How a cause-of-death reduction can be compensated in presence of heterogeneity?

A population dynamics approach

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- Context : increase in life expectancy due to mortality improvements over the past years
  - explained by changes in cause-of-death mortality
    - medical advances, changes in lifestyle

⇒ How populations would evolve if a cause-specific mortality could be reduced/eliminated ?

  - Impacts on populations structure : possible issues for intergenerational balance and governmental purposes

- Heterogeneity in populations : age, gender, social status, etc.
  - Mortality is known to be strongly dependent on socioeconomic circumstances, as well as cause-specific mortality :
    e.g. Raleigh and Kiri (1997), Singh and Hiatt (2006), Longevity basis risk report (2014)
  - Mortality gaps increase between socioeconomic categories :
    e.g. Elo (2009), Villegas (2015)
  - Individual characteristics give additional information when modelling mortality and population evolution

- Socioeconomic differences in mortality : challenge for pension systems and annuity portfolios
  - Can lead to inadequate funding of annuity and pension obligations :
    e.g. Meyricke and Sherris (2013)
  - Modelling mortality differentials between subgroups and a reference population :
    e.g. Bensusan (2010), Jarner and Kryger (2011), Villegas and Haberman (2014)
- Population dynamics give additional information :
  - Structural relationship between the reference population mortality and subgroups mortality generated by the population dynamics

- Global population central death rate :

$$ {}_1m_{xt} = \frac{D_{xt}}{E_{xt}} = \sum_{j=1}^{n} \frac{D^j{}_{xt}}{E_{xt}} = \sum_{j=1}^{n} \frac{D^j_{xt} E^j_{xt}}{E^j_{xt} E_{xt}} = \sum_{j=1}^{n} {}_1m^j_{xt} \frac{E^j_{xt}}{E_{xt}} = \sum_{j=1}^{n} {}_1m^j_{xt} W^j_{xt} $$

at age x during calendar year t, comprising n risk classes.

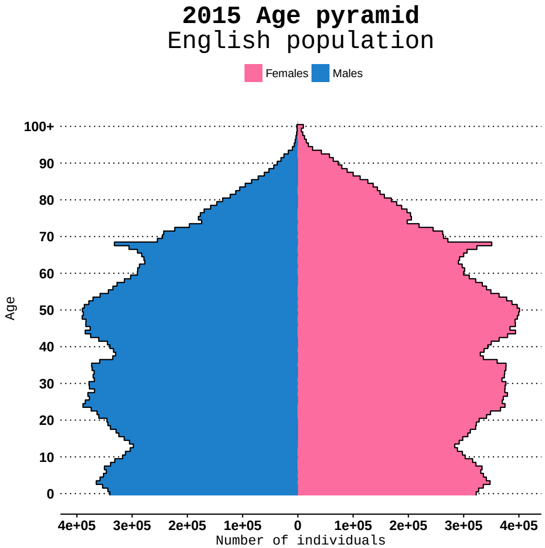
  - The relative exposure $W^j_{xt} = \frac{E^j_{xt}}{E_{xt}}$ is link to the proportion of individual of age x in risk class j.
  - Evolution of these quantities are determined by the population dynamics and can vary significantly depending on the age x and time t.

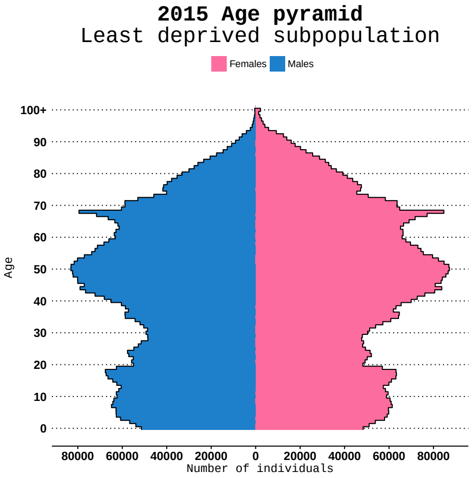
- Relation even more complicated for the life expectancy which depends on the whole age pyramids of the risk classes

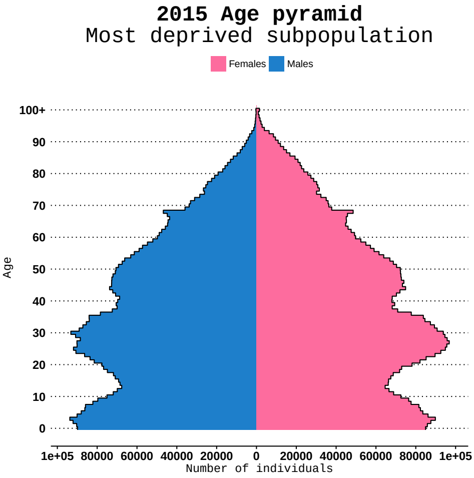
Research problem

1. What are the impacts of changes in the socioeconomic composition of the population on aggregated indicators such as the life expectancy ?
2. How does these structural changes interact with cause specific reduction of mortality ?

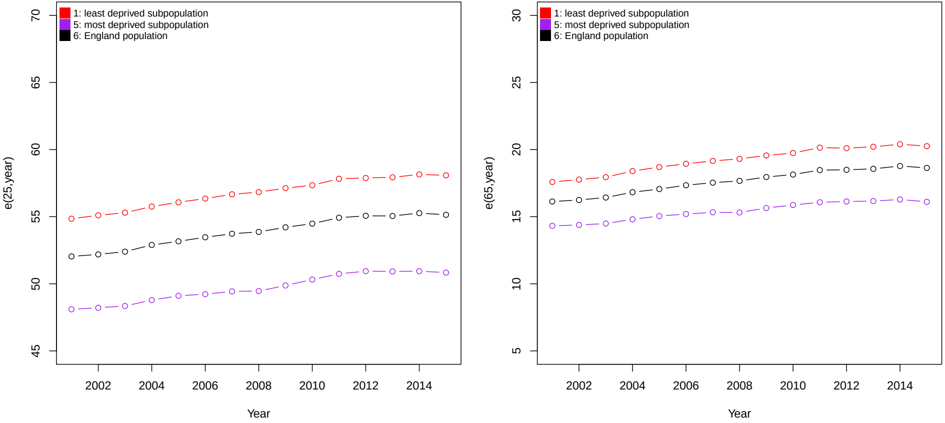
⇒ **How a cause-of-death reduction can be compensated in presence of heterogeneity ?**


2015 Age pyramid
English population
Females
Males
100+
90
80
70
60
50
40
30
20
10
0
Age
4e+05
3e+05
2e+05
1e+05
0
1e+05
2e+05
3e+05
4e+05
Number of individuals

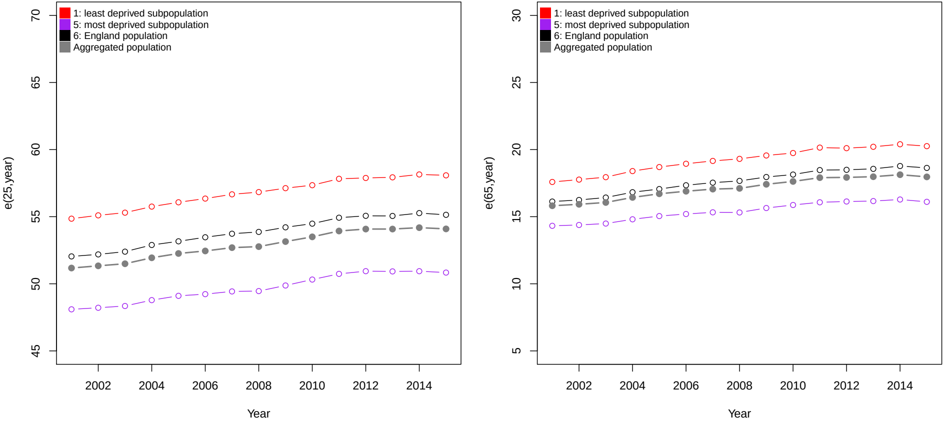

2015 Age pyramid
Least deprived subpopulation
Females
Males
Age
100+
90
80
70
60
50
40
30
20
10
0
80000 60000 40000 20000 0 20000 40000 60000 80000
Number of individuals


2015 Age pyramid
Most deprived subpopulation
Females
Males
Age
100+
90
80
70
60
50
40
30
20
10
0
1e+05 80000 60000 40000 20000 0 20000 40000 60000 80000 1e+05
Number of individuals

Males period life expectancy at age 25 (left) and 65 (right) over the years



Males period life expectancy at age 25 (left) and 65 (right) over the years



Aggregated population= 50% least deprived subpopulation + 50% most deprived subpopulation

Let us consider a subpopulation with socio-economic category j :

- Deterministic approach, McKendrick-Von Foerster equation :
  - Aging law : $(\partial_a + \partial_t) g_j^{\epsilon}(a,t) = -\mu_j^{\epsilon}(a,t) g_j^{\epsilon}(a,t)$
  - Births law : $g_j^{\epsilon}(0,t) = \int_0^{a^{\dagger}} p^{\epsilon} g_j^{f}(a,t) b_j^{f}(a,t) da$
  - Initial Pyramid : $g_j^{\epsilon}(a,0)$
- Short term behavior : depends on the initial population
- Long term behavior : characterized by birth and survival functions
- References : McKendrick (1926), Von Foerster (1959), Bensusan (2010), Arnold et al. (2016)

Let us consider a heterogeneous population with two homogeneous subpopulations :

$$g^{\epsilon}(a,t) = g_1^{\epsilon}(a,t) + g_2^{\epsilon}(a,t) \quad (1)$$

Balance law for the global population :

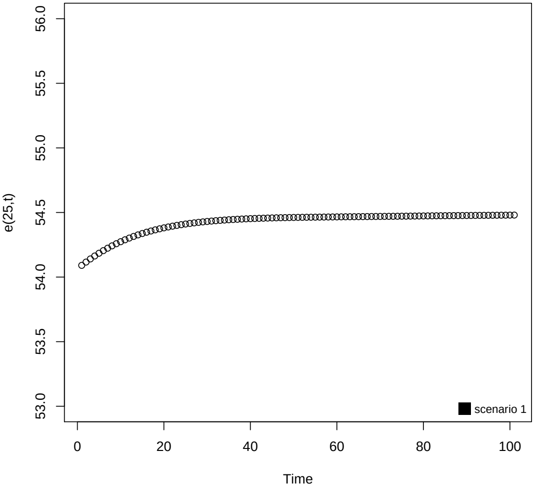
$$(\partial_a + \partial_t) g^{\epsilon}(a,t) = -d^{\epsilon}(a,t) g^{\epsilon}(a,t) \quad (2)$$

- Aggregated death rate :
  - $d^{\epsilon}(a,t) = w_1^{\epsilon}(a,t)\mu_1^{\epsilon}(a,t) + w_2^{\epsilon}(a,t)\mu_2^{\epsilon}(a,t)$

    where $w_j^{\epsilon}(a,t) = \frac{g_j^{\epsilon}(a,t)}{g^{\epsilon}(a,t)}$
  - d depends non-linearly on $g_1, g_2, \mu_1, \mu_2, b_1, b_2$
  - Even with rates time-independent : $\mu_j^{\epsilon}(a, \not{t})$ and $b_j^{\epsilon}(a, \not{t})$

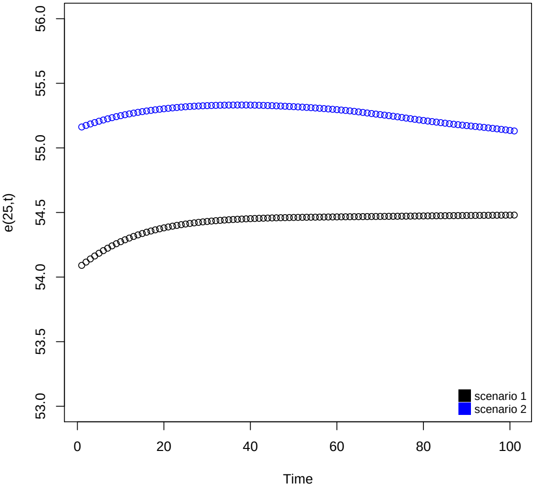
    $\Rightarrow$ the aggregate death rate depends on time : $d^{\epsilon}(a, \mathbf{t})$

Males period life expectancy at age 25 over time (initial year = 2015)



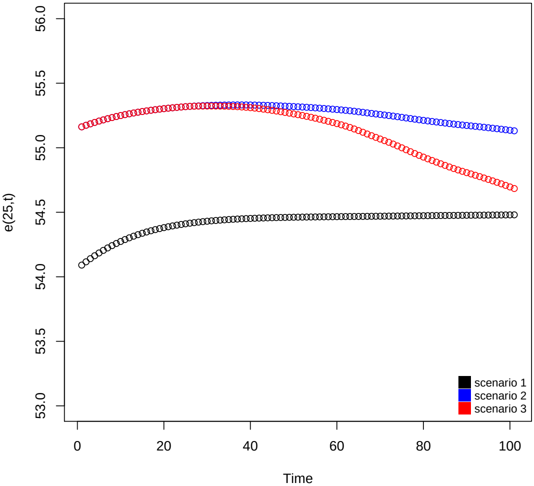
Scenario 1 = 50% least deprived subpopulation + 50% most deprived subpopulation and same birth rate (England,2015)

Males period life expectancy at age 25 over time (initial year = 2015)



Scenario 2 = Scenario 1 + external causes removal for the most deprived and same birth rate (England,2015)

Males period life expectancy at age 25 over time (initial year = 2015)



Scenario 3 = Scenario 2 + changes in birth rates for each subpopulations (-10%,+20%)

Concluding remarks

- With a population dynamics model, we study impacts of cause-of-death changes on a heterogenous population, comprising different social status :
  - Population composition and birth patterns affect also the aggregated death rates

  $\Rightarrow$ **Depending on the population composition, impacts of cause-of-death mortality changes can be compensated and misinterpreted**

- To go further :
  - Deepen cause-of-death mortality and fertility compensation
  - Social status changes and migrations
  - Time-dependent rates
  - Study the bias of the deterministic model

Short Bibliography

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Thank you!