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Risk estimation model on epidemic outbreaks for an insurer

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1 Introduction

Epidemic outbreaks are challenging issues for modern society.

For examples: Zika fever, Ebola virus diseases, SARS (Severe Acute Respiratory Syndrome), MERS (Middle East Respiratory Syndrome)

Primary importance: Scientific research
Insurance companies also provide insurances so that a kind of mitigation is obtained for the impact of such epidemic outbreaks.

1 Introduction (2)

Our aim is to introduce a simple stochastic model of the risk for an insurer of such epidemic bursts.

Our proposed model makes it possible to estimate the risk for an insurer of such epidemic outbreaks.

1 Introduction (3)

Our strategy:

Epidemic model: the use of classical Kermack-Mckendric model is made to estimate the total number of removals.

Onset of epidemics: the use of stochastic processes is made to estimate the onset of epidemic outbreaks.

Market risk: Once an epidemic outbreak takes place, a depression will follow. We simply use a standard Black-Scholes-Merton type model to evaluate the depreciation process.

1 Introduction (4)

Combining all together, we are then able to estimate the risk of the considered epidemics bursts by taking the expectation.

Remarks: Our modeling is motivated by that of catastrophic events, such as huge earthquakes and/or extreme flood, and so on.

2 Epidemic model

We recall the classical deterministic epidemic model due to Kermack-Mckendrick.

Let $x(t)$: the number of susceptibles;

$y(t)$: the numebr of infectives;

$z(t)$: the number of removals.

In this model, there holds

$N = x(t) + y(t) + z(t)$ is preserved for $t \geq 0$.

2 Epidemic model (2)

The model system of equations is expressed as

$$\frac{dx(t)}{dt} = -\beta x(t)y(t) \qquad (1)$$

$$\frac{dy(t)}{dt} = \beta x(t)y(t) - \gamma y(t) \qquad (2)$$

$$\frac{dz(t)}{dt} = \gamma y(t). \qquad (3)$$

with $\big(x(0), y(0), z(0)\big) = (x_0, y_0, z_0)$

2 Epidemic model (3)

Here

β : the infection parameter representing the strength of epidemics,

γ: the removal parameter indicating the rate of infectives becoming immune.

We dene the critical quantity:

$\rho := \gamma/\beta$ as the relative removal rate.

2 Epidemic model (4)

The famous threshold theorem is stated as follows. See D.J. Dalley and J. Gani (1999).

Threshold theorem. (i) *Let* $x_\infty = \lim_{t\to\infty} x(t)$ *and* $z_\infty = \lim_{t\to\infty} z(t)$. *Then, when infection ultimately ceases spreading, it follows that*

$$N - z_\infty = x_0 + y_0 - z_\infty = x_0 e^{-\frac{z_\infty}{\rho}},$$

where $y_0 < z_\infty < x_0 + y_0 = N$.

(ii) *A major outbreak occurs if and only if* $x_0 > \rho$.

2 Epidemic model (5)

(iii) *If* $x_0 = \rho + \nu$ *with small* $\nu > 0$ *and* y_0 *is small relative to* ν, *then the total number of susceptible left in the population are approximately*

$$\rho - \nu \quad \text{and} \quad 2\nu ,$$

respectively.

We employ this theorem to estimate the ultimate number of removals z_∞.

2 Epidemic model (6)

Remark

Kendall (1956) has succeeded in an exact handling and deduced that $z_\infty = \zeta$ where ζ is a positive root of

$$N - z - (\rho + \nu)e^{-\frac{z}{\rho}} = 0\,.$$

Remark

Other types of epidemic models may be also employed.

3 Doubly stochastic Poisson processes

For the modeling of the onset of epidemics outbreaks, we employ the so-called doubly stochastic Poisson process.

Let $\Lambda = \{\lambda(t)\}_{t\geq 0}$ be an intensity process; that is, a nonnegative, measurable, and locally integrable stochastic process.

3 Doubly stochastic Poisson processes (2)

A counting process $\{N(t;\Lambda)\}_{t\geq 0}$ is called a Cox process or a doubly stochastic Poisson process with intensity Λ if for each sequence $\{k_j\}_{j=1,2,3,\cdots,n}$ of nonnegative integers, and for $0<t_1\leq s_1\leq t_2\leq s_2\leq\cdots\leq t_{n-1}\leq s_{n-1}\leq t_n\leq s_n$, there holds

$$P\left(\cap_{j=1}^{n}\{N(s_j;\Lambda)-N(t_j;\Lambda)=k_j\}\right)=$$

$$\prod_{j=1}^{n} E\left[\frac{1}{k_j!}\left(\int_{t_j}^{s_j}\lambda(u)du\right)^{k_j}\exp\left(-\int_{t_j}^{s_j}\lambda(u)du\right)\right].$$

4 Our model and the estimate of risk

Now we introduce our model for an insurer.

We assume that the risk $R(t)$ due to epidemic bursts is described as

$$R(t) := E^{Q}\left[e^{-r(T-t)} 1_{\left\{N(T) > \frac{\rho}{\varepsilon}\right\}} \cdot\ 2\nu \max\{S(T) - K, 0\}\right],$$

for the Kermack-McKendrick original approximation and

4 Our model and the estimate of risk (2)

$$R(t) := E^Q[e^{-r(T-t)} 1_{\{N(T) > \rho/\varepsilon\}} \cdot \zeta \max\{S(T) - K, 0\}],$$

where ζ is a positive root of

$$N - z - (\rho + \nu) e^{-\frac{z}{\rho}} = 0\,.$$

Here $\varepsilon>0$ is a small parameter.

The meaning is as follows.

4 Our model and the estimate of risk (3)

Insurer is exposed to a market risk as well as the epidemic risk.

The market is assumed to be governed by the Black-Scholes-Merton type movement:

$$S(t) = S_0 \exp\left[-\alpha N(t) + \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W(t)\right],$$

where $\alpha > 0$, $\mu > 0, S_0 > 0$ are positive constants and the volatility σ is taken to be also positive constant.

4 Our model and the estimate of risk (4)

Here $N(T)$ denotes the above mentioned doubly stochastic Poisson process and $W(t)$ the standard Brownian motion.

The term $1_{\{N(T)>\rho/\varepsilon\}}$ indicates the trigger variable of the onset of epidemics. Here

$$1_A(\omega) = \begin{cases} 1 & (\omega \in A) \\ 0 & (\omega \notin A) \end{cases}$$

4 Our model and the estimate of risk (5)

The term 2ν and/or ζ means the ultimate number of removals and the term $\max\{S(T)-K,0\}$ shows that the insurer takes the stop-loss strategy with the retention level K.

To summarize, the risk for an insurer is the discount value of the combined effect of the epidemic risk and the market risk. Here $r > 0$ is the risk-free interest rate.

4 Our model and the estimate of risk (6)

Now our main results reads as follows, which gives the pricing formula for this risk.

Theorem. The above risk $R(t)$ for an insurer is

$$R(t) = \sum_{l=\nu+1}^{\infty} 2\nu \left[S(t) e^{-\alpha l + \log\left(M_{(\Lambda_T - \Lambda_t)}(k)\right)} \cdot \Phi\left(d_l + \sigma\sqrt{T-t}\right) \right.$$

4 Our model and the estimate of risk (7)

$$k := 1 - e^{-\alpha}$$

$$d_l = \frac{\log\left(\frac{S(t)}{K}\right) + r(T-t) - \alpha l + \log(M_{(\Lambda_T - \Lambda_t)}(k))}{\sigma\sqrt{T-t}} - \frac{1}{2}\sigma\sqrt{T-t}\,.$$

Moreover we have defined

$$M_{\Lambda_t}(k) = E[\exp(k\Lambda_t)]$$

5 Sketch of proof

Here we give a sketch of proof. Next lemma is important.

Lemma. *Let* $\{N(t) = N(t; \Lambda)\}_{t \geq 0}$ *be a doubly stochastic Poisson process with intensity* $\Lambda = \{\lambda(t)\}_{t \geq 0}$ *and* $M_{\Lambda_t}(k)$ *denotes the moment generating function of the aggregated process* $\Lambda_t := \int_0^t \lambda(s) ds$*; namely*

$$M_{\Lambda_t}(k) = E[\exp(k\Lambda_t)].$$

5 Sketch of proof (2)

Then, for $k := 1 - e^{-\alpha}$*, the process*

$$\left\{\exp\{-\alpha N(t) + \log(M_{\Lambda_t}(k))\}\right\}_{t\geq 0}$$

gives a martingale with respect to the minimum augmented filtration $\mathcal{F}_t^{\Lambda}$ *of* $\sigma\{N(s) \mid 0 \leq s \leq t\}$.

Given this lemma, we are able to compute the proposed risk through the standard change of measure technique.

6 Hedging parameters

Hedging parameters, Delta, Gamma, Rho, and Vega of $R(t)$ are also computed.

For example, the Delta $\Delta(t) = \frac{\partial R(t)}{\partial S}$ is expressed as follows in the case of Kermack-McKendrick original approximation.

6 Hedging parameters (2)

$$\Delta(t) = \frac{\partial R(t)}{\partial S}$$

$$= \sum_{l=\nu+1}^{\infty} 2\nu \, e^{-\alpha l + \log\left(M_{(\Lambda_T - \Lambda_t)}(k)\right)} \Phi\left(d_l + \sigma\sqrt{T-t}\right)$$

$$\cdot \, E\left[\frac{(\Lambda_T - \Lambda_t)^l}{l!} e^{-l(\Lambda_T - \Lambda_t)}\right]$$

7 Conclusion

- The model of estimating the risk for an insurer due to epidemic outbreaks is introduced and Its closed pricing formula is obtained.
- The model is based on the combination of the epidemic modeling and the stochastic process.
- It should be compared with the real data as well as the improvement of the model should be undertaken.

References

[1]D.J. Dalley and J. Gani, Epidemic Modelling, Cambridge University Press, 1999, ch. 2.

[2]D.G. Kendall, "Deterministic and stochastic epidemics in closed populations," in "Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability," vol. 4, Ed. J. Neyman, University of California Press, Berkeley, pp. 149-165, 1956.

Obrigado pela sua atenção !