HOCHSCHULE LANDSHUT
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A Model based Analysis of German Interest Rates

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ASTIN Colloquium 2016, Lisbon

Outline

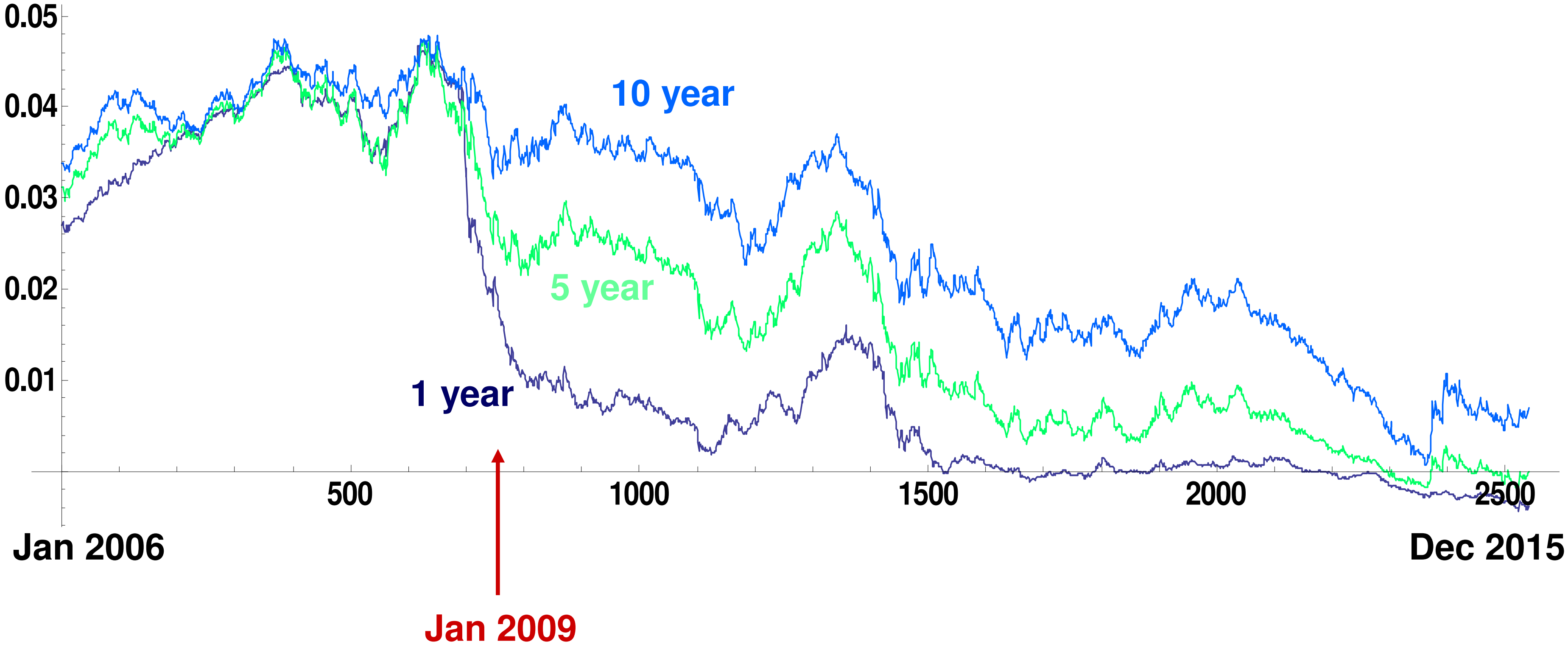
- Introduction: Interest rate data
- Interest rate models
- Data analysis: Parameter estimation
- Results

Interest rate data

Interest Rate Data Germany: Source “Deutsche Bundesbank”, Statistics

→ Interest term structure (1- 15 year rates), daily Data:

Analysis here: 1, 5 and 10 year rates:



Interest rate models

Standard Models: Langevin – type

- Drift term: Mean reverting
- Stochastic term

Cox – Ingersoll – Ross Model (CIR)

$$dr(t) = \kappa(\mu - r(t))\,dt + \sigma\sqrt{r(t)}\,d\tilde{W}$$

Vasicek Model (stationary Hull-White; Ornstein-Uhlenbeck-Process)

$$dr(t) = \kappa(\mu - r(t))\,dt + \sigma\,d\tilde{W}$$

with the **Parameters**: μ (mean reverting level),
κ ("velocity" of reverting),
σ (standard deviation, stochastic term)

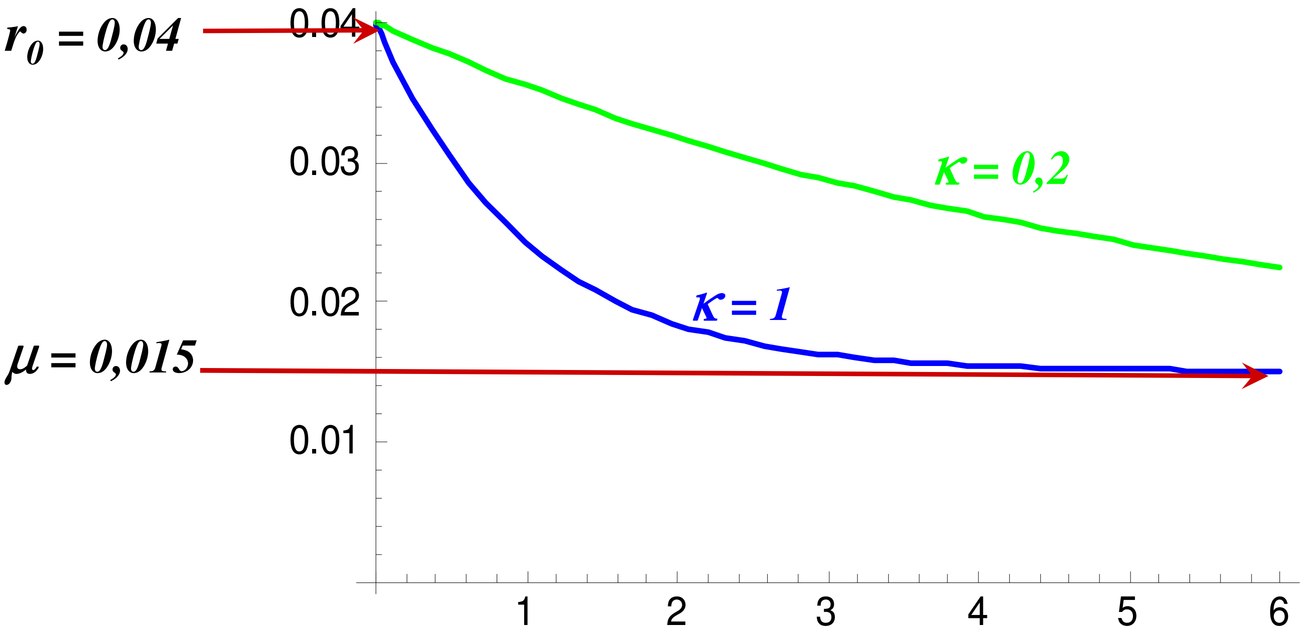
$d\tilde{W}$ is a **Wiener – process** with mean 0, standard deviation 1 and independent increments.

Interest rate models

Deterministic Part (σ → 0):

- Equal for both models
- Differential equation with initial condition $r(t=0) = r_0$

$$\frac{dr(t)}{dt} = \kappa\left(\mu - r(t)\right) \rightarrow r(t) = \mu - \left(\mu - r_0\right)e^{-\kappa t}$$



Parameter Estimation

Parameter Estimation

Parameters: μ (mean reverting level),
κ ("velocity" of reverting),
σ (standard deviation, stochastic term)

→ Linear Regression: Interest – model equation can be written as:

$$r_{t+\Delta} = \kappa \mu \Delta + (1 - \kappa \Delta) r_t + \sigma \sqrt{\Delta}\, r_t^{\alpha} \varepsilon_t$$

with: Δ: Time step between the data (Time scaling behaviour!)
ε : Random variable (standard normal, independent)
α : Vasicek for $\alpha = 0$; CIR for $\alpha = 0.5$

Parameter Estimation

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→ **Linear Regression**: Interest – model equation can be written as:

$$r_{t+\Delta} = \kappa\,\mu\,\Delta + (1-\kappa\,\Delta)\,r_t + \sigma\sqrt{\Delta}\,r_t^{\alpha}\,\varepsilon_t$$

off - set and **slope** of **regression line**

residuals

with: Δ: Time step between the data (Time scaling behaviour!)
ε : Random variable (standard normal, independent)
α : Vasicek for $\alpha = 0$; CIR for $\alpha = 0.5$

Parameter Estimation

Linear Regression with weights indirect proportional to data variance.

Bayes Framework:
"Degree of belief"
of single data
(Ref.: D.S. Sivia, Data Analysis, Oxford U.P., 2006)

→Vasicek: Equally weighted (standard linear regression)

→CIR: Weights ~ 1/r for CIR

Problem for r→0.

Solution here:
Weights ~ 1/ Max(|r|;0,001) → Cut-off!

Parameter Estimation

We analyse….

Time – scaling behaviour of the estimated parameters:

Δ: 1 day, 10, 20, 30, 40, 50 days.

consistent model → parameters independent of time scale Δ.

"Stationarity":

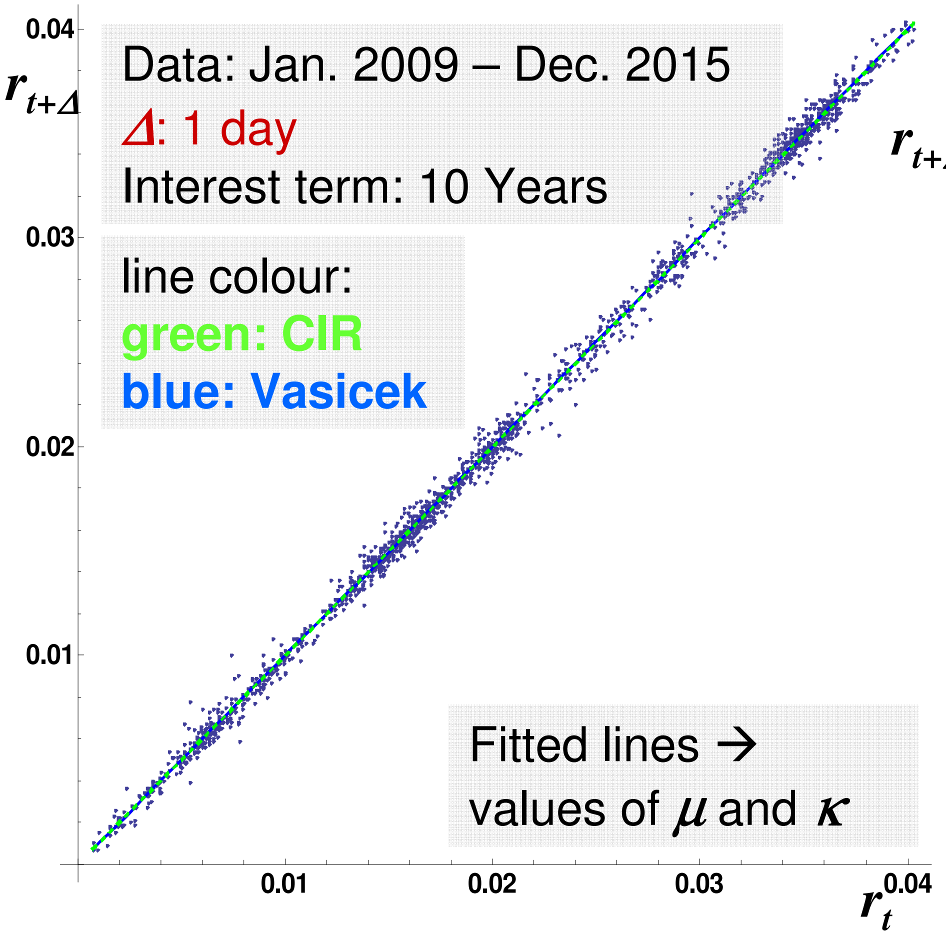
two different data sets: Jan 2006 – Dec 2015 and
Jan 2009 – Dec 2015 ("younger data").

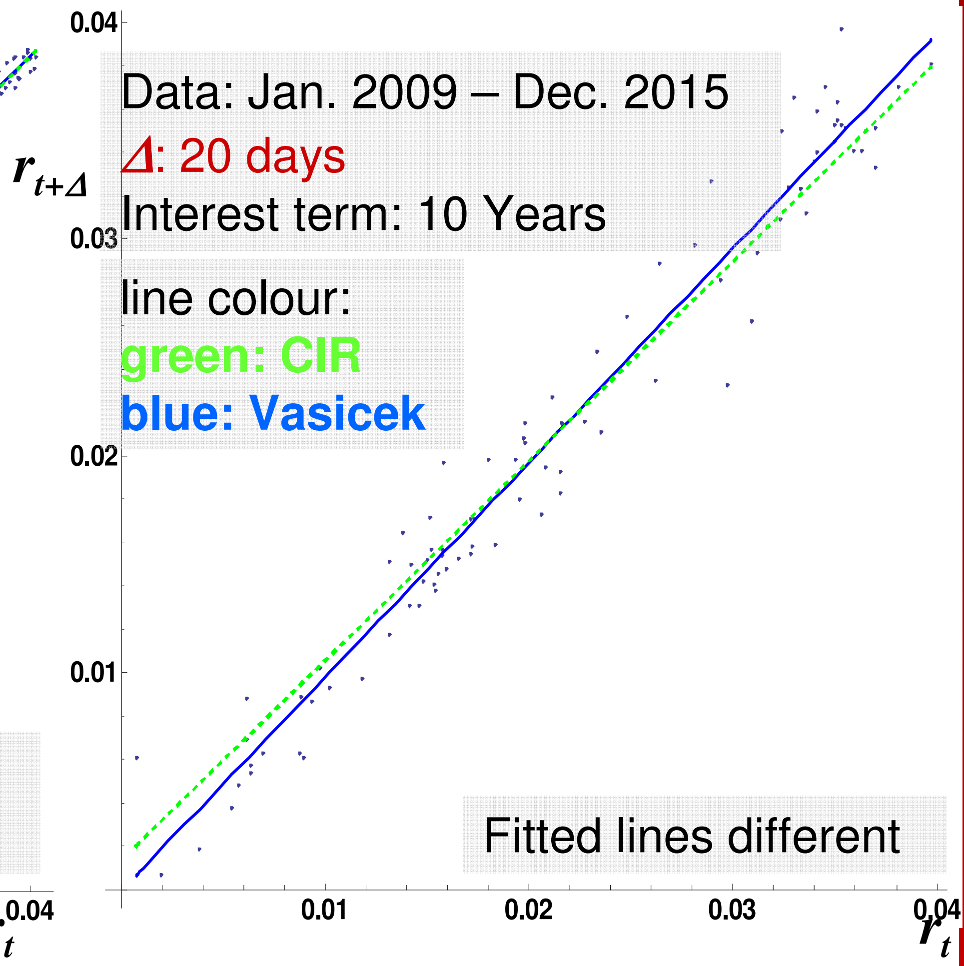
"Better" statistics vs. "relevant", present data.

Parameter Estimation - Results

Interest rate data and fitted lines (Vasicek and CIR)

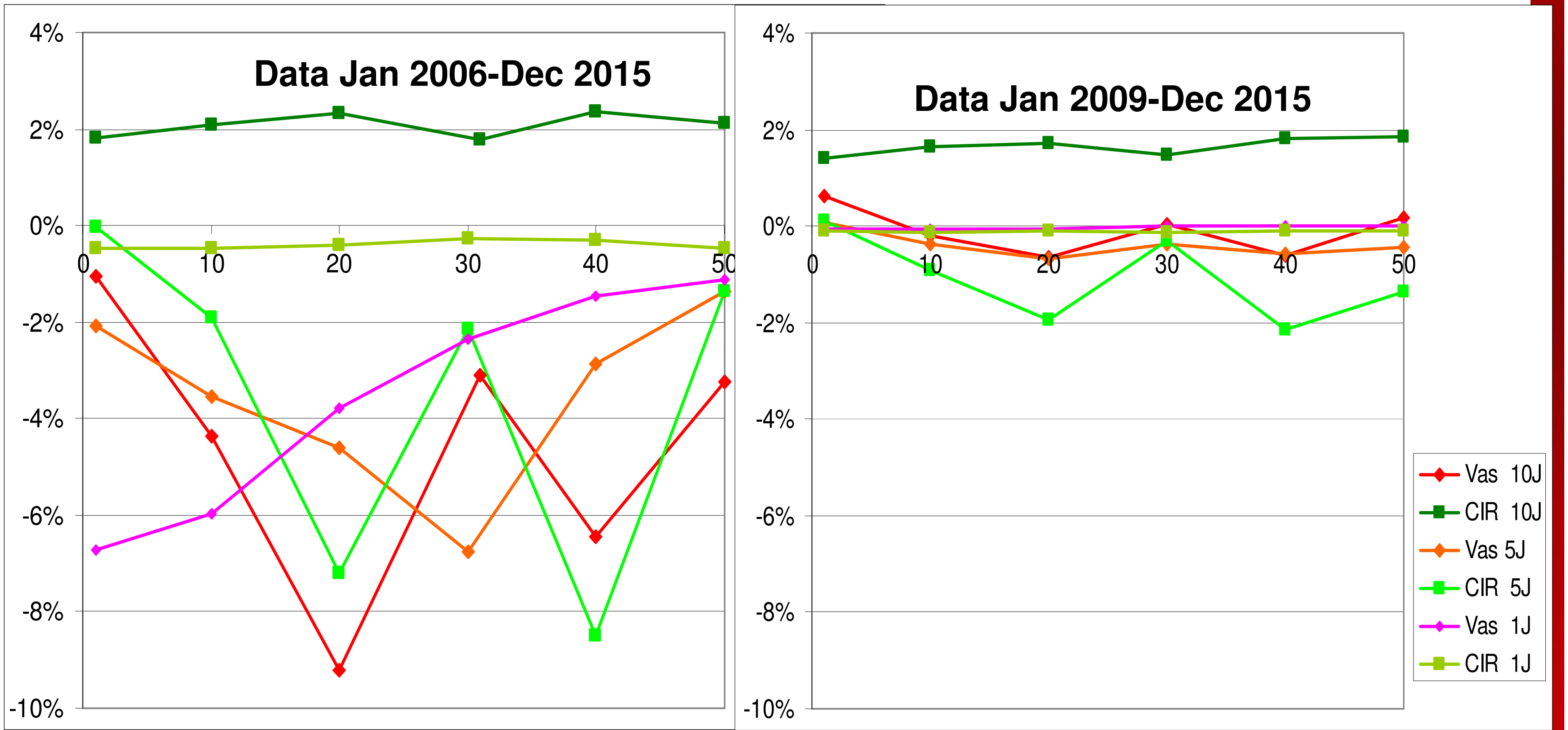
From line parameters → μ and κ:





Parameter Estimation - Results

Parameter μ : Parameter value dependent on Δ (daily, 10, 20, …, 50 days) for CIR (green) and Vasicek (red); for 1-, 5-, 10-year interest rate



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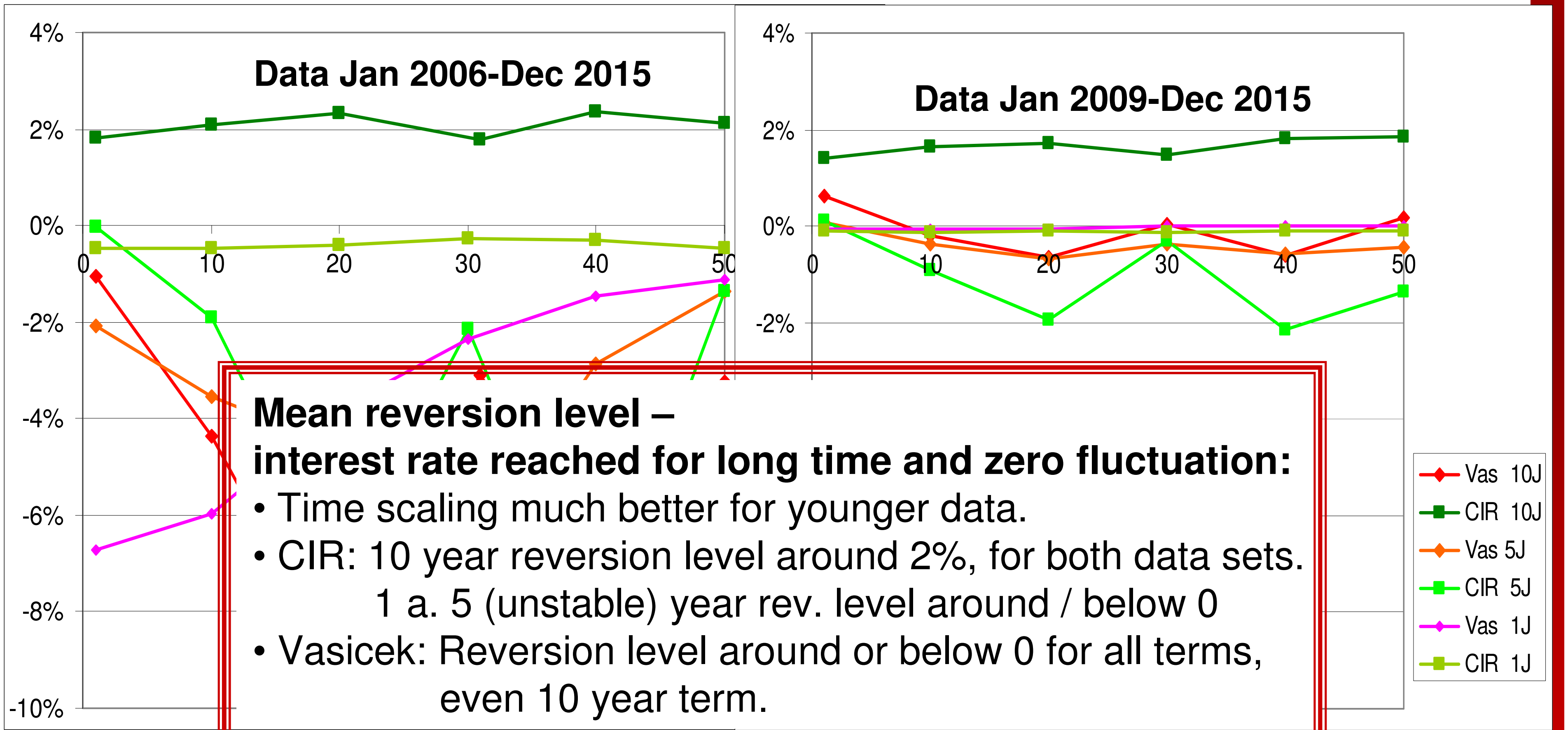
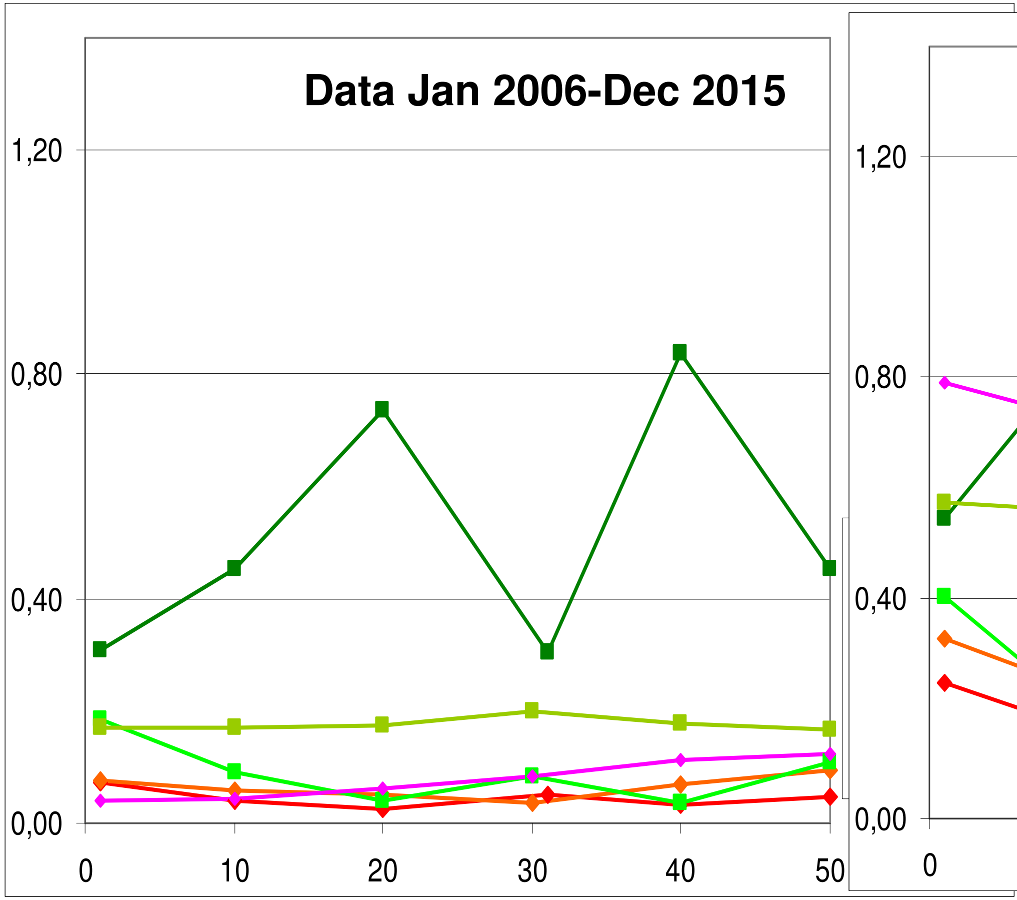


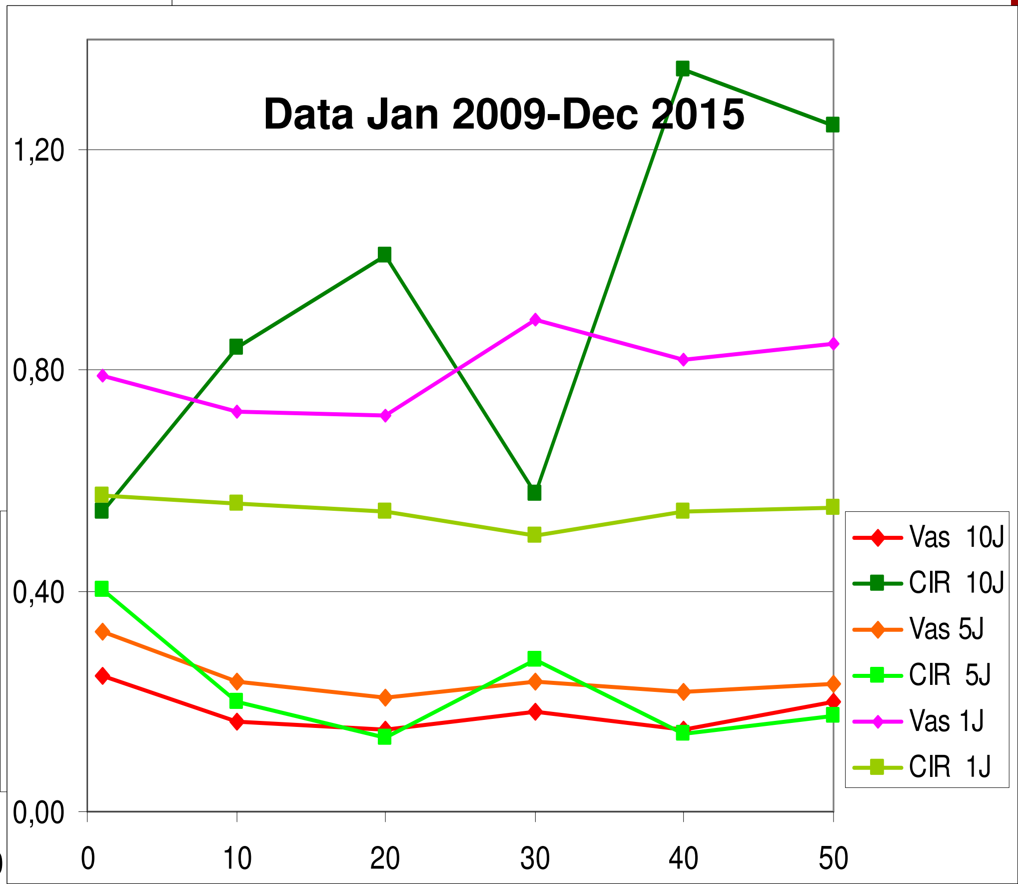
Mean reversion level –
interest rate reached for long time and zero fluctuation:

- Time scaling much better for younger data.
- CIR: 10 year reversion level around 2%, for both data sets.
  1 a. 5 (unstable) year rev. level around / below 0
- Vasicek: Reversion level around or below 0 for all terms,
  even 10 year term.

Parameter Estimation - Results

Parameter κ : Parameter value dependent on Δ (daily, 10, 20, …, 50 days) for CIR (green) and Vasicek (red); for 1-, 5-, 10-year interest rate





Parameter Estimation - Results

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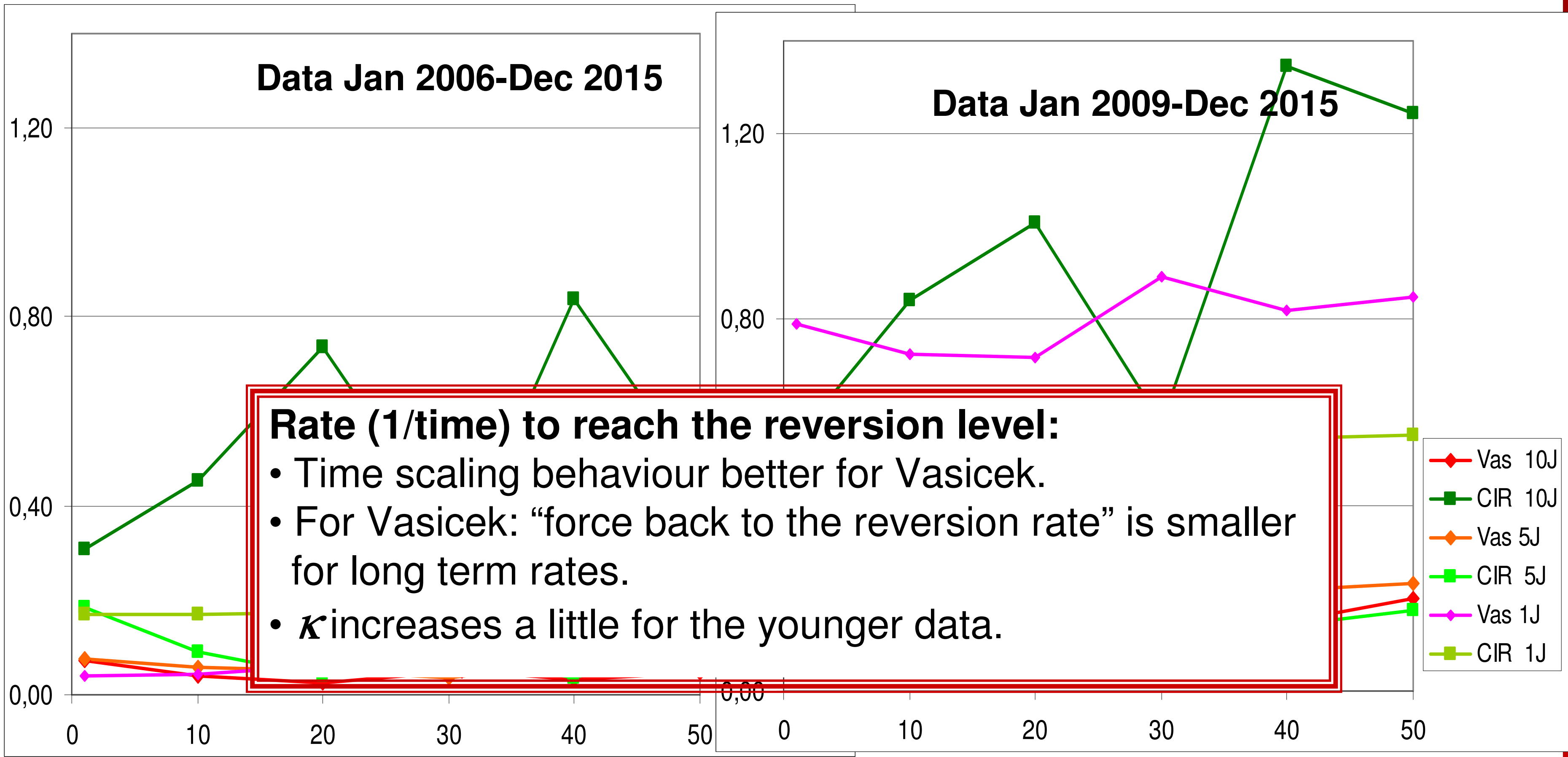
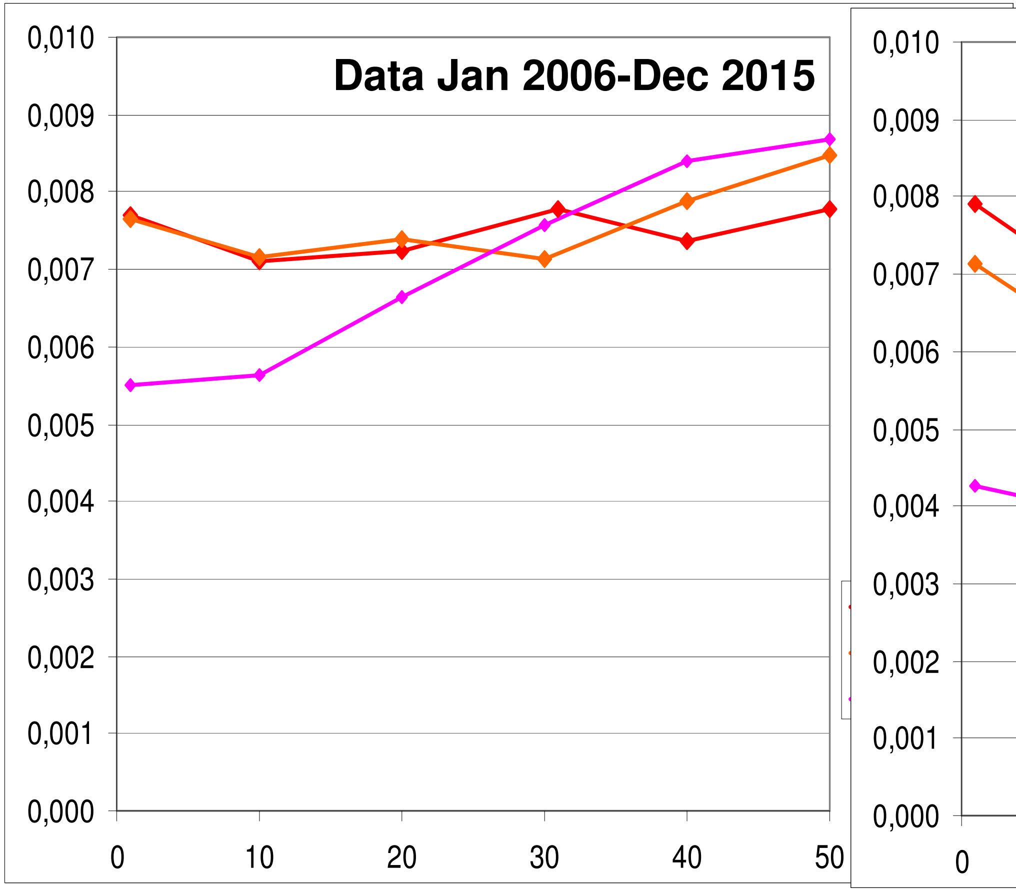
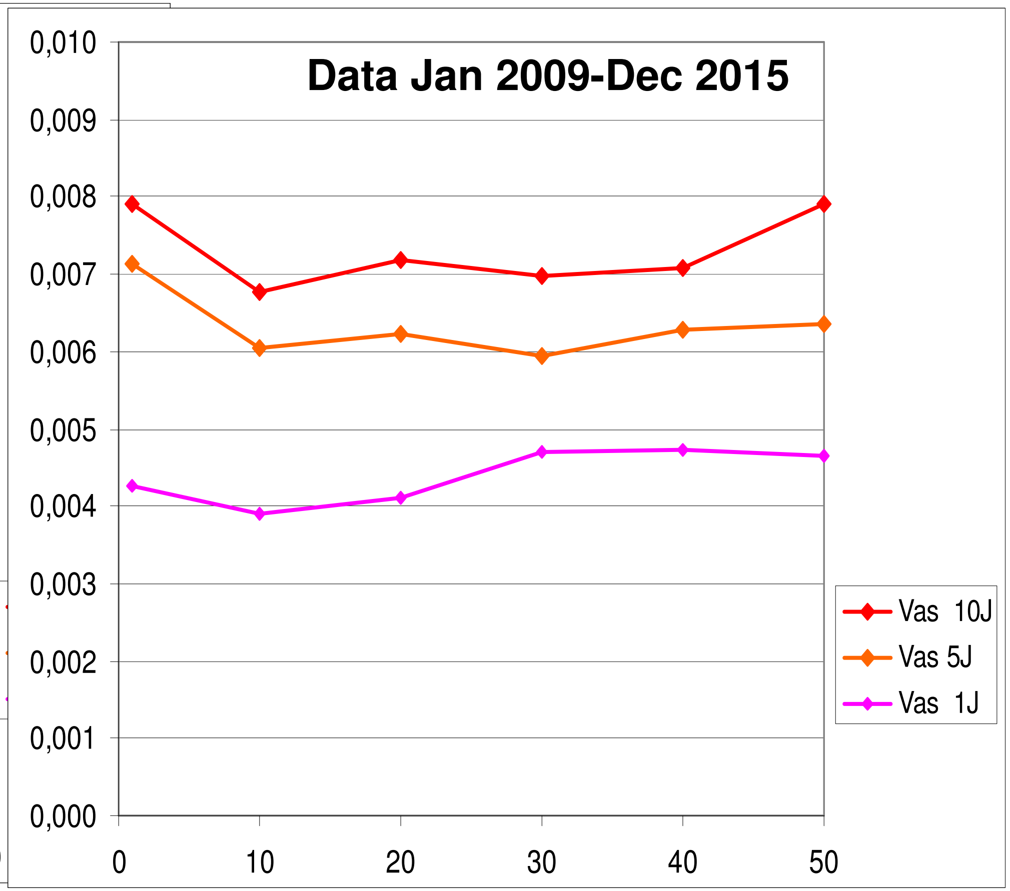


Rate (1/time) to reach the reversion level:
- Time scaling behaviour better for Vasicek.
- For Vasicek: “force back to the reversion rate” is smaller for long term rates.
- κ increases a little for the younger data.

Parameter Estimation - Results

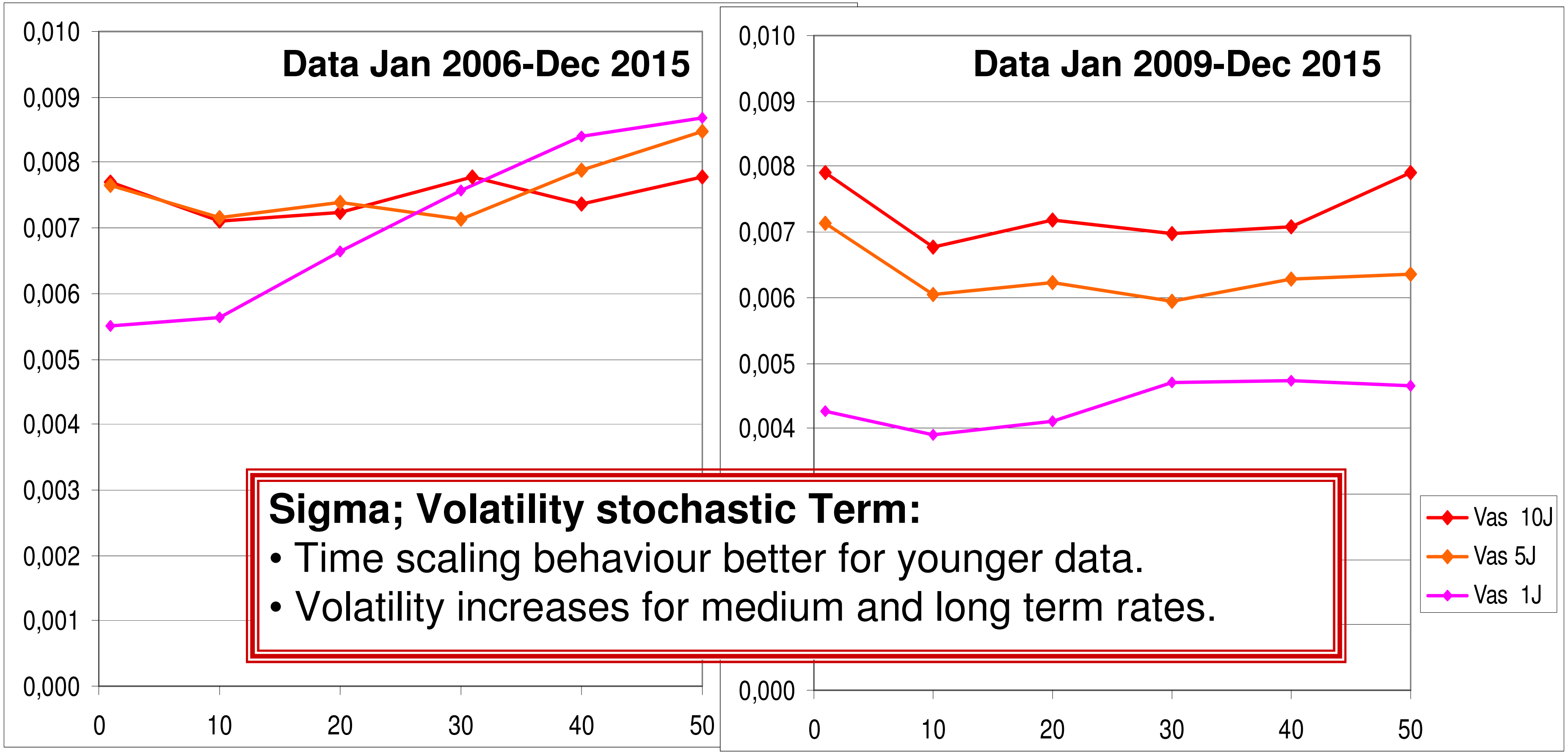
Parameter σ : Parameter value dependent on Δ (daily, 10, 20, …, 50 days) for **Vasicek**; for 1-, 5-, 10-year interest rate





Parameter Estimation - Results

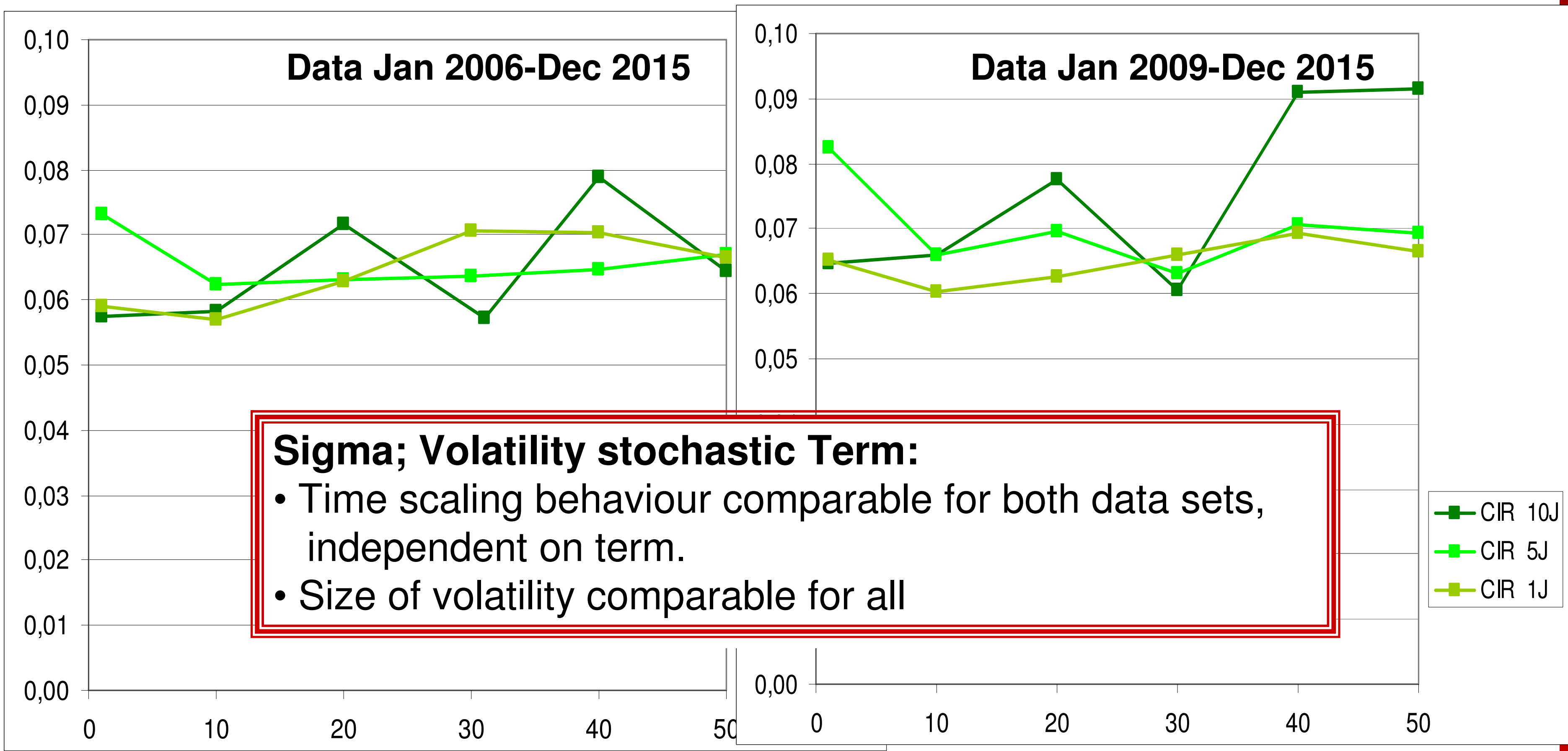
Parameter σ : Parameter value dependent on Δ (daily, 10, 20, …, 50 days) for **Vasicek**; for 1-, 5-, 10-year interest rate



Sigma; Volatility stochastic Term:
- Time scaling behaviour better for younger data.
- Volatility increases for medium and long term rates.

Parameter Estimation - Results

Parameter σ : Parameter value dependent on Δ (daily, 10, 20, …, 50 days) for **CIR**; for 1-, 5-, 10-year interest rate



Sigma; Volatility stochastic Term:

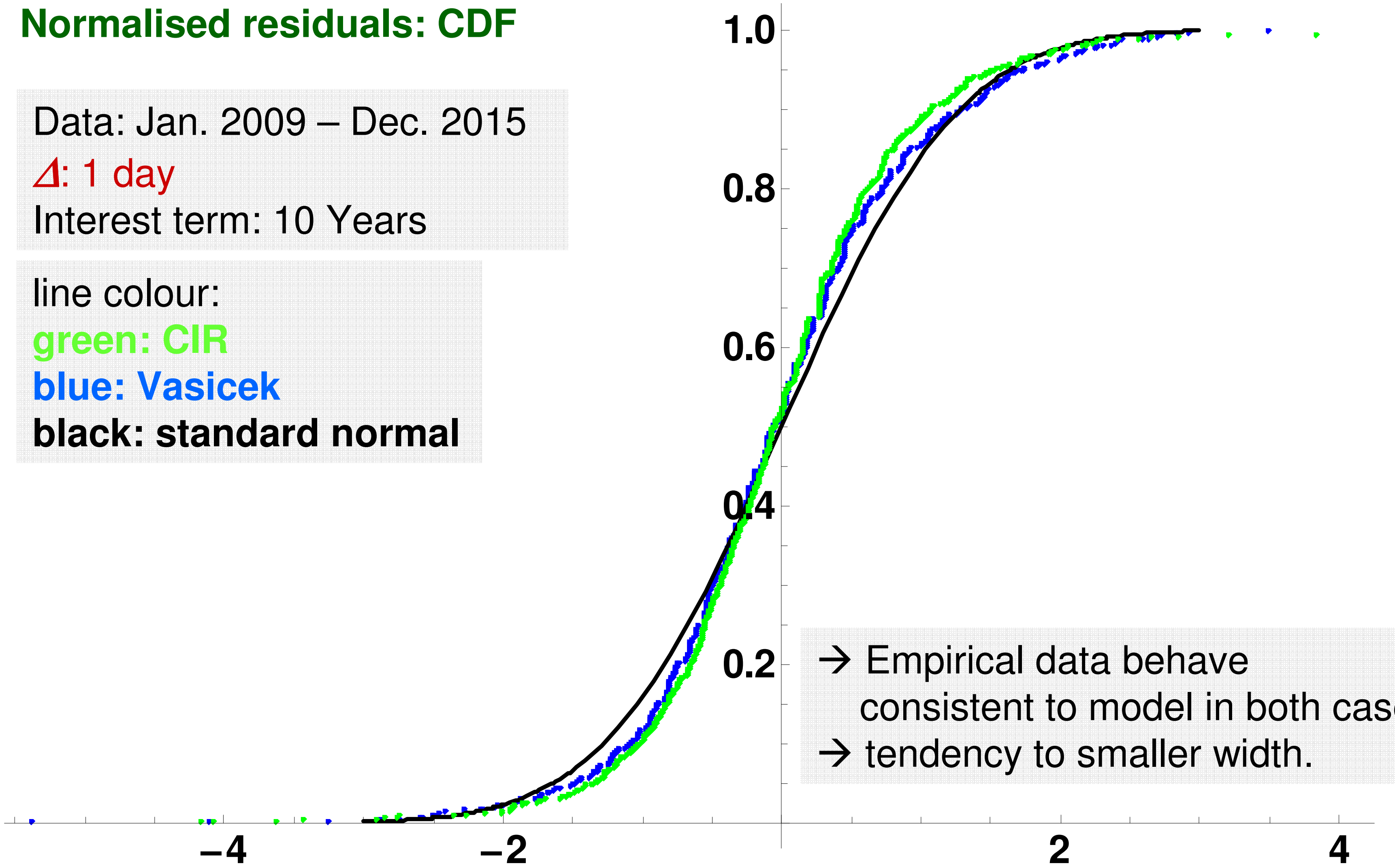
- Time scaling behaviour comparable for both data sets, independent on term.
- Size of volatility comparable for all

Parameter Estimation - Results

Normalised residuals: CDF

Data: Jan. 2009 – Dec. 2015
Δ: 1 day
Interest term: 10 Years

line colour:
green: CIR
blue: Vasicek
black: standard normal



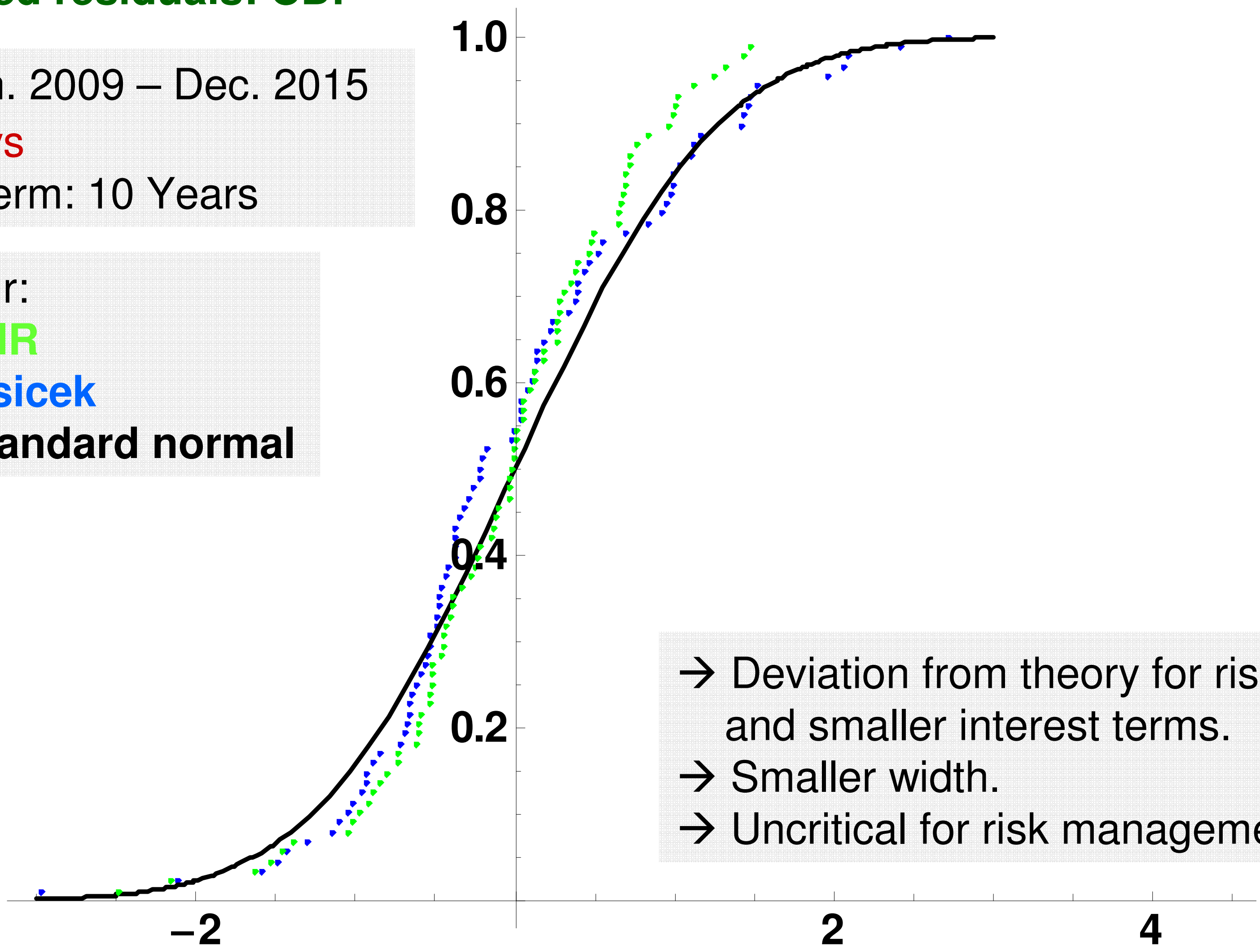
→ Empirical data behave consistent to model in both cases.
→ tendency to smaller width.

Parameter Estimation - Results

Normalised residuals: CDF

Data: Jan. 2009 – Dec. 2015
Δ: 20 days
Interest term: 10 Years

line colour:
green: CIR
blue: Vasicek
black: standard normal



→ Deviation from theory for rising Δ and smaller interest terms.
→ Smaller width.
→ Uncritical for risk management!

Parameter Estimation - Results

Conclusion

Time series 2009-2015 (short series, younger data) give better results.

CIR: → μ estimator for long term rates around 2%.
→ weights for r → 0 ? => κ estimator problematic
→ σ estimator: time scaling behaviour acceptable,
value independent on term structure.

Vasicek → μ estimator around / below 0 for all rates.
→ κ estimator: very good time scaling behaviour.
value decreases for long term rates
(-> stronger relaxation to μ for short time rate)
→ σ estimator: time scaling behaviour very good.
value increases for long term rates.

Residuals in any case fine or at least acceptable and uncritical for risk management.

Thank you for your attention !

I look forward to your questions and our discussion!

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