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DEFINING AMBITION

Valuation of Bermudan-DB-Underpin Plan

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Defined Benefit Plans

- Actuarial Liability (AL) can be calculated using **Accrued Benefit Obligation** (ABO) or **Projected Benefit Obligation** (PBO).

- $ABO_t = b \times t \times L_{t-1} \times \ddot{a}_T \times e^{-r(T-t)}$
  - b : The accrual rate for benefit, ex. 1.6% of salary.
  - L_t: The salary at time t.
  - t : Time, number of past year in service.
  - T : Retirement time.
  - $\ddot{a}_T$: Annuity factor at retirement.
  - r : Discount rate.
- Retirement Benefit = $b \times T \times L_{T-1} \times \ddot{a}_T$ for final salary DB plan.

Defined Contribution Plans

- The DC benefit at time t is

$$DC_t = \sum_{u=1}^{t-1} cL_u \frac{S_t}{S_u}$$

  - S_t: Fund accumulation process at time t.
  - c : The contribution rate, ex, 9% of salary

DB-Underpin

- Also known as the “Floor Offset” plan.
- Main benefit is DC plan.
- Guaranteed minimum benefit as DB plan.
- Example: Wilfrid Laurier University

Money Purchase Account	Minimum Guaranteed Pension
Employer Contribution = 7%	1.65% of final salary for every year of service.
Employee Contribution = 9%	

* The rates are calculated from an employee with annual income $100,000 throughout his/her career.

DB-Underpin Cost

- The cost to the employer is

$$E^Q\left[e^{-rT}\sum_{t=0}^{T-1} cL_t + e^{-rT}(ABO_T - DC_T)_+\right]$$

  - $e^{-rT}\sum_{t=0}^{T-1} cL_t$: Cost of employer contribution to DC account
  - $(ABO_T - DC_T)_+$: Price of DB-underpin guarantee.

- Chen and Hardy [2009] has studied the valuation and funding of DB-Underpin plans.

Second Election

- Initial plan can be either DC or DB.
- Before retirement, the employee has option to transfer to DB (DC) plan.
- For DC-to-DB option:
  - The “buy-in” cost for the transfer is the ABO of the DB plan.
  - The excess of DC balance over the “buy-in” cost will kept in the investment account until the retirement.
  - If DC balance is less than the “buy-in” cost, the ***employee*** is responsible to fund up the difference.
- For DB-to-DC option:
  - The ABO of DB plan will be the opening investment account balance.
- Example: Florida Retirement System.

Second Election Cost

- The cost to the employer

$$\sup_{\tau\in[0,1,\cdots,T]} E^{Q}[e^{-r\tau}\sum_{t=0}^{\tau-1} cL_t + e^{-rT}ABO_T - e^{-r\tau}ABO_\tau]$$

  - $e^{-r\tau}\sum_{t=0}^{\tau-1} cL_t$: Cost of employer contribution to DC account before the transfer.
  - $e^{-rT}ABO_T$: Cost of DB plan.
  - $e^{-r\tau}ABO_\tau$: Present value of "buy-in" cost paid by the employee at time τ.

- Milevsky and Promislow [2004] studied the value of the second election option under the deterministic assumptions.

Bermudan-DB-Underpin

- Purpose of our project
  - To study different risk-sharing arrangements between employer and employee.
  - The second election does not protect employees against downside market risk. Employees may end up with a benefit lower than the minimum of DC and DB plans.
- Methodology
  - Adding a guarantee to the second-election option, similar to the DB-underpin plan.
  - Price the new option with market consistent valuation method.

Bermudan-DB-Underpin

- Employee has initially enrolled in DC plan.
- Before retirement, the employee has option to transfer into DB plan.
- The “buy-in” cost for the transfer is the ABO of the DB plan.
- The excess of DC balance over the “buy-in” cost will be kept in the investment account until the retirement.
- If DC balance is less than the “buy-in” cost, the ***employer*** is responsible to fund up the difference.

Bermudan-DB-Underpin

- The cost to the employer

$$\sup_{\tau\in[0,1,\cdots,T]} E^{Q}\left[e^{-r\tau}\sum_{t=0}^{\tau-1} cL_t + e^{-rT}ABO_T - e^{-r\tau}ABO_\tau + e^{-r\tau}(ABO_\tau - DC_\tau)_+\right]$$

  - $e^{-r\tau}\sum_{t=0}^{\tau-1} cL_t$: Cost of employer contribution to DC account.
  - $e^{-rT}ABO_T - e^{-r\tau}ABO_\tau$: Cost of DB plan since the transfer.
  - $e^{-r\tau}(ABO_\tau - DC_\tau)_+$: Cost of the guarantee.

Problem Formulation

- Let C(t,w) be the employer cost to sponsor the Bermudan-DB-Underpin plan at time t, with employee investment account balance w.

- We can reformulate the problem (by the Optional Sampling Theorem)

$$C(t,w) = E^{Q}\left[e^{-r(T-t)}ABO_T\right] - w$$

$$+ \sup_{\tau\in[0,1,\cdots,T-t]} E^{Q}[e^{-r\tau}\,(DC_{t+\tau} - ABO_{t+\tau})_{+}|DC_t = w]$$

- DC Benefit Process : $DC_{t+\tau} = DC_t\,\frac{S_{t+\tau}}{S_t} + \sum_{u=t}^{\tau-1}\frac{S_{t+\tau}}{S_u}cL_u$
- Salary Process : $L_{t+\tau} = L_t e^{\mu_L \tau}$
- Fund Return Process : $dS_t = S_t(rdt + \sigma dZ_t^{Q})$

Value Function

- Define the value function as

$$v(t,w) = \sup_{\tau \in [0,1,\cdots,T-t]} E^{Q}[e^{-r\tau}(DC_{t+\tau} - ABO_{t+\tau})_{+} | DC_t = w]$$

- Admissible exercising dates are 0, 1, … , T.
- Bermudan Call Option with time-dependent strike value.

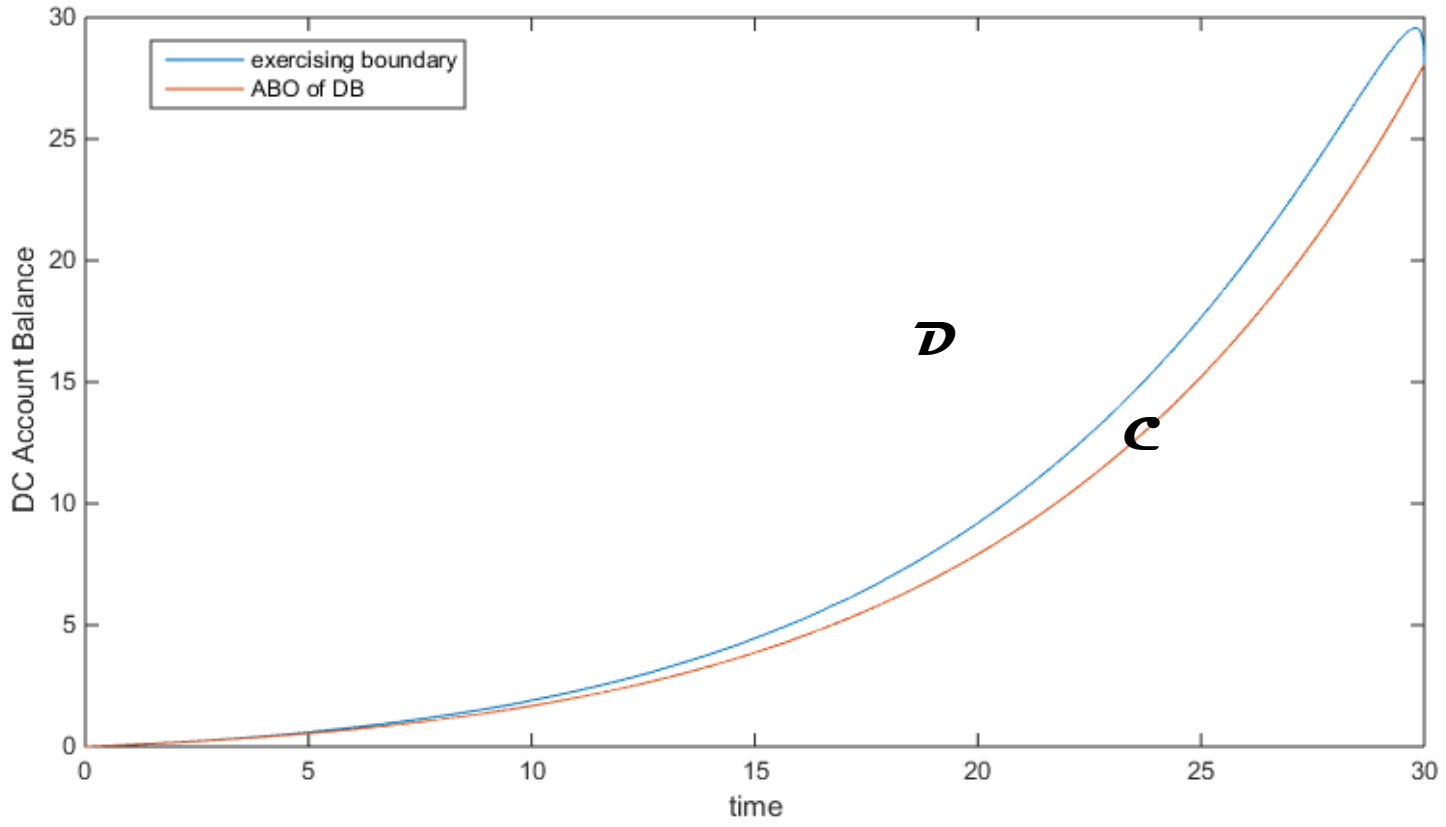
Theoretical Results

- Let **C** be the continuation region, where it is optimal for employees to stay in the DC plan.
  - Denote $v^h(t,w) = E^Q[e^{-r}v(t+1, DC_{t+1})|DC_t = w]$ the continuation value.

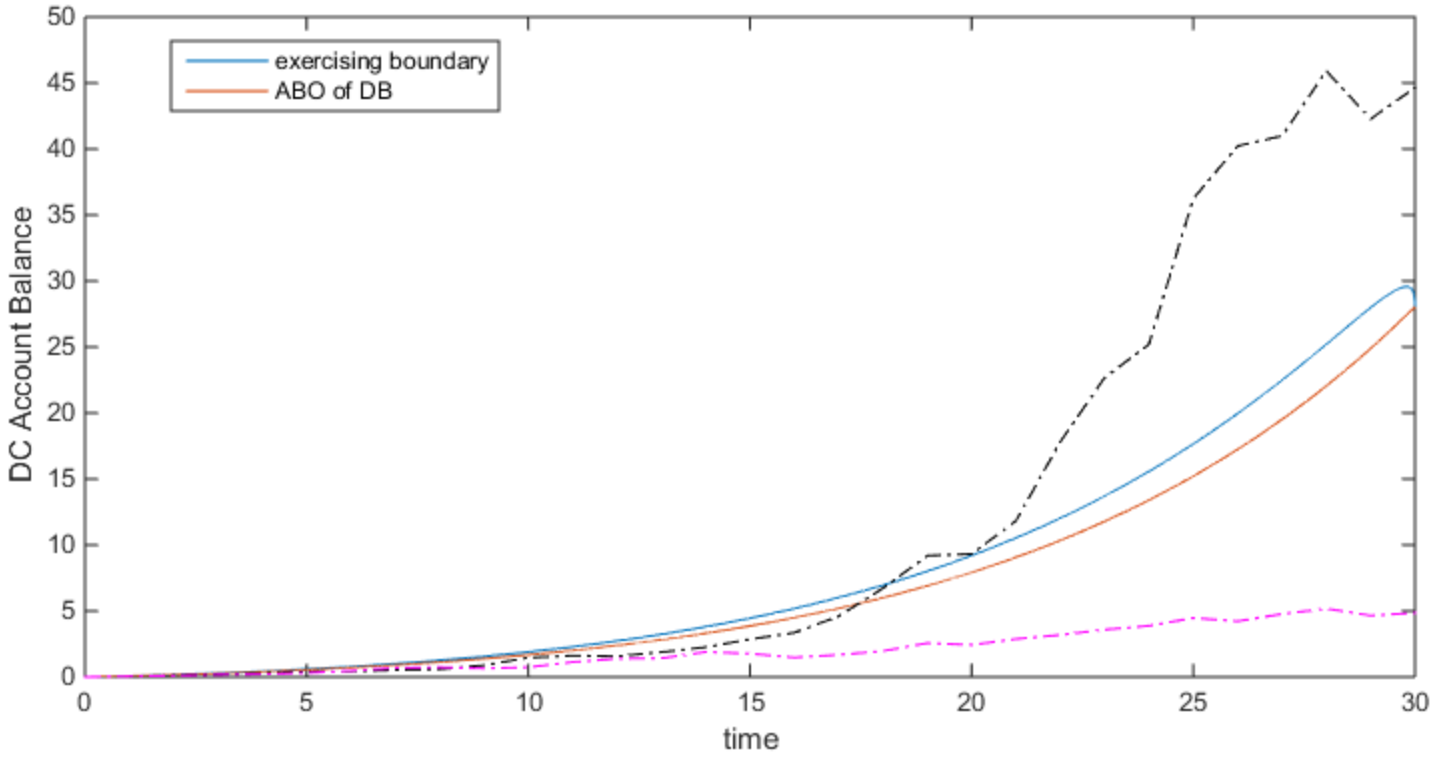
- Let **D** be the stopping region, where it is optimal for the employees to transfer to the DB plan.
  - Denote $v^e(t,w) = (w - ABO_t)_+$ the exercising value.

- $v(t,w) = \max(v^e(t,w), v^h(t,w))$

Theoretical Results



Theoretical Results



Theoretical Results

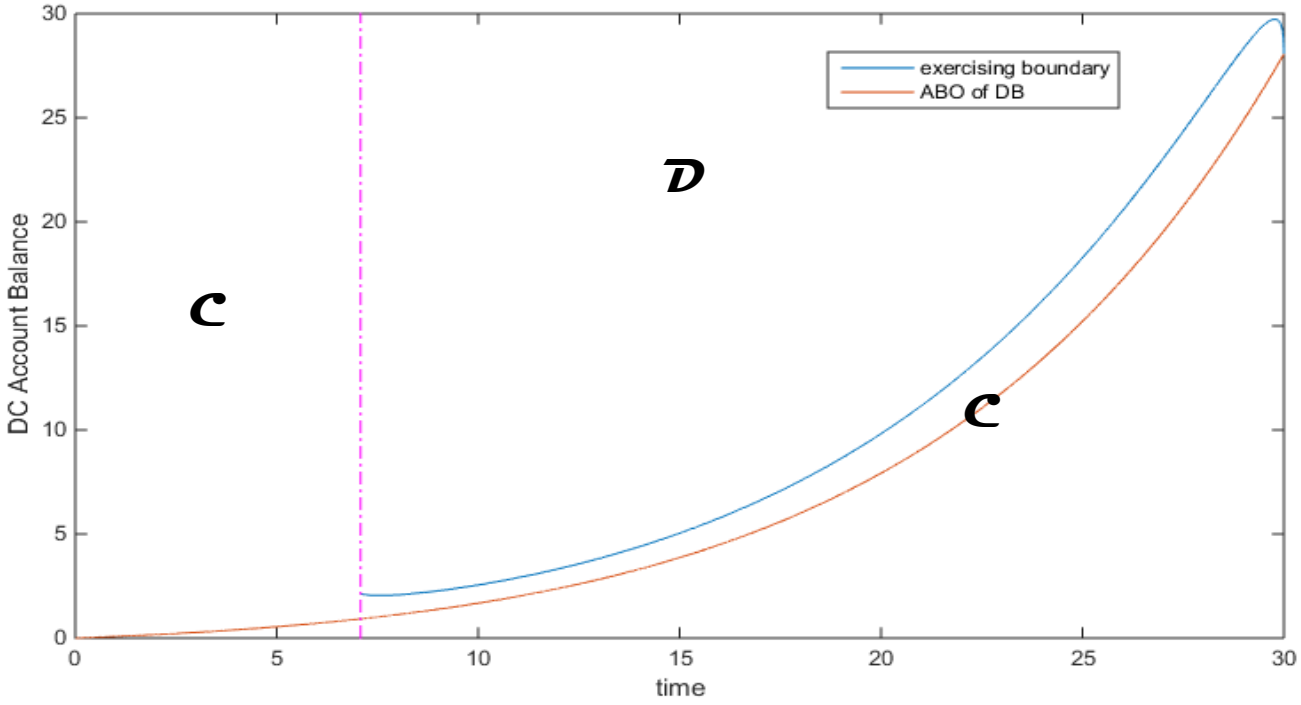
- Denote $\varphi(t)$ the exercise boundary.
  - If $\varphi(t) < \infty$, employee should transfer to DB plan if his/her DC balance is above $\varphi(t)$.

$$v(t,w) = \begin{cases} v^e(t,w) & if\ w \geq \varphi(t) \\ v^h(t,w) & if\ w < \varphi(t) \end{cases}$$

  - If $\varphi(t) = \infty$, employee should stay in DC plan regardless his/her DC balance.

Theoretical Results

- If $\frac{c}{b\ddot{a}_T e^{-rT}} \geq 1$, then there exists a $t_* \in [0, 1, \ldots, T-1]$, such that $\varphi(t) = \infty, \forall t \leq t_*$ and $\varphi(t) < \infty, \forall t > t_*$

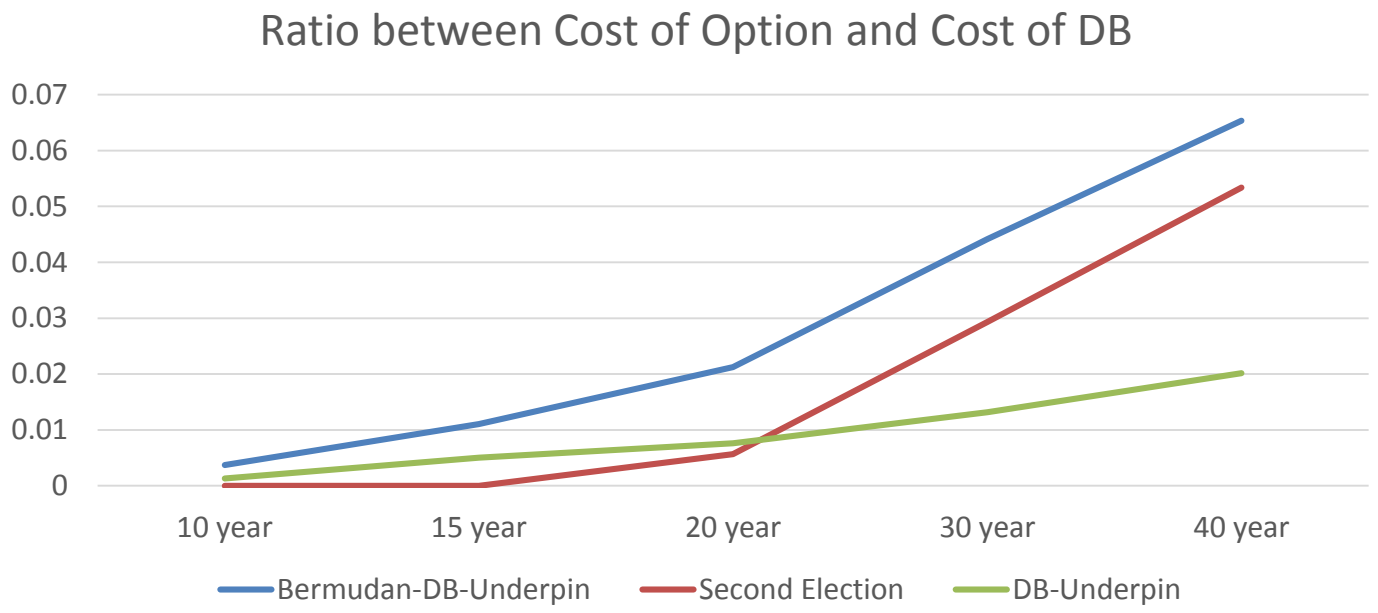


Numerical Examples

- Evaluate the option using the Least-Square method (Longstaff and Schwartz [2001]).

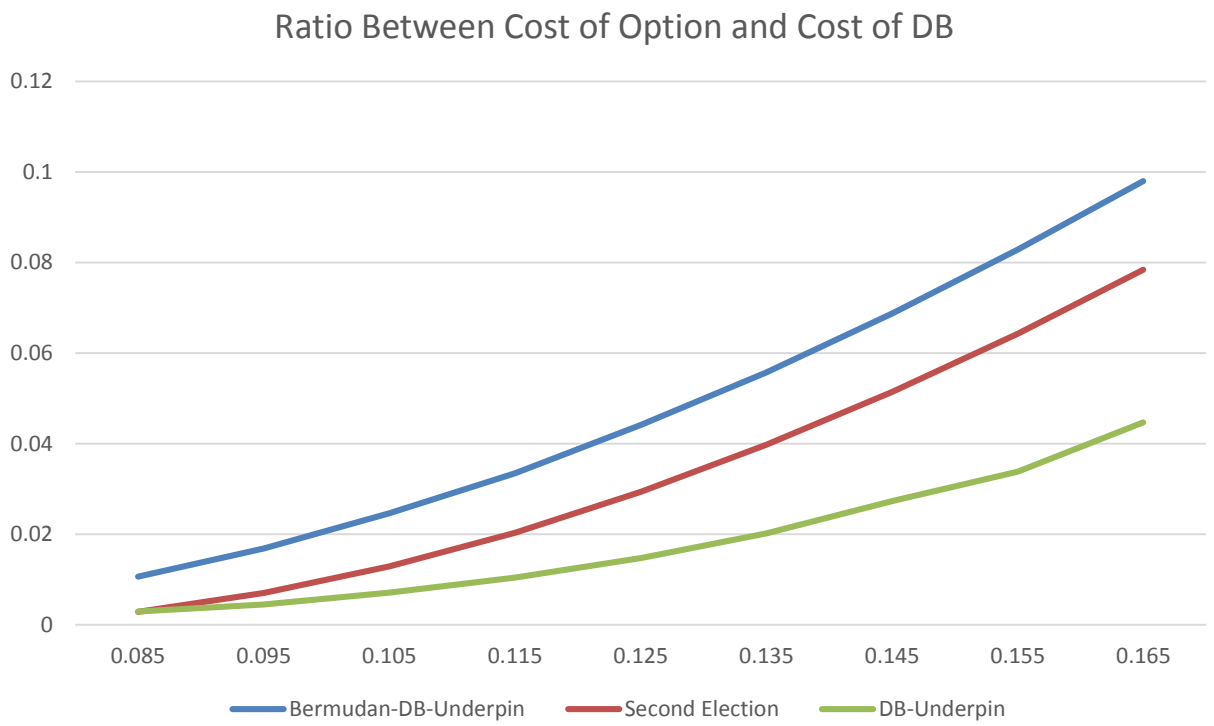
Parameter	
b	0.016
c	0.125
μ_L	0.0459
σ_S	0.15
$\ddot{a}_T$	14.75
r	0.04
L_0	1
W_0	0

Numerical Examples

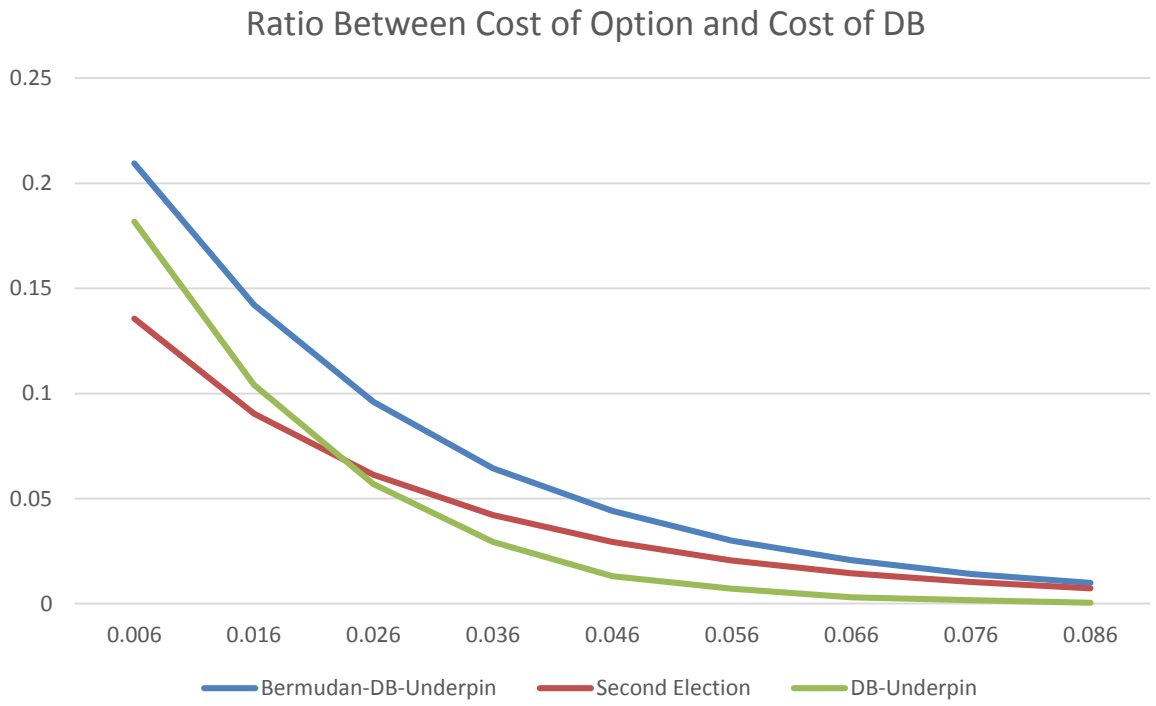


Bermudan-DB-Underpin, Second Election, and DB-Underpin option are expressed as additional relative cost to DB plan.

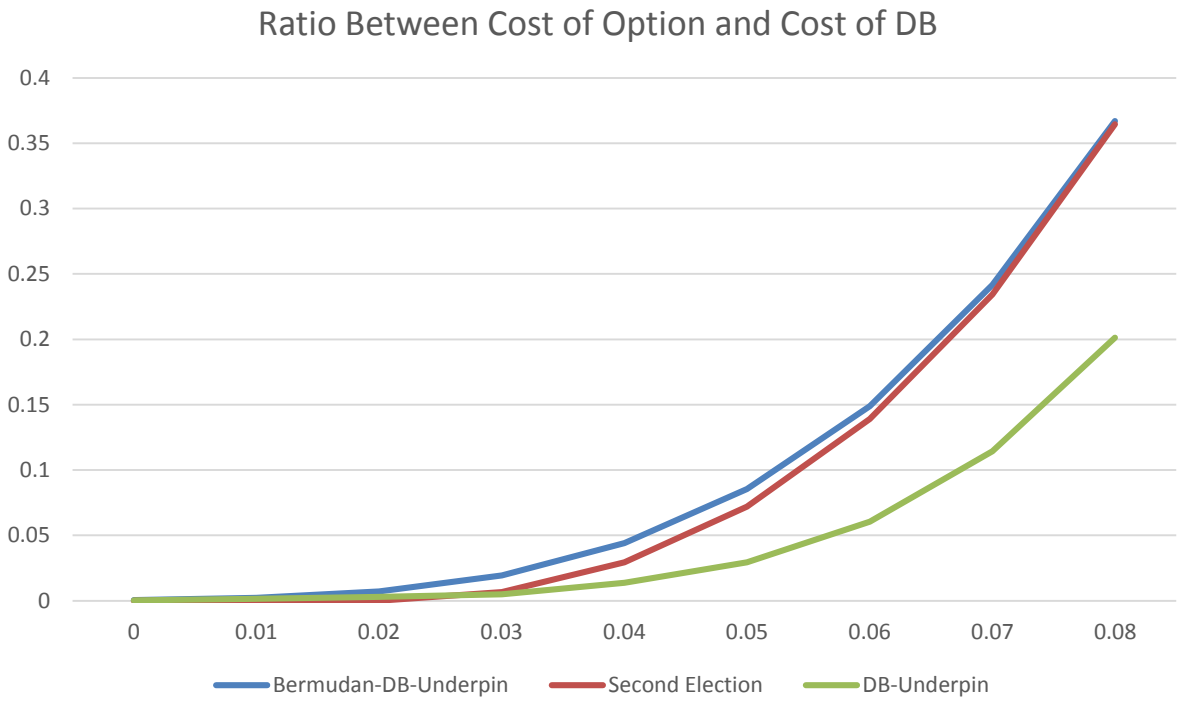
Sensitivity on DC Contribution Rate



Sensitivity on Salary Growth μ_L



Sensitivity on Interest Rate r



Modified Assumptions

- Different Discount Rate for ABO
  - Let ρ be the discounting rate for ABO calculation only.
  - $ABO_t = t \times b \times L_{t-1} \times \ddot{a}_T \times e^{-\rho(T-t)}$
- Use PBO to Calculate the Liability and Transfer Cost
  - $PBO_t = t \times b \times L_{T-1} \times \ddot{a}_T \times e^{-r(T-t)}$
- Include Employee Contribution.

Modified Assumptions

- Modify the option to reduce employer costs.
  - The employee must “trade-in” all his/her DC balance to make the transfer.
  - DC shortfall will still be covered by employer.
  - Excess in DC account will be forfeited.
  - Employer Cost is

$$\sup_{\tau\in[0,1,\cdots,T]} E^{Q}[e^{-r\tau}\sum_{t=0}^{\tau-1} cL_t + (ABO_T e^{-rT} - e^{-r\tau}DC_\tau)I_{\tau<T} + (ABO_T - DC_T)_+ I_{\tau=T}]$$

Conclusion

- This project is to study different risk-sharing schemes in pension plans.
- Bermudan-DB-Underpin costs more than both Second-Election and DB-Underpin plan.
- Under most scenarios, Bermudan-DB-Underpin costs less than 10% of DB plan.
- When discounting rate is assumed to be high, the relative cost of Bermudan-DB-Underpin is high, but very close to Second-Election plan.

Future Work

- Theoretical Results on $\varphi(t)$.
- Risk Management.
- Stochastic Salary Process.
- Stochastic Interest Rate Process.
- Incorporate Mortality Assumption.

THANK YOU FOR YOUR ATTENTION!