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Cause-of-Death Mortality: A Study of a Heterogeneous Portfolio Dynamics

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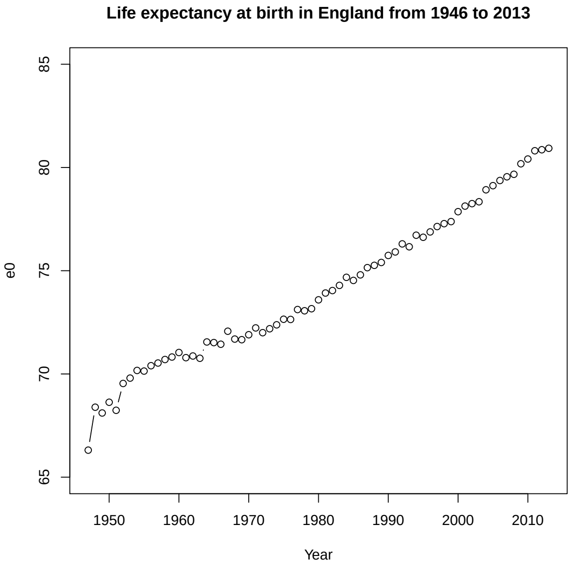
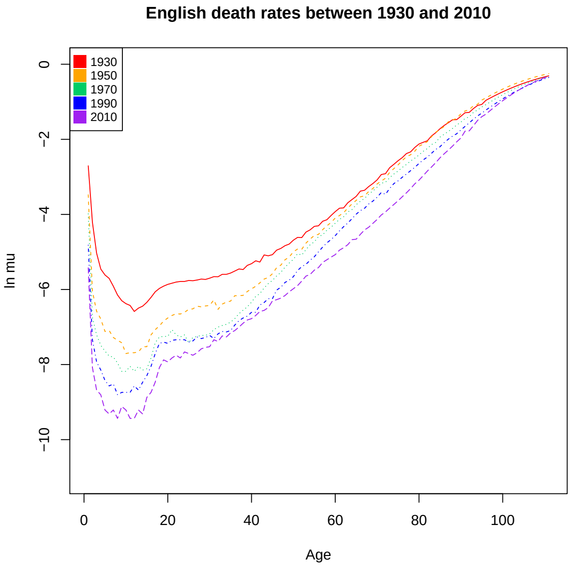
Context



Source : The Human Mortality Database (HMD)

Context



Source : The Human Mortality Database (HMD)

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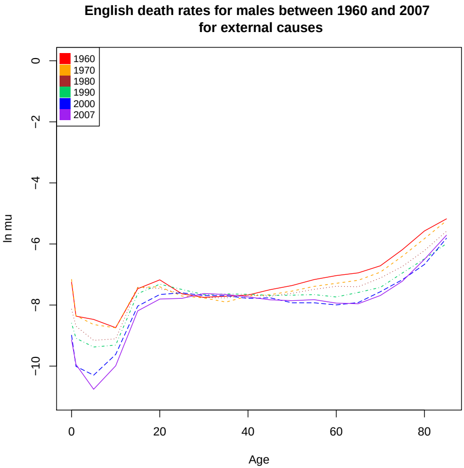

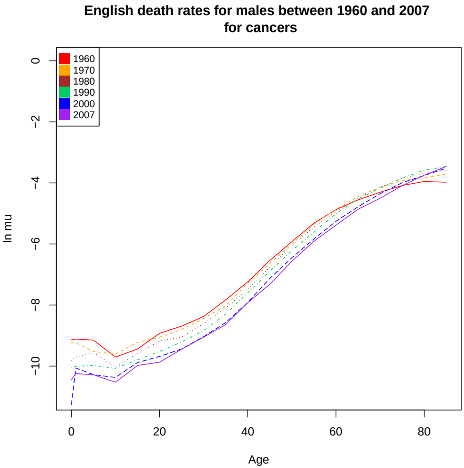

Source : The World Health Organization (WHO)

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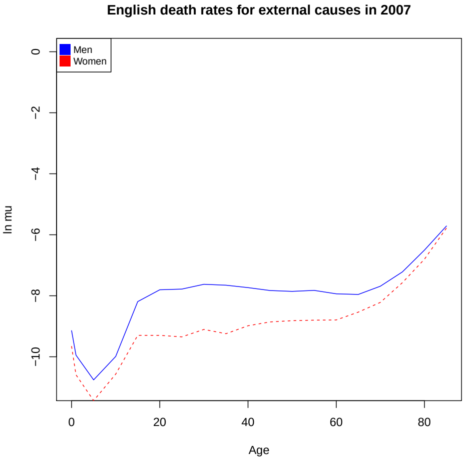

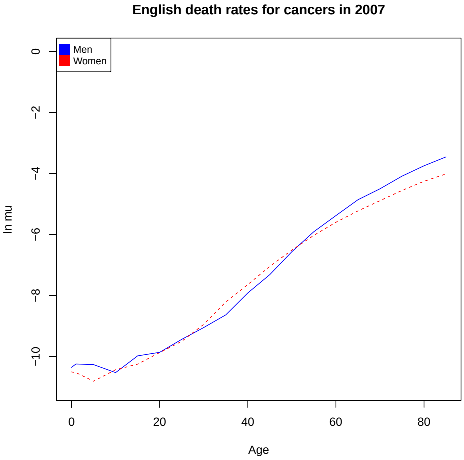

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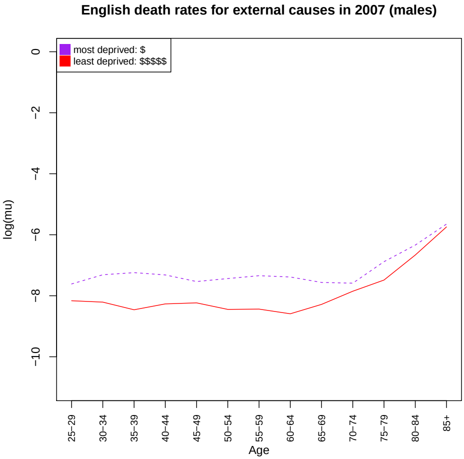
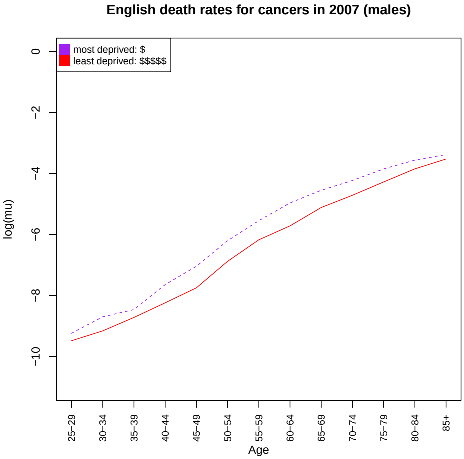




Source : The Office for National Statistics (ONS)

Objective

- What could be impacts of mortality changes following the socio-economic composition ? for an insurance portfolio ?

⇒ Study impacts of changes in cause-of-death mortality on an insurance portfolio composed with different socio-economic category

- Model portfolio dynamics
  - By taking into account deaths and arrivals :
    - with cause-of-death rates depending on age, time, gender and socio-economic category
  - Provide a general framework to address the issue

- Let us characterized the policyholders by the gender ϵ, the year of birth y and the socio-economic status j :

$$g^{\epsilon}(t) = \sum_{y_{min}}^{y_{max}} g^{\epsilon}(y,t) = \sum_{y_{min}}^{y_{max}} \sum_{j=1}^{m} g_j^{\epsilon}(y,t)$$

  - $g^{\epsilon}(t)$ is the total number of policyholders at time t with gender ϵ
  - $g^{\epsilon}(y,t)$ is the total number of policyholders at time t with gender ϵ and year of birth y
  - $g_j^{\epsilon}(y,t)$ is the total number of policyholders at time t with gender ϵ, year of birth y and socio-economic status j

- Model the heterogeneous cohort dynamics and the aggregated cohort death rate : $g^{\epsilon}(y,t)$, $d^{\epsilon}(y,t)$

- For a closed portfolio, policyholders in the sub-cohort with socio-economic category j evolve only according to deaths :

$$\frac{\mathrm{d}g_j^{\epsilon}(y,t)}{\mathrm{d}t} = g_j'^{\epsilon}(y,t) = -\mu_j^{\epsilon}(y,t)g_j^{\epsilon}(y,t). \qquad (1)$$

- In this sense, the aggregated cohort dynamics in a closed portfolio is also defined only by deaths :

$$\frac{\mathrm{d}g^{\epsilon}(y,t)}{\mathrm{d}t} = g'^{\epsilon}(y,t) = -d^{\epsilon}(y,t)g^{\epsilon}(y,t) \qquad (2)$$

$$\Rightarrow d^{\epsilon}(y,t) = \frac{\sum_{j=1}^{m}\mu_j^{\epsilon}(y,t)g_j^{\epsilon}(y,t)}{\sum_{j=1}^{m}g_j^{\epsilon}(y,t)}$$

  - $g_j^{\epsilon}(y,t)$: survivors from the initial sub-cohort

- For an opened portfolio, policyholders in the sub-cohort with socio-economic category j evolve according to deaths, arrivals and cancellations (B_j) :

$$\frac{dg_j^{\epsilon}(y,t)}{dt} = g_j'^{\epsilon}(y,t) = -\mu_j^{\epsilon}(y,t)g_j^{\epsilon}(y,t) + B_j^{\epsilon}(y,t). \qquad (3)$$

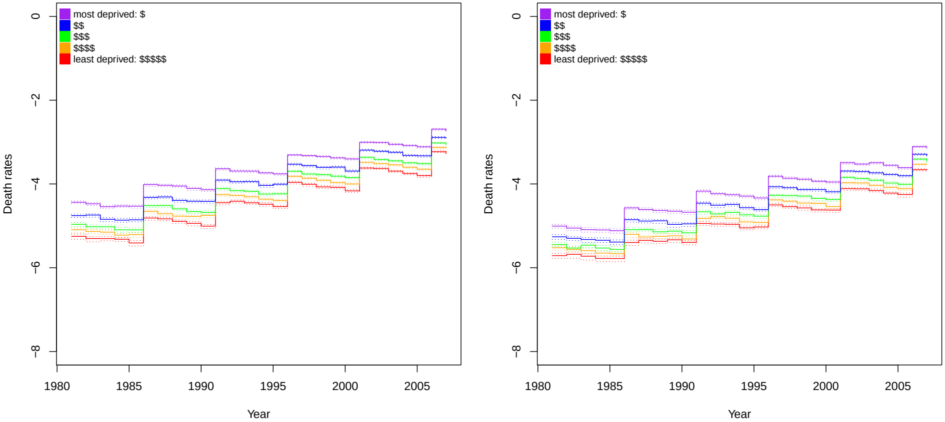
- By summing, the aggregated cohort dynamics in an opened portfolio is also defined by deaths, arrivals and cancellations :

$$\frac{dg^{\epsilon}(y,t)}{dt} = g'^{\epsilon}(y,t) = -d(y,t)g^{\epsilon}(y,t) + B^{\epsilon}(y,t) \qquad (4)$$

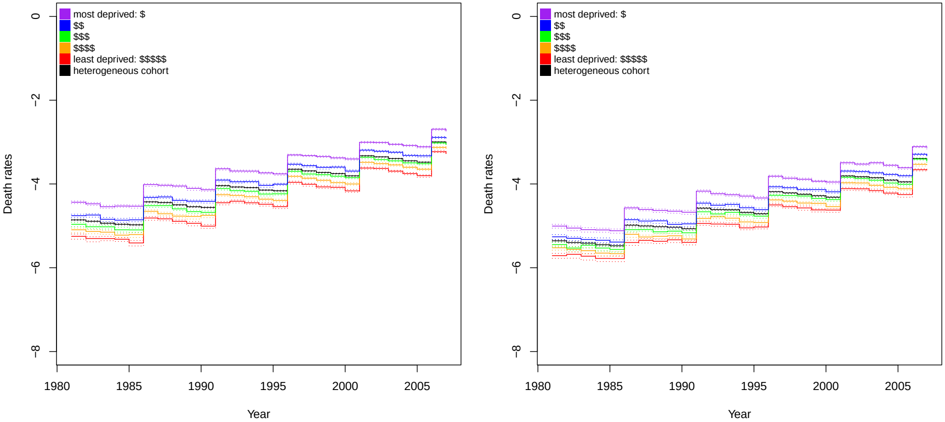
$$\Rightarrow d^{\epsilon}(y,t) = \frac{\sum_{j=1}^{m} \mu_j^{\epsilon}(y,t)g_j^{\epsilon}(y,t)}{\sum_{j=1}^{m} g_j^{\epsilon}(y,t)}$$

  - $g_j^{\epsilon}(y,t)$: survivors from the initial sub-cohort
    + survivors from arrivals

English cohort death rates per socio-economic category for age 50 in 1981 with 95% confidence intervals (left for males, right for females)

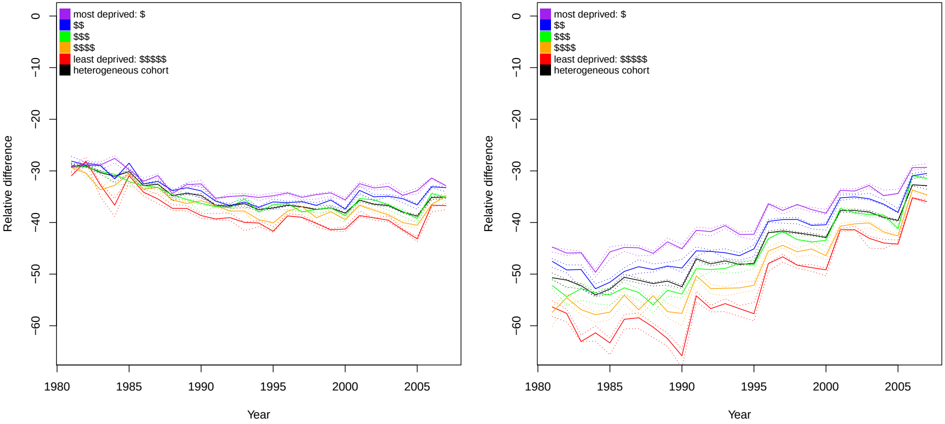


Cohort death rates per socio-economic composition for age 50 in 1981 with 95% confidence intervals (left for males, right for females)



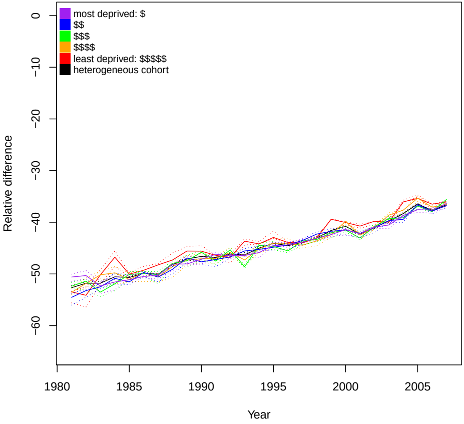
Cancer removal

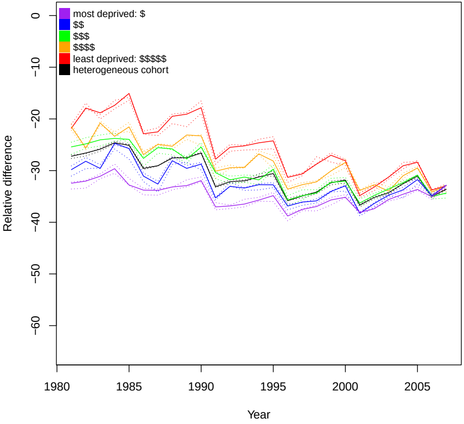
Relative difference of death rate per socio-economic composition for age 50 in 1981 with 95% confidence intervals (left for males, right for females)



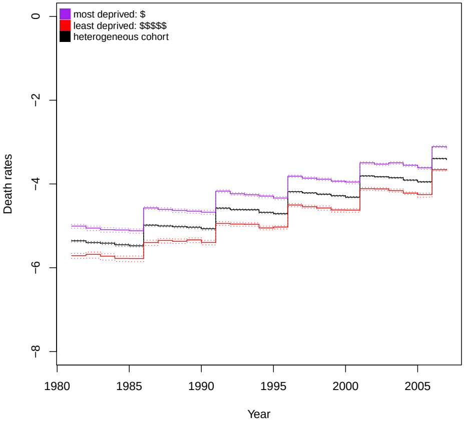
Circulatory diseases removal

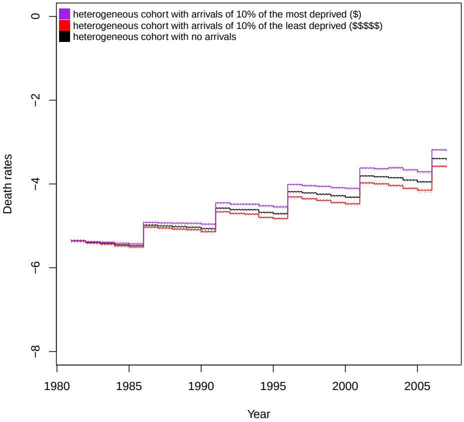
Relative difference of death rate per socio-economic composition for age 50 in 1981 with 95% confidence intervals (left for males, right for females)





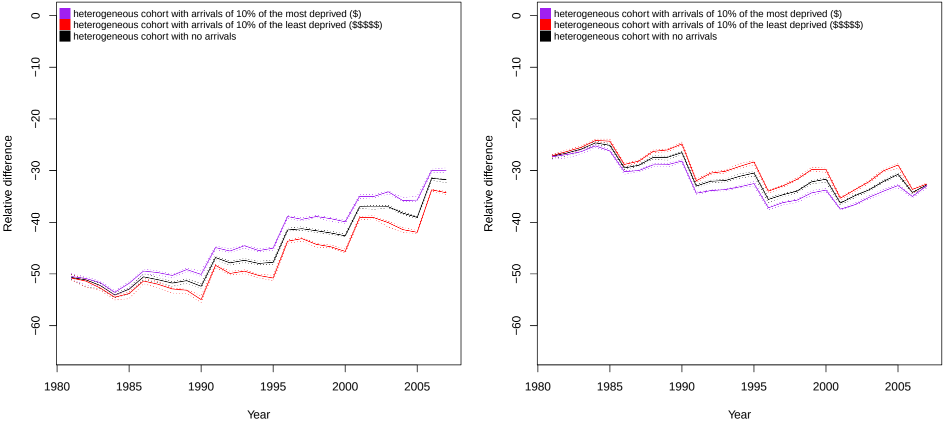
Cohort death rates per socio-economic composition for females of age 50 in 1981 with 95% confidence intervals (left for closed portfolios, right for opened portfolios)





Causes removal

Relative difference of death rate per socio-economic composition for females of age 50 in 1981 with 95% confidence intervals (left for cancer removal, right for circulatory diseases removal)



Followings

- Portfolio dynamics with arrivals and seniority of policyholders :
  - $g_j^{\epsilon}(y,t)$ is the sum over all seniorities : $\int_0^{t-t_0} g_j^{\epsilon}(y,t,u)\mathrm{d}u$

$$\Rightarrow d^{\epsilon}(y,t) = \frac{\sum_{j=1}^{m}\left(\int_0^{t-t_0}\mu_j^{\epsilon}(y,t,u)g_j^{\epsilon}(y,t,u)\mathrm{d}u\right)}{\sum_{j=1}^{m}\left(\int_0^{t-t_0}g_j^{\epsilon}(y,t,u)\mathrm{d}u\right)}$$

    - $g_j^{\epsilon}(y,t,u)$ depends on the survivors from the initial sub-cohort and survivors from arrivals with different death rates ($\mu_j^{\epsilon}(y,t,u)$)

- Application : impacts of cause-of-death mortality changes on a heterogeneous portfolio with medical selection

Concluding remarks

- With a population dynamics model, we study impacts of cause-of-death changes on a portfolio mortality comprising different socio-economic categories :
  - for a closed and an opened portfolio

  ⇒ **Following the portfolio structure, cause-of-death mortality changes can have different impacts on the aggregated mortality**

- To go further
  - arrivals processes
  - modeling medical selection
  - dependence assumptions between causes of death
  - study aggregated mortality of a population composed with different socio-economic categories

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thank you!