The life insurance on several heads: sensitivity analysis to increases in life expectancy

The single head insurance

- Risk of death = premium.
- In the beginning the individual is alive (deterministic state).
- In each period the risk occurs (indeterministic state).
- At some point, he dies (deterministic state).
- Once a deterministic state is reached, the individual can not escape from it.
- Financially:
  - There are n possible paths (tree of probabilities).
  - Each path has a financial valuation.
  - The PV is different.

The single head insurance

- The procedure:
  - Reproduce the complete tree of individual states (from its first initial state to all possible deterministic states) over time.
  - Use of the interest rate curve (not necessarily a single interest rate).
  - Calculation of the probability of each path $\Pi_{\alpha_{t+1}} = \Pi_{\alpha_t} \times P(\alpha_t \to \alpha_{t+1})$
  - Calculating the NPV for each possible path.
  - Probability & Value = Probability distribution function.
  - The classical value of the mathematical provision coincides with the average value of the previous probability distribution function.
  - In addition, for being a distribution function we are aware of the dispersion (standard deviation, etc.).

The single head insurance: algorithm

Algorithm 1 Cálculo de la función de probabilidad

Procedure *Evolucion* (*estado*, *probabilidad*, VAN, t)
for $EstadoNuevo = 1$ **to** $PosiblesEstado$ **do**
  $VAN_{acumulado} \leftarrow VAN + CashFlow(EstadoNuevo, t)$
  $Prob_{acumulado} \leftarrow probabiilidad \times P(estado \rightarrow EstadoNuevo)$
  if $EstadoNuevo\ is\ terminal$ **then**
    $resultado.Add(VAN_{acumulado}, Prob_{acumulado})$
  else
    $Evolucion(EstadoNuevo, Prob_{acumulado}, VAN_{acumulado}, t + 1)$
  end if
end for
End Procedure

The insurance on several heads

- We have N individuals, where each individual can have ε states (alive, dead, invalid, etc.).
- At each moment we will have ε ^ N possibilities, states that will determine the group.
- We need to know the probability of transition from one state to another, not at the individual level, but at the group level.
- In short, a group of several heads is an individual with many states ε^N, of which we must know the probability of transition from one state to another. The method of calculation is, therefore, identical to that of single head insurance.

Transitions for two heads with two states

$$
\begin{aligned}
P(vivo \otimes vivo \to vivo \otimes vivo) &= p_{x_1} \times p_{x_2} \\
P(vivo \otimes vivo \to vivo \otimes muerto) &= p_{x_1} \times q_{x_2} \\
P(vivo \otimes vivo \to muerto \otimes vivo) &= q_{x_1} \times p_{x_2} \\
P(vivo \otimes vivo \to muerto \otimes muerto) &= q_{x_1} \times q_{x_2} \\
P(vivo \otimes muerto \to vivo \otimes vivo) &= 0 \\
P(vivo \otimes muerto \to muerto \otimes vivo) &= 0 \\
P(vivo \otimes muerto \to vivo \otimes muerto) &= p_{x_1} \\
P(vivo \otimes muerto \to muerto \otimes muerto) &= q_{x_1} \\
P(muerto \otimes vivo \to vivo \otimes vivo) &= 0 \\
P(muerto \otimes vivo \to muerto \otimes vivo) &= p_{x_2} \\
P(muerto \otimes vivo \to vivo \otimes muerto) &= 0 \\
P(muerto \otimes vivo \to muerto \otimes muerto) &= q_{x_2} \\
P(muerto \otimes muerto \to vivo \otimes vivo) &= 0 \\
P(muerto \otimes muerto \to muerto \otimes vivo) &= 0 \\
P(muerto \otimes muerto \to vivo \otimes muerto) &= 0 \\
P(muerto \otimes muerto \to muerto \otimes muerto) &= 1
\end{aligned}
$$

Two-Headed Life Annuity Analysis

- Father pays to the insurance periodically while the child lives.
- If father dies, the child earns a life annuity.
- If child dies, the product is extinguished.
- This product provides a higher yield than a financial income, because of the actuarial factor.
- It is a suitable product to constitute a pension to a child with dependency (disability).

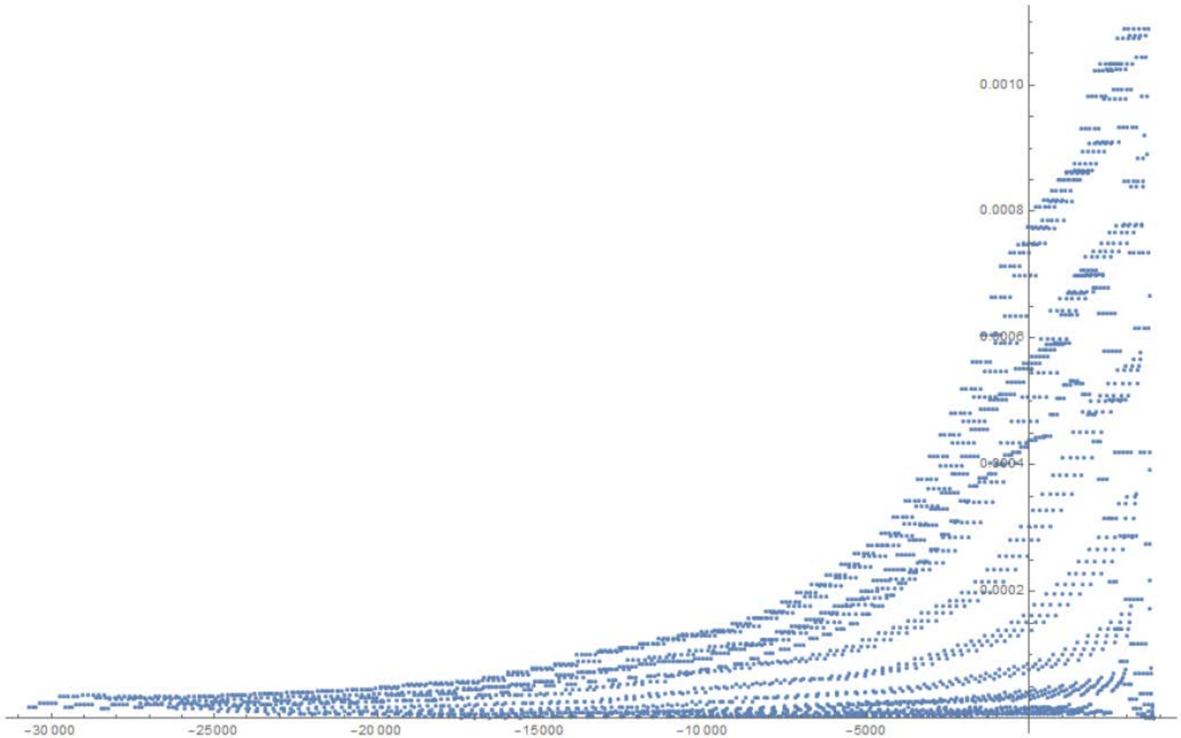
Particular case: Down Syndrome

- We assume that the child has Down Syndrome.
- Much shorter life expectancy due to the propensity of diseases and premature aging.
- The table shows a strong infant mortality, and is assembled every 5 years (same mortality).

Cuadro 1: Descripción del caso

Parámetro	Valor
Edad del padre	35
Edad del hijo	0
Tabla de mortalidad del padre	PERM2000C/gen 1980 (Esperanza 85 años)
Tabla de mortalidad del hijo	Strauss-Eyman Down severo (Esperanza 47 años)
Tipo de interés	3 %
Cuota Anual a pagar por el padre	120€
Rescate en caso de fallecer el hijo	0€
Renta anual a cobrar por el hijo	1000€

Result of the NPV: Probability Function



Analysis of the probability function

- Each point corresponds to a specific case (a single complete path, from the beginning to the termination of the contract).

- Different overlapping curves are observed, each curve corresponds to a determined pattern.

Analysis of the probability function

- Bias to the right.
- The father is young and will live for a long time.
- The peak moves to the left if we consider rents of longer periods.

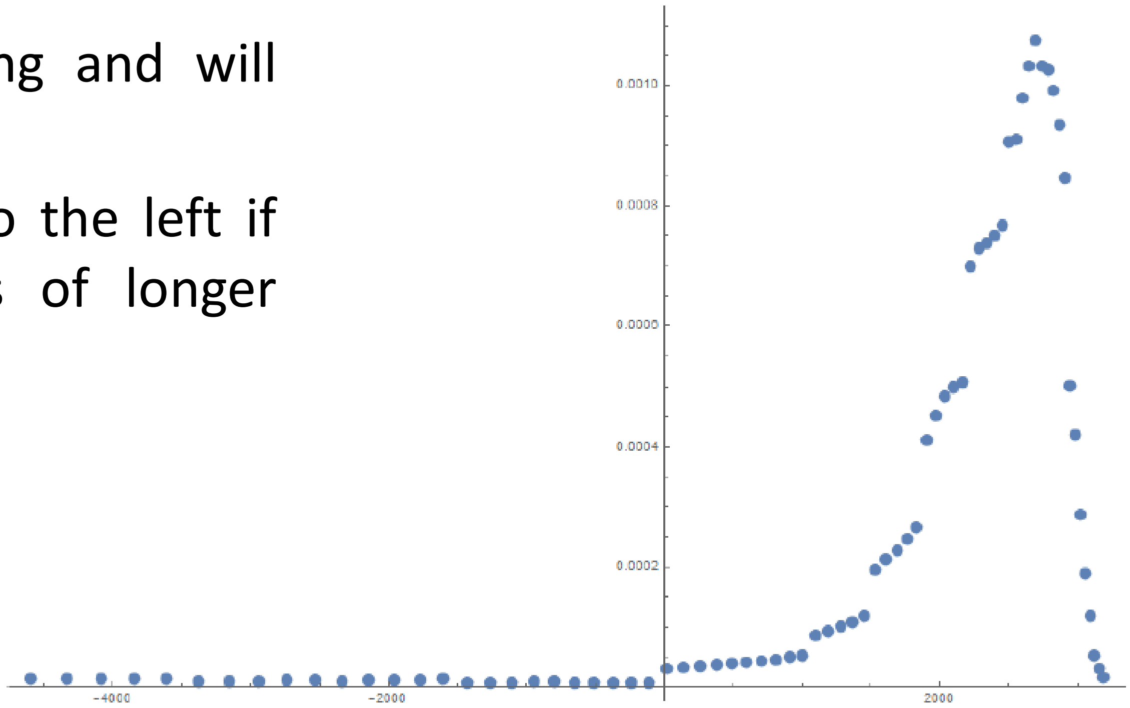


Figura 2: VAN con renta de 5 periodos

Analysis of the probability function

- Always on the right.
- In the beginning the probability is high due to infant mortality.
- It is reduced at the beginning as child improves at the survival.
- It increases as the "speed" of growth of the child's mortality exceeds that of the father.

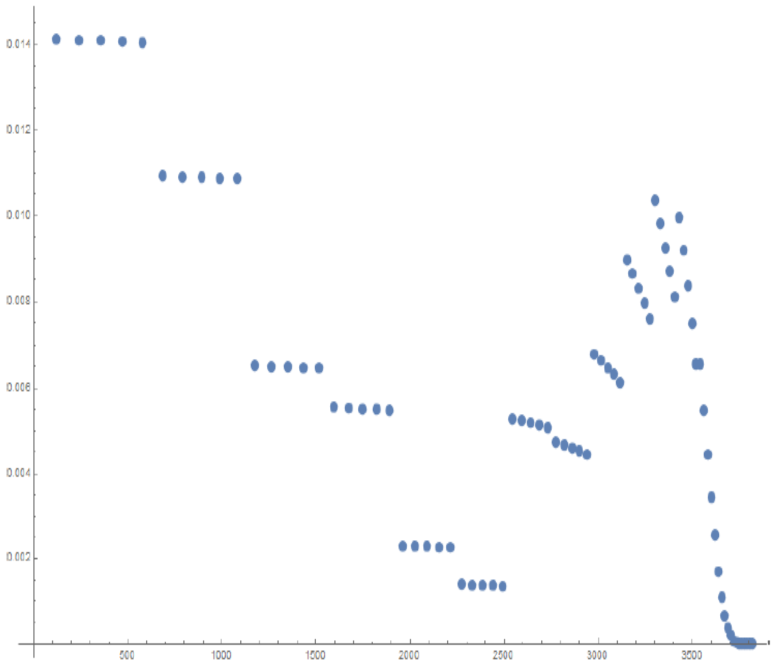


Figura 3: VAN: padre sobrevive al hijo

Analysis of the probability function

- On the left, except the case where the child dies before (only one point is appreciated by the ensemble of the mortality table).
- As we demand more survival to the father, it moves to the right.
- It is reduced at the beginning as child improves his survival.
- It increases as the "speed" of growth of the child's mortality exceeds that of the father.

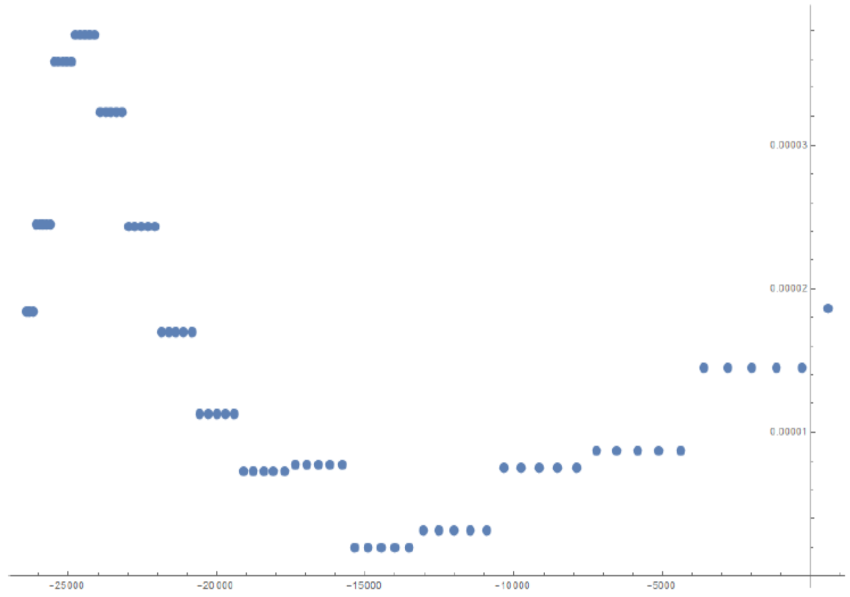


Figura 4: VAN: padre sobrevive 5 periodos

Analysis of the probability function

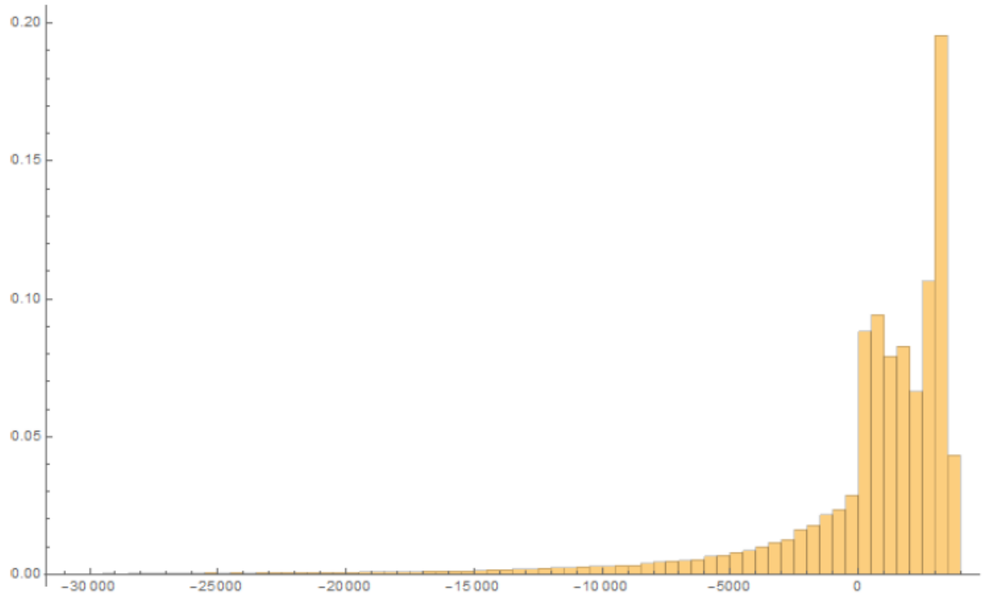


Figura 5: VAN agrupado por cada 500€

Sensitivity Analysis

- Falling of interest rates.
- Improvement in the survival of the disability.

Sensitivity to Fall Interest Rates

- If the rates fall, it is lost (obviously), and volatility increases as the discount factor increases (possible increase of NPV values).
- It is necessary to adjust the income multiplier to sustainable values (or at least to agree on an adjustment to interest rate drops).

Cuadro 2: Impacto de la provisión ante caídas de tipos de interés

Tipo de interés	Media aritmética	Desviación típica
3.0 %	311.40	4325.69
2.5 %	-34.91	5088.90
2.0 %	-508.87	6051.69
1.5 %	-1154.72	7277.10
1.0 %	-2032.48	8850.40
0.5 %	-3223.74	10887.40
0.0 %	-4839.80	13546.40

Parámetro	Valor(€)
Cuota del padre	120.00
Renta del hijo	480.00
Media	154.15
desviación típica	6877.84

Interes rate: 0%

Sensitivity to improvements in survival

- Improvement of the survival for the entire population.
- More improvement in those who are at greater risk of dying.
- Therefore, more improvement in the Down.
- We define the improvement factor as:

$$q_x^{mejorada} = q_x^{padre} \times \lambda + q_x^{down} \times (1 - \lambda)$$

Sensitivity to improvements in survival

- For the example in question...

Cuadro 3: Esperanza de Vida al nacer

Factor de mejora	Esperanza de vida al nacer (años)
0 %	49.3
10 %	50.7
20 %	52.6
30 %	54.4
40 %	56.4
50 %	58.6
60 %	61.2
70 %	64.2
80 %	67.9
90 %	73.5
100 %	85.5

Cuadro 4: Provisión matemática según factor de mejora

Factor de mejora	Media aritmética	Desv. Típica
0 %	311.40	4325.69
10 %	221.69	4399.51
20 %	119.39	4475.16
30 %	2.00	4552.53
40 %	-133.81	4631.44
50 %	-292.81	4711.63
60 %	-482.42	4792.84
70 %	-715.718	4875.01
80 %	-1020.64	4959.20
90 %	-1473.71	5049.92
100 %	-2354.10	5145.08

Sensitivity to improvements in survival

- If we calculate the income with multiplier 4 (resistant to the fall in interest rates), we obtain enormous losses facing with the improvement of the survival of Down.

Cuadro 5: Provisión Matemática según factor de mejora al 0 %

Factor de mejora	Media aritmética	DEsv. Típica
0 %	154.15	6877.84
10 %	-92.16	7066.84
20 %	-380.60	7265.19
30 %	-720.91	7473.06
40 %	-1126.61	7690.68
50 %	-1617.81	7918.75
60 %	-2227.48	8160.08
70 %	-3018.16	8424.84
80 %	-4135.57	8750.19
90 %	-6024.29	9266.11
100 %	-10505.80	10117.20

Sensitivity to improvements in survival

- If we assume that the medicine also improves the survival of the father in 5 years, an income with multiplier 4 resists an improvement factor of 60% (11 years of life expectancy of the child), and fall to 0% (provision = 51.65 €).

Conclusions

- Product very sensitive to the interest rate, forcing the product to recharge without necessarily losing interest on it.
- The product is quite stable to increases in survival, since both father and son obtain this benefit.
- Finally, the presented method gives us the ability to perform analysis in more complex situations (non-flat interest rate curves, death with disability, etc.).

THANK YOU SO MUCH