 284-311.
- Blake, D., Burrows, W. (2001): Survivor Bonds – Helping to Hedge Mortality Risk. *Journal of Risk and Insurance*, 68 (2), 339-348.
- Blake, D., Cairns, A. J. G., Dowd, K., MacMinn, R. (2006): Longevity Bonds – Financial Engineering, Valuation, and Hedging. *Journal of Risk and Insurance*, 73 (4), 647-672.
- Box, G. E. P., Jenkins, G. M. (1976): Time Series Analysis - Forecasting and Control. Revised Edition. Prentice-Hall, Englewood-Cliffs, New Jersey.
- Brouhns, N., Denuit, M., Vermunt, J. K. (2002a): A Poisson log-bilinear Regression Approach to the Construction of Projected Lifetables. *Insurance: Mathematics and Economics*, 31, 373-393.
- Brouhns, N., Denuit, M., Vermunt, J. K. (2002b): Measuring the Longevity Risk in Mortality Projections. *Bulletin of the Swiss Association of Actuaries*, 2, 105-130.
- Cairns, A. J. G. (2011): Modelling and Management of Longevity Risk – Approximations to Survivor Functions and Dynamic Hedging. *Insurance: Mathematics and Economics*, 49, 438-453.
- Cairns, A. J. G., Blake, D., Dowd, K. (2006a): A Two-Factor Model for Stochastic Mortality with Parameter Uncertainty – Theory and Application. *Journal of Risk and Insurance*, 73 (4), 687-718.
- Cairns, A. J. G., Blake, D., Dowd, K. (2006b): Pricing Death – Frameworks for the Valuation and Securitization of Mortality Risk. *ASTIN Bulletin*, 36 (1), 79-120.
- Cairns, A. J. G., Blake, D., Dowd, K. (2008): Modelling and Management of Mortality Risk – a Review. *Scandinavian Actuarial Journal*, 2/3, 79-113.
- Chandra, A., Samwick, A. A. (2005): Disability Risk and the Value of Disability Insurance. *National Bureau of Economic Research (NBER), Working Paper 11605*.
- Christiansen, M. C. (2012): Multistate Models in Health Insurance. *Advances in Statistical Analysis*, 96, 155-186.
- Cox, S. H., Lin, Y. (2007): Natural Hedging of Life and Annuity Mortality Risks. *Northern American Actuarial Journal*, 11 (3), 1-15.

- Cox, S. H., Lin, Y., Pedersen, H. (2010): Mortality Risk Modeling – Applications to Insurance Securitization. *Insurance: Mathematics and Economics*, 46, 242-253.
- Dadkhah, K. (2011): Foundations of Mathematical and Computational Economics. Second Edition. Springer-Verlag, Berlin.
- Dahl, M. (2004): Stochastic Mortality in Life Insurance – Market Reserves and Mortality-Linked Insurance Contracts. *Insurance: Mathematics and Economics*, 35, 113-136.
- Dahl, M., Melchior, M., Møller, T. (2008): On Systematic Mortality Risk and Risk-minimization with Survivor Swaps. *Scandinavian Actuarial Journal*, 2-3, 114-146.
- Dahl, M., Møller, T. (2006): Valuation and Hedging of Life Insurance Liabilities with Systematic Mortality Risk. *Insurance: Mathematics and Economics*, 39, 193-217.
- D’Amico, G., Guillen, M., Manca, R. (2009): Full Backward Non-Homogeneous Semi-Markov Processes for Disability Insurance Models: A Catalunya Real Data Application. *Insurance: Mathematics and Economics*, 45, 173-179.
- Dowd, K., Blake, D., Cairns, A. J. G., Dawson, P. (2006): Survivor Swaps. *Journal of Risk and Insurance*, 73 (1), 1-17.
- Gatzert, N., Wesker, H. (2011): Managing Longevity Risk under Adverse Selection – The Influence of Policyholder Age and Contract Duration. *Zeitschrift für die gesamte Versicherungswissenschaft*, 100, 695-706.
- Gatzert, N., Wesker, H. (2012a): Mortality Risk and Its Effect on Shortfall and Risk Management in Life Insurance. *Journal of Risk and Insurance* (forthcoming).
- Gatzert, N., Wesker, H. (2012b): The Impact of Natural Hedging on a Life Insurer’s Risk Situation. *Journal of Risk Finance*, 13 (5), 396-423.
- Goodman, L. A. (1979): Simple Models for the Analysis of Association in Cross-Classification Having Ordered Categories. *Journal of the American Statistical Association*, 74, 537-552.
- Graf von der Schulenburg, J.-M. (2005): Versicherungsökonomik – Ein Leitfaden für Studium und Praxis. Verlag Versicherungswirtschaft, Karlsruhe.
- Gründl, H., Post, T., Schulze, R. N. (2006): To Hedge or not to Hedge – Managing Demographic Risk in Life Insurance Companies. *Journal of Risk and Insurance*, 73 (1), 19-41.
- Haberman, S., Pitacco, E. (1999): Actuarial Models for Disability Insurance. Chapman & Hall/CRC, London.
- Hanewald, K. (2011): Explaining Mortality Dynamics – The Role of Macroeconomic Fluctuations and Cause of Death Trends. *North American Actuarial Journal*, 15 (2), 290-314.

- Hanewald, K., Post, T., Gründl, H. (2011): Stochastic Mortality, Macroeconomic Risks and Life Insurer Solvency. *Geneva Papers on Risk and Insurance – Issues and Practice*, 36, 458-475.
- Hagen, C., Himmelreicher, R. K., Kemptner, D., Lampert, T. (2011): Soziale Ungleichheit und Risiken der Erwerbsminderung. *WSI Mitteilungen*, July 2011, 336-344.
- Hári, N., De Waegenaere, A., Melenberg, B., Nijman, T. E. (2008): Longevity Risk in Portfolios of Pension Annuities. *Insurance: Mathematics and Economics*, 42, 505-519.
- Hull, J. C. (2009): Options, Futures, and Other Derivatives. Seventh Edition. Prentice Education, Upper Saddle River, New Jersey.
- Hull, J. C. (2011): Risikomanagement – Banken, Versicherungen und andere Finanzinstitutionen. Second Edition. Pearson Studium, München.
- Lee, R. D. (2000): The Lee-Carter Method of Forecasting Mortality, with Various Extensions and Applications. *North American Actuarial Journal*, 4, 80-93.
- Lee, R. D., Carter, L. R. (1992): Modeling and Forecasting U.S. Mortality. *Journal of the American Statistical Association*, 87 (419), 659-671.
- Luciano, E., Regis, L., Vigna, E. (2011): Natural Delta Gamma Hedging of Longevity and Interest Rate Risk. *International Center for Economic Research, Working Paper No. 21/2011*.
- Luciano, E., Regis, L., Vigna, E. (2012): Delta-Gamma Hedging of Mortality and Interest Rate Risk. *Insurance: Mathematics and Economics*, 50, 402-412.
- Macdonald, A. S. (1996): An Actuarial Survey of Statistical Models for Decrement and Transition Data – I: Multiple State, Poisson and Binomial Models. *British Actuarial Journal*, 2 (1), 129-155.
- Milevsky, M. A., Promislow, S. D. (2002): Can Life Insurance Be Used to Hedge Payout Annuities? Part 1 – Modeling Longevity Risk. *Working Paper, York University*.
- Ngai, A., Sherris, M. (2011): Longevity Risk Management for Life and Variable Annuities – The Effectiveness of Static Hedging Using Longevity Bonds and Derivatives. *Insurance: Mathematics and Economics*, 49, 100-114.
- Pandit, S. M., Wu, S. (1990): Time Series and System Analysis with Applications. Reprinted Edition. Krieger Publishing, Malabar, Florida.
- Pitacco, E. (2004): Disability Insurance. In Teufels, J. L., Sundt, B. (Eds.), *Encyclopedia of Actuarial Science*, Volume 1, 530-541. John Wiley & Sons, Chichester.
- Ribeiro, R., di Pietro, V. (2009): Longevity Risk and Portfolio Allocation. *Investment Strategies*, 57, J.P. Morgan.
- Richter, A., Weber, F. (2011): Mortality-Indexed Annuities – Managing Longevity Risk via Product Design. *North American Actuarial Journal*, 15 (2), 212-236.

- Rogers, R. (2002): Will Mortality Improvements Continue? *National Underwriter*, 106 (34), 11.
- Schwark, P. (2011): Die Bedeutung der Berufsunfähigkeitsversicherung. *Versicherungsmedizin*, 63 (3), 129-131.
- Segerer, G. (1993): The Actuarial Treatment of the Disability Risk in Germany, Austria and Switzerland. *Insurance: Mathematics and Economics*, 13, 131-140.
- Stenberg, F., Manca, R., Silvestrov, D. (2007): An Algorithmic Approach to Discrete Time Non-Homogeneous Backward Semi-Markov Reward Processes with an Application to Disability Insurance. *Methodology and Computing in Applied Probability*, 9, 497-519.
- Tsai, J. T., Wang, J. L., Tzeng, L. Y. (2010): On the Optimal Product Mix in Life Insurance Companies Using Conditional Value at Risk. *Insurance: Mathematics and Economics*, 46, 235-241.
- Wang, J. L., Huang, H. C., Yang, S. S., Tsai, J. T. (2010): An Optimal Product Mix for Hedging Longevity Risk in Life Insurance Companies – The Immunization Theory Approach. *Journal of Risk and Insurance*, 77 (2), 473-497.
- Wetzel, C., Zwiesler, H. (2008): Das Vorhersagerisiko der Sterblichkeitsentwicklung – Kann es durch eine geeignete Produktzusammensetzung minimiert werden? *Blätter der Deutschen Gesellschaft für Versicherungs- und Finanzmathematik (DGVMF)*, 29 (1), 73-107.
- Wills, S., Sherris, M. (2010): Securitization, Structuring and Pricing of Longevity Risk. *Insurance: Mathematics and Economics*, 46, 173-185.

## APPENDIX

**Figure A.1:** Estimated values of  $\exp(\alpha_x)$  and  $\beta_x$  over all ages**Figure A.2:** Estimated and predicted mortality trend