HYBRID SOCIAL SECURITY PENSION SCHEMES
risk sharing and stochastic optimal control

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Agenda

1. Hybrid Pension Schemes
2. PAYG Risk sharing model
3. Example 1 : Musgrave rule
4. Example 2 : Convex invariant
5. Optimal hybridization by stochastic control
1. Hybrid Pension Schemes

- Financial viability of classical Pay As You Go (PAYG) social security pension schemes
- Most of them using a Defined Benefit (DB) structure
- Important risk factors:
  - Ageing
  - Fertility
  - Baby boom effect
Some classical solutions

- **Parametric** reforms (*retirement age, early retirement, indexation,...*)

- Move from DB schemes to DC schemes (*Notional Accounts, NDC*)

- Introduction of **Automatic Balance Mechanisms** as an answer to risk exposure (DB and DC) to avoid any form of “Pension Populism”
Sustainability and Adequacy

• ...But Pension reform is not just a matter of financial sustainability

• The mission of the social security is also social adequacy

• Fairness between generations and between categories of workers is a key point
Hybrid pension plans

• Development of Hybrid pension plans between DB and DC as well for public as for occupational pension schemes
• **Sweden**: NDC with automatic adjustment
• **Netherlands**: conditional indexation, collective DC plans
• **Belgium**: project of reform of the first pillar (points system with Musgrave rule)
2. PAYG Risk Sharing Model

**Incomes:**

\[ A(t) = \text{number of contributors at time } t \]
\[ W(t) = \text{mean wage} \]
\[ \pi(t) = \text{contribution rate} \]

\[ \text{IN}(t) = A(t).\pi(t).W(t) \]
Equilibrium equation

Outcomes:

\[ B(t) = \text{number of retirees at time } t \]
\[ P(t) = \text{mean pension} \]
\[ \delta(t) = \text{replacement rate} \]

\[ \text{OUT}(t) = B(t).P(t) = B(t).\delta(t).W(t) \]
Equilibrium equation

Actuarial equilibrium:

\[ \text{IN}(t) = \text{OUT}(t) \]

\[ A(t) \cdot \pi(t) \cdot W(t) = B(t) \cdot P(t) \]

\[ \pi(t) = \frac{B(t)}{A(t)} \cdot \frac{P(t)}{W(t)} \]

\[ D(t) = \frac{B(t)}{A(t)} = \text{dependence ratio} \]

\[ \delta(t) = \text{replacement rate} \]
Equilibrium equation

\[
\pi(t) = \frac{\text{number of retirees}}{\text{number of contributors}} \times \frac{\text{mean pension}}{\text{mean wage}}
\]

Demographic risk

Financing the system

*Generation of active people*

Social quality of the system

*Generation of retirees*

\[
\pi(t) = D(t).\delta(t)
\]
Automatic Adjustment

\[ \pi(t) = \frac{B(t)}{A(t)} \cdot \frac{P(t)}{W(t)} = D(t) \cdot \frac{P(t)}{W(t)} = D(t) \cdot \delta(t) \]

Risk factor

**Automatic Adjustment:**
How to maintain automatically this equilibrium in case of change of \( D(t) \) (Ageing = Increase !)
Automatic Adjustment

\[ \pi(t) = \frac{B(t)}{A(t)} \cdot \frac{P(t)}{W(t)} = D(t) \cdot \frac{P(t)}{W(t)} = D(t) \cdot \delta(t) \]

Risk factor

Constant in pure DC

Adjustment of \( \delta \)

Constant in pure DB

Adjustment of \( \pi \)

Social risk

Financial risk
Automatic Adjustment

\[
\frac{d\pi(t)}{\pi(t)} = \frac{d\delta(t)}{\delta(t)} + \frac{dD(t)}{D(t)}
\]

**DB**
\[
\begin{align*}
d\delta(t) &= 0 \\
d\pi(t) &= \frac{dD(t)}{D(t)}
\end{align*}
\]
\[\alpha = 0\]

**DC**
\[
\begin{align*}
d\delta(t) &= -\frac{dD(t)}{D(t)} \\
d\pi(t) &= 0
\end{align*}
\]
\[\alpha = 1\]

**Hybrid / risk sharing**
\[
\begin{align*}
d\delta(t) &= -\alpha(t) \cdot \frac{dD(t)}{D(t)} \\
d\pi(t) &= (1 - \alpha(t)) \cdot \frac{dD(t)}{D(t)}
\end{align*}
\]
\[0 \leq \alpha(t) \leq 1\]
3. Example 1: the Musgrave rule

Goal:

To keep constant the replacement rate *but net of contributions*

\[ \delta_0 = \delta(t) = \frac{P(t)}{W(t)} \]

\[ M = \frac{P(t)}{W(t)(1 - \pi(t))} = \frac{\delta(t)}{1 - \pi(t)} \]
The Musgrave rule

Musgrave Condition

\[
\frac{dP(t)}{P(t)} - \frac{dW(t)}{W(t)} = \frac{d(1-\pi(t))}{1-\pi(t)} = -\frac{\pi(t)}{1-\pi(t)} \cdot \frac{d\pi(t)}{\pi(t)}
\]

Equilibrium Condition

\[
\frac{d\pi(t)}{\pi(t)} = \left( \frac{dP(t)}{P(t)} - \frac{dW(t)}{W(t)} \right) + \frac{dD(t)}{D(t)}
\]

Musgrave adjuster

\[
\frac{d\pi(t)}{\pi(t)} = (1-\pi(t)) \cdot \frac{dD(t)}{D(t)}
\]

\[\alpha(t) = \pi(t)\]
4. Example 2: Convex invariant

\[ \beta \delta(t) + (1 - \beta) \pi(t) = \text{constant} \]
with \( 0 \leq \beta \leq 1 \) constant

A fixed proportion of the replacement rate and of the contribution rate has to remain constant.

\[ \begin{align*}
\beta = 0 & : \text{DB} \\
\beta = 1 & : \text{DC}
\end{align*} \]
Convex invariant

Musgrave is a particular case of convex invariant

\[ M = \frac{\delta(t)}{1 - \pi(t)} \]

Or:

\[ \frac{1}{1 + M} \cdot \delta(t) + \frac{M}{1 + M} \cdot \pi(t) = \frac{M}{1 + M} \]

Or:

\[ \beta \cdot \delta(t) + (1 - \beta) \cdot \pi(t) = \frac{M}{1 + M} = \text{constant} \]

\[ \beta = \frac{1}{1 + M} \quad \pi = 20\% ; \delta = 50\% \rightarrow \beta = 0.62 \]
Convex invariant

Other example: min variance

Difference between the net wage and the replacement rate

\[ V(t) = (1 - \pi(t)) - \delta(t) = 1 - (\pi(t) + \delta(t)) \]

Goal: stability of this spread!

We can try to minimize the variance of \( V \)

\[ \text{var} V(t) = 0 \quad \text{if} \quad \pi(t) + \delta(t) = K \]

\[ \beta = 1/2 \]
Numerical illustration

Mean reverting dependence ratio

\[ D(t) = D_0 e^{-\gamma t} + \bar{D}(1 - e^{-\gamma t}) \quad (D_0 < \bar{D}) \]
\[ \delta(0) = \delta_0 \]
\[ \pi(0) = \pi_0 = D_0 \delta_0 \]

\[ D_0 = 40\% \quad \bar{D} = 66\% \quad \gamma = 5\% \]
\[ \delta(0) = 50\% \quad \alpha = 50\% \]
\[ \pi(0) = 20\% \]

\[ \delta(t) = ? \quad \pi(t) = ? \]
Numerical illustration

![Replacement rate graph](image)
Numerical illustration
5. Optimal hybridization by stochastic control

- Optimal choice for the risk sharing level between DB and DC

Stochastic optimal control

- State variable + equation
- Control variable
- Optimization criterion
Stochastic optimal control

State variable: Geometric Brownian Motion for the Dependence ratio

Solution:

\[ dD(t) = \gamma D(t) dt + \sigma D(t) dw(t) \]

With:

\[ w(.) = \text{standard Brownian motion} \]

\[ D(t) = D_0 \exp((\gamma - \sigma^2/2)t + \sigma w(t)) \]
Stochastic optimal control

Criterion: joined stability of:

- the replacement rate (retirees point of view)
- the contribution rate (active point of view)

Stability around a target value:

for $\pi(t) \rightarrow \bar{\pi}$

for $\delta(t) \rightarrow \bar{\delta}$
Stochastic optimal control

Value function to minimize (cf. Cairns (2000))

\[
J = \mathbb{E}\left( \int_{t}^{T} e^{-r(s-t)} ((1 - \rho)\left(\frac{\delta(s)}{\delta} - 1\right)^2 + \rho\left(\frac{\pi(s)}{\bar{\pi}} - 1\right)^2) \, ds \right)
\]

where \(0 \leq \rho \leq 1\): weight to fix

(DC: \(\rho = 1\) / DB: \(\rho = 0\))

Basic Equations:

\[
d\bar{D}(t) = \gamma . D(t) \, dt + \sigma . D(t) \, dw(t)
\]

\(\pi(t) = D(t) . \delta(t)\)
**Stochastic optimal control**

**Solution**

Optimal replacement rate

\[
\delta^*(t) = \frac{(1 - \rho) \frac{1}{\delta} + \rho \frac{D(t)}{\pi}}{(1 - \rho) \frac{1}{\delta^2} + \rho \frac{D^2(t)}{\pi^2}}
\]

Optimal contribution rate

\[
\pi^*(t) = \frac{(1 - \rho) \frac{D(t)}{\delta} + \rho \frac{D^2(t)}{\pi}}{(1 - \rho) \frac{1}{\delta^2} + \rho \frac{D^2(t)}{\pi^2}}
\]
Stochastic optimal control

Calibration of the target values

2 natural constraints:
- initial conditions: \( \delta^*(0) = \delta_0 \)
- link between the targets:
  \( \pi = \bar{D} \bar{\delta} \)

Solution:
\[
\bar{\delta} = \delta_0 \frac{(1 - \rho) + \rho \frac{D_0^2}{\bar{D}^2}}{(1 - \rho) + \rho \frac{D_0}{\bar{D}}} \]
CONCLUSION

• Different ways to build hybrid PAYG pension schemes with automatic adjustment

• Possibility to take into account simultaneously adequacy and sustainability

• Risk sharing between generations; DB and DC are extreme solutions

• Example of the Musgrave rule (Belgium)
Future research

- Optimal choice for the risk sharing level
- NDC with risk sharing
- Model with conditional indexation


References


E.Schokkaert et al. (2017) : Towards an equitable and sustainable points system : A proposal for pension reform in Belgium, *DPS 17.03, KULeuven*