Extension, Compression, and Beyond
A Unique Classification System for Mortality Evolution Patterns

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- Martin Genz
- Joint work with Matthias Börger and Jochen Ruß
- Institute for Finance and Actuarial Sciences and University of Ulm, Germany

www.ifa-ulm.de
Agenda

Key question

Classification of mortality evolutions in the past

Shortcomings

A new classification framework

Requirements

Details

Application

Summary
Key question:

- Life expectancy increases in many countries.
- But changes in life expectancy (and other typically used statistics) are only a consequence of the underlying change of the age distribution of deaths.

Key question: How does the shape of these curves change over time?
There exists a variety of literature on the question how the age distribution of deaths changes over time. We have identified some shortcomings there:

- Different notions for certain observations have been established but sometimes these scenarios were defined imprecisely, e.g.:
  - compression (≈ vertical deformation of the deaths curve)
  - extension (≈ horizontal deformation of the deaths curve)
  - rectangularization (≈ survival curve becomes more and more rectangular)
  - ...

- Some of these scenarios were supposed to be mutually exclusive, but there are counterexamples.

- Several often used statistics are insufficient or even misleading.

- Often effects caused by the choice of a certain age range under observation were not considered.

In what follows, we discuss some examples for these shortcomings.
Classification of mortality evolutions in the past

Shortcomings

- **Imprecise scenario definitions:**
  - E.g., **rectangularization** is defined by a final state.

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**Rectangularization:** Present state (blue line) and final state (yellow line)

Overall rectangularization, but two different evolutions between State 1 and 3 and State 3 and 5, respectively.
Classification of mortality evolutions in the past

Shortcomings

- **Exclusiveness of scenarios:**
  - E.g., **compression** and **shifting mortality** are assumed to be opposing scenarios.

Neither compression nor shifting mortality prevail.  
Compression and shifting mortality coexist.
Classification of mortality evolutions in the past

Shortcomings

- **Insufficient or misleading statistics:**
  - Example 1: compression cannot always be detected by an exclusive analysis of $M$ and $SD(M+)$.
  - Example 2: compression cannot always be detected with $IQR$.
Classification of mortality evolutions in the past

Shortcomings

- The choice of the **age range matters:**
  - The age range should be chosen depending on the question at hand.

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Compression into the higher ages
No compr. within the higher ages
```
A new classification framework
Requirements

In light of these shortcomings of previous approaches, a new classification system should...

- ... uniquely categorize every material change in mortality patterns,
- ... allow for mixed scenarios,
- ... be applicable to different age ranges,
- ... build on statistics that can be feasibly calculated and easily interpreted,
- ... be extendable by additional components if needed.

Our new approach:

- We use the deaths curve as basis for the framework.
- We define 4 characteristics of the deaths curve for a unique classification of observed mortality evolutions.
A new classification framework

Details

- Number of deaths in the modal age at death $d(M)$: concentration or diffusion
- Degree of inequality DoI: compression or decompression
- Upper bound of the death curve’s support UB: extension or contraction
- Modal age at death $M$: right or left shift
A new classification framework

Details

Each scenario is defined by a **4-dimensional vector** where each component can have three specifications:

<table>
<thead>
<tr>
<th>component</th>
<th>attainable states</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>right shift / neutral / left shift</td>
</tr>
<tr>
<td>UB</td>
<td>extension / neutral / contraction</td>
</tr>
<tr>
<td>DoI</td>
<td>compression / neutral / decompression</td>
</tr>
<tr>
<td>d(M)</td>
<td>concentration / neutral / diffusion</td>
</tr>
</tbody>
</table>

- This allows for $3^4 = 81$ different scenarios (some of which might not be relevant in practice)
- The framework satisfies the requirements:
  - Each observed mortality evolution can uniquely be classified in one of those scenarios.
  - Pure and mixed scenarios are included.
  - The framework can be applied to age ranges starting at any given age up to UB.
  - Feasible and easily interpretable statistics are used.
  - The framework is extendable by additional statistics if needed.

In the paper, we discuss different issues in estimating these statistics, e.g. how to estimate UB, and methods for the detection of trends.
A new classification framework
Application: UB for US females, starting age 10

- For each calendar year, we estimate the four statistics. This gives us four time series.
- Example: UB for US females

- The time series for each statistic...
  - ... are “noisy” → We need to eliminate the noise.
  - ... have outliers → We need to eliminate the outliers.
  - ... in parts have unclear trends → We need a method to identify periods of stable trends.

- We fit a polygonal curve to the data and allow for changes in slope with or without jumps. Thus we achieve a decomposition of the time range into periods, where the time series follows a linear trend.

- For each period we detect whether a trend is increasing, neutral, or decreasing using a statistical test.
A new classification framework
Application: The mortality evolution of US females

Each component of the vector develops independently from the others (no redundant information).

We observe mixed scenarios (rather the rule than an exception).

We observe different scenarios for different age ranges (age range matters).

In the paper, we analyze this application in more detail.

Moreover, in Genz (2017) we analyze such patterns for several different countries.
Summary

In the paper, we have...

- ... identified **shortcomings** of previous approaches for classification of mortality scenarios,
- ... derived **requirements** for a new framework,
- ... identified 4 central **characteristics** of the deaths curve,
- ... derived a **new classification framework** based on these characteristics, which
  - ... builds on clear scenario definitions,
  - ... provides a unique classification for each mortality evolution,
  - ... allows for mixed scenarios,
  - ... is applicable for different age ranges,
- ... discussed **methods** we suggest for the determination of prevailing states, and
- ... **applied the framework** to concrete data for US females.
Thank you for your attention!

Martin Genz (M.Sc.)
+49 (731) 20 644-264
m.genz@ifa-ulm.de
References