Collective Defined Contribution Plans - Backtesting based on German capital market data 1955 - 2015

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Abstract

The Pension Schemes Act of 2015 in the UK introduced *shared risk schemes* (also called *defined ambition schemes*) which are designed as a compromise between *defined contribution* (*DC*) and d*efined benefit* (*DB*) *schemes*. The concept of *shared risk schemes* includes the design of pension plans with *collective benefits*, in particular so called *collective defined contribution schemes* (*CDC*) schemes. They are already quite popular in Denmark and The Netherlands. *CDC* pension plans combine two objectives: From the perspective of the plan sponsor (the employer) it looks like a *DC* plan, since after having paid the *defined contribution* the sponsor has no further obligations - unlike *DB* plans. From the perspective of the employee a *CDC* plan offers collective benefits given by certain types of collective risk sharing mechanism so that she or he is not exposed to the typical risks of an individual *DC* plan.

Our contribution is to check the advantage of *CDC* plans over individual *DC* plans on the basis of real market data for the German capital market 1955 – 2015. To the best knowledge of the author, so far only Wesbroom et al. (2013) have presented a comparable analysis bases on historical market data.

Our *CDC* plan design is taken from (Goecke 2013), which is appealingly simple to implement in a real world scenario. *DC* plans with a high equity ratio had been badly hit by the dotcom crisis of 2000 and by the financial crises of 2008. Our calculations give strong evidence that the collective component in *CDC* plans can effectively protect against equity market crashes.

Classification Codes G11, G17, G22

Key Words: self-financing pension fund; collective defined contribution plans; intergenerational risk transfer; asset liability management; resilience.

Highlights

- We describe a collective saving model and test it with real market data for Germany 1955 to 2015.
- We show that *CDC* plans would have systematically outperformed individual saving plans.

- We demonstrate how collective risk sharing arrangements would have behaved during equity market crashes (2000, 2008).
- We show how intergenerational risk transfer improves the risk-return profile.
- We measure the return smoothing effect of collective saving.

1. Introduction

Occupational pension plans are usually categorized into *defined benefit* (*DB*-) and *defined contribution* (*DC*-) plans.[1] In a *DC* plan the employer is only obliged to a pay a certain (*defined*) contribution into a pension fund and has no obligations with respect to the benefits. A typical *DC* plan provides an individual investment account, which is paid off on retirement. The accrued capital can then used be to purchase an annuity. *DB* plans typically promise a pension linked to salary and years of service in the company. Volatile capital markets, a long lasting low interest rate environment combined with market oriented accounting standards have led to a situation that employers abandon *DB* plans and, if at all, replaced them with *DC* plans leaving the fundamental pension risks (in particular the investment risk) to the employees.

Against this background the British Government has recently introduced a new category of pension plans, called *shared risk schemes* or *defined ambition schemes*.[2] In this article we want to explore *collective defined contribution* (*CDC*-) schemes, which can be regarded as a possible design within the shared risk framework. The basic idea behind *CDC* plans is that investment risks are shared between the different generations of savers. The vehicle to enable the risk-sharing mechanism is a *collective reserve*. These are assets which are not allocated to the individual accounts of the savers but belong to the savers collectively. If the assets perform better than expected then the collective reserve will build up. If assets perform worse than expected (e.g. after a stock market crash) the assets attributed to the collective reserve are used to stabilize the performance of the individual accounts.

We want to demonstrate how a properly designed *CDC* plan would have performed in the past compared to individual saving plans. Our backtesting procedure is based on German capital market data for the time period Jan. 1955 to Sept. 2015.

The discussion in the UK [3] and other countries on reforming private pension systems has boosted the literature on alternatives to *DC* and *DB* plans. There is a long list of science articles on intergenerational risk sharing in public or private pension systems. A list of more technical papers on this subject can be found in (Goecke 2013), the reference list of (Turner 2014) is a compilation of more practice-oriented papers and reports on risk sharing arrangements.

The following literature review is confined to contributions toward the question to what extent risk sharing pension plans are in some sense better or worse than individual arrangements. One approach to compare pension plan designs is to apply the utility methodology. A utility function is used to measure the welfare gains or losses. Probably the first who applied a utility function approach to intergenerational risk sharing for funded pension schemes were Gordon/ Varain (1988), whose work has inspired hundreds of

[1] International Accounting Standards (IAS) 19.7
[2] Pension Schemes Act 2015, section 3.
[3] A broad discussion for the UK can be found in (DWP, 2008), the US- and the international perspective are set out in (Mitchell, Shea, 2016).

publications. For example, Cui et al. (2005 and 2011) consider *DB* schemes where benefits and / or contributions are adjusted. The authors combine generational accounting methods with fair-value principles to evaluate future contingent cash flows. The intergenerational transfer helps to smooth consumption during retirement. The main finding of Cui et al. (2011) is that only hybrid *DB* schemes with simultaneous adjustment of contributions *and* benefits are able to improve welfare. Gollier (2008) also uses a utility function approach to measure the welfare effect of intergenerational risk transfer. He compares individual *DC* plans with *CDC* plans with a one-off payment at retirement. He determines a *first-best* intergenerational risk sharing (with no restriction on funding ratio and borrowing) and a *second-best* intergenerational risk sharing under more realistic assumptions. Gollier (2008) concludes that even in the second-best case intergenerational risk transfer is clearly improving welfare. Gollier (2008) and Cui et al. (2011) cannot be compare directly since Gollier allows for path-depent asset allocation while Cui et al. assume a constant equity share

In the same spirit as Cui et al. (2005, 2011) is the work of Hoevenaars/ Ponds (2008). They examine real life *DB* pension plans with collective components. The primary purpose of the paper is demonstrate how certain collective components influence future funding ratios.

All studies mentioned above require an obligatory pension scheme. If this requirement is dropped, the intergenerational transfer is at stake. The new generation will enter the collective scheme only if the expected utility is not diminished. This problem is analyzed in detail by Westerhout (2011). He shows that if the transfer between generations is limited to a certain degree then the new entrants will continue to participate in the collective scheme.

The DWP consultation paper (DWP 2008, pp. 89) reports on some comparative results for *DC* and *CDC* plans based on Monte-Carlo simulations. The evaluating of 5,000 scenarios for 30 years of saving shows that the underlying *CDC* plan has higher median pension with a smaller range compared to an individual *DC* plan. The higher median level results from the fact that the underlying *CDC* plan is fully invested in equities while the individual *DC* plan has a life cycle component with a gradual switching to bond investment during the last 5 year before retirement. The smaller rage of outcomes for the *CDC* plan can be attributed the risk sharing component. Wesbroom et al. (2013) is so far the only thorough analysis of *CDC* plans based on historical market data. These authors contrast the pension outcomes of standard *DC* plans with a sample *CDC* plan on the basis of historic capital market data for the UK (1930 to 2012). In addition to this, Wesbroom et al. (2013) also present results for the prospective behaviour over the period 2013 to 2062. In contrast to the DWP Consultation Paper of 2008, Wesbroom et al. (2013) give a detailed description of the management rules for the the sample *CDC* scheme. The control variable in the sample *CDC* scheme is the funding level. A funding level in the range between 90% and 110% requires no action. If funding level is out of this range, then a revaluation or a one-off reduction of benefits is effected. Wesbroom et al. (2013) calculations show a clear superiority of *CDC* plans with respect to size and stability of replacement rates.

Our paper is organized as follows: In Chapter 2 we present a simple model for *CDC* plans and describe the return smoothing mechanism. Chapter 3 contains a short description of the methods we use to compare saving plans. The results of our back testing calculations are summarized in Chapter 4, we end with a short comment of the results and an outlook (Chapter 5). A description of the database for the German capital market can be found in the Appendix.

2. A simple *CDC* model for backtesting

2.1. Model description

In Goecke (2013) a simple time continuous model for a collective saving arrangement was introduced. This serves as the basis for our discrete time *CDC* model with monthly adjustments. We consider a pension fund with the following balance sheet – cf. Fig. 1.

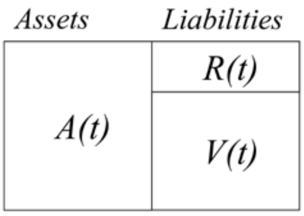

Fig. 1. Balance sheet of a *CDC* pension funds

We assume that the pension fund is a pure saving vehicle, i.e. it collects saving contributions and invests the money in the capital market. Each saver has an individual account, which is liquidated at retirement age. The total of the deposits of all individuals at time t constitute the liabilities $V(t)$ of the pension fund. The time value of assets $A(t)$ differs from $V(t)$. That makes the fundamental difference to a mutual fund for which by definition $V(t) \equiv A(t)$. In our *CDC* pension fund the assets should properly back the liabilities $V(t)$, i.e. $A(t) \geq V(t)$. However, under certain circumstances a temporary deficit could be acceptable or even unavoidable. The difference $R(t) := A(t) - V(t)$ we call *collective reserve.* We measure the reserve status by the logarithm of the funding ratio, $\rho(t) := \ln(A(t)/V(t))$, which turns out to be a more practical ratio than $R(t)/A(t)$, the usual reserve ratio. We call $\rho(t)$ the *log-reserve ratio* or simply *reserve ratio*; note that $R(t)/A(t) \leq \rho(t) \leq R(t)/V(t)$.

To rule out dynamic effects from a growing or shrinking population, we assume that the payouts from due saving plans are exactly compensated by the total of all saving contributions. Therefore the temporal development of $A(t)$ only depends on the performance of the investment.

Finally, we assume that the pension fund is fully *self-financing*: There is no external source of income for the fund other than contributions and investment returns. There is also no external entity who can withdraw money from the pension fund.

Asset-Liability-Management strategies

We assume that there is a person who manages the pension fund altruistically, i.e. she or he is acting solely for the benefit of the savers. The pension manager can influence the fund in two directions: Firstly, she or he can determine the asset allocation (*asset management*), and secondly, can decide to what extend assets are attributed to $V(t)$ or to $R(t)$ (*liability management*). More specifically, we assume that at time t the pension manager determines the *risk exposure* $\sigma(t)$ of the assets for the next time interval $[t, t + \Delta]$. A risk exposure of $\sigma(t) = 0$ would correspond to a risk free investment, say money market instruments with *term* = Δ.

The liability management is performed in such a way that at the beginning of every time interval $[t, t + \Delta]$ the pension manager *declares* an interest rate on the individual accounts. If $\eta(t)$ notes the declared (compound) interest rate, then $V(t + \Delta) = V(t)\exp(\Delta\,\eta(t))$. Following the wording used for with-profit life insurance contracts we call $\eta(t)$ the *profit participation* or *bonus declaration*. However, we should keep in mind that the pension fund does not provide any interest rate guaranties and that we allow for negative values for $\eta(t)$.

Asset-Liability-Management (ALM) is the simultaneous decision on $\sigma(t)$ and $\eta(t)$ on the basis of all information up to time *t*. If $\mu^{(r)}(t) := \ln(A(t+\Delta)/A(t)) / \Delta$ denotes the *realized rate of return* , then

$$\rho(t+\Delta) - \rho(t) = \Delta\left(\mu^{(r)}(t) - \eta(t)\right). \tag{1}$$

Eq. (1) can be regarded as the *fundamental ALM-equation* of a *CDC* plan. The actual return $\mu^{(r)}(t)$ and the profit participation $\eta(t)$ are not necessarily identical. But any difference between the two must be absorbed by the reserve. An increasing reserve is clearly in the interest of future generations, while a reduction of the reserve is advantageous especially for savers whose plans mature in near future. In the case of pure *defined contribution* (*DC*-) with no risk sharing mechanism we have $\eta(t) \equiv \mu^{(r)}(t)$. Then, in view of Eq. (1), $\rho(t)$ will be constant; actually a non-zero reserve makes no sense for a *DC* plans. Eq. (1) also indicates the welfare effect of *CDC* plans: The individual saving accounts can be protected against capital market risks - at least to a certain degree.

Goecke (2013) discusses a simple (linear) ALM-model based on a Black- Scholes-type capital market model. There $\sigma(t) = \beta(t)\,\sigma_M$, where $\beta(t)$ is the equity ratio at time *t* and σ_M is the constant volatility of the underlying geometric Brownian motion.

The proposed ALM- strategy is characterized by three essentials:

1. The pension manager seeks to keep $\rho(t)$ close to a time independent *strategic reserve ratio* $\rho_S > 0$. If $\rho(t) = \rho_S$ then the pension manager chooses $\sigma(t) = \sigma_S$ we; we call σ_S the *strategic risk exposure*.

2. The pension manager pursuits a "fair" profit participation. This means that in general the profit participation follows the *expected rate of return*, which in turn depends on the chosen risk exposure.

3. In case of an imbalance (i.e. $\rho(t) \neq \rho_S$) the pension manager takes measures to restore equilibrium by adjusting profit participation *and* risk exposure. More specifically

   Asset management rule: $\sigma(t) = \sigma_s + a\left(\rho(t) - \rho_s\right)$ (2)

   Liability management rule: $\eta(t) = \mu^{(e)}(t) + \theta\left(\rho(t) - \rho_s\right)$, (3)

   where $\mu^{(e)}(t)$ is the *expected* return on investment, depending on $\sigma(t)$. Constants a and θ are *adjustment parameters* that govern the speed of resilience.

Supposing we have always estimated favourably with respect to the asset returns, i.e. $\mu^{(e)}(t) = \mu^{(r)}(t)$, then combining Eq. (1) and Eq. (3) we get

$$\rho(t+\Delta) - \rho_s = (1 - \Delta\theta)\left(\rho(t) - \rho_s\right). \tag{4}$$

This means that the liability management rule Eq. (3) ensures that the reserve gap ($\rho(t)$ - ρ_S) is reduced with the constant rate, i.e. we have an exponential damping with factor θ. For the continuous time model one can show that $(\sigma(t))_{t \geq 0}$ and $(\rho(t))_{t \geq 0}$ are mean reverting processes with $\sigma(t) \geq 0$ and $\rho(t) \geq \rho_S - \sigma_S / a$ provided $a > 0$.[4]

To apply the above model to real capital market data we have to make a few rather obvious adjustments. Similar to the Black-Scholes model we assume that our pension fund can invest into two assets, an equity fund (*DAX*-portfolio) and a fixed income fund (*REXP*-portfolio). *DAX* and *REXP* are performance indices mapping a broadly invested German blue chip stock and a portfolio of German government bonds, resp.[5] Our backtesting is performed on a month-on-month basis, i.e. $\Delta = {}^{1}/_{12}$; starting 01.01.1955. At the beginning of each month, the pension manager determines the equity share $\beta(t) \in [0, 1]$ for the following month. We assume a one-to-one correspondence between the risk exposure and equity ratio: $\sigma(t) = \sigma_{DAX}\, \beta(t)$, where σ_{DAX} is the time independent risk exposure of a 100% *DAX*-investment. To apply the asset management rule in Eq. (2) we need a suitable calibration for the strategic risk exposure σ_S and the adjustment parameter a. To apply the liability management rule in Eq. (3) to real data, we have to determine the *expected return on assets* $\mu^{(e)}(t)$ as a function of the risk exposure $\sigma(t)$. We define the *estimated return* by

[4] cf. (Goecke, 2013), Proposition A.2
[5] For details see Appendix.

$$\mu^{(e)}(t) := \mu_m(t) + \frac{\sigma(t)}{\sigma_{DAX}} ERP - \tfrac{1}{2}\sigma^2(t), \tag{5}$$

where $\mu_m(t)$ is the (historic) 1-month money market rate, observed at time t for an investment periode $[t, t+\Delta]$.[6] In Eq (5) *ERP* denotes the *equity risk premium*, the extra return for a 100% stock portfolio. One should note that a *REXP*-investment is not really risk free since, depending on the duration, volatile interest rates induce price gains and losses. That is the reason why in Eq. (4) we take money market rates as a proxy for a risk free investment.

2.2. Calibration and *CDC*-variants

Generally spoken a capital funded pension system should enable a fair participation of the savers in the *production factor capital*, whereas pay-as-you go systems are designed to enable a fair participation of the retirees in the *production factor labor*. An easy access to the production factor capital is to buy shares of a wide range of companies. An investment into corporate bonds only indirectly secures a share in production factor capital since part of the profit generated by the company, namely a risk premium, goes to the shareholders. Similar considerations apply to an investment into government bonds. Thus - at least in theory - only a 100% real investment would insure a full participation in the production factor capital. But a 100% equity investment means an unacceptable risk for most savers. The idea of *CDC* plans is to take a high risk exposure on the investment side and to distribute the risk between the saver generations.

Another important feature of *CDC* plans is that they are *self-financing*. A *CDC* plan can only redistribute to the savers what the asset side has earned. In the long run a *CDC* plan cannot disassociate from fundamental capital market trends. Therefore, a sustainable ALM-strategy must automatically adapt to capital market trends, i.e. a *CDC* plan needs a “breathing” ALM-strategy. With respect to our model, this means that we should not use time independent parameters, which are calibrated with historic capital market data. However, in Eq. (4) we do not comply with this rule since we take *ERP* = 5% and $\sigma_{DAX} = 0.2$. [7] All other parameters (see Table 1 below) are calibrated by direct calculations (cf. Goecke 2013) or Monte Carlo simulation (cf. Goecke 2012). Actually, we could make Eq. (4) “breathing” by replacing the time independent values σ_{DAX} and *ERP* by moving averages of observed market values. Then the *ALM*-rules would automatically adjust to structural capital market shifts. In other words, the *ALM*-rules (2) and (3) ensures that the pension system is *resilient* against capital market shocks and structural shifts.

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[6] The data base is explained in the Appendix.
[7] In view of literature on calculation and estimation of the *ERP* a value of 5% seems to be a reasonable estimation – cf. (Stehle/ Schmidt 2015), (Damodaran 2015), (Dimson et al. 2011). The annualized volatility of a *DAX*-investment between 01.1955 and 09.2015 was 19.69%.

Overview of the backtesting model and calibration

Backtesting period: 1 Jan. 1955 ($t = 0$) – 30 Sept. 2015 ($t = T := 60.75$ *years* = 729 *months*)

Input data: *DAX*, *REXP* and money market interest rates $(\bar{\mu}(t))$

Parameters:
- ρ_s: strategic (log-) reserve ratio
- σ_s: strategic equity share (risk exposure)
- θ: adjustment (speed) parameter for profit participation
- a: adjustment (speed) parameter for asset allocation
- ρ_0: (log-) reserve ratio at time $t = 0$

ALM-rules:

$$\sigma(t) = \sigma_s + a\big(\rho(t) - \rho_s\big),\;\; 0 \le \sigma(t) \le \sigma_{DAX} \qquad \textit{Asset allocation}$$

$$\mu^{(e)}(t) = \mu_m(t) + 0.25\,\sigma(t) - \tfrac{1}{2}\sigma^2(t) \qquad \textit{Expected return on assets}$$

$$\eta(t) = \mu^{(e)}(t) + \theta\big(\rho(t) - \rho_s\big) \qquad \textit{Profit participation}$$

The following Table 1 gives an overview of the variants of CDC plans. We take *CDC*1 as our reference model. *CDC*2 - *CDC*9 are variants of *CDC1* to illustrate the effect of different calibration of the ALM-parameters. Deviations from the reference calibration of *CDC1* are given in bold figures For all variants we have calculated some key figures with respect to the (log-) reserve ratio, the equity ratio and the profit participation:

	CDC1	CDC2	CDC3	CDC4	CDC5	CDC6	CDC7	CDC8	CDC9
ρ_0	0.2	**0.0**	**0.8**	0.2	0.2	0.2	0.2	0.2	0.2
ρ_s	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
σ_s	0.1	0.1	0.1	0.1	0.1	0.1	0.1	**0.04**	**0.16**
θ	0.3	0.3	0.3	**0.1**	**0.5**	0.3	0.3	0.3	0.3
a	0.6	0.6	0.6	0.6	0.6	**0.1**	**1.4**	0.6	0.6
Reserve ratio in %									
Mean	26.74	26.04	29.77	29.78	25.27	23.58	29.25	28.19	24.69
Min	-0.08	-0.08	-0.08	-4.76	2.79	-7.73	1.34	3.60	-4.82
Max	115.91	112.63	129.65	127.46	104.02	90.11	118.22	108.99	117.83
Std. dev.	19.24	19.12	22.10	23.50	16.20	15.49	19.79	15.60	21.63
last value	24.45	24.45	24.45	31.70	20.42	28.35	26.32	26.43	25.04
Equity ratio in %									
Mean	56.86	55.28	60.97	59.06	56.78	51.78	57.41	39.25	71.04
Min	0	0	0	0	0	36.14	0	0	5.53
Max	100	100	100	100	100	85.06	100	100	100
Std. dev.	30.69	31.94	31.40	33.39	29.35	7.74	39.62	30.64	27.77
Profit participation (rates per month) in %									
Mean	0.71	0.69	0.80	0.63	0.76	0.63	0.76	0.70	0.69
Min	-0.12	-0.26	-0.12	0.07	-0.31	-0.34	-0.04	0.07	-0.32

Max	3.09	3.01	3.44	1.59	4.20	2.43	3.15	2.92	3.14
Std. dev.	0.50	0.50	0.57	0.26	0.68	0.37	0.52	0.42	0.55

Table 1: Overview of *CDC*-variants,

- *CDC*2 and *CDC* 3 show the effect of different initial reserve ratios. *CDC*2 will be of special interest since this calibration represents a cold start of the model. Since the ALM rules require the building up of reserves in the initial phase, the first saver generation will systematically subsidize the reserve.
- *CDC*4 and *CDC* 5 show the effect of varying parameter θ. A small θ will result in a more stable profit participation (cf. standard deviation) but also increases the probability of a negative reserve.
- *CDC*6 and *CDC*7 show the effect of varying parameter a. If a is small then the asset allocation will only slowly adjust to changing capital market data. This might result in higher risk of underfunding but could also result in missing the chance of higher returns (if risk exposure is too low).
- *CDC*8 and *CDC*9 show the effect of different strategic risk exposures σ_S. *CDC*8 (σ_S = 0.04) represents a target equity ratio of only 20%, which results in a less volatile reserve ratio and also in a more stable, but lower profit participation. In *CDC*9 (σ_S = 0.16) the pension manager wants to realize an equity ratio of 80%. In the long run this allows for a higher profit participation but also jeopardizes the funding ratio. However, it turns out that the back testing results for these two variants do not differ too much from each other. This is due to the fact that the *average equity ratio* for *CDC8* is about 40% (twice the target ratio) and for *CDC9* about 70%.

Our aim is to compare *CDC* plans with individual savings plans. However, we start with an illustration of how the *CDC*-model works (cf. Fig. 2a-c). For the sake of clarity, we exhibit only the first 10 years for the *CDC*-variants 1,4,5,6 and 7. The effect of the speed parameter a

becomes clear in Fig. 2b by comparing *CDC*6 (a = 0.1) and *CDC*7 (a = 1.4). In *CDC*6 the equity ratio is only gradually adjusted while *CDC*7 very much resembles a stop-and-go asset strategy. Comparing *CDC*4 (θ = 0.1) and *CDC*5(θ = 0.5) reveals that a timely adjustment of profit participation helps to keep the reserve level high (cf. solid blue line in Fig. 2a).

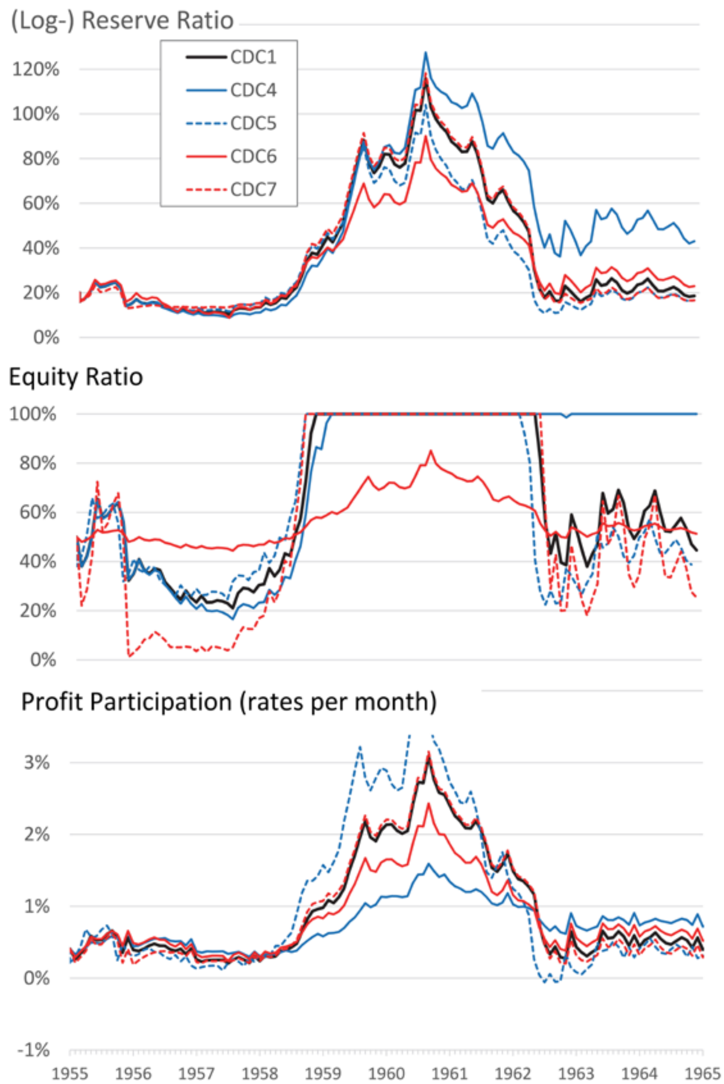

Fig. 2a-c. Effect of the parameters θ and a on reserve ratio (Fig. 2a: top), equity ratio (Fig. 2b: middle) and profit participation (Fig. 2c: bottom)

All *CDC* variants except for *CDC*2 and *CDC*3 start with the initial reserve ratio of ρ_0 = 0.2; this corresponds to a funding ratio of 122%. This means, that the first generation of savers has inherited assets that can use used as risk buffer against capital market volatility. The question arises of what happens if no initial reserve exists. Actually, the problem comes up whenever a new generation of savers enters a *CDC* plan. They will ask themselves whether an individual saving plan would be more favorable than entering an underfunded *CDC* plan.

We will discuss this issue in more detail in section 3.4. One aspect of this question is illustrated in Fig. 3 – see below. Here we compare *CDC*1 (ρ_0 = 0.2), *CDC*2 (ρ_0 = 0) and *CDC*3 (ρ_0 = 0.8) with respect to the reserve ratio. It turns out that after a transitional period of about 60 month the initial reserve ratio plays no further role. If we look at long term saving plans, we see that the initial reserve ratio has almost no influence on the final performance (cf. Fig. 3 below).

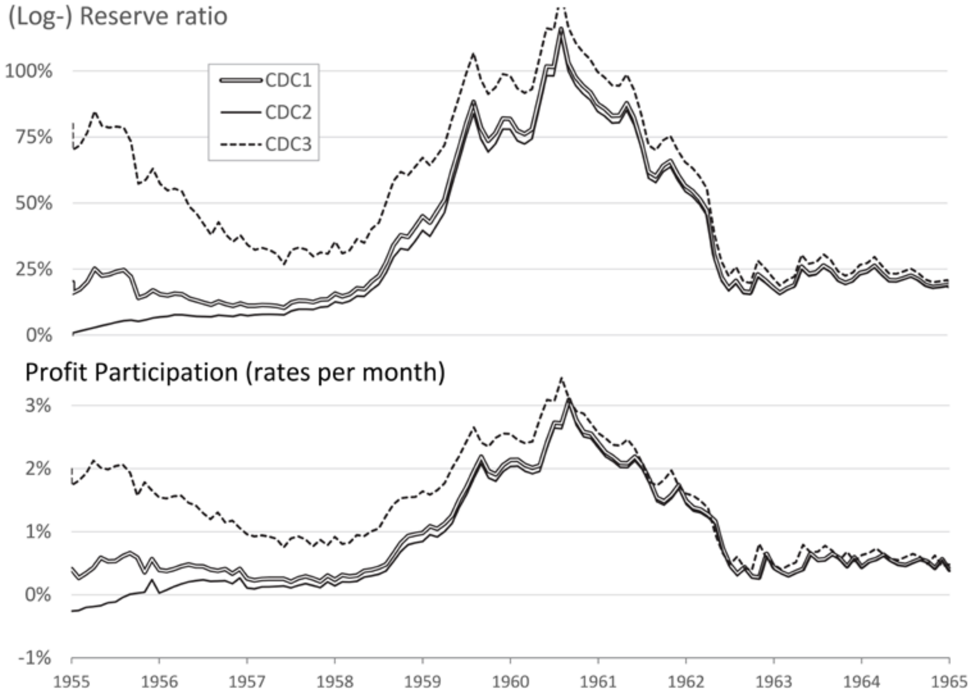

Fig. 3. Reserve ratio and profit participation during the first 10 years for different initial reserve ratios

3. Results

3.1. Risk measures for long term saving plans

Before we present the results, we have to explain how to evaluate the different saving schemes. Our point of view is that of an employee who sets aside money from her or his regular salary to build up a pension capital. Upon retirement the pension capital is withdrawn. We assume that she or he has the choice between

- an individual saving plan where the saving rates are invested into a *DAX* Portfolio (*DAX* plan)
- an individual saving plan where the saving rates are invested into a *REXP* Portfolio (*REXP* plan)
- an individual saving plan where the saving rates are invested in a money market account (*MM* plan)

- a *CDC* plan which is managed as stated above. We analyze the reference *CDC* plan (*CDC*1) and some variants (*CDC*2 - *CDC*9).

We neglect any cost components (acquisition cost, administration charge), keeping in mind that a collective plan might incur fewer administrative costs due to economies of scale resulting from the collective asset management.[8]

Figure 4 shows the performance of the different saving vehicles. Except for the first 3 years investment into a *DAX* portfolio was always better than investing into a *REXP* portfolio which in turn was much better than a money market investment. One should note that in Fig. 4 we have a log-scale; the final value for *DAX* (123.72) is roughly factor 3 higher than final *REXP* value (44.84) which in turn is about 3 times the value for *MM* plans (15.41).

Fig. 4. Performance (log scale) of a 1€-investment in 1 Jan. 1955 up to 30 Sept. 2015 in different saving vehicles: equity (*DAX*), government bonds (*REXP*), one month money (*MM*), *CDC* pension fund with initial reserve ratio of 20% (*CDC*1) or 0% (*CDC*2)

Fig. 4 also illustrates the risk of a lump sum investment into stocks. There is a twofold timing risk, the risk of *choosing the wrong time to start* the investment and the risk of *choosing the wrong time to withdraw the money*. A few months starting or ending earlier or later can make a tremendous difference!

We restrict ourselves to the analysis of long term saving plans with constant saving rates. Clearly we have to evaluate the return *and* the risk. We will measure the return as the annualized rate of return. There is a long list of possible risk indicators measuring the

[8] (DWP 2008, p. 91) reports that a collective *DC* schemes might incur expenses of 0.5 per cent less than for individual *DC* plans.

investment risk – cf. (Bacon, 2012). However, none of these is specially tailored to long term saving plans. Our choice of risk indicators is motivated heuristically and therefore the choice is in no way compelling. We use two sets of risk indicators. The first contains risk indicators that measure the degree of uncertainty of the wealth at maturity. The second set comprises risk indicators that measure the degree of unsteadiness of the saving process.

3.2. Risk return profiles with respect to the final capital

When evaluating the series of saving plans, we first look from the perspective of savers who are solely interested in her/ his wealth at maturity. This group of savers does not bother *how* their savings evolves - intermediate ups and downs do not cause any stress.

We evaluate 10-, 20-, 30- and 40-year saving plans with respect to the (annualized) yield at maturity. If we interpret the observed yield at maturity as the realization of a random variable, then the *standard deviation* is a natural risk indicator. The minimum and maximum of observed yields also measure aspects of the risk. As pointed out, the investment into a stock portfolio means a twofold timing risk. In our context, the timing risk is more or less the *risk of belonging to the wrong generation* since the saving period coincides with the working life.

To illustrate this aspect we look at all possible 40-years saving plans.[9] Each 40-year period represents a generation of employees. From a macro economic viewpoint a capital funded pension system should ensure a fair participation in the production factor capital. In view of Fig. 5 (see below) it is obvious that the generation of savers who retired in April of 2003 will definitely not feel fairly treated. To measure the risk of belonging to the wrong generation we introduce a risk indicator we call *intergenerational imbalance*. We calculate this as the maximum difference between the rates of return between "neighboring" generations. To be more precise, we define

$$\textit{intergenerational imbalance} := \max\left(y_t - y_s : |t-s| \le 1,\ 0 \le s,t \le T \right),$$

where y_t is the yield at maturity of a saving plan maturing at time t. An intergenerational imbalance of zero means that all generations have the same provision capital at maturity. The idea behind this risk indicator is twofold. Firstly, we think that within 1 year period the "value" of a lifelong participation in production factor capital should not change too much.

[9] Between Jan. 1995 and Sept. 2015 there are 250 generations of savers entering work life at age 25 and retiring at 65.

Secondly, we believe that in the long run the fundamental economic data can change. Therefore the value of participation for one generation can be substantially different from that of a generation born 10 years or more later. If for example, we observed that the *yield at maturity* steadily decreases over time, then the *standard deviation* of these values would indicate a high risk. However, the steady decline could also be a consequence of a steady decline of economic growth and it is only fair that pensions are affected. Thus, in this case the *standard deviation* would be misleading. In contrast, in this situation the risk parameter *intergenerational imbalance* would rightly indicate a low risk.

Fig. 5. Final wealth of 40-years saving plans maturing between 31 Dec 1994 and 30 Sept. 2015 for a monthly rate of 100€.

The following table is based on the evaluation of all 10-, 20-, 30- and 40-year saving plans with in the time span Jan. 1955 to Sept. 2015.

	1-MM	REXP	DAX	CDC1	CDC2
	10y saving plans (610 samples)				
Minimum	0.78%	3.88%	-5.91%	5.55%	5.55%
Maximum	7.46%	9.32%	23.09%	14.26%	14.26%
Mean	**4.99%**	**6.69%**	**7.79%**	**8.83%**	**8.77%**
Median	5.59%	6.63%	6.82%	8.39%	8.35%
Standard deviation	1.80%	1.37%	5.71%	2.32%	2.27%
Intergenerational imbalance	1.19%	2.49%	17.66%	2.64%	2.64%
	20y saving plans (490 samples)				
Minimum	1.73%	4.68%	2.08%	6.66%	6.65%
Maximum	6.83%	8.37%	16.70%	12.04%	12.04%
Mean	**5.29%**	**6.96%**	**8.20%**	**8.94%**	**8.92%**
Median	5.73%	7.15%	7.79%	9.28%	9.28%
Standard deviation	1.45%	0.91%	3.43%	1.36%	1.37%
Intergenerational imbalance	0.54%	1.09%	8.29%	1.26%	1.26%
	30y saving plans (370 samples)				
Minimum	2.75%	5.49%	6.21%	8.17%	7.98%
Maximum	6.67%	8.04%	13.61%	10.76%	10.76%
Mean	**5.37%**	**7.07%**	**8.98%**	**9.25%**	**9.23%**
Median	5.81%	7.28%	8.59%	9.06%	9.02%
Standard deviation	1.11%	0.63%	1.61%	0.70%	0.71%
Intergenerational imbalance	0.31%	0.59%	4.53%	0.66%	0.68%
	40y saving plans (250 samples)				
Minimum	3.61%	6.07%	6.32%	8.75%	8.68%
Maximum	6.23%	7.67%	11.04%	9.81%	9.81%
Mean	**5.27%**	**7.03%**	**8.77%**	**9.19%**	**9.17%**
Median	5.40%	7.22%	8.69%	9.19%	9.19%
Standard deviation	0.76%	0.41%	0.91%	0.31%	0.32%
Intergenerational imbalance	0.23%	0.40%	3.19%	0.38%	0.39%

Table 2: Evaluation the yield at maturity of 10-, 20-, 30- and 40-year saving plans.

It is worth noting that the *intergenerational imbalance* is not proportional to the *standard deviation*. In particular, *MM* plans are more risky than *REXP* plans with respect to *standard deviation* but are less risky with respect to *intergenerational imbalance*. For all durations the risk of *CDC* plans is considerably lower than that of *DAX* plans. For durations of 30 and 40 years the risk of *CDC* plans is roughly at the level of *REXP* plans. For shorter durations the *standard deviation* of *REXP* plans is slightly lower than that of *CDC* plans. It should be noted that in Table 2 the difference between *CDC1* ($\rho_0 = 0.2$) and *CDC2* ($\rho_0 = 0$) can be neglected.

That means that the generation of savers who entered the *CDC* scheme realized nearly identical pension wealth at maturity.

3.3.Path dependent risk measures for saving plans

Compared with an employee who pays contributions and then patiently awaits retirement, in this section, we take the view of an employee who permanently checks her or his pension wealth. It is just a matter of transparency that at least once a year a pension fund should inform its clients about the performance. This is also demanded by consumer organizations. The rational behind this is that the employee should have the opportunity to react if things evolve differently from her or his expectations. From this perspective we need path-dependent risk measures to evaluate the risk of a pension arrangement. Again, there is no single undisputed path dependent risk measure. We take a rather pragmatic selection of risk indicators:

- *Path volatility*: For each investment vehicle we consider the time series of the monthly (log-) returns. The path volatility is then the annualized standard deviation of this time series. Note that a steady decrease of the returns (e.g. due to declining interest rates) will indicate a risk with respect to path volatility.
- *Number of months with negative returns*
- *Maximum Drawdown* (*MDD*): If S_k denotes the accrued capital after k months of saving, then $MDD := \max\left(\frac{S_k - S_{k+d}}{S_k} : 1 \le k \le k+d \le n\right)$. *MDD* is the maximum relative loss during the saving time. *MDD* is a common risk indicator for investment strategies. However, it is mostly used for lump sum investments and measured as absolute drawdown; we prefer the relative drawdown.
- *Maximum Recovery Time* (*MRT*): $MRT := \max\left(k - l : S_k > S_{k+d} \text{ for all } d = 1, ..., l-k\right)$. *MRT* is the maximal time it takes to recover from losses after a prior peak. It is clear that *MRT* = 0 if and only if *MDD* = 0.

Path volatility and *number of negative returns* are path dependent risk measure that only depend on the monthly returns of the underlying vehicle and give the same value for lump sum investments and for saving plans with constant rates.

MDD and *MRT* can be defined for wide range of continuous time stochastic processes.[10] In practice, *MDD* and *MRT* are usually applied to lump sum investments.[11] We think that *MDD* and *MRT* are good indicators of the savers frustration. We illustrate both concepts in Fig. 6. The example given is the worst case with respect to *MDD* among all 30-year *DAX* plans. A saver of this generation (starting April 1973) lost 67.8% of the wealth (≈ 230,000€) within the last 3 years. In this case the *MRT* is 40 months. If this generation invested in a *DAX* plan it had to bear the full impact of the dotcom crisis. If this generation had invested in a *REXP* plan it would not have been affected by the dotcom crises but would have ended up not much better. An investment into a *CDC* plan would have definitely been the better choice!

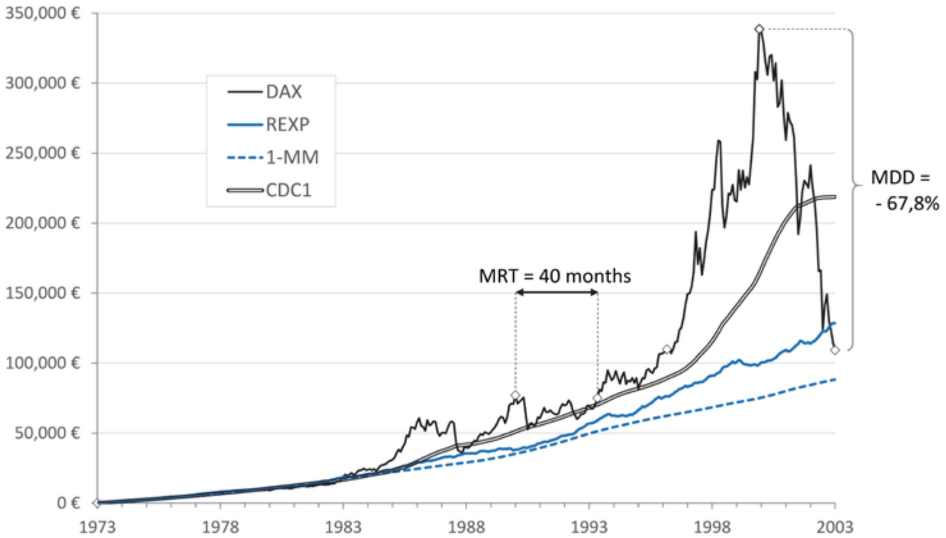

Fig. 6 Performance of a single 30-year saving plan (01.04.1973 - 31.03.2003, monthly rate = 100€). *MDD* and *MRT* are indicated for the *DAX* plan.

We calculate the mentioned path dependent risk indicator for all 370 samples of 30-year saving plans – cf. Fig. 7a-d below. We notice that the volatility of the *DAX* has increased gradually from about 17% to 22%. The *REXP*-volatility fluctuates around 3.5%. The *path volatility* of money market saving (*MM*-) plans is more or less constant at 0.7%. The average path volatility of a *CDC1* plan is 1.5%. It is interesting to note that the number of months with negative returns does not differ so much between *DAX*- and *REXP* plans (cf. Fig. 7b, chart top

[10] (Mahmoud 2015) presents a mathematical framework for continuous time path-dependent risk measures. *MDD* for geometric Brownian motion is studied in (Goldberg, Mahmoud 2014), (Zhang et al. 2013).
[11] (Bacon 2012) pp. 97ff.

right). Though the ALM rule (3) allows for negative profit participation for *CDC* plans, the historic backtesting shows that negative returns were extremely rare events for *CDC* plans.

With respect to the *maximum drawdown* (cf. Fig. 7c, chart bottom left) *MM*- and *CDC1* plans had been risk free. None of the 370 generations of savers had to suffer from a single drawdown. In other words, every single contribution increased the pension wealth. The situation for *DAX* plans is totally different: On average an employee with an individual *DC* plan fully invested in German equities would have suffered an intermediate loss of more than 50%.

The *MRT* for 30-year saving plans for a pure *DAX*-investment ranges between 61 and 147 months with an average of 91 months (cf. Fig. 7d, chart bottom right). Thus every long term saver in a *DAX* plan experienced a time span of at least 61 months during which she or he continuously paid in without having one cent more. The two indicators *MDD* and *MLT* apparently measure different aspects of risk and might help to explain why savers very often terminate their saving plans prematurely.

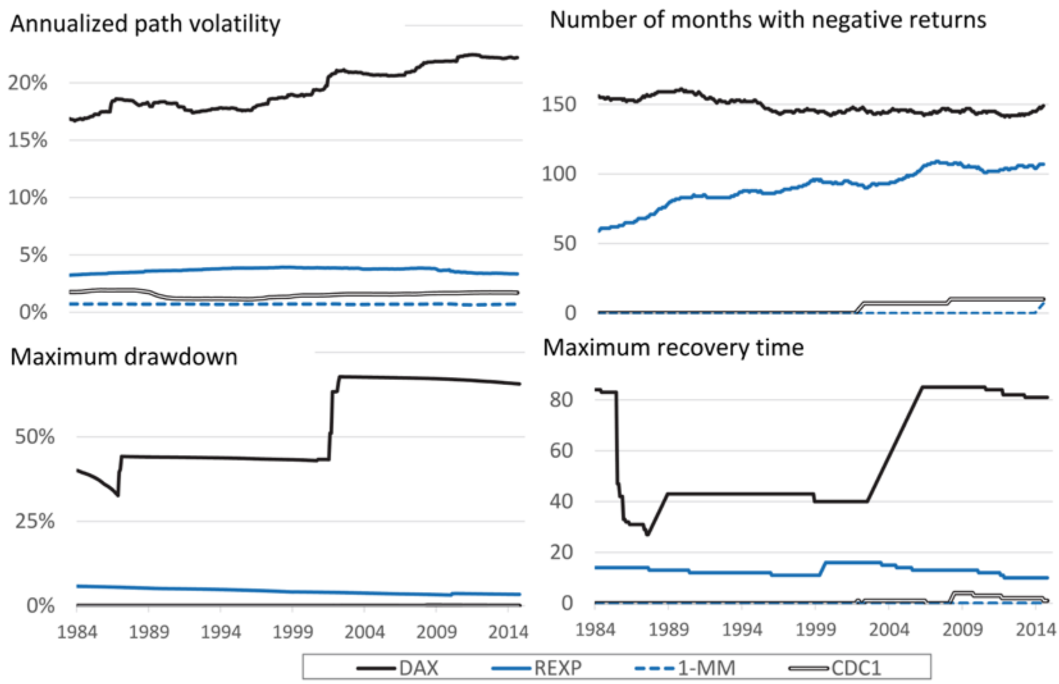

Fig. 7a-d. Path dependent risk indicators of 30-year saving plans, comparison between individual plans (*DAX*-, *REXP*-, *1-MM* plans) and collective plans (*CDC*1). The calculations are based on 370 saving plans maturing between 31.12.1984 and 30.09.2015.

Fig. 7a-d show that with respect to all path-depending risk indicators the *CDC* plan ranges at the level of money market instruments. From that point of view, a *CDC* plan is an extremely safe investment. It turns out that the initial reserve ratio has alomost no effect on the risk parameters – cf. Table 3.

	1-MM	REXP	DAX	CDC1	CDC2
	Evaluation of 30y saving plan (370 samples)				
Path volatility:					
Minimum	0.64%	3.23%	16.69%	1.14%	1.14%
Maximum	0.72%	3.93%	22.46%	1.92%	1.87%
Mean	0.70%	3.66%	19.52%	1.53%	1.52%
Months neg. returns:					
Minimum	0	59	141	0	0
Maximum	7	109	161	10	10
Mean	0.1	90.3	148.5	3.5	3.6
Max. drawdown:					
Minimum	0	3.18%	32.63%	0	0
Maximum	0	5.78%	67.75%	0.11%	0.11%
Mean	0	4.30%	52.93%	0.02%	0.02%
Max. recovery time:					
Minimum	0	10	27	0	0
Maximum	0	16	85	4	4
Mean	0	12.8	57.2	0.7	0.7

Table 3: Path dependent risk indicators for 30-year saving plan – cf. Fig. 7a-d

To visualize the dependency between risk and return, we sketch the relation between the four path dependent risk indicators (average over all 370 saving plans) and the return, measured as the average yield at maturity – see Fig. 8a-d. We have included calculations for all variants of *CDC* plans, for *MM* plans and for *DAX*/ *REXP* mixed portfolios (0%, 10%, …, 100% *DAX*-share). It turns out that all variants of *CDC* plans clearly outperform all variants of individual saving plans. It should be emphasized that the initial reserve endowment has barely any on the risk return profile - cf. *CDC1*, *CDC2* and *CDC3* in Fig. 8a-d.

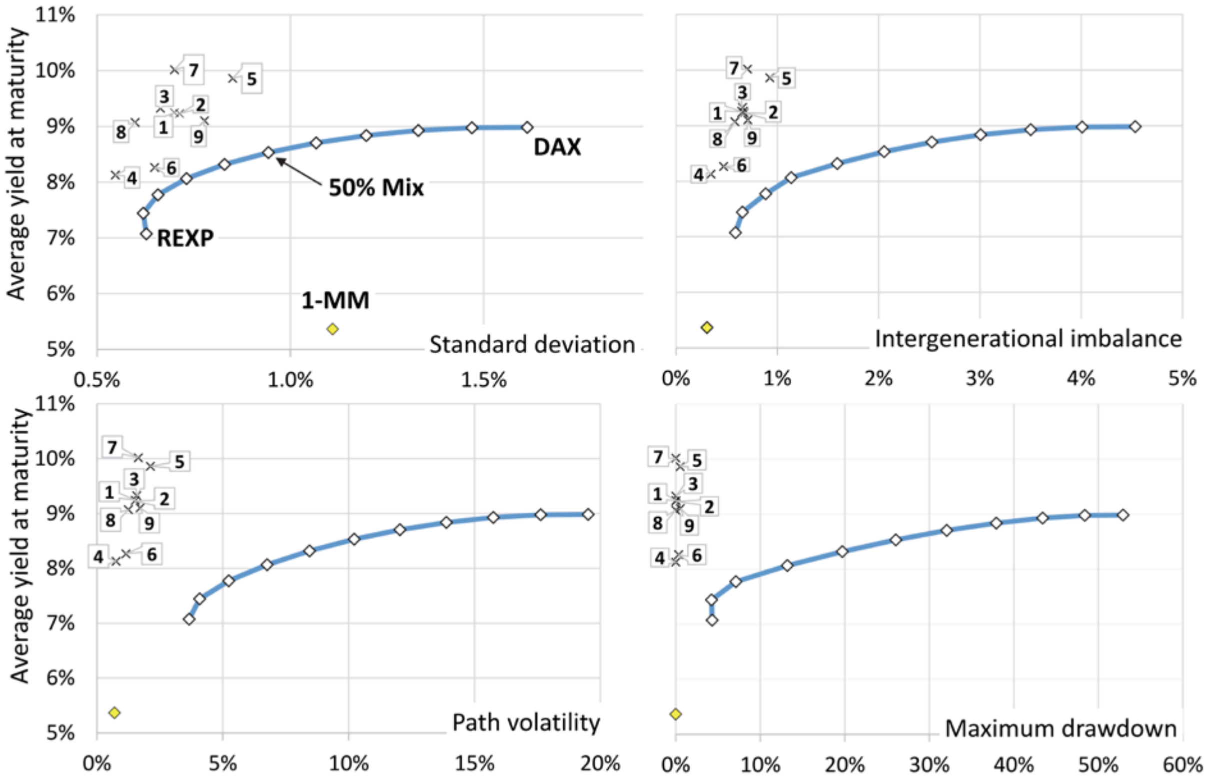

Fig. 8a-d. Risk return-profile of 30-years saving plans: Mean value of annualized yield at maturity versus four risk indicators: *standard deviation* (Fig. 8a, top left), *intergenerational imbalance* (Fig. 8b, top right), *path volatility* (Fig. 8c, bottom left) and *maximum drawdown* (Fig. 8d, bottom right). We compare individual saving plans (*DAX* - *REXP* mixed portfolios, *MM* plan) and with *CDC*-variants (*CDC*1-*CDC*9, numbered 1-9 in the diagrams).

3.4. Worst case analysis

The reserve ratio (or equivalently the funding ratio) is an indicator of the burden (or the wealth) one generation hands over to the next generation. It is clear that a supervisory authority has to step in if the funding ratio (or equivalently the reserve ratio) falls below a critical value. In this section we examine the temporal development of the reserve ratio. Our focus will be to check whether the adjustment components of the ALM rules (2) and (3) work effectively for real market data. Furthermore, we conduct a worst case analysis by calculating the minimal reserve ratio for all possible starting times. We do this for our reference model *CDC1* ($\rho_0 = 0.2$) and for the *cold start* variant *CDC2* ($\rho_0 = 0$).

The results are shown in Fig. 9. It transpires that if the *CDC* plans started with an initial reserve ratio of 0.2, the reserve never falls substantially below zero. Actually, for all starting times after 1980 the minimal reserve ratio remains well above 5%, which indicates that from that date onwards a more risk seeking ALM-strategy could be have been applied. However, if the *CDC* plan were to start without reserves (model *CDC2*) then for certain starting times the minimum reserve ratio would fall below zero - cf. Fig. 9. The worst of all possible cases is a starting time on 1st Aug. 1980. If the *CDC*-model started at that date with no initial reserve, then the strict application of the ALM rules would lead to a negative reserve of nearly 10%, which corresponds to a funding ratio of 90.5%.

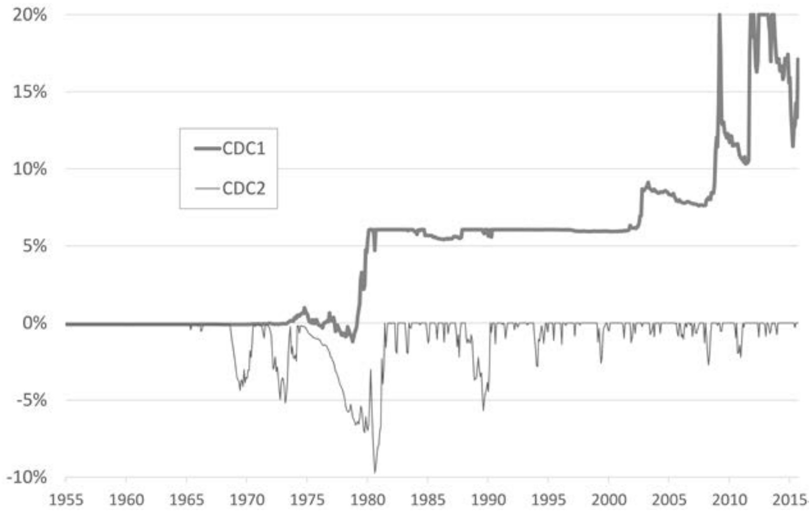

Fig. 9. Minimum (log-) reserve ratio depending on the starting time (1st Jan. 1955 earliest, 1st Sept. 2015 latest) for *CDC1* (ρ_0 = 0.2) and *CDC2* (ρ_0 = 0).

A closer look at this worst case scenario for *CDC2* shows that during the first 10 months (Aug. 1980 until June 1980) the reserve ratio reduces from 0 (starting level) to -9.71% while at the same time interest rates shot up (money-market rates from 9.16% to 12.94%, yields of government bonds from 7.80% to 10.80%). This had then two effects on our *CDC2*-model. Firstly, *REXP*-asset lost 7% and secondly, the profit participation went up because of rule (3). Following rule (3) the profit participation is set to $\eta(t) = \mu^{(e)}(t) + \theta(\rho(t) - \rho_s)$. The extreme money market rates which effect $\mu^{(e)}(t)$ outweigh the stabilizing factor $\theta(\rho(t) - \rho_s)$.

This observation might be a clue as how to modify rule (3) for a real world implementation of the *CDC*-model. However, for our backtesting analysis we stick to the mechanic rule (3) which was derived from the theoretical model. It should be stressed that neither the dotcom crisis nor the financial crisis had any substantial impact on the stability of the *CDC* pension fund.

We close this section with the calculation of the risk return profiles in the worst case scenario, supposing that the *CDC* plan had started on 1st Aug. 1980. Again we compare all 30-year saving plans for individual saving vehicles (*DAX*-, *REXP*-, *MM* plans) with the *CDC* variants.[12]

The following Fig. 10a-d corresponds to Fig. 8a-d. Apparently, the risk return profiles in the worst case scenario do not differ substantially from those in Fig. 8a-d. Only with respect to the *standard deviation* the *CDC* plans are riskier than *REXP* plans. However, if one looks at the time span between Aug. 2010 and Sept. 2015, we see that interest rates steadily decreased and so did the yields at maturity of *MM*- and *REXP* plans. This explains the high values for the standard deviation of *MM*- and *REXP* plans in Fig. 8a. The same holds for *CDC* plans in that time period. The decreasing yields demonstrate that the *CDC* plan steadily adjusts to a changing economic environment of lower interest rates.

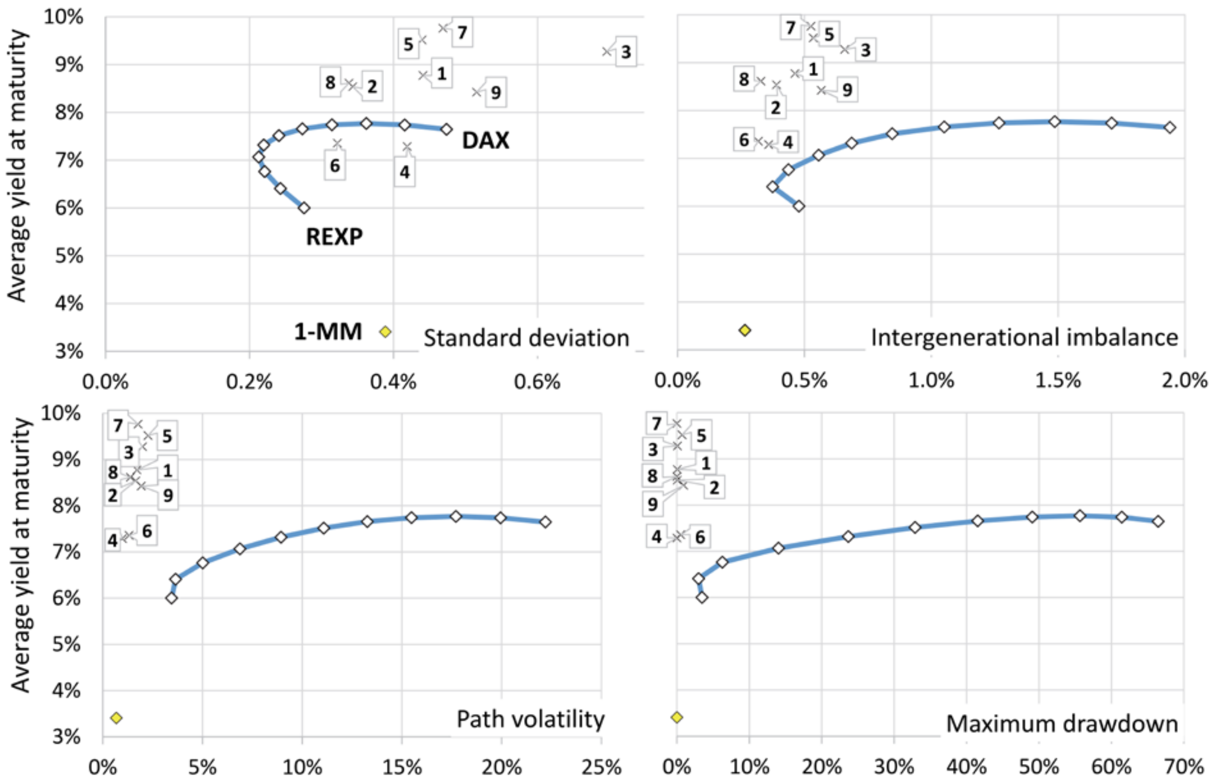

Fig. 10a-d. Risk return-profile of 30-year saving plans in the worst case scenario: Mean value of annualized yield at maturity versus four risk indicator: *standard deviation* (Fig. 10a, top left), *intergenerational imbalance* (Fig. 10b, top right), *path volatility* (Fig. 10c, bottom left) and *maximum drawdown* (Fig. 10d, bottom right). We compare individual saving plans (*DAX* - *REXP* mixed portfolios, *MM* plan) and with *CDC*-variants (*CDC*1-*CDC*9, numbered 1-9 in the diagrams).

[12] There are in total 62 saving plans between 01.08.1980 and 30.09.2015.

4. Conclusion and outlook

Our analysis shows that *collective defined contribution* (*CDC*) schemes are more than just a nice idea. Based on the theoretical model presented in (Goecke, 2013) we could prove that the risk return profile for *CDC* plans is much better than that for *DC* plan. Even in a worst case scenario *CDC* plans perform better than individual *DC* plans. Clearly, there is no guarantee that an excellent performance observed in the past will recur in future. But it should be stressed that our analysis is based on observations of 60 years. Within this time span, we have observed extreme situations such as the oil price crisis 1973, stock market crashes and bubbles, high inflation rates, and extreme short and long term interest rates.[13] The strength of the *CDC* plan is that it is self adjusting due to resilience factors in the *ALM* rules. With respect to the fundamental objective to ensure a fair participation in production factor capital, a *CDC* plan is superior to a pension plan with an interest rate guarantee. Looking back into the economic history, we should realize that long term interest rates either are worthless or are unbearable for the warrantor. Pension systems must be adjustable to the economic reality.

It has been criticized that a *CDC* plan is just a zero sum game: the advantage for one generation of savers is the disadvantage for the other. This view totally ignores the principle of risk sharing and insurance. A *CDC* scheme is an “insurance” contract where the saver pays a premium (in form of contributions into the collective reserve) and receives benefits (in form of payments out of the collective reserve). In *A Theory of Justice* John Rawls introduced the concept of *fairness* under the *veil of ignorance*.[14] Following Rawls’ theory a transgenerational contract between savers will be regarded as *fair*, if the savers were to agree upon this contract provided they did not know in advance which generation they belong to – i.e. *under the veil of ignorance*. From this perspective a *CDC* plan is fair. However, it is also clear that a saver might feel unfairly treated if she or he was obliged to enter a heavily underfunded *CDC* scheme.

CDC schemes are not an all-purpose answer to the pension challenge in an aging society. There are apparent limitations. For example a *CDC* plan can only work if the participants cannot withdraw money at will. Contractual compliance is essential for intergenerational risk transfer. That does not mean that *CDC* plans are Ponzi plans. Whenever the flow of new entrants stops, a *CDC* plan can be converted into a simple *DC* plan by setting the target reserve ratio to zero.

Our backtesting analysis considered only the accumulation phase. We assumed that the wealth at retirement is paid out and reinvested outside the fund. An apparent extension of the model is to include a decumulation phase and then analyze the pension risk, e.g. the volatility of pensions in payment. It would be equally interesting to analyze population dynamics (growing or shrinking generations, winding off) in a *CDC* scheme.

[13] The 1-month money market rates ranged between 13.33% (Sept. 1973) and – 0.11% (Sept. 2015).

[14] Cf. (Rawls 1991), in particular Section 24 (pp. 118ff.) and Section 44 (pp. 251ff.)

We hope at least, that our analysis has added one more argument in favour of *CDC* plans as a good alternative to *DC* and *DB* plans.

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Appendix: Capital market data for Germany 1955 - 2015

German equity performance index (*DAX*): The *DAX* started on 30 Dec. 1987 with a base value of 1000. It is a performance index calculated on the basis of the 30 largest (measured according the market value of free floating equities) German listed companies.[15] End of month values for the *DAX* (from 12.1987 onwards) are provided by Deutsche Bundesbank[16]. Backward calculations for the *DAX* are taken from (Stehle et al. 1996)[17]. It should be noted that due to the rules of German corporate tax laws for the years 1977 to 1993 (1994 to 2001) dividend payments included a 36% (30%) corporate income tax credit. Thus, for these years the *DAX* slightly underestimates the pre-tax performance of a German blue chip stocks portfolio.

German fixed income performance index (*REXP*): The *REXP* started on 30 Dec. 1987 with a base value of 100. It is a performance index calculated on the basis synthetic German Government bonds with remaining terms between 6 month and 10.5 years. [18] End of month values for the *REXP* (including a backward calculation from 01.1967 onwards) are provided by Deutsche Bundesbank.[19] To the best knowledge of the author there are no published backward calculations for months before Jan. 1967. There we calculate a *REXP*-proxy for the time period 12.1954 to 12.1966, based on Government bond interest rates reported by Deutsche Bundesbank.[20]

Month Money market interest rates $\mu_m(t)$ for t: 01.1955 to 09.2015: The 1-month money market interest rates series is a concatenation of data provided by Deutsche Bundesbank.[21]

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[15] For details c.f. (Deutsche Börse AG, 2015a)
[16] Time series BBK01.WU3141
[17] Table 3, p. 296
[18] For details c.f. (Deutsche Börse AG, 2015b)
[19] Time series BBK01.WU046A
[20] Time series BBK01.WU0115 (03.1960- 01.1966), BBK01.WU0018 (05.1956- 02.1960) and 6.10% for the 5 months 12.1954 - 04.1955.
[21] Time series BBK01.SU0310 (from 01.1999 onwards), BBK01.SU0104 (12.1959- 12.1998) and values taken from the Statistical Supplements of the Monthly Reports of Deutsche Bundesbank (01.1955 – 12.1958).