HYBRID SOCIAL SECURITY PENSION SCHEMES
risk sharing and stochastic optimal control

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Agenda

1. Hybrid Pension Schemes
2. PAYG Risk sharing model
3. Example 1 : Musgrave rule
4. Example 2 : Convex invariant
5. Optimal hybridization by stochastic control

1.Hybrid Pension Schemes

- Financial viability of classical Pay As You Go (PAYG) social security pension schemes

- Most of them using a Defined Benefit (DB) structure

- Important risk factors :
  - Ageing
  - Fertility
  - Baby boom effect

Some classical solutions

- Parametric reforms (*retirement age, early retirement , indexation,…*)

- Move from DB schemes to DC schemes (*Notional Accounts* , NDC)

- Introduction of Automatic Balance Mechanisms as an answer to risk exposure (DB and DC) to avoid any form of" Pension Populism"

Sustainability and Adequacy

- …But Pension reform is not just a matter of financial sustainability

- The mission of the social security is also social adequacy

- Fairness between generations and between categories of workers is a key point

Hybrid pension plans

- Development of Hybrid pension plans between DB and DC as well for public as for occupational pension schemes
- ***Sweden*** : NDC with automatic adjustment
- ***Netherlands*** : conditional indexation, collective DC plans
- ***Belgium :*** project of reform of the first pillar (points system with Musgrave rule)

2. PAYG Risk Sharing Model

Incomes :

$A(t)$ = number of contributors at time t

$W(t)$ = mean wage

$\pi(t)$ = contribution rate

$$IN(t) = A(t).\pi(t).W(t)$$

Equilibrium equation

Outcomes :

$B(t)$ = number of retirees at time t

$P(t)$ = mean pension

$\delta(t)$= replacement rate

$$OUT(t) = B(t).P(t) = B(t).\delta(t).W(t)$$

Equilibrium equation

Actuarial equilibrium :

$$IN(t) = OUT(t)$$

$$A(t).\pi(t).W(t) = B(t).P(t)$$

$$\pi(t) = \frac{B(t)}{A(t)} \cdot \frac{P(t)}{W(t)}$$

$$D(t) = \frac{B(t)}{A(t)} = \text{dependence ratio}$$

$$\delta(t) = \text{replacement rate}$$

Equilibrium equation

$$\pi(t) = \frac{\text{number of retirees}}{\text{number of contributors}} \times \frac{\text{mean pension}}{\text{mean wage}}$$

Demographic risk

Financing the system

Generation of active people

Social quality of the system

Generation of retirees

$$\pi(t) = D(t).\delta(t)$$

Automatic Adjustment

$$\pi(t) = \frac{B(t)}{A(t)} \cdot \frac{P(t)}{W(t)} = D(t).\frac{P(t)}{W(t)} = D(t).\delta(t)$$

Risk factor

Automatic Adjustment :

How to maintain automatically this equilibrium in case of change of D(t) (*Ageing = Increase* !)

Automatic Adjustment

$$\pi(t) = \frac{B(t)}{A(t)} \cdot \frac{P(t)}{W(t)} = D(t) \cdot \frac{P(t)}{W(t)} = D(t) \cdot \delta(t)$$

Risk factor

Constant in pure DC

Constant in pure DB

Adjustment of δ

Adjustment of π

Social risk

Financial risk

Automatic Adjustment

$$\frac{d\pi(t)}{\pi(t)} = \frac{d\delta(t)}{\delta(t)} + \frac{dD(t)}{D(t)}$$

Ageing effect → Risk factor

DB

$$d\delta(t) = 0$$

$$\frac{d\pi(t)}{\pi(t)} = \frac{dD(t)}{D(t)}$$

$$\alpha = 0$$

DC

$$\frac{d\delta(t)}{\delta(t)} = -\frac{dD(t)}{D(t)}$$

$$d\pi(t) = 0$$

$$\alpha = 1$$

Hybrid / risk sharing

$$\frac{d\delta(t)}{\delta(t)} = -\alpha(t).\frac{dD(t)}{D(t)}$$

$$\frac{d\pi(t)}{\pi(t)} = (1-\alpha(t)).\frac{dD(t)}{D(t)}$$

$$0 \leq \alpha(t) \leq 1$$

Control variable ← automatic adjuster

3.Example 1 : the Musgrave rule

Goal:

To keep constant the replacement rate ***but net of contributions***

DB

$$\delta_0 = \delta(t) = \frac{P(t)}{W(t)}$$

→

Musgrave

$$M = \frac{P(t)}{W(t).(1-\pi(t))} = \frac{\delta(t)}{1-\pi(t)}$$

The Musgrave rule

Musgrave Condition

$$\frac{dP(t)}{P(t)} - \frac{dW(t)}{W(t)} = \frac{d(1-\pi(t))}{1-\pi(t)} = -\frac{\pi(t)}{1-\pi(t)} \cdot \frac{d\pi(t)}{\pi(t)}$$

Equilibrium Condition

$$\frac{d\pi(t)}{\pi(t)} = \left(\frac{dP(t)}{P(t)} - \frac{dW(t)}{W(t)}\right) + \frac{dD(t)}{D(t)}$$

$$\frac{d\pi(t)}{\pi(t)} = (1-\pi(t)).\frac{dD(t)}{D(t)}$$

Musgrave adjuster

$$\alpha(t) = \pi(t)$$

4.Example 2 : Convex invariant

$$\beta.\delta(t) + (1-\beta).\pi(t) = \text{constant}$$
$$\text{with } 0 \leq \beta \leq 1 \text{ constant}$$

A fixed proportion of the replacement rate and of the contribution rate has to remain constant.

$$\beta = 0 \quad : \quad \text{DB}$$
$$\beta = 1 \quad : \quad \text{DC}$$

Convex invariant

Musgrave is a particular case of convex invariant

$$M = \frac{\delta(t)}{1-\pi(t)}$$

Or :

$$\frac{1}{1+M}.\delta(t) + \frac{M}{1+M}.\pi(t) = \frac{M}{1+M}$$

Or :

$$\beta.\delta(t) + (1-\beta).\pi(t) = \frac{M}{1+M} = \text{constant}$$

$$\beta = \frac{1}{1+M}$$

$\pi = 20\% \; ; \; \delta = 50\% \rightarrow \; \beta = 0.62$

Convex invariant

Other example : min variance

Difference between the net wage and the replacement rate

$$V(t) = (1 - \pi(t)) - \delta(t) = 1 - (\pi(t) + \delta(t))$$

Goal : *stability of this spread* !

We can try to minimize the variance of V

$$\text{var}\, V(t) = 0 \quad \text{if} \quad \pi(t) + \delta(t) = K$$

$$\beta = 1/2$$

Numerical illustration

Mean reverting dependence ratio

$$D(t) = D_0.e^{-\gamma t} + \bar{D}.(1 - e^{-\gamma t}) \qquad (D_0 < \bar{D})$$

$$\delta(0) = \delta_0$$

$$\pi(0) = \pi_0 = D_0.\delta_0$$

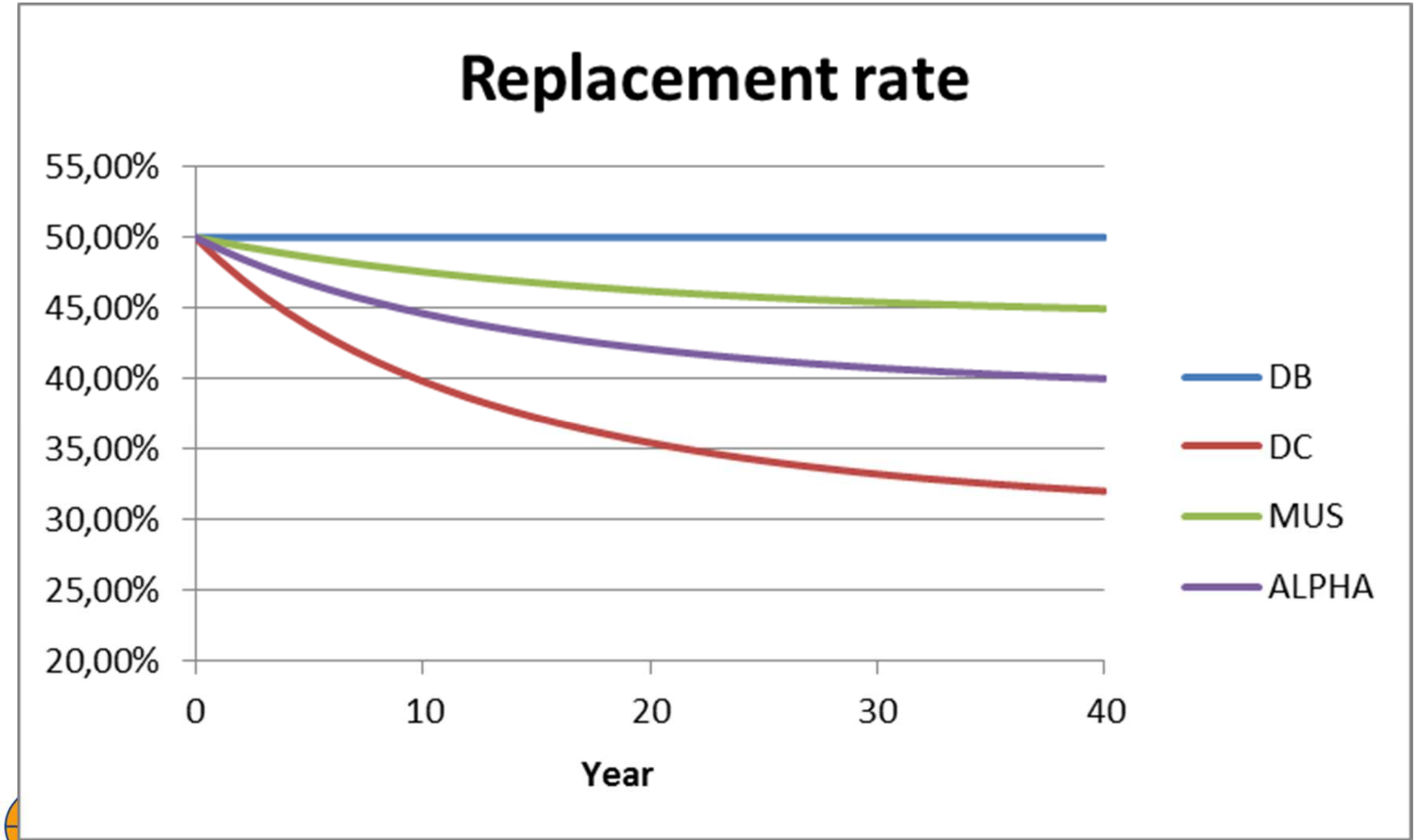
$$D_0 = 40\% \qquad \bar{D} = 66\% \qquad \gamma = 5\%$$

$$\delta(0) = 50\% \qquad \alpha = 50\%$$

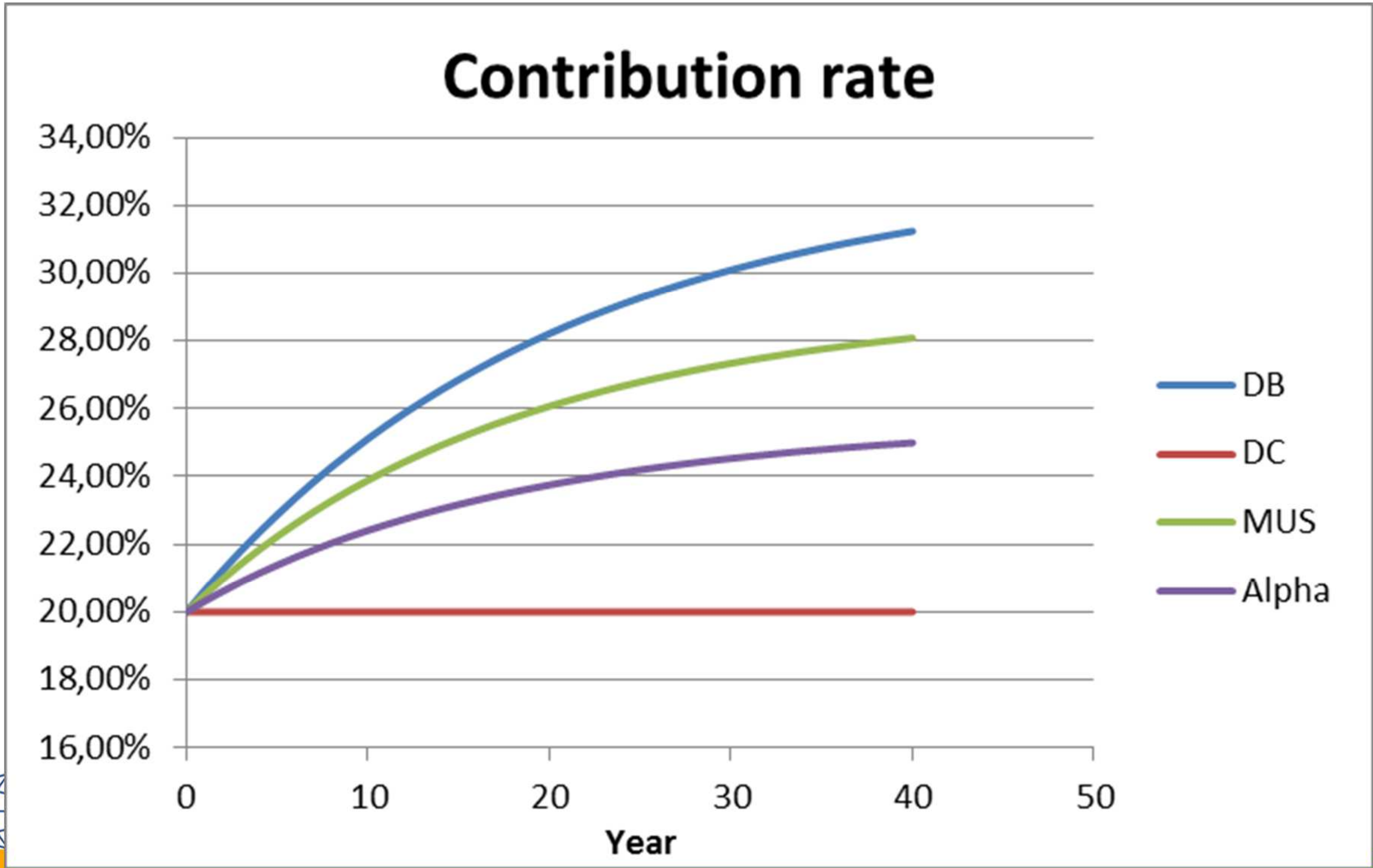
$$\pi(0) = 20\%$$

$$\delta(t) = ? \qquad \pi(t) = ?$$

Numerical illustration



Numerical illustration



5.Optimal hybridization by stochastic control

- Optimal choice for the risk sharing level between DB and DC

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Stochastic optimal control

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- *State variable + equation*
- *Control variable*
- *Optimization criterion*

Stochastic optimal control

State variable :Geometric Brownian Motion for the Dependence ratio

$$dD(t) = \gamma.D(t)dt + \sigma.D(t)dw(t)$$

With :

$$w(.) = \text{standard Brownian motion}$$

Solution :

$$D(t) = D_0.\exp((\gamma - \sigma^2 / 2)t + \sigma.w(t))$$

Stochastic optimal control

Criterion : joined **stability** of:

- the replacement rate (*retirees point of view*)
- the contribution rate (*active point of view*)

Stability around a target value :

$$\text{for} \quad \pi(t) \rightarrow \overline{\pi}$$

$$\text{for} \quad \delta(t) \rightarrow \overline{\delta}$$

Stochastic optimal control

Value function to minimize (*cf. Cairns (2000)*)

$$J = E\left(\int_t^T e^{-r.(s-t)} ((1-\rho)(\frac{\delta(s)}{\bar{\delta}} - 1)^2 + \rho.(\frac{\pi(s)}{\bar{\pi}} - 1)^2).ds \right)$$

where $0 \le \rho \le 1$: weight to fix

$(DC: \rho = 1 / \ DB: \rho = 0)$

Basic Equations :

$$dD(t) = \gamma.D(t)\,dt + \sigma.D(t)\,dw(t)$$

$$\pi(t) = D(t).\delta(t)$$

Stochastic optimal control

Solution

Optimal replacement rate

$$\delta^*(t) = \frac{(1-\rho)\frac{1}{\overline{\delta}} + \rho\frac{D(t)}{\overline{\pi}}}{(1-\rho)\frac{1}{\overline{\delta}^2} + \rho\frac{D^2(t)}{\overline{\pi}^2}}$$

Optimal contribution rate

$$\pi^*(t) = \frac{(1-\rho)\frac{D(t)}{\overline{\delta}} + \rho\frac{D^2(t)}{\overline{\pi}}}{(1-\rho)\frac{1}{\overline{\delta}^2} + \rho\frac{D^2(t)}{\overline{\pi}^2}}$$

Stochastic optimal control

Calibration of the target values

2 natural constraints :

- initial conditions : $\delta^*(0) = \delta_0$

- link between the targets:

$$\bar{\pi} = \bar{D}.\bar{\delta}$$

Solution :

$$\bar{\delta} = \delta_0 \frac{(1-\rho) + \rho \frac{D_0^2}{\bar{D}^2}}{(1-\rho) + \rho \frac{D_0}{\bar{D}}}$$

CONCLUSION

- Different ways to build ***hybrid PAYG*** pension schemes with automatic adjustment

- Possibility to take into account simultaneously ***adequacy*** and ***sustainability***

- Risk sharing between generations; DB and DC are extreme solutions

- Example of the ***Musgrave rule*** (*Belgium*)

IACA PBSS

Future research

- Optimal choice for the risk sharing level
- NDC with risk sharing
- Model with conditional indexation

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