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How to infer the layer variance from the expectation

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When do you need that?

- **Mock reporting:** You did a *quick and dirty* rating, but have to report the outputs of a *sound* rating.
- **Portfolio extrapolation:** You need to simulate the impact of e.g. a Variance loading for a large number of layers, but don't have rating details in your data base.

Approach

Stick to your expected loss, construct key figures – or a whole model – being consistent to it.

- Collective model
- Use market knowledge to "infer" some parameters.

It is surprising how high/low model uncertainty can be.

Layer notation

l xs d per loss, then **AAL xs AAD** per year

X_i layer losses

λ layer loss frequency (*entry*)

λ^* total loss frequency (*exit*)

$$S = \sum_{i=1}^{N} X_i$$

$E(S)=\lambda E(X)$

$RoL=E(S)/l$

$\lambda \geq RoL \geq \lambda^*$

$S^*=\min[(S-AAD)^+, AAL]$

Reparametrize loss count: Contagion

$$c := Ct(N) := \left(\frac{Var(N)}{E(N)} - 1 \right) \Big/ E(N) = CV^2(N) - \frac{1}{E(N)}$$

- We have $Var(N)=\lambda+c\lambda^2$
- c indicates overdispersion
- *Poisson*: c=0, *Binomial*: c=-1/n, *NegBin / Mixed Poisson*: c=Var(mixing distribution)
- c is the same for all layers!
- c is **hard** to estimate

Collective model

$$CV^2(S) = c + \frac{1}{\lambda}\frac{E(X^2)}{E^2(X)}$$

non-"diversifiable"

severity shape

Define $\tau_X := \frac{E(X^2)}{lE(X)} \leq 1$ X heavy tailed: ≈

$$CV^2(S) = c + \frac{\tau_X}{RoL}$$ ≈1/RoL (last resort)

Example GPD / Pareto

$$\tau_X = \tau(\alpha,\gamma) = \frac{2}{\gamma - 1}\left(\frac{\alpha - 1}{\alpha - 2}\frac{1 - \gamma^{2-\alpha}}{1 - \gamma^{1-\alpha}} - 1\right)$$

- **GPD:** α=1/ξ, γ=1+ξl/σ
  ξ=0, 0.5, 1 via L'Hopital's rule

- **Pareto:** special case ξ>0, σ=ξd
  γ=1+l/d *relative layer length*

Model selection

- **N:** select plausible c (not always 0!)

High model risk for *low* layers

- **X:** according to available info

RoL	Pareto, plausible **α**
RoL, λ	Pareto, calculate **α** (works!)
RoL, λ, λ*	GPD, calculate ξ, **σ** (works!)

Low model risk for *not-too-long* layers

After AAD/AAL

AAD=0: E(S*)≤E(S), CV(S*)≤CV(S)

- usually ≈ (AAL high, low impact)

AAD>0 (comprises StopLoss!)

- collective model for layer AAL xs AAD
- count Bernoulli $\chi_{S'>0}$ severity S*|S*>0
- c'=–1, RoL'=E(S*)/AAL
- Pareto/GPD

$$CV^2(S^*) = -1 + \frac{\tau_{S^*|S^*>0}}{RoL'}$$

Remarks

- For *unlimited* layers similar parametric approach possible, e.g. GPD
- If λ and λ* are known, τ_X can be assessed parameter-free:

$$1 - \frac{\lambda - RoL}{\lambda - \lambda^*}\frac{RoL - \lambda^*}{RoL} \le \tau_X \le 1$$

Narrower bounds for concave cdf, e.g.

$$\tau_X \le 1 - \frac{1}{3}\frac{RoL - \lambda^*}{RoL}$$

Thanks

Feedback welcome, now or later.

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Literature

Loss count

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