Monitoring actuarial assumptions in life insurance

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Typical paths with change of regime at date 3

(a) Processes $N$ and $V_t$

(b) Processes $N$ and $Y_t$

Figure: Sample paths, for $\rho = 1.5$, of the cusum processes $N, V^\rho$ (left) and $N, Y^\rho_t$ for $\rho = 0.5$ (right).
Quick Outline

• Why is quick detection important in insurance?

• Quick version of quickest detection

• Monitoring populations in practice

• Monitoring other insurance portfolios (quickly if time permits)
\[ \Delta P(t < \theta) \]

FALSE ALARM PROB

CHANGE POINT

\[ \theta \text{ detection delay} \]

drift $\mu > 0$ (known)

detection
$K_t$ (Lee-Carter-type model)

CHANGE POINT

1993 2005

TPRV 93

TGH-TGF 05

$\uparrow +8\%$
Longevity risk components 2/4

The trend

The mortality improvement is not a diversifiable risk: it affects the whole portfolio and can thus not be managed using the law of large numbers.

Actual mortality estimation

Estimation error for an estimate of mortality based on actual experience: the error is larger for small populations (or for poorly represented age groups).

Doubled improvements

Mortality level at 80% of the expected

<table>
<thead>
<tr>
<th>Age</th>
<th>Females</th>
<th>Males</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Improvement</td>
<td>Improvement</td>
</tr>
<tr>
<td></td>
<td>Pension Value</td>
<td>Interest Rate</td>
</tr>
<tr>
<td>55</td>
<td>+5.4%</td>
<td>+32bp</td>
</tr>
<tr>
<td>65</td>
<td>+5.76%</td>
<td>+43bp</td>
</tr>
<tr>
<td>75</td>
<td>+5.2%</td>
<td>+55bp</td>
</tr>
<tr>
<td>85</td>
<td>+3.6%</td>
<td>+60bp</td>
</tr>
</tbody>
</table>

**Table:** TGH05/TGF05 with flat interest rate of 3%
Bayesian setup for random change-point

Brownian framework with abrupt change in the drift

- Based on the conditional distribution of the time of change,
- Formulated as an optimal stopping problem
- Page (1954), Shiryaev (1963), Roberts (1966), Beibel (1988), Moustakides (2004), and many others...

Poisson framework with abrupt change in intensity

- Based on the conditional distribution of the time of change, with exponential or geometric prior distribution
**Mathematical settings**

We consider a portfolio of insured population:

- Let $N = (N_t)_{t \geq 0}$ be a counting process indicating the deaths of policyholders and $\lambda = (\lambda_t)_{t \geq 0}$ its intensity.

- The counting process $N_t$, is available sequentially through the filtration $\mathcal{F}_t = \sigma\{N_s, 0 < s \leq t\}$.

- We suppose that the insurance company relies on a Cox-like model to project her own experienced mortality:

  $$\lambda_t = \rho \lambda^0_t,$$

  - $\lambda^0_t$ is a reference intensity and $\rho$ is a positive parameter.
  - $\lambda^0$ is considered deterministic and may refer whether to a projection of national population/best estimate...

Model risk/parameter uncertainty: **Change-point**

$$\lambda_t = 1_{\{t < \theta\}} \rho \lambda^0_t + 1_{\{t \geq \theta\}} \bar{\rho} \lambda^0_t.$$

Without loss of generality we can assume that $\underline{\rho} = 1$ and let $\rho = \bar{\rho} > 1$. 
Let $P_\theta$ (resp. $E_\theta[\cdot]$) be the probability measure (resp. expectation) induced when the change takes place at time $\theta$.

**Example**

- For $\theta = 0$, the process is *out-of-control*.
- For $\theta = \infty$, the process is *in-control*.

Detect the change-point $\theta$ as quick as possible while avoiding false alarms.

**Optimality Criteria, Lorden (1971)-like**

- The detection delay $E_\theta \left[ (N_\tau - N_\theta)^+ \middle| \mathcal{F}_\theta \right]$.
- The frequency of false alarm $E_\infty [N_\tau]$. 
Optimization Problem

Find $\tau^*$ such that $C(\tau^*) = \inf_{\tau} \sup_{\theta \in [0, \infty]} \text{ess sup}_{\theta} \mathbb{E}_{\theta} \left[ (N_{\tau} - N_{\theta})^+ \bigg| \mathcal{F}_{\theta} \right]$
subject to $\mathbb{E}_{\infty}[N_{\tau}] = \omega$.

Assumption

1. $\int_0^t \lambda_s ds < \infty$, $\mathbb{P}_{\infty}, \mathbb{P}_0$-a.s.
2. $N_{\infty} = \infty$ $\mathbb{P}_{\infty}, \mathbb{P}_0$-a.s.
Optimality of the Cusum Procedure (1/7)

Let the Radon-Nikodym density of $\mathbb{P}_0$ with respect to $\mathbb{P}_\infty$ be defined as

$$
\frac{d\mathbb{P}_0}{d\mathbb{P}_\infty} \big|_{\mathcal{F}_t} = \exp U_t,
$$

where $U_t = \log(\rho) N_t + (1 - \rho) \int_0^t \lambda_s^0 \, ds$ is the log-likelihood ratio.

Let $V(x)$ be the CUSUM process; with head-start $0 \leq x < m$; defined as

$$
V_t(x) = U_t - (-x) \wedge U_t
$$

where $U_t$ is the running infimum of $U$, i.e. $U_t = \inf_{s \leq t} U_s$.

The process $V(x)$ measures the size of the drawup, comparing the present value of the process $U$ to its historical infimum $U$.

Let $\tau_m(x)$ be the fist hitting time of $V(x)$ of the barrier $m$, i.e.

$$
\tau_m(x) = \inf \{ t \geq 0, V_t(x) \geq m \}.
$$

Theorem

If $\mathbb{E}_\infty [N_{\tau_m(0)}] = \omega$ then $\tau_m(0)$ is optimal, i.e. $\inf_{\tau} C(\tau) = C(\tau_m(0))$. 

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Figure: Sample paths, for $\rho = 1.5$, of the cusum processes $N, V^\rho$ (left) and $N, Y_t^\rho$ for $\rho = 0.5$ (right).
**Detection Procedure – Algorithm**

Step 1: Fix the input parameters: The post-change intensity through the specification of $\rho$ and the false alarm constraint $\omega$.

Step 2: Determine the threshold $m$ as the solution of the equation $E_{\infty}[N_{\tau_m}] = \omega$.

Step 3: For each new observation at time $t$ compute the value of the CUSUM process $V$ given by the iterative relation $V_{t+1} = (V_{t-1} + U_t)^+$.

Step 4: Compare the current value of $V$ to the threshold $m$ and stop the procedure once $V_t \geq m$ and sound an alarm. Hence $\tau_m(0) = t$. 
We consider the Continuous Mortality Investigation assured lives dataset and England & Wales national population. We split data into two periods:

- We consider the period 1947-1969 as a training period.
- The Cox model is estimated over this period using the MLE.

Hence we monitor the sequentially the dataset over the period 1970-2005 and look for changes on the mortality of assured lives.
Figure: Detection scheme for age groups 50 – 59 (right) and 80 – 89 (left). The post-change is set to $\rho = 15\%$ and the false alarm constraint to $\omega = 100\bar{\lambda}$. 
### Detection Procedure – Real World (4/4)

<table>
<thead>
<tr>
<th>Age</th>
<th>( \tau_m )</th>
<th>( \rho = 1.50 )</th>
<th>( \rho = 1.15 )</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>70 – 79</td>
<td>1988</td>
<td>1984</td>
<td></td>
<td>1974</td>
</tr>
<tr>
<td>80 – 89</td>
<td>1983</td>
<td>1978</td>
<td></td>
<td>1973</td>
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</table>

**Table:** Detection of mortality change with a post-change ratio of \( \rho = 1.15 \) and an average run length (false alarm) constraint of 100. The right column reports the detected change-point using an off-line procedure.
Monitoring Mortality
Sounding an alarm for the change $\rho^{\text{Hyp}} \rightarrow \rho^{\text{Targer}}$

- We simulate deaths on the portfolio with different levels $\rho^{\text{Targer}} = 95\%, 90\%$ and $85\%$ s.t.

$$D(x, t) \sim \text{Pois}(\rho^{\text{Targer}} \times L(x, t) \times \mu^{\text{ERM00}}(x, t))$$

- We suppose that the actuary made an assumption of $\rho^{\text{Hyp}} = 100\%$

- We set-up the monitoring/surveillance on the observed deaths and try to detect a change from $\rho^{\text{Hyp}} = 100\%$ to $\rho^{\text{Targer}} = 95\%, 80\%$ and $85\%$ respectively.

- We test different sizes of the portfolio small sized 1000, 5000 and a (relatively) large 10000 and compare the results
Monitoring Mortality

Sounding an alarm for the change $\rho^{\text{Hyp}} = 100\% \rightarrow \rho^{\text{Targ}} = 95\%$
## Detection Delay

**Impact of Portfolio Size and Age Tranches**

<table>
<thead>
<tr>
<th></th>
<th>1000</th>
<th></th>
<th>5000</th>
<th></th>
<th>10000</th>
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<tbody>
<tr>
<td><strong>Size</strong></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Ages</td>
<td>60-90</td>
<td>60-75</td>
<td>76-90</td>
<td>60-90</td>
<td>60-75</td>
<td>76-90</td>
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<tr>
<td>Hyp.</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td><strong>deaths</strong></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>100% → 95%</td>
<td>596</td>
<td>710</td>
<td>498</td>
<td>246</td>
<td>99</td>
<td>107</td>
</tr>
<tr>
<td>100% → 90%</td>
<td>244</td>
<td>320</td>
<td>186</td>
<td>106</td>
<td>55</td>
<td>59</td>
</tr>
<tr>
<td>100% → 85%</td>
<td>92</td>
<td>122</td>
<td>100</td>
<td>58</td>
<td>35</td>
<td>36</td>
</tr>
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<tr>
<td><strong>time</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100% → 95%</td>
<td>1086</td>
<td>1130</td>
<td>1120</td>
<td>576</td>
<td>617</td>
<td>422</td>
</tr>
<tr>
<td>100% → 90%</td>
<td>931</td>
<td>1124</td>
<td>947</td>
<td>276</td>
<td>373</td>
<td>241</td>
</tr>
<tr>
<td>100% → 85%</td>
<td>707</td>
<td>980</td>
<td>734</td>
<td>161</td>
<td>247</td>
<td>159</td>
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Detection Time
26-Dec-2003
Perspectives

• LoLitA closing international conference, Paris, Jan. 15-16-17, 2018

• LoLitA Lecture Notes

• Own Longevity of LoLitA: after 2018