# Nonfinancial Defined Contribution Pension Schemes:

*Construction of Annuity Divisors Using Direct Method Technique*

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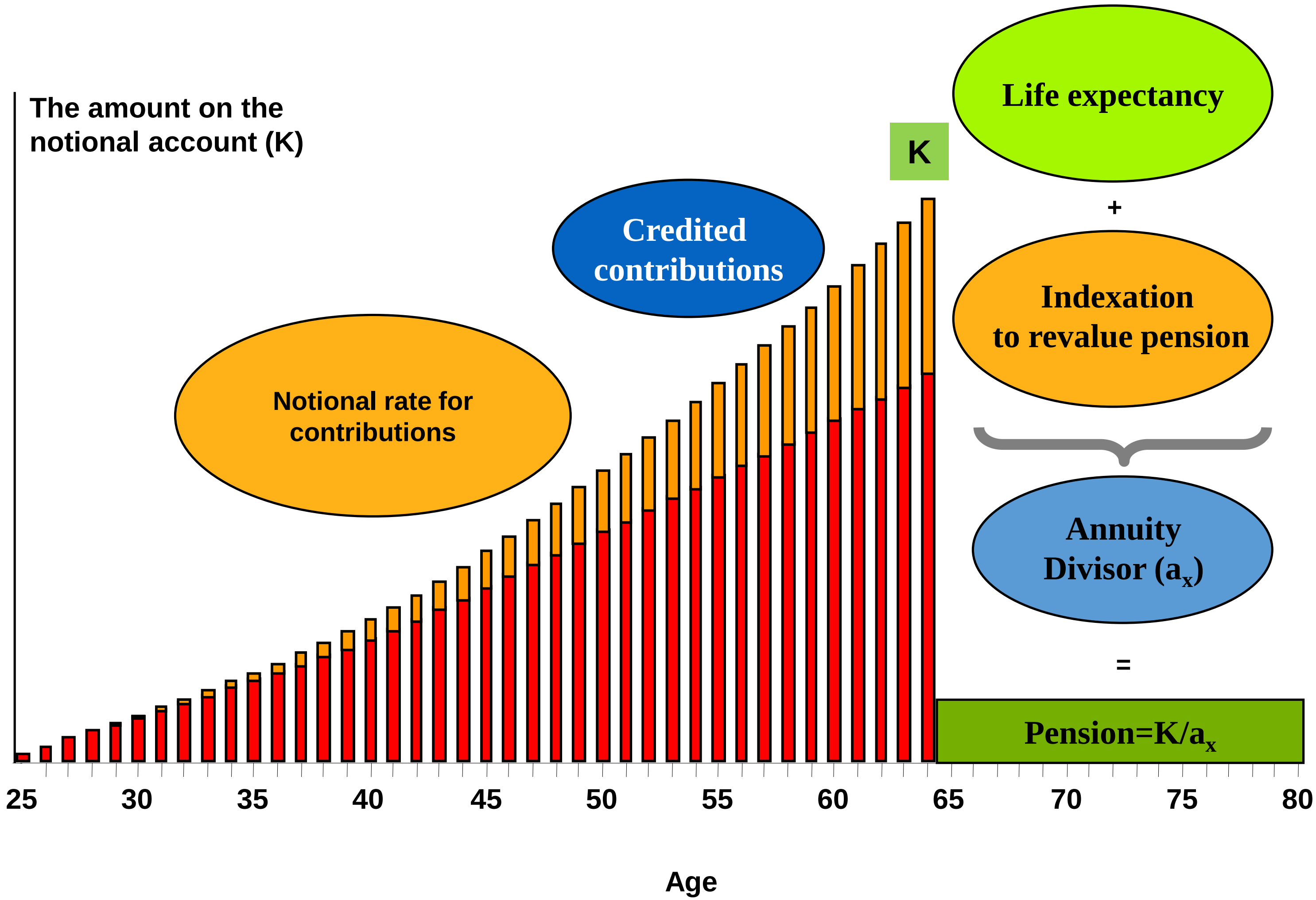
# Outline

- NDC pension system
- Transformation coefficients of the Italian NDC system
- Construction of mean present value of pension benefits (annuity divisor)
- Numerical illustration
- Conclusion

# What is  an NDC scheme?

- Italy (1995), Latvia (1996), Poland (1999), Sweden (1999)
- a state pension scheme
- **defined contribution + PAYG**
  - a fixed contribution rate on earnings into an individual account
  - pay-as-you-go financing: current contributors pay for current pensioners
- notional/ fictitious interest rate (wage bill or GDP growth rate)
- use of unisex life expectancy to convert the accumulated capital into an annuity

# An NDC scheme



# The Italian NDC system (1)

- Two main factors to determine the amount of pension:
  - the notional capital (contributions) accumulated with the nominal GDP growth (in line with a five-year moving average) and
  - the transformation coefficient whose calculation is mainly based on the probabilities of death, the probabilities of leaving any widow or widower and the number of years that a survivor's benefit will be withdrawn
- Pension benefits cover both old-age and survivors.
- The NDC formula to compute pension benefit at retirement age $x_r$:

$$P(x_r) = \left[ \sum_{t=x_e}^{x_r-1} c_t W_t (1+r)^{x_r-1} \right] \cdot \delta_{x_r} \qquad (1)$$

where $x_e$ is the age of entry to the labour market; $x_r$ is the retirement age; $g$ is the GDP growth rate; $c_t$ is the contribution rate for a participant at age $t$; $W_t$ is the gross earnings at age $t$; and $\delta_{x_r}$ represents the transformation coefficient for retirement at age $x_r$.

# The Italian NDC system (2)

- The transformation coefficient (age-related conversion factor) $\delta_{x_r}$

$$\delta_{x_r} = \left( \frac{\sum_{s=m,f} dir_{x_r,s} + ind_{x_r,s}}{2} - k \right)^{-1} \qquad (2)$$

  - The present value of pension benefits paid to pensioner

$$dir_{x_r,s} = \sum_{t=0}^{\omega - x_r} \frac{l_{x_r+t,s}}{l_{x_r,s}} \left( \frac{1+r}{1+\sigma} \right)^{-t} \qquad (3)$$

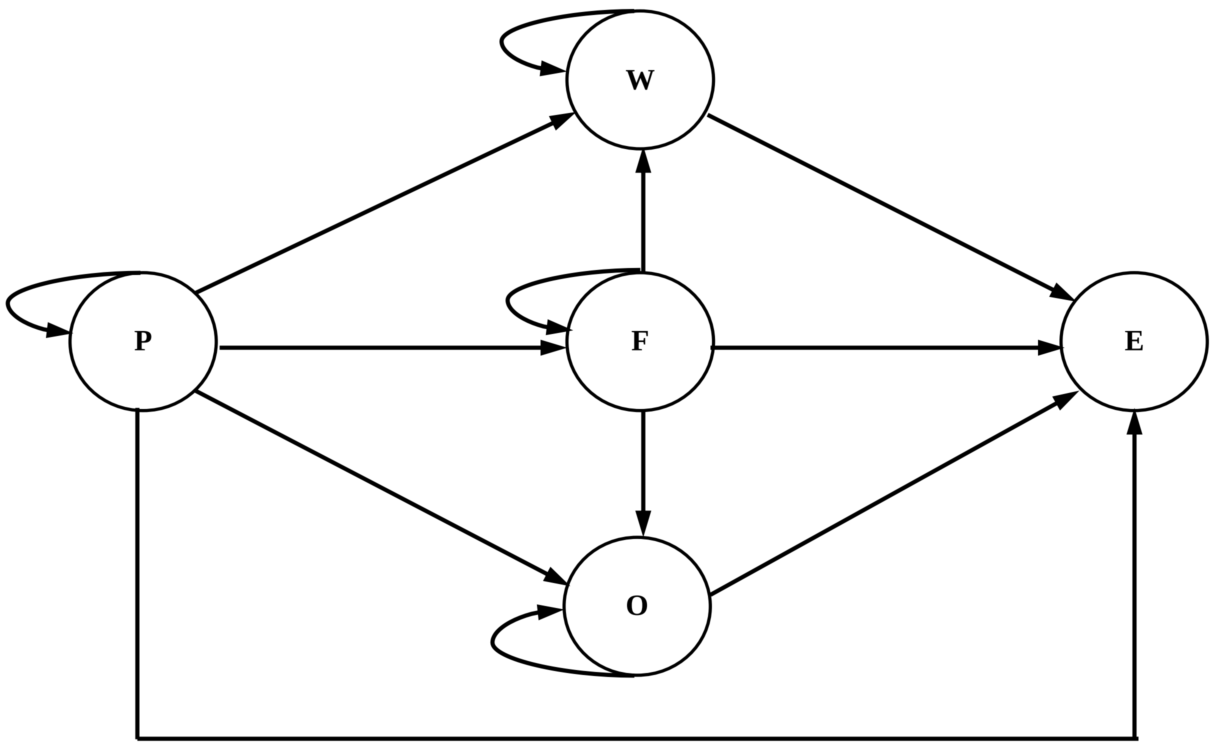
  - The present value of pension benefits paid to widow or widower in the case of pensioner’s death

$$ind_{x_r,s} = \theta \sum_{t=0}^{\omega - x_r} \frac{l_{x_r+t,s}}{l_{x_r,s}} \left( 1 - \frac{l_{x_r+t+1,s}}{l_{x_r+t,s}} \right) \left( \frac{1+r}{1+\sigma} \right)^{-(t+1)} a^W_{x_r+t+1} \qquad (4)$$

where $l_{x_r+t,s}$ is the number of people who are alive at age $x_r + t$ for the specific gender $s$ ($m = male, f = female$); $k$ is an actuarial adjustment factor to take into account different frequencies in pension payment; $r$ is the internal rate of return; $\sigma$ is the inflation rate; $a^W_{x_r+t+1}$ is the expected present value of a unitary annuity paid to the widow or widower at age $x_r + t + 1$ ; and $\theta$ is the quota of pension revertible to the widow or widower

# The multiple state model

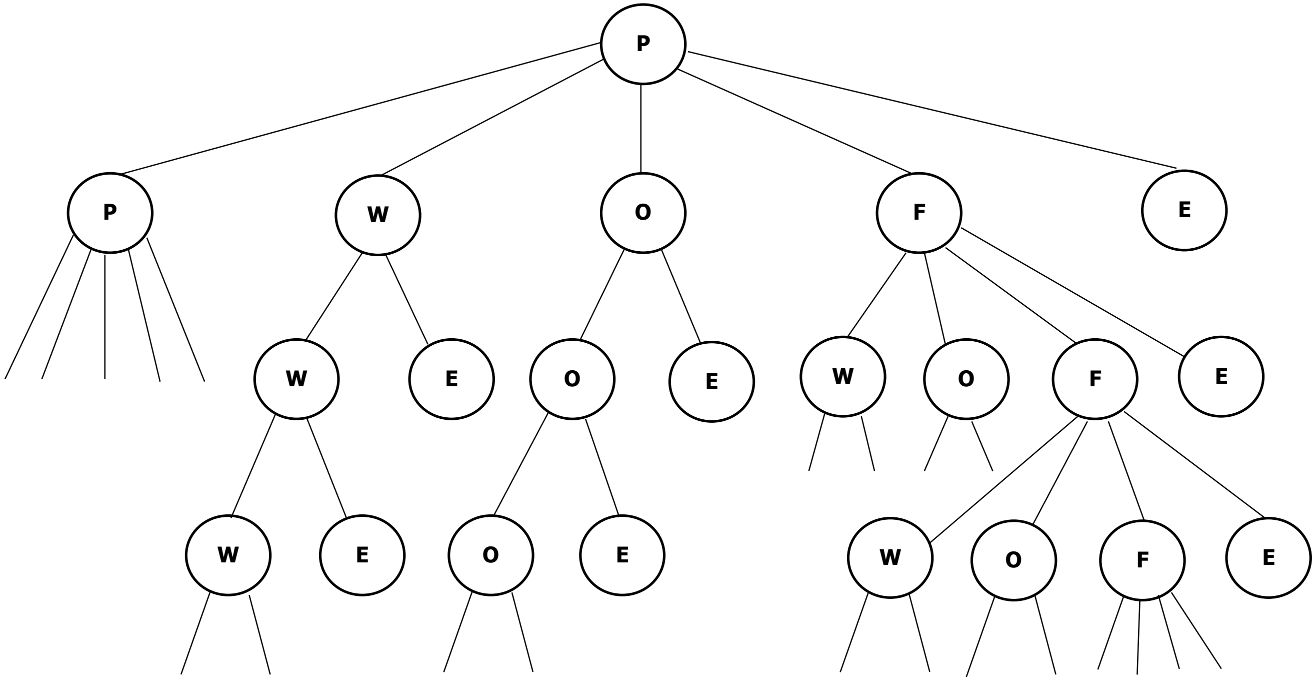
- The pension transition consisting of;
  - P : pensioner
  - F: complete family (children and spouse)
  - W: widow
  - O: offspring
  - E: absorbing state representing the exit from the pension system



# The direct method

- A simulation technique works on the mean values of the variables, rather than the distribution of random variables as in the Monte Carlo.
- This method follows all possible trajectories that pension benefits will be paid to either pensioner or his/her dependents.

**Tree of the possible trajectories**



# Construction of mean present value of pension benefits (1)

**<u>year 0</u>**; the amount of pension paid to a person of age $x$ is given by

$$P(x,0) = p^{PP}(x) + \frac{1}{2}\left(p^{PF}(x) + p^{PW}(x) + p^{PO}(x) + p^{PE}(x)\right) + \frac{1}{2}\left(p^{PF}(x)RP^{FP} + p^{PW}(x)RP^{WD} + p^{PO}(x)RP^{OP}\right) \quad (5)$$

where $p^{PP}(x) + p^{PF}(x) + p^{PW}(x) + p^{PO}(x) + p^{PE}(x) = 1$ ;

$RP^{FP}$ represents the percentage of pension paid to survivors in the case of complete family;

$RP^{WP}$ represents the percentage of pension paid to the only widow/widower;

$RP^{OP}$ represents the percentage of pension paid to only children

# Construction of mean present value of pension benefits (2)

**year 1**; the amount of pension paid to a person of age $x$ from year 0 is

$$P(x,1) = p^{PP}(x) \cdot P(x+1,0) + p^{PF}(x) \cdot RP^{FP} \cdot C^{F}(x^{F}+1) + p^{PW}(x) \cdot RP^{WP} \cdot C^{W}(x^{W}+1) + p^{PO}(x) \cdot RP^{OP} \cdot C^{O}(n_x) \quad (6)$$

where $C^{F}(x^{F})$ represents the mean present value of a unitary pension paid to the survivors in the case of complete family, the value is as a function of the age of the widow/widower;

$C^{W}(x^{W})$ represents the mean value of a unitary pension paid to the only widow/widower and the value is as a function of the widow's age;

$C^{O}(n_x)$ represents the mean value of a unitary pension paid to the only children and this value is a function of a mean number and mean age of children

# Construction of mean present value of pension benefits (2)

**<u>At year *h*</u>**; a person of age x was at time 0 will receive pension:

$$P(x,h) = p^{PP}(x) \cdot P(x+1, h-1) \qquad (7)$$

- The recursive approach is connected to this equation.
- The pension disbursement is obtained by multiplying the probability of remaining the direct pensioner at time 0 for a person of age x and all the other information is given by $P(x+1, h-1)$.

**The mean present value of the unitary pension (annuity divisor):**

$$\boxed{C(x) = \sum_{h=x}^{\omega} V(x,h)} \qquad (8)$$

where $V(x,h) = P(x,h) \cdot \prod_{j=t}^{t+h-1} (1+r_j)^{-1} (1+r_{t+h})^{-0.5}$

# Construction of mean present value of pension benefits (3)

**The algorithm path**

$$
\begin{array}{cccccccc}
 & 0 & 1 & 2 & \cdots & h & \cdots & \omega-x \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \iddots \\
x & P(x,0) & P(x,1) & P(x,2) & \cdots & P(x,h) & \cdots & P(x,\omega-x) \\
x+1 & P(x+1,0) & P(x+1,1) & P(x+1,2) & \cdots & P(x+1,h) & \iddots & \\
x+2 & P(x+2,0) & P(x+2,1) & P(x+2,2) & \cdots & \iddots & & \\
\vdots & \vdots & \vdots & \vdots & \iddots & & & \\
\omega-1 & P(\omega-1,0) & P(\omega-1,1) & & & & & \\
\omega & P(\omega,0) & & & & & &
\end{array}
$$

# Numerical illustration (1)

- **Mean present value of pension benefits** obtained by Italian rules

| Year | Retirement age | | | | |
|---|---|---|---|---|---|
| | **57** | **59** | **61** | **63** | **65** |
| **1995** | 21.90964 | 20.69965 | 19.47685 | 18.25450 | 17.02128 |
| **1996** | 22.03080 | 20.82162 | 19.60400 | 18.37999 | 17.14384 |
| **1997** | 22.08920 | 20.87726 | 19.65332 | 18.42333 | 17.19217 |
| **1998** | 22.08676 | 20.87378 | 19.64791 | 18.41315 | 17.17829 |
| **1999** | 22.24001 | 21.02607 | 19.79884 | 18.56217 | 17.31842 |
| **2000** | 22.41047 | 21.20396 | 19.97523 | 18.74239 | 17.49812 |
| **2001** | 22.54334 | 21.34198 | 20.12032 | 18.88396 | 17.63980 |

- **Mean present value of pension benefits** obtained by Direct Method

| Year | Retirement age | | | | |
|---|---|---|---|---|---|
| | **57** | **59** | **61** | **63** | **65** |
| **1995** | 21.46107 | 20.25275 | 19.03420 | 17.81864 | 16.59035 |
| **1996** | 21.58242 | 20.37448 | 19.16076 | 17.94366 | 16.71235 |
| **1997** | 21.64127 | 20.43026 | 19.21045 | 17.98723 | 16.76081 |
| **1998** | 21.63846 | 20.42692 | 19.20492 | 17.97688 | 16.74705 |
| **1999** | 21.79171 | 20.57910 | 19.35584 | 18.12546 | 16.88647 |
| **2000** | 21.96161 | 20.75636 | 19.53125 | 18.30463 | 17.06485 |
| **2001** | 22.09408 | 20.89484 | 19.67613 | 18.44576 | 17.20608 |

# Numerical illustration (2)

- **Transformation coefficients** obtained by Italian rules

| Year | Retirement age | | | | |
|---|---|---|---|---|---|
| | 57 | 59 | 61 | 63 | 65 |
| **1995** | 0.045642 | 0.048310 | 0.051343 | 0.054781 | 0.058750 |
| **1996** | 0.045391 | 0.048027 | 0.051010 | 0.054407 | 0.058330 |
| **1997** | 0.045271 | 0.047899 | 0.050882 | 0.054279 | 0.058166 |
| **1998** | 0.045276 | 0.047907 | 0.050896 | 0.054309 | 0.058213 |
| **1999** | 0.044964 | 0.047560 | 0.050508 | 0.053873 | 0.057742 |
| **2000** | 0.044622 | 0.047161 | 0.050062 | 0.053355 | 0.057149 |
| **2001** | 0.044359 | 0.046856 | 0.049701 | 0.052955 | 0.056690 |

- **Transformation coefficients** obtained by Direct Method

| Year | Retirement age | | | | |
|---|---|---|---|---|---|
| | 57 | 59 | 61 | 63 | 65 |
| **1995** | 0.046596 | 0.049376 | 0,052537 | 0.056121 | 0.060276 |
| **1996** | 0.046334 | 0.049081 | 0.052190 | 0.055730 | 0.059836 |
| **1997** | 0.046208 | 0.048947 | 0.052055 | 0.055595 | 0.059663 |
| **1998** | 0.046214 | 0.048955 | 0.052070 | 0.055627 | 0.059712 |
| **1999** | 0.045889 | 0.048593 | 0.051664 | 0.055171 | 0.059219 |
| **2000** | 0.045534 | 0.048178 | 0.051200 | 0.054631 | 0.058600 |
| **2001** | 0.045261 | 0.047861 | 0.050823 | 0.054213 | 0.058119 |

# Conclusion

- The transformation coefficients constructed by the direct method are slightly greater than the ones by Italian rules.
- The difference is not big, more than around 2% and tends to increase with the age.
- The values of transformation coefficients have been continuously declining over the last decades due to the improvement in mortality.
- The transformation coefficient is an important factor in the NDC pension system. Thus, in the construction of coefficients, it is necessary to adopt a truly reliable method.