New copulas based on general partitions-of-unity and their applications to risk management

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1. Introduction

- Construction of new multivariate copulas on the basis of a generalized infinite partition-of-unity approach
- Allows for tail-dependence as well as for asymmetry
- Can be easily implemented for risk management purposes
- Can be extended to an uncountable infinite partition-of-unity approach

2. Main Results

Let $\mathbb{Z}^+ = \{0,1,2,3,\cdots\}$ denote the set of non-negative integers and suppose that $\{\varphi_i(u)\}_{i\in\mathbb{Z}^+}$ and $\{\psi_j(v)\}_{i\in\mathbb{Z}^+}$ are non-negative maps defined on the interval $(0,1)$ each such that

$$\sum_{i=0}^{\infty}\varphi_i(u) = \sum_{j=0}^{\infty}\psi_j(v) = 1 \tag{2.1}$$

and

$$\int_0^1 \varphi_i(u)du = \alpha_i > 0,\ \int_0^1 \psi_j(v)dv = \beta_j > 0 \text{ for } i,j \in \mathbb{Z}^+. \tag{2.2}$$

2. Main Results

The maps $\varphi_i(u)$ and $\psi_j(v)$ can be thought of representing discrete distributions over the non-negative integers $\mathbb{Z}^+$ with parameters u and v, resp.

The sequences $\{\alpha_i\}_{i\in\mathbb{Z}^+}$ and $\{\beta_j\}_{j\in\mathbb{Z}^+}$ represent the probabilities of the corresponding mixed distributions.

2. Main Results

Let $\{p_{ij}\}_{i,j\in\mathbb{Z}^+}$ represent the probabilities of an arbitrary discrete bivariate distribution over $\mathbb{Z}^+\times\mathbb{Z}^+$ with marginal distributions given by $p_{i\bullet}=\sum_{j=0}^{\infty}p_{ij}=\alpha_i$ and $p_{\bullet j}=\sum_{i=0}^{\infty}p_{ij}=\beta_j$ for $i,j\in\mathbb{Z}^+$. Then

$$c(u,v):=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{p_{ij}}{\alpha_i\beta_j}\varphi_i(u)\psi_j(v),\ u,v\in(0,1) \tag{2.3}$$

defines the density of a bivariate copula, called *generalized partition-of-unity copula*.

2. Main Results

The function c is in fact is the density of a bivariate copula:

$$\int_0^1 c(u,v)\,dv = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{p_{ij}}{\alpha_i\beta_j}\,\varphi_i(u)\int_0^1 \psi_j(v)\,dv = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{p_{ij}}{\alpha_i\beta_j}\,\beta_j\,\varphi_i(u)$$

$$= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{p_{ij}}{\alpha_i}\,\varphi_i(u) = \sum_{i=0}^{\infty}\frac{\varphi_i(u)}{\alpha_i}\sum_{j=0}^{\infty}p_{ij} = \sum_{i=0}^{\infty}\frac{\varphi_i(u)}{\alpha_i}\,\alpha_i$$

$$= \sum_{i=0}^{\infty}\varphi_i(u) = 1, \qquad (2.4)$$

likewise for $\int_0^1 c(u,v)\,du$.

2. Main Results

From a „dual" point of view, we can rewrite (2.3) as

$$c(u,v) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} p_{ij} f_i(u) g_j(v),\; u,v \in (0,1) \qquad (2.5)$$

where $f_i(\bullet) = \frac{\varphi_i(\bullet)}{\alpha_i}$, $g_j(\bullet) = \frac{\psi_j(\bullet)}{\beta_j}$, $i,j \in \mathbb{Z}^+$ denote the densities induced by $\{\varphi_i(u)\}_{i\in\mathbb{Z}^+}$ and $\{\psi_j(v)\}_{i\in\mathbb{Z}^+}$.

This means that the copula density c can also be seen as a mixture of product densities, which possibly allows for a simple way for a stochastic simulation.

2. Main Results

An extension of this approach to d dimensions with $d > 2$ is obvious: assume that $\{\varphi_{ki}(u)\}_{i\in\mathbb{Z}^+}$ for $k = 1,\cdots,d$ represent discrete probabilities with

$$\sum_{i=0}^{\infty} \varphi_{ki}(u) = 1 \text{ for } u \in (0,1) \tag{2.6}$$

and

$$\int_0^1 \varphi_{ki}(u)du = \alpha_{ki} > 0 \text{ for } i \in \mathbb{Z}^+. \tag{2.7}$$

2. Main Results

Let $\{p_{\mathbf{i}}\}_{\mathbf{i}\in\mathbb{Z}^{+d}}$ represent the distribution of an arbitrary discrete d-dimensional random vector $\mathbf{Z}$ over $\mathbb{Z}^{+d}$ where, for simplicity, we write $\mathbf{i}=(i_1,\cdots,i_d)$, i.e.

$$P(\mathbf{Z}=\mathbf{i})=p_{\mathbf{i}},\ \mathbf{i}\in\mathbb{Z}^{+d}. \tag{2.8}$$

Suppose further that for the marginal distributions, there holds

$$P(Z_k=i)=\alpha_{ki},\ i\in\mathbb{Z}^{+},k=1,\cdots,d. \tag{2.9}$$

2. Main Results

Then

$$c(\mathbf{u}) := \sum_{\mathbf{i}\in\mathbb{Z}^{+d}} \frac{p_{\mathbf{i}}}{\prod_{k=1}^{d} \alpha_{k,i_k}} \prod_{k=1}^{d} \varphi_{k,i_k}(u_k),\ \mathbf{u} = (u_1,\cdots,u_d) \in (0,1)^d \qquad (2.10)$$

defines the density of a d-variate copula, which is also called *generalized partition-of-unity copula*.

Alternatively, we can rewrite (2.10) again as

$$c(\mathbf{u}) = \sum_{\mathbf{i}\in\mathbb{Z}^{+d}} p_{\mathbf{i}} \prod_{k=1}^{d} f_{k,i_k}(u_k),\ \mathbf{u} = (u_1,\cdots,u_d) \in (0,1)^d \qquad (2.11)$$

with densities $f_{ki}(\bullet) = \dfrac{\varphi_{ki}(\bullet)}{\alpha_{ki}},\ i \in \mathbb{Z}^+, k = 1,\cdots,d.$

3. The symmetric case (diagonal dominance)

For simplicity, we restrict ourselves to the two-dimensional case in the sequel. The generalization to higher dimensions is obvious.

Let $\varphi_i = \psi_i$ for $i \in \mathbb{Z}^+$ and $\int_0^1 \varphi_i(u)du = \alpha_i > 0$. Define

$$p_{ij} := \begin{cases} \alpha_i, & \text{if } i = j \\ 0, & \text{otherwise.} \end{cases} \tag{3.1}$$

Then

$$c(u,v) := \sum_{i=0}^{\infty} \frac{\varphi_i(u)\varphi_i(v)}{\alpha_i} = \sum_{i=0}^{\infty} \alpha_i f_i(u) f_i(v), \; u,v \in (0,1) \tag{3.2}$$

defines the density of a *generalized partition-of-unity copula with diagonal dominance*.

3. The symmetric case (diagonal dominance)

Example 1 (binomial distributions - Bernstein copula).

For a fixed integer $m \geq 2$, consider the family of binomial distributions given by their point masses

$$\varphi_{m,i}(u) = \begin{cases} \binom{m-1}{i} u^i (1-u)^{m-1-i}, & i = 0, \cdots, m-1 \\ 0, & i \geq m. \end{cases} \tag{3.3}$$

3. The symmetric case (diagonal dominance)

Example 1 (binomial distributions - Bernstein copula).

We have, for $i = 0, \cdots, m-1$,

$$\alpha_{m,i} = \int_0^1 \varphi_{m,i}(u)du = \binom{m-1}{i} \int_0^1 u^i (1-u)^{m-1-i} du$$

$$= \frac{(m-1)!}{i!(m-1-i)!} \cdot \frac{\Gamma(i+1)\Gamma(m-i)}{\Gamma(m+1)} = \frac{(m-1)!}{i!(m-1-i)!} \cdot \frac{i!(m-1-i)!}{m!} = \frac{1}{m} \quad (3.4)$$

and hence

$$c_m(u,v) = m \sum_{i=0}^{m-1} \binom{m-1}{i}^2 (uv)^i \left((1-u)(1-v)\right)^{m-1-i}, \; u,v \in (0,1), \quad (3.5)$$

which is the density of a bivariate *Bernstein copula*.

3. The symmetric case (diagonal dominance)

Example 2 (negative binomial distributions).

Consider, for fixed $\beta > 0$, the family of negative binomial distributions given by their point masses

$$\varphi_{\beta,i}(u) = \binom{\beta + i - 1}{i}(1-u)^{\beta} u^{i}, \; i \in \mathbb{Z}^{+}. \tag{3.9}$$

3. The symmetric case (diagonal dominance)

Example 2 (negative binomial distributions).

Here we have, for $i \in \mathbb{Z}^+$,

$$\alpha_{\beta,i} = \int_0^1 \varphi_{\beta,i}(u)\,du = \binom{\beta+i-1}{i} \int_0^1 u^i (1-u)^\beta\,du$$
$$= \frac{\Gamma(\beta+i)}{i!\,\Gamma(\beta)} \cdot \frac{\Gamma(i+1)\Gamma(\beta+1)}{\Gamma(\beta+i+2)} = \frac{\beta}{(\beta+i)(\beta+i+1)} \qquad (3.10)$$

and hence for $u,v \in (0,1)$,

$$c_\beta(u,v) = (\beta+1)\big((1-u)(1-v)\big)^\beta \sum_{i=0}^{\infty} \binom{\beta+i-1}{i}\binom{\beta+i+1}{i}(uv)^i. \qquad (3.11)$$

3. The symmetric case (diagonal dominance)

Example 2 (negative binomial distributions)**.**

For integer values of β, this expression can be explicitly evaluated as a finite sum.

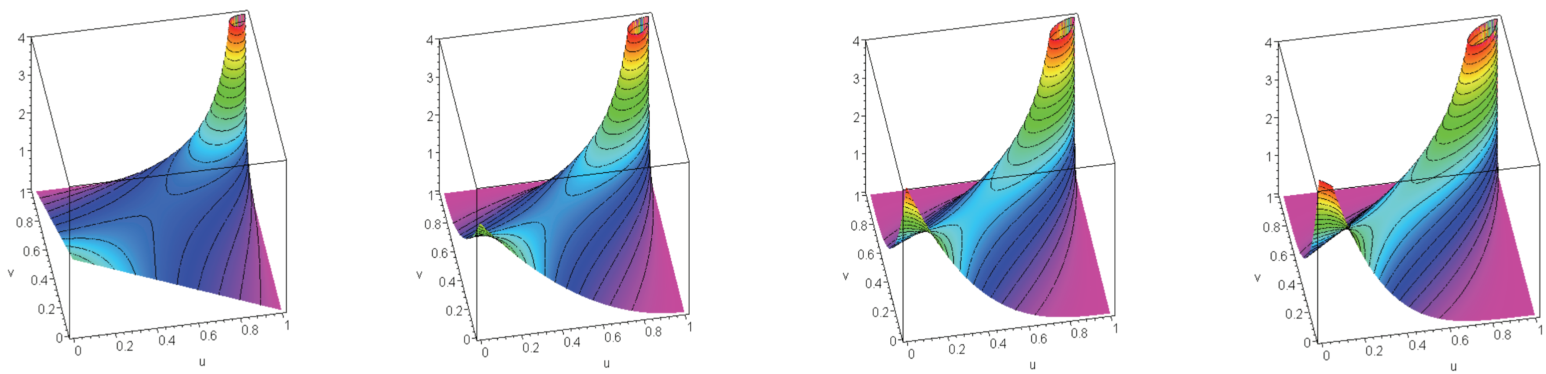
Lemma 1. For $\beta \in \mathbb{N}$ and $u, v \in (0,1)$, there holds

$$c_\beta(u,v) = (\beta+1)\frac{\left((1-u)(1-v)\right)^\beta}{(1-uv)^{2\beta+1}}\sum_{i=0}^{\beta-1}\binom{\beta-1}{i}\binom{\beta+1}{i}(uv)^i. \qquad (3.12)$$

3. The symmetric case (diagonal dominance)

Example 2 (negative binomial distributions).

The following graphs show the negative binomial copula densities c_β for $\beta = 1, \cdots, 4$.



$\beta = 1$ $\beta = 2$ $\beta = 3$ $\beta = 4$

3. The symmetric case (diagonal dominance)

Example 2 (negative binomial distributions).

Negative binomial copulas typically show an upper tail dependence, as can be seen from the following exemplary table.

β	1	2	3	4	5	6	7	8	9	10
$\lambda_U(\beta)$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{93}{128}$	$\frac{193}{256}$	$\frac{793}{1024}$	$\frac{1619}{2048}$	$\frac{26333}{32768}$	$\frac{53381}{65536}$	$\frac{215955}{262144}$

3. The symmetric case (diagonal dominance)

Example 2 (negative binomial distributions).

A closed formula for the tail dependence coefficients for integer values of β is given in the following result.

Lemma 2. For $\beta \in \mathbb{N}$, there holds

$$\lambda_U(\beta) = \lim_{t\uparrow 1} \frac{\int_t^1 \int_t^1 c_\beta(u,v)\,du\,dv}{1-t} = \frac{2\Gamma(2\beta)}{\Gamma^2(\beta)} \cdot \int_0^1 \int_0^1 \frac{x^\beta y^\beta}{(x+y)^{2\beta+1}}\,dx\,dy$$

$$= \frac{2\Gamma(2\beta)}{\Gamma^2(\beta)} \cdot \left(\sum_{k=0}^{\beta} \binom{\beta}{k} \frac{(-1)^k}{\beta+k} \sum_{j=0}^{\beta+k-2} \binom{\beta+k}{j+2} \frac{(-1)^{j+1}}{j+1} \left(1 - \frac{1}{2^{j+1}}\right) \right). \quad (3.19)$$

3. The symmetric case (diagonal dominance)

Example 3 (Poisson distributions).

Consider the family of Poisson distributions given by their point masses

$$\varphi_{\gamma,i}(u) = (1-u)^{\gamma} \frac{\gamma^i L(u)^i}{i!}, \; i \in \mathbb{Z}^+ \tag{3.27}$$

where $L(u) = -\ln(1-u) > 0$, $u \in (0,1)$ and $\gamma > 0$.

3. The symmetric case (diagonal dominance)

Example 3 (Poisson distributions).

Here for $i \in \mathbb{Z}^+$, with the substitutions $z = L(u)$ and $y = (1+\gamma)z$,

$$\alpha_{\gamma,i} = \int_0^1 \varphi_{\gamma,i}(u)du = \int_0^1 (1-u)^\gamma \frac{\gamma^i L(u)^i}{i!}du = \int_0^\infty \frac{\gamma^i z^i}{i!} e^{-(1+\gamma)z}\,dz$$

$$= \frac{\gamma^i}{(1+\gamma)^{i+1}} \int_0^\infty \frac{y^i}{i!} e^{-y}\,dy = \frac{\gamma^i}{(1+\gamma)^{i+1}} = \left(\frac{\gamma}{1+\gamma}\right)^i \left(1 - \frac{\gamma}{1+\gamma}\right), \qquad (3.28)$$

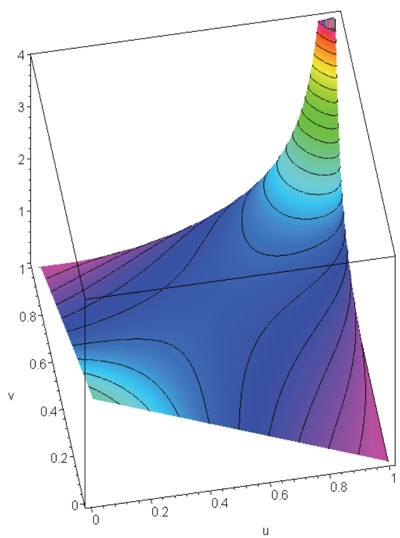
indicating that the $\alpha_{\gamma,i}$ correspond to a geometric distribution with mean γ, and hence for $u,v \in (0,1)$,

$$c_\gamma(u,v) = (1+\gamma)(1-u)^\gamma (1-v)^\gamma \sum_{i=0}^\infty \frac{\left(\gamma(1+\gamma)\ln(1-u)\ln(1-v)\right)^i}{i!^2}. \qquad (3.29)$$

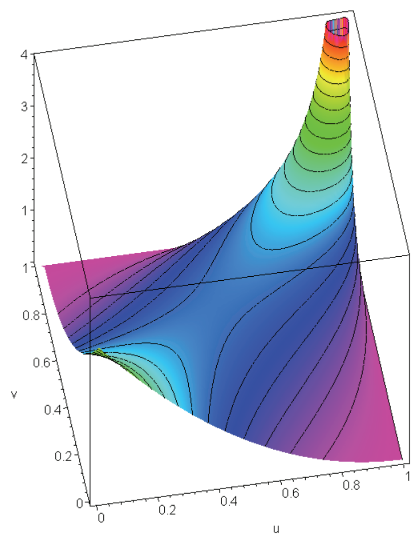
3. The symmetric case (diagonal dominance)

Example 3 (Poisson distributions).

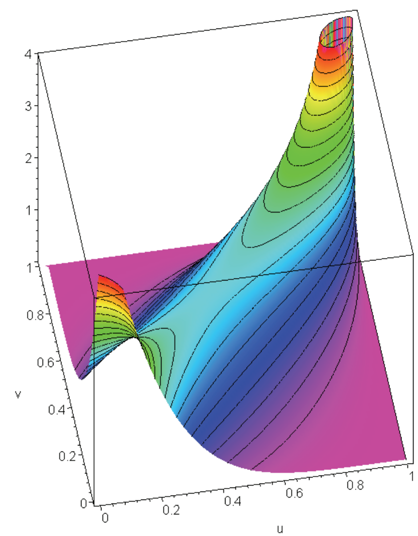
The following graphs show some of these copula densities for different choices of γ.



$\gamma = 1$



$\gamma = 2$

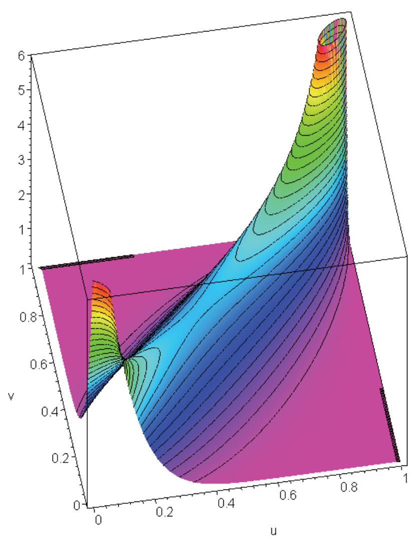


$\gamma = 5$

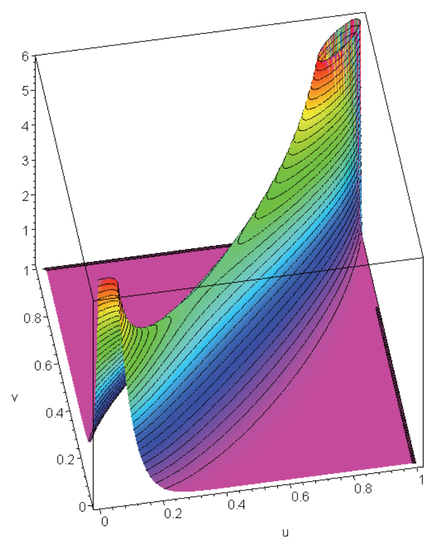
3. The symmetric case (diagonal dominance)

Example 3 (Poisson distributions).

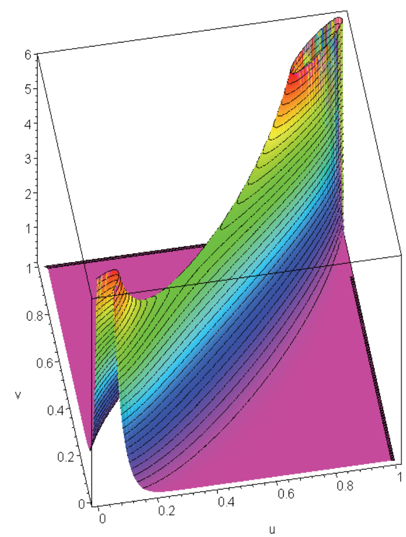
The following graphs show some of these copula densities for different choices of γ.



$\gamma = 10$



$\gamma = 20$



$\gamma = 30$

3. The symmetric case (diagonal dominance)

Example 3 (Poisson distributions).

The corresponding copula C cannot be calculated explicitly. However, in contrast to the visual impression, the coefficient $\lambda_U(\gamma)$ of upper tail dependence is zero here for all $\gamma > 0$, although we have a singularity in the point (1,1) in all cases.

3. The symmetric case (diagonal dominance)

Example 4 (log series distribution).

Consider the family of log series distributions given by their point masses

$$\varphi_i(u) = \frac{u^i}{i \cdot L(u)}, \; i \in \mathbb{Z}^+ \qquad (3.33)$$

where again $L(u) = -\ln(1-u), u \in (0,1)$. Here we get

$$\alpha_i = \int_0^1 \varphi_i(u)du = \frac{\beta_i}{i} = \frac{1}{i}\sum_{j=1}^{i}\binom{i}{j}(-1)^{j+1}\ln(j+1) \text{ for } i \in \mathbb{N}. \qquad (3.34)$$

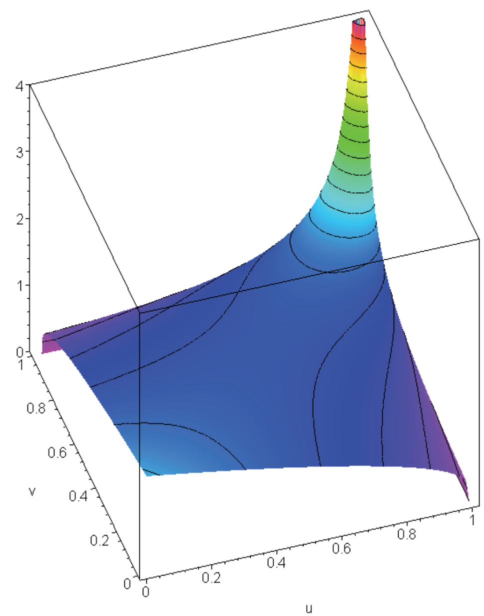
The density of the bivariate log series copula is given by

$$c(u,v) = \sum_{i=1}^{\infty}\frac{1}{\alpha_i}\varphi_i(u)\varphi_i(v) = \frac{1}{\ln(1-u)\ln(1-v)}\sum_{i=1}^{\infty}\frac{(uv)^i}{i\beta_i}, u,v \in (0,1). \qquad (3.42)$$

3. The symmetric case (diagonal dominance)

Example 4 (log series distribution).

The following graph shows the corresponding copula density. The log series copula also has no tail dependence.



plot of $c(u,v)$

4. The asymmetric case

Specifying the probabilities p_{ij} in a non-symmetric way we obtain asymmetric copula densities even if the maps $\varphi_i(\bullet)$ and $\psi_j(\bullet)$ are identical. A very simple approach to this problem is a specification of a suitable non-symmetric $(n+1)\times(n+1)$-matrix $M_n = \left[p_{ij}\right]_{i,j=0,\cdots,n}$ for $n \in \mathbb{Z}^+$ with

$$\sum_{k=0}^{n} p_{ik} = \sum_{k=0}^{n} p_{ki} = \alpha_i \quad \text{for } i = 0,\cdots,n \tag{4.1}$$

and

$$p_{ij} := \begin{cases} \alpha_i, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases} \quad \text{for } i,j > n. \tag{4.2}$$

4. The asymmetric case

Example 5 (negative binomial distributions).

We consider negative binomial distributions with $\beta = 1$. Then $\alpha_i = \int_0^1 \varphi_{1,i}(u)du = \frac{1}{(1+i)(2+i)}$ for $i \in \mathbb{Z}^+$. With $n = 4$ and

$$M_4 := \frac{1}{60}\begin{bmatrix} 18 & 5 & 5 & 0 & 2 \\ 10 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 2 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.3}$$

we obtain

4. The asymmetric case

Example 5 (negative binomial distributions).

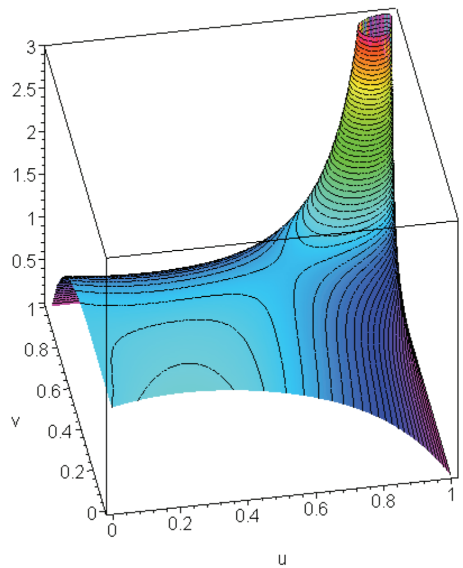
$$c(u,v)=\frac{(1-u)(1-v)}{5(1-uv)^3}H(u,v),\ u,v\in(0,1) \qquad (4.5)$$

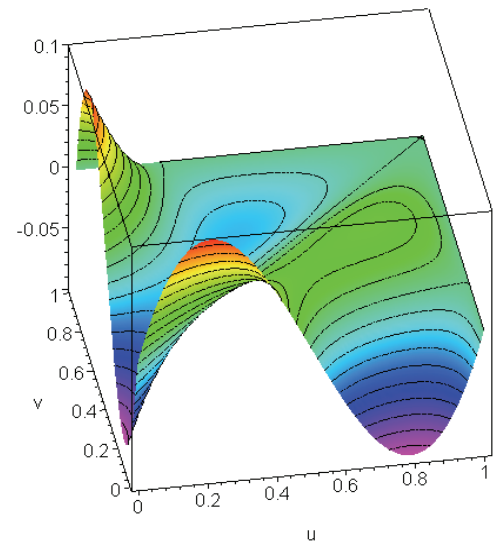
with the polynomial

$$\begin{aligned}H(u,v)={}&150u^7v^7-450u^6v^6-10u^7v^3+510u^5v^5-10u^3v^7-\dots\\&\dots-30u^5v^4-10u^3v^5+30u^2v^6-300u^4v^4+30u^6v^2-\dots\\&\dots-5u^3v^4+80u^4v^3-30u^5v+94u^3v^3+30u^2v^4-\dots\\&\dots-30uv^5-60u^3v^2+15u^2v^3+10u^4+18u^2v^2-30uv^3+\dots\\&\dots+10v^4-15uv^2+10v^2-18uv+10u+5v+6.\end{aligned} \qquad (4.6)$$

4. The asymmetric case

Example 5 (negative binomial distributions).





plot of $c(u,v)$ plot of $c(u,v)-c(v,u)$

The corresponding copula C has a coefficient of upper tail dependence $\lambda_U = \dfrac{1}{2}$ as in the symmetric case.

4. The asymmetric case

Example 6 (different negative binomial distributions).

For $\beta = 1$ and $\beta = 2$, resp. we get

$$\alpha_i = \int_0^1 \varphi_{1,i}(u)\,du = \frac{1}{(1+i)(2+i)} \quad \text{and}$$

$$\beta_j = \int_0^1 \varphi_{2,j}(v)\,dv = \frac{2}{(2+j)(3+j)} = 2\alpha_{j+1} \text{ for } i,j \in \mathbb{Z}^+.$$

Let further
$$\left[p_{ij}\right]_{i,j\in\mathbb{Z}^+} = \begin{bmatrix} \beta_0 & \beta_1 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \beta_2 & \beta_3 & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \beta_4 & \beta_5 & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \beta_6 & \beta_7 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \end{bmatrix} \quad (4.8)$$

where $\cdots$ stands for zero.

4. The asymmetric case

Example 6 (different negative binomial distributions).

Then $p_{i\bullet} = \sum_{j=0}^{\infty} p_{ij} = \alpha_i$ and $p_{\bullet j} = \sum_{i=0}^{\infty} p_{ij} = \beta_j$ for $i, j \in \mathbb{Z}^+$ since

$$\beta_{2i} + \beta_{2i+1} = \frac{2}{(2+2i)(3+2i)} + \frac{2}{(3+2i)(4+2i)} = \frac{1}{(1+i)(2+i)} = \alpha_i. \quad (4.9)$$

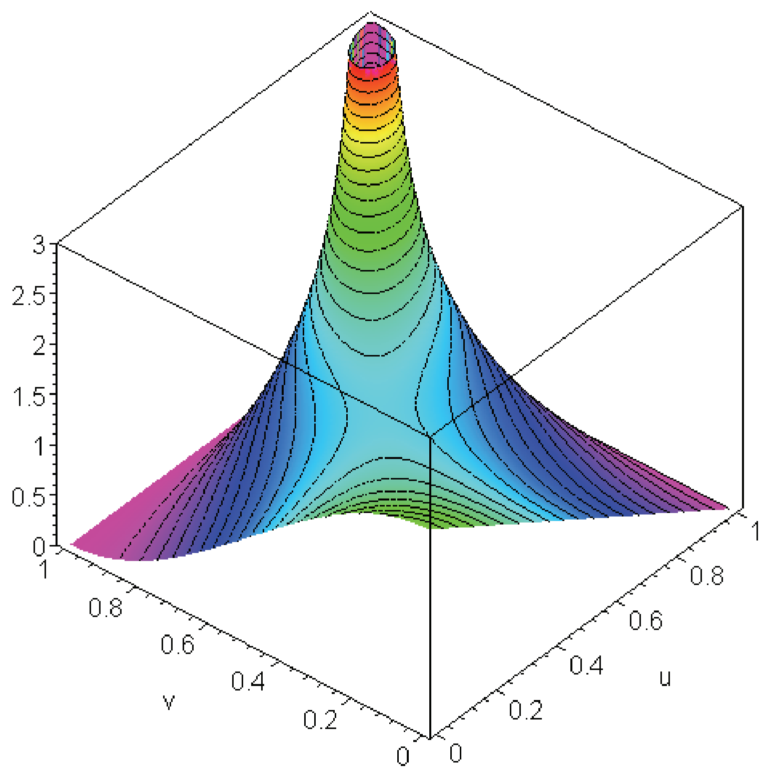
It now follows that

$$c(u,v) = \frac{2(1-u)(1-v)^2\left(1+2v+5uv^2+4uv^3\right)}{\left(1-uv^2\right)^4},\ u,v \in (0,1) \qquad (4.11)$$

which obviously is asymmetric.

4. The asymmetric case

Example 6 (different negative binomial distributions).



plot of $c(u,v)$

4. The asymmetric case

Example 6 (different negative binomial distributions).

The corresponding copula C is, for $x, y \in (0,1)$, given by

$$C(x,y) = \frac{xy}{\left(1 - xy^2\right)^4}\left(2 - x - 2xy^3 + xy^4 + x^2y^3 - 2y^2 + y^3\right) \qquad (4.12)$$

This copula has a coefficient of upper tail dependence of

$$\lambda_U = \frac{5}{9} \qquad (4.13)$$

5. Conclusions and Applications

Remark 1: Negative binomial copulas can easily be simulated through the alternative representation formula (2.5) involving mixed product-Beta distributions. Poisson copulas can be simulated using the transformation $z \mapsto 1-e^{-z}$ applied to Gamma distributed random variables Z with a random shape parameter α, where $\alpha-1$ is generated by the geometric distribution shown in (3.28), and scale parameter $1+\gamma$.

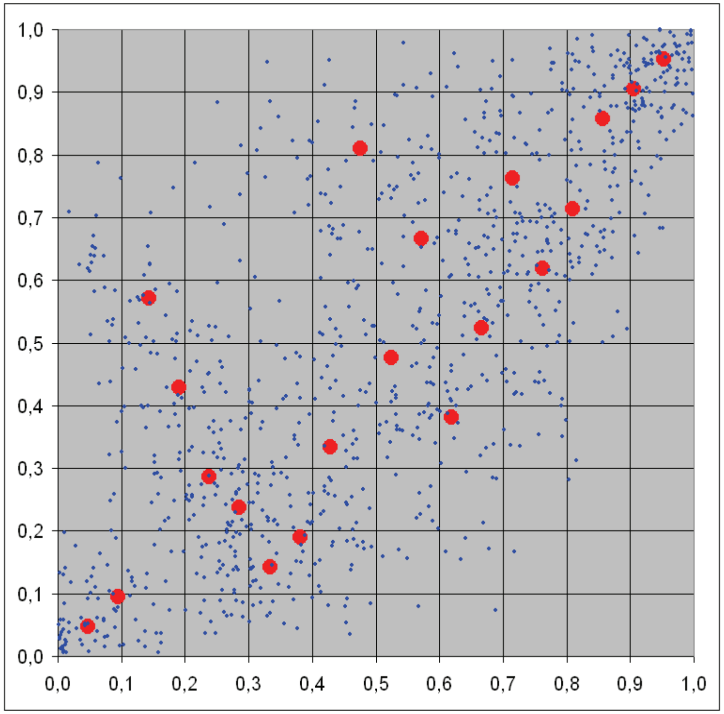
5. Conclusions and Applications

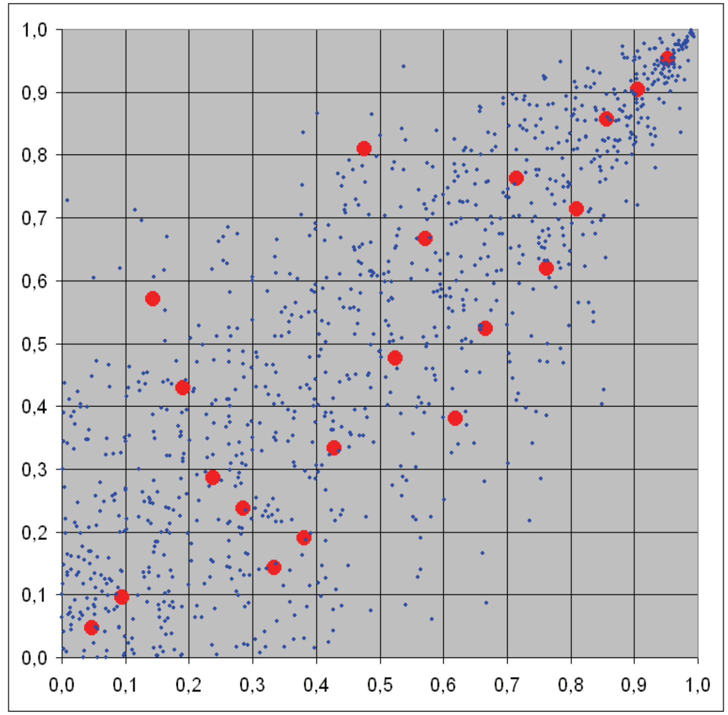
Remark 2: For practical applications in quantitative risk management, it seems reasonable to fit the required probabilities $\left[p_{ij}\right]_{i,j\in\mathbb{Z}^+}$ to empirical data via their empirical copula, for instance as was proposed in PFEIFER, STRASSBURGER AND PHILIPPS (2009). In the particular case of Bernstein copulas such a procedure can be very easily implemented, even in higher dimensions (cf. COTTIN AND PFEIFER (2014)).

5. Conclusions and Applications

As a practical exercise, we refer to Example 4.2 in COTTIN AND PFEIFER (2014) where the empirical copula from an original data set was fitted to a Bernstein copula. The following two graphs show the scatter plot from the empirical copula (big red dots) superimposed by 1000 simulated points of that Bernstein copula (left) and of a negative binomial copula of type (3.11), with $\beta = 5$.

5. Conclusions and Applications





Bernstein copula fit   negative binomial copula fit

The Bernstein copula represents the local asymmetry of the empirical copula better, but shows no tail dependence, as does the negative binomial copula.

5. Conclusions and Applications

The fit to the negative binomial copula was, for the sake of simplicity, performed by a numerical match between the theoretical correlation for the negative binomial copula and the correlation of the empirical copula, which is 0.815. Note that the theoretical correlation $\rho(\beta)$ for the negative binomial copula of type (3.11) can be explicitly calculated as

$$\rho(\beta) = 12\beta\left(\sum_{i=0}^{\infty} \frac{(i+1)^2}{(\beta+i)(\beta+i+1)(\beta+i+2)^2}\right) - 3$$
$$= 3\beta\left(2(\beta+1)^2\Psi(1,\beta+2) - 2\beta - 1\right)$$

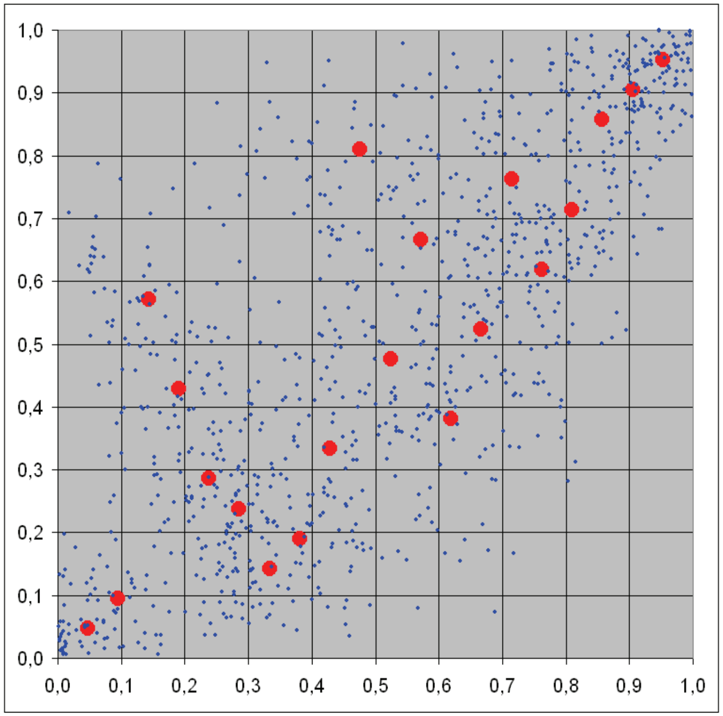
where $\Psi(1,z)$ denotes the first derivative of the digamma function, or $\Psi(1,z) = \frac{d^2}{dz^2}\ln\Gamma(z),\ z>0.$

5. Conclusions and Applications

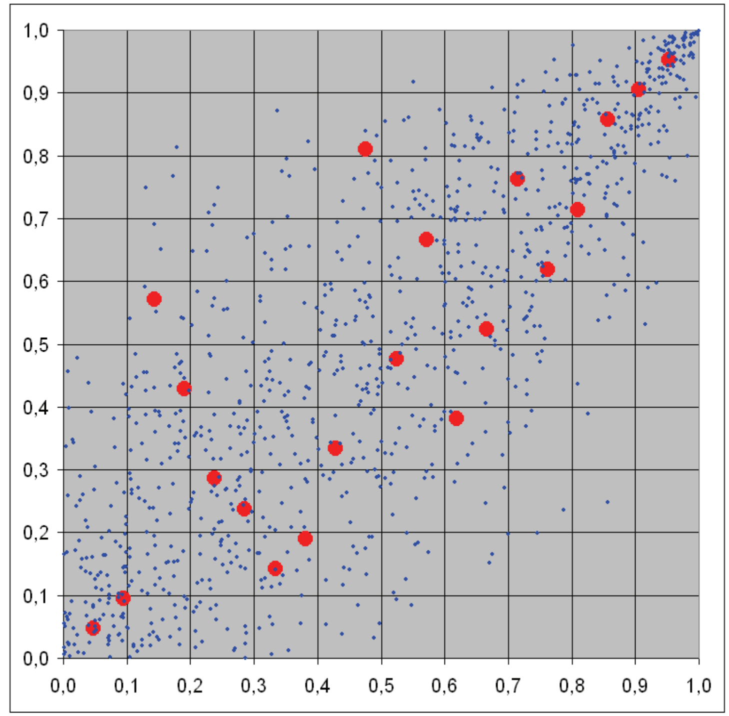
β	1	2	3	4	5	6	7
$\rho(\beta)$	0.4784	0.6529	0.7410	0.7937	0.8288	0.8537	0.8723

For the sake of completeness, we finally show a comparison between the Bernstein copula fit and a Poisson copula fit with parameter $\gamma = 6$. The empirical correlation for the Poisson copula here is 0.814.

5. Conclusions and Applications



Bernstein copula fit



Poisson copula fit

Note that although the empirical plot for the Poisson copula might suggest some tail dependence here this is actually not true in the light of (3.32).

6. Selected References

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