ASTIN COLLOQUIUM LISBOA 2016

31 May – 03 June 2016
at ISEG – Lisbon School of Economics
and Management

Fishing for scenarios

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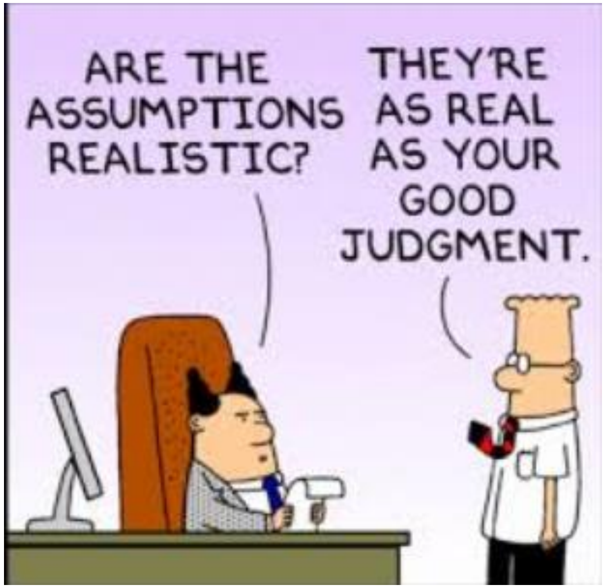
SCOR ©

… the usual question…

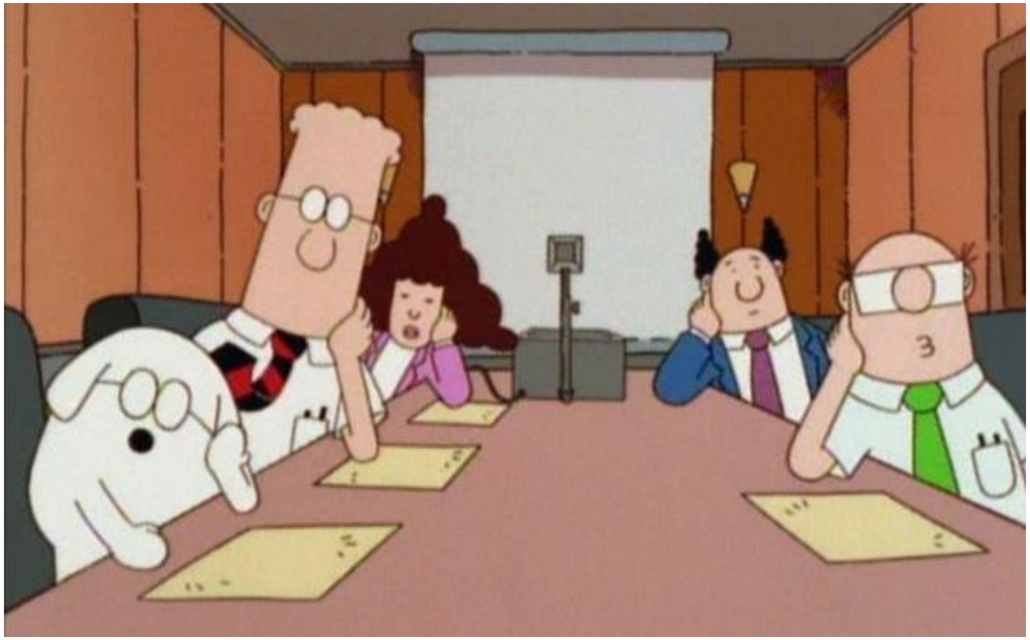
- In order to back-test our assumptions in the internal model, we need a scenario.
  - Can you provide a scenario ?



… the usual answer…

- Ok !
- … what scenario do you need ?

(Brainstorming)

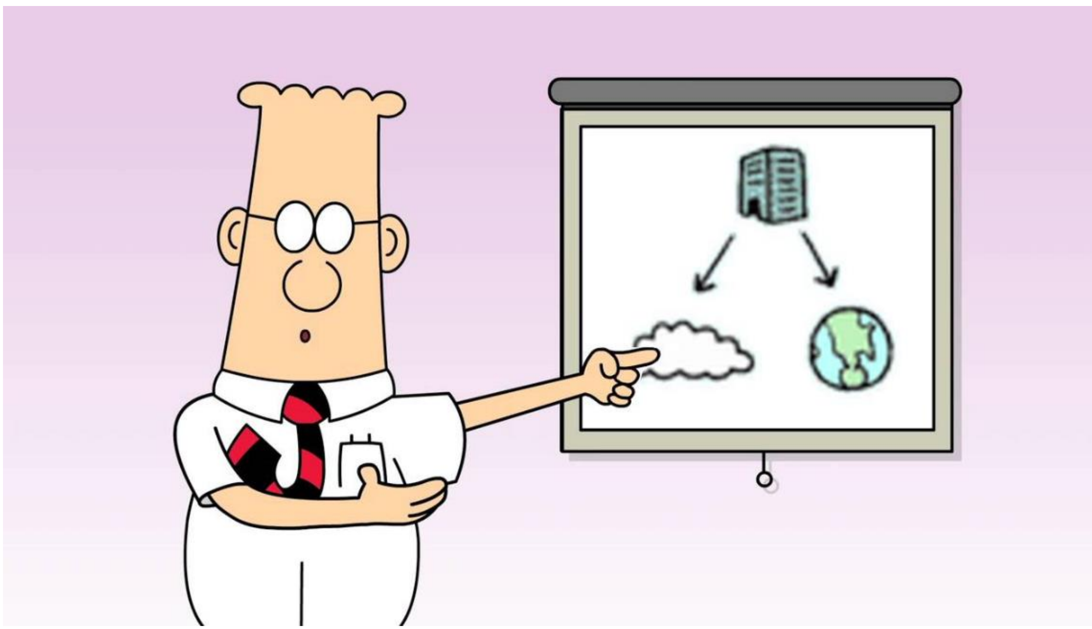


… the usual answer…

- Why not an inflation scenario ?
- … Good idea !

Scenario work

- Work on the scenario…



- **And the impact of an increase of the inflation on the reserves is EUR 100m**

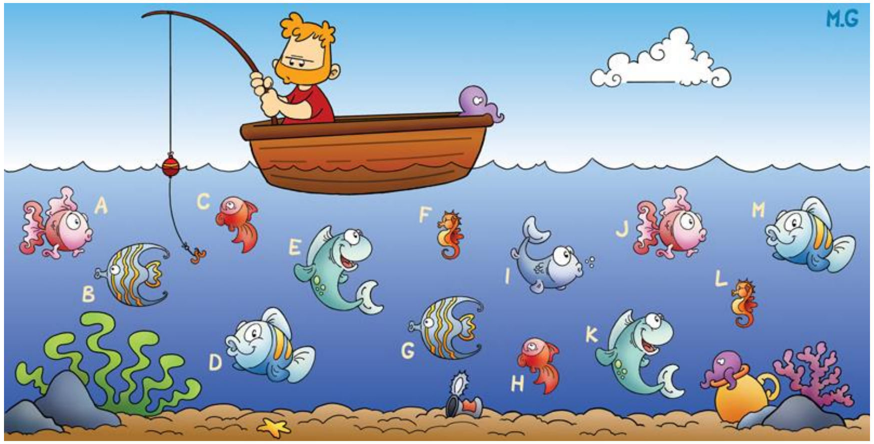
… and the next question …

- Great work !

- … **What is the return period of this scenario ?**

… the next question…

- …Well, I need to fish for this scenario in the internal model …



- … and then start the problems …

Scenarios in solvency frameworks

- Swiss Solvency Test
- Quadrant scenario essentially for market risks
- Extreme financial stress test (financial market stress associated with impacts on life insurance – lapse risk)
- Pure financial market stress test
- Reinsurer default
- Longevity stress
- Invalidity stress
- Lapse stress
- Pandemia
- Panic in a stadium
- Industrial accident
- Under-reserving
- Terrorism

Scenarios in solvency frameworks

- Solvency 2 – EIOPA Stress test 2014
- Market stress scenarios
  - Adverse 1: EU equity market distress
  - Adverse 2: Non-financial corporate bond market distress
- Single-factor Insurance Stresses
  - Undertaking specific natural or man-made event stress
  - Market wide defined events
  - Provisions Deficiency Stress
  - Proposal for life insurance stresses
  - Longevity Stress
  - Mortality Stress
  - Lapse Stress
- Low Yield Scenarios

Scenarios in solvency frameworks

- ORSA – IAA 2015

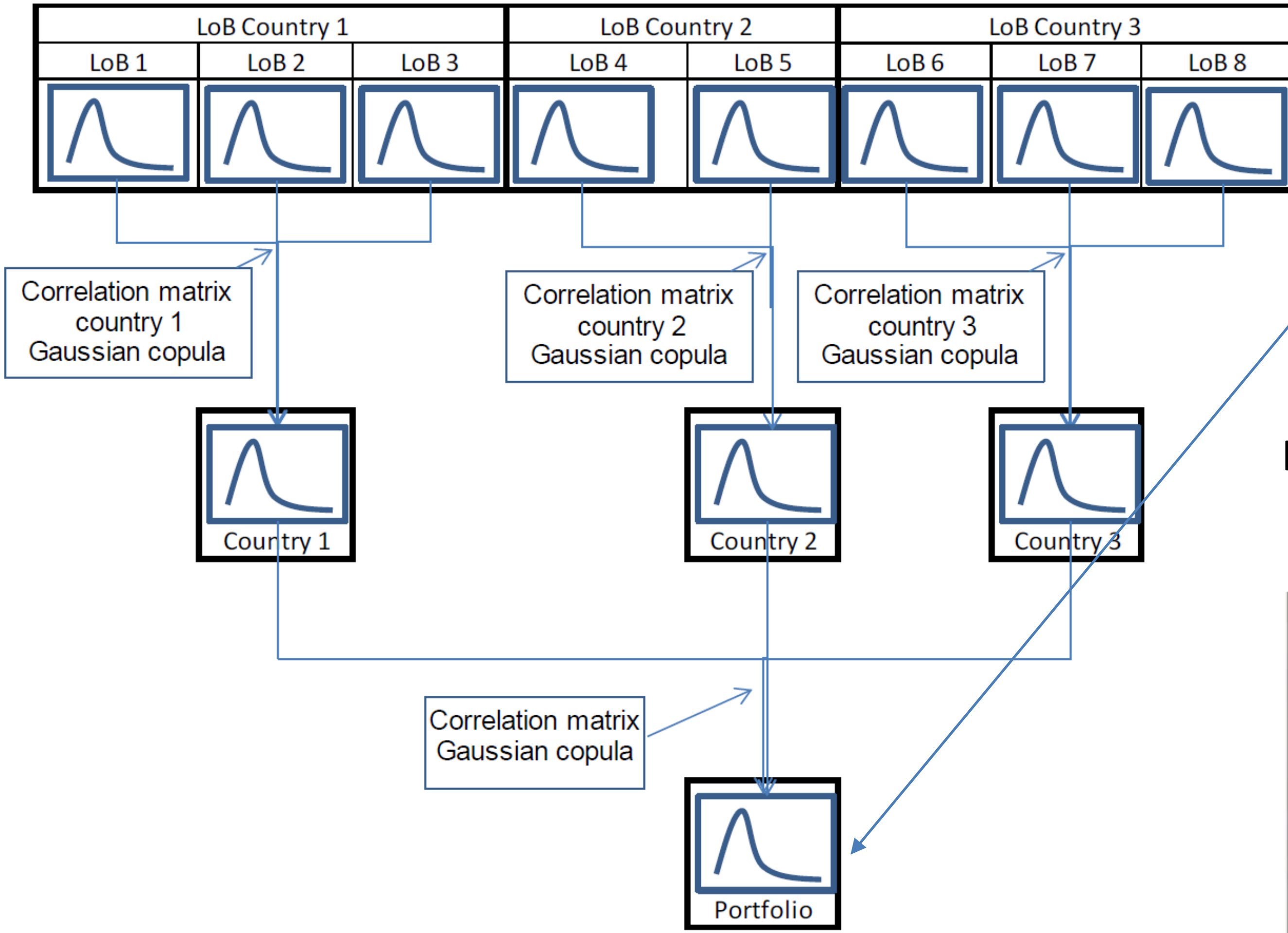
"Deriving Value from ORSA - Board Perspective" April 2015 (actuaries.org)

- What-if analyses

- SCOR = "Footprint scenarios"

Context

- Internal model



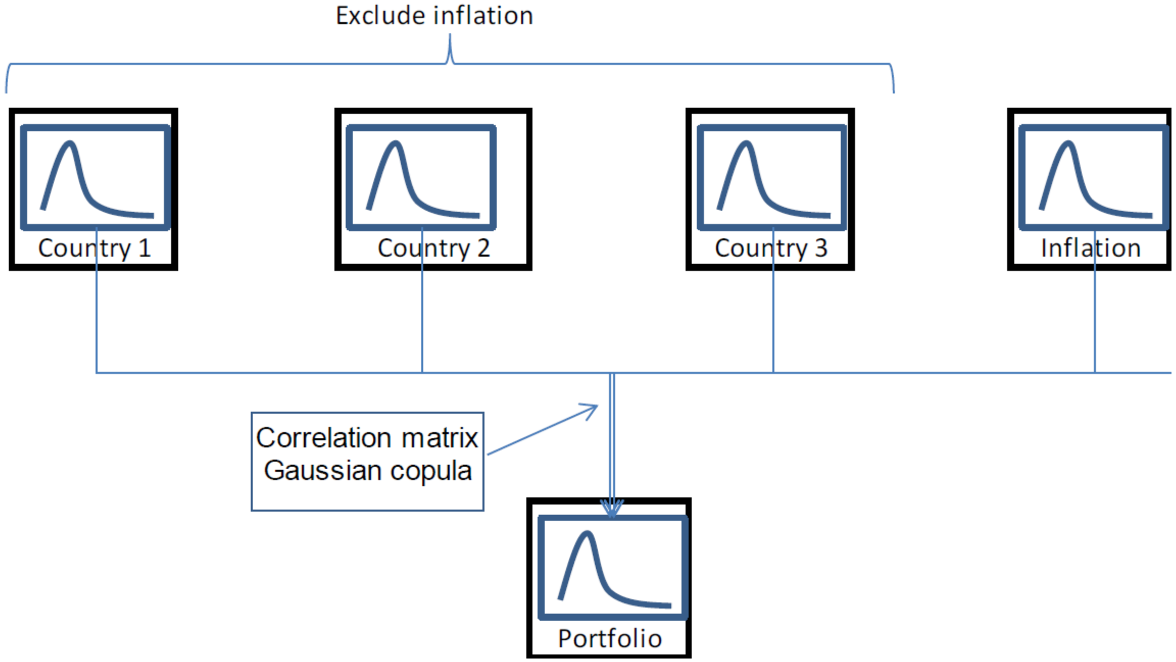
Scenario	Loss
1	45
2	47
…	
750	250
…	
1000	1200

Inflation scenario of 250

$$\text{Return period} = \frac{1}{1-75\%} = 4$$

Context

- Internal model – Reviewed



- Separate inflation = Choice of the Lines of Business most exposed to inflation (e.g. medical malpractice)
- The distributions for each country and for inflation is characterized by:
  - their best estimates,
  - their coefficients of variations,
  - their skewness.
- USE of Cornish-Fisher expansion

The Cornish-Fisher expansion

- In 1938, MM. Cornish and Fisher published a paper allowing the approximate calculation of different VaR based on the knowledge of the first 4 moments.

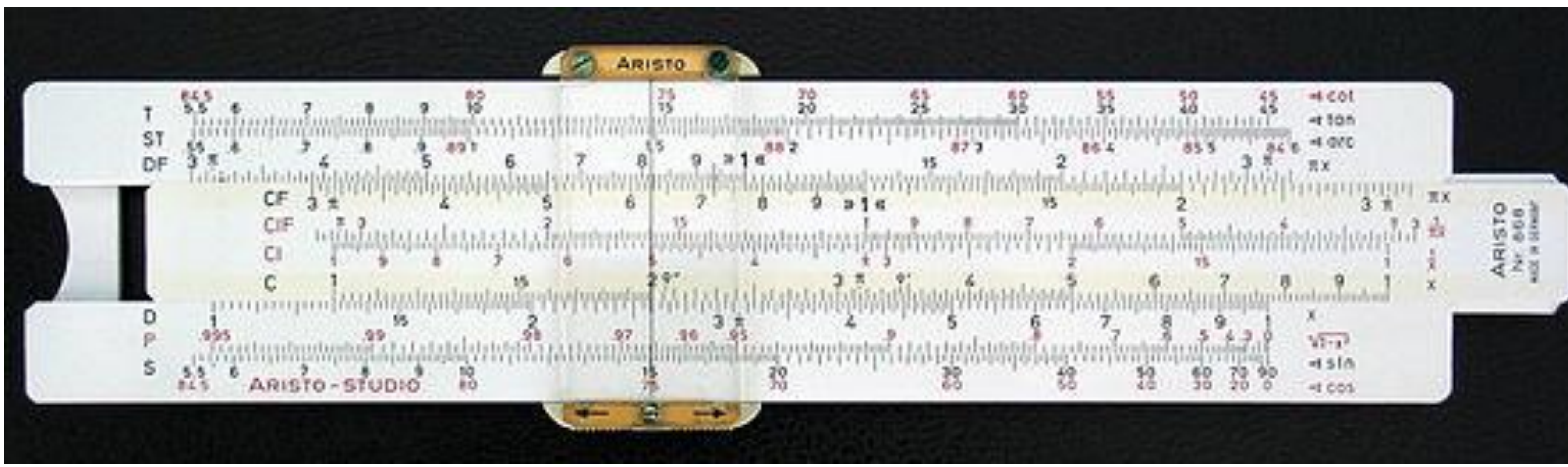
Edmund Alfred Cornish

Ronald Aylmer Fisher

The Cornish-Fisher expansion

Slide rule or “slipstick”

- In 1938, why publish a paper about an approximation of the VaR ?



The Cornish-Fisher expansion

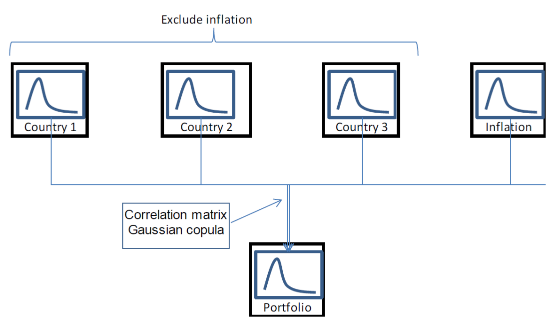
- Let X be a random variable with density function with mean 0 and variance 1. Let β_1 be the skewness of this distribution. Let Z be a normally distributed random variable and let z_α be the αth quantile of this distribution. Then the αth quantile ω_α of the distribution X can be approximated by:

$$\omega_\alpha = z_\alpha + \frac{1}{6}\left(z_\alpha^2 - 1\right)\beta_1$$

- Example of X:
  - Mean = 1474, Std Deviation = 300, Skewness = 0.1
  - Lognormal fitted distribution: Mu = 7.275, Sigma = 0.2015
  - VaR 99% (Cornish-Fisher) = 2 193
  - VaR 99% (Lognormal) = 2 308

The solution

- The overall portfolio characteristics are:
  - Mean: $M = \mu_1 + \mu_2 + \mu_3 + \mu_{\text{inflation}}$
  - Standard deviation: $\Sigma = \sqrt{\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_{\text{inflation}} \end{pmatrix}^t \begin{pmatrix} 1 & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & 1 & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & 1 & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & 1 \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_{\text{inflation}} \end{pmatrix}}$
  - Skewness Ω given from the distribution

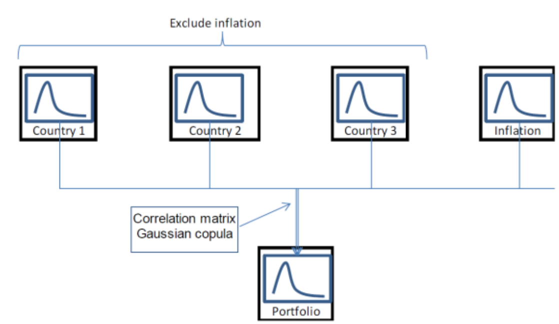


The solution

- Looking for scenario L_{scen} and using Cornish-Fisher, we have:

$$L_{scen} = \mathrm{M} + \left(\Phi^{-1}(\alpha) + \frac{1}{6}\left(\Phi^{-1}(\alpha)^2 - 1\right)\Omega \right)\Sigma$$

Scenario	Loss
1	45
2	47
...	
750	250
...	
1000	1200



Inflation scenario of 250

Return period = $\dfrac{1}{1-\alpha} = \dfrac{1}{1-75\%} = 4$

The solution

- Proxy derived from:
  - we assume that the factor neutralization corresponds to the VaR 50% (Value at risk) on the country risk distribution,
  - and we are looking for the quantile β on the inflation risk distribution

$$
\begin{aligned}
L_{scen} = \mu_1 &+ \left(\Phi^{-1}(50\%) + \frac{1}{6}\left(\Phi^{-1}(50\%)^2 - 1\right)\omega_1 \right)\sigma_1 \\
&+ \mu_2 + \left(\Phi^{-1}(50\%) + \frac{1}{6}\left(\Phi^{-1}(50\%)^2 - 1\right)\omega_2 \right)\sigma_2 \\
&+ \mu_3 + \left(\Phi^{-1}(50\%) + \frac{1}{6}\left(\Phi^{-1}(50\%)^2 - 1\right)\omega_3 \right)\sigma_3 \\
&+ \mu_{\text{inflation}} + \left(\Phi^{-1}(\beta) + \frac{1}{6}\left(\Phi^{-1}(\beta)^2 - 1\right)\omega_{\text{inflation}} \right)\sigma_{\text{inflation}}
\end{aligned}
$$

This is a second degree equation very easy to solve.

Numerical application

- Lognormal distributions

	Lognormal					
	m	s	CoV	Skewness ω	Mean μ	Std deviation σ
Inflation	2.301	0.050	0.05	0.15	10.000	0.500
Country 1	3.000	0.060	0.06	0.18	20.122	1.207
Country 2	2.500	0.040	0.04	0.12	12.192	0.488
Country 3	2.000	0.070	0.07	0.21	7.407	0.518

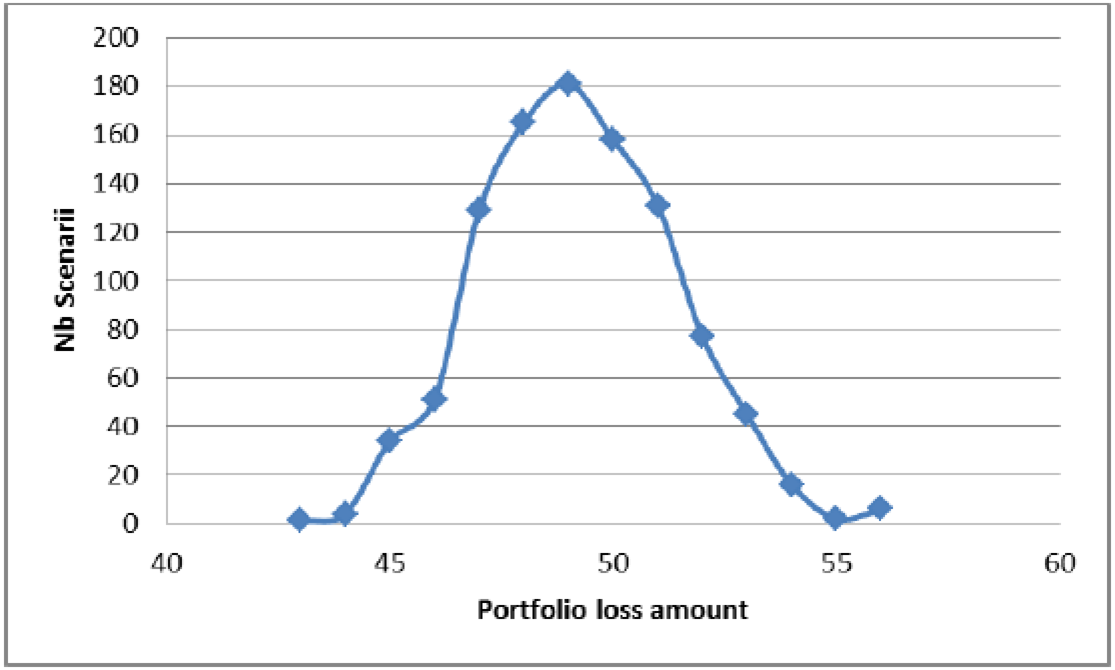
- Correlations:

	Inflation	Country 1	Country 2	Country 3
Inflation	100%	50%	50%	50%
Country 1	50%	100%	50%	50%
Country 2	50%	50%	100%	50%
Country 3	50%	50%	50%	100%

We are looking for a scenario of 49.91

Numerical application

- Aggregated distribution:



On the aggregated distribution, the scenario is at the 55% quantile.

With the proxy, the scenario is at the 70% quantile of the inflation distribution.

Application - Excel

An excel sheet developed to estimate the presented formula is available on the URL:

https://drive.google.com/file/d/0B6piPKdUSkYIbEs5QmdFQ2xBdGM/view?usp=sharing

Questions ?