It Takes Two: Why Mortality Trend Modeling is more than modeling one Mortality Trend

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- Joint work with Matthias Börger and Jochen Russ

- IAA Life Section Colloquium, Barcelona, 23th-24th October 2017

Introduction

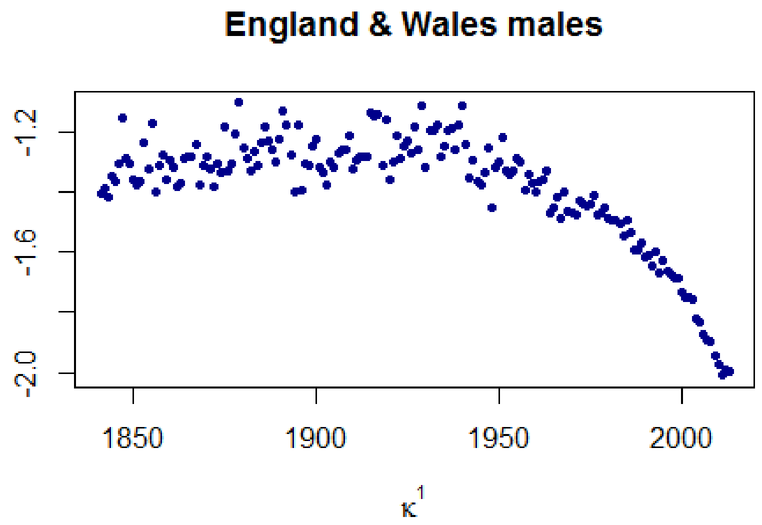
Uncertainty about the evolution of mortality

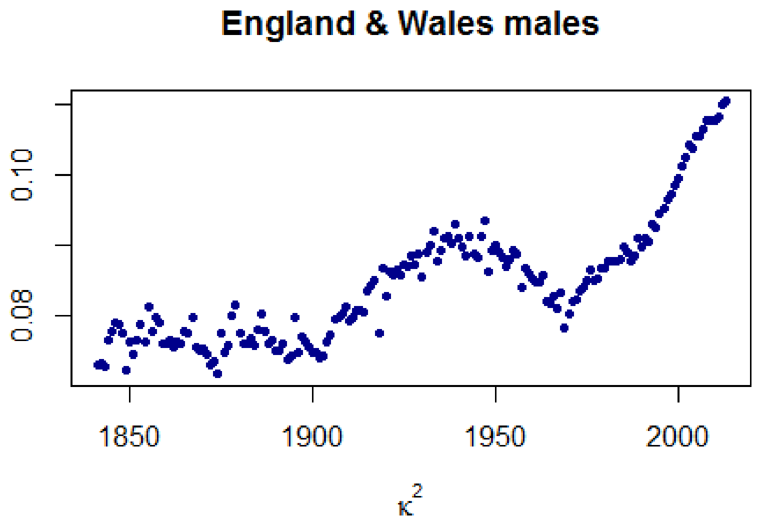
- Decrease in mortality rates and increase in life expectancy
- Similar patterns for most countries
- Increasing attention on longevity risk
- Measure longevity risk in pension or annuity portfolios with stochastic mortality models

- Parametric mortality models: Lee-Carter model, Cairns-Blake-Dowd model, APC model, etc.
  - Estimate the current speed of improvements in mortality
  - Stochastic forecasts of future mortality

Introduction

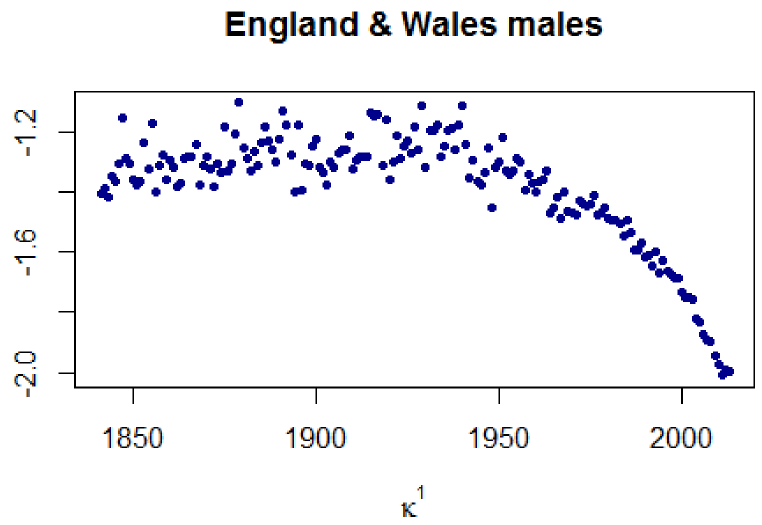
- Two parameter processes (Cairns et al. (2006))
  - $\log\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^1 + \kappa_t^2 \cdot (x - \bar{x})$
  - Parameter processes calibrated for English and Welsh males older than 65 years
- In principle, our approach can be applied to any parametric mortality model
- Popular choice: a (multivariate) random walk with drift for stochastic forecasts
- Historic trend changed once in a while
  - Only a piecewise linear trend with random changes in the trends slope
  - Random fluctuation around the prevailing trend
- Extrapolating only the most recent trend, systematically underestimates future uncertainty, see e.g. Sweeting (2011), Li et al. (2011), Börger et al. (2014)

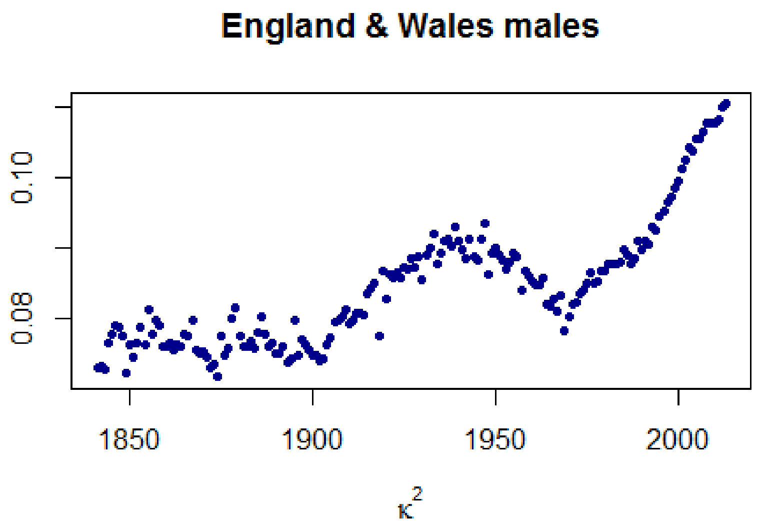




Introduction

- We don't know the current mortality trend for sure
- But the estimate for the current trend seems a good best estimate for the future evolution
- Possible future changes of the trend in both directions
- One model for the actual mortality trend
- One model for the estimation of the current trend at some point in time, that is the estimated mortality trend
- In many situations, both components are necessary





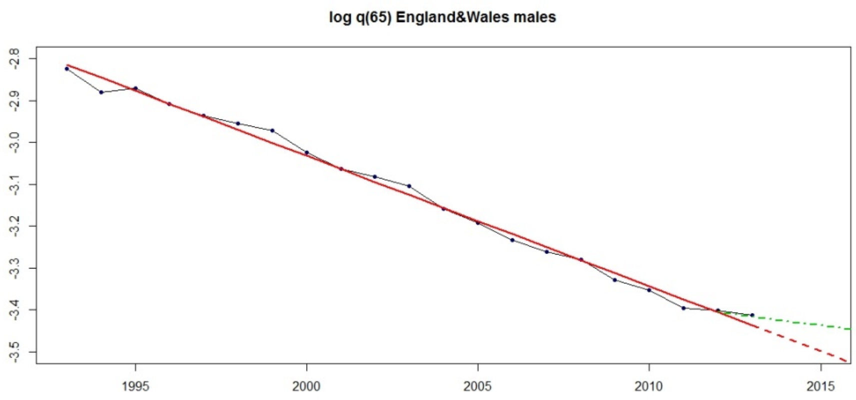
Agenda

- **Why two mortality trends?**
  - Actual mortality trend (AMT)
  - Estimated mortality trend (EMT)
  - Some examples
- **A combined model for AMT & EMT**
  - AMT component
  - EMT component
- **Conclusion**

Why two mortality trends?

Actual Mortality Trend (AMT)

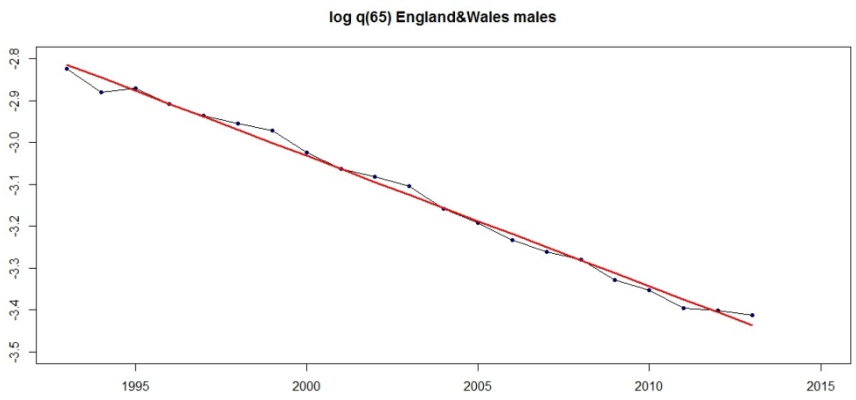
- The AMT describes realized mortality trends
  - Core of most existing mortality models
  - Time and magnitude of changes in the AMT and the error structure around the trend process need to be modeled
- We have an idea of the historic AMT but it’s not fully observable!
- We can’t always distinguish between a recent trend change and “normal” random fluctuation around the prevailing trend → possible undetected trend change in the recent years
- Unknown current value of the AMT and unknown current value of the trend process



Why two mortality trends?

Estimated Mortality Trend (EMT)

- The EMT describes actuary's/demographers expectation about the AMT, i.e. the current slope of the mortality trend at some point in time
- Based on most recent historical, observed mortality evolution and updated as soon as new observations become available
- The EMT is the basis for mortality projections, (generational) mortality tables, reserves, etc.



Why two mortality trends?

Some examples

Why another trend?

- Requirement for AMT and/or EMT depends on application:
  - Reserves for a portfolio → EMT today
  - Capital for a portfolio run-off → AMT over the run-off
  - Reserves for a portfolio after 10 years → AMT over the 10 years, EMT after 10 years
  - Payout of a mortality derivative → AMT up to maturity, EMT at maturity
  - Analyse the hedge effectiveness of the previous derivative → EMT at maturity, AMT beyond

A Combined model for AMT / EMT

AMT component

- Continuous piecewise linear trend, with random changes in the slope and random fluctuation around the trend
- AMT model specification:
  - Model the trend process with random noise → $\kappa_t = \hat{\kappa}_t + \epsilon_t;\ \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$
  - Extrapolate the most recent actual mortality trend → $\hat{\kappa}_t = \hat{\kappa}_{t-1} + AMT_t$
  - In every year, there is a possible change in the mortality trend with probability p

    → $AMT_t = \begin{Bmatrix} AMT_{t-1} & with\ probability\ 1-p \\ AMT_{t-1} + \lambda_t & with\ probability\ p \end{Bmatrix}$
  - In the case of a trend change → $\lambda_t = M_t \cdot S_t$
    - With absolute magnitude of the trend change $M_t \sim \mathcal{LN}(\mu, \sigma^2)$
    - Sign of the trend change S_t bernoulli distributed with values -1, 1 each with probability $\frac{1}{2}$

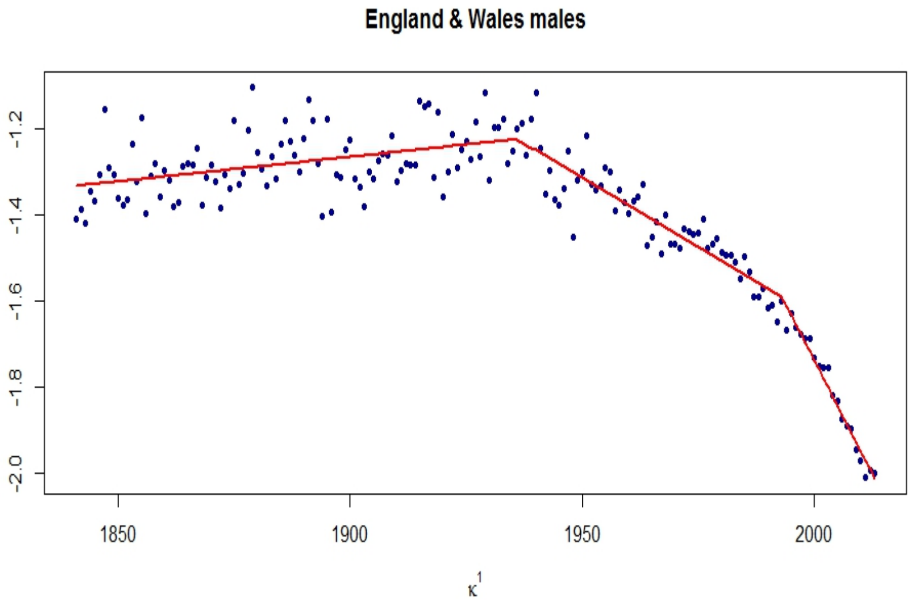
Parameters to be estimated for projections:

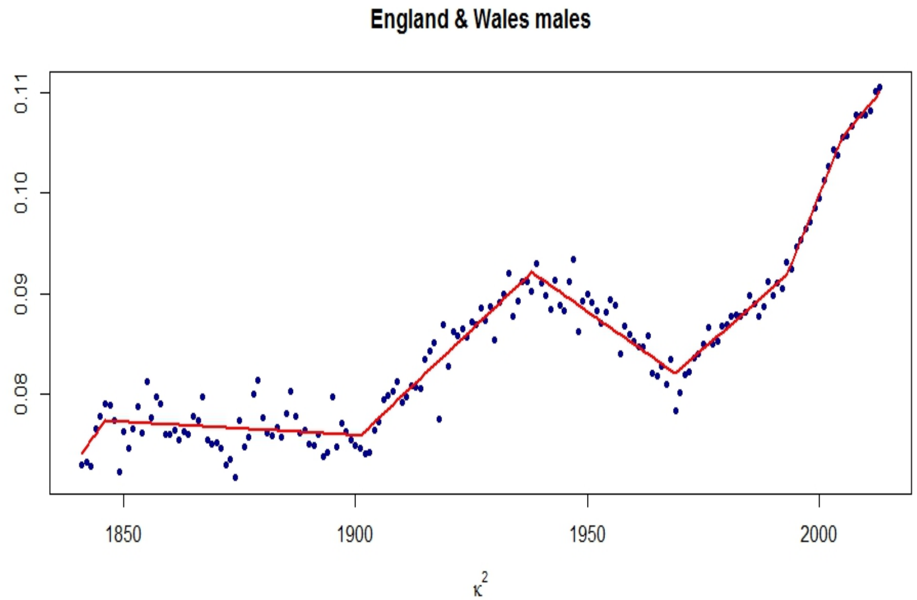
$p, \sigma_\epsilon^2, \mu, \sigma^2, AMT_n, \hat{\kappa}_n$

A Combined model for AMT / EMT

AMT component

Idea: Use historic trends to estimate the parameters $p, \sigma_\epsilon^2, \mu, \sigma^2, AMT_n, \hat{\kappa}_n$





For details on the calibration we refer to Börger and Schupp (2015) and Schupp (2017). Parameter uncertainty is included. See Appendix for a comparison with other AMT approaches.

A Combined model for AMT / EMT

EMT component

- We see random changes in the future AMT according to the symmetric density function of the trend change intensity ($\lambda_i = M_i \cdot S_i$ in each year i with a trend change)

→ Symmetric density function of future AMT_s , $s > t$ with mean AMT_t

→ $\mathbb{E}(AMT_s) = AMT_t$, $s > t$ arbitrary

- Choose EMT_t as the expected AMT_t given realized mortality up to this point in time
  - $EMT_t = \mathbb{E}(AMT_t)$
  - Difficult in a simulation, as the path-dependent calculation of the EMT_t is complex (see Börger and Schupp (2015)). In each path the complete trend process needs to be recalibrated
- Possible, but not feasible from a practical point of view
- Piecewise linear trend process with symmetric changes in the AMT
  - → Calibrate the EMT with a linear regression on most recent data

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A Combined model for AMT / EMT

EMT component

- Higher influence of most recent data in the estimation of the regression
  - Weighted regression in year s: $w_i(s,t) = 1/(1+\frac{1}{h_i})^{s-t}$

    for both parameter processes $i = 1{,}2$ and $t \leq s$

- Other possible methods:
  - Linear regression with data from the last 5/10/20 years (in the spirit of Cairns et al. (2014))

- How many years should be included in the regression?
  - Too many → delayed reaction of EMT on trend changes in the AMT
  - Too little → EMT is vulnerable to random noise in the AMT

A Combined model for AMT / EMT

EMT component

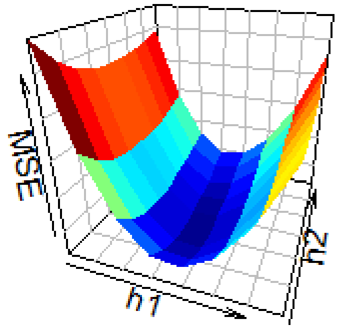
Calibration of the weights based on a practical application

- Consider a portfolio of 45 year old males. Calculate the required reserves when the portfolio retires (at age 65). Fixed interest rate of 2%.

- Calibrate the AMT model for 65 year old males (England and Wales)
- Simulate the future evolution of the AMT 100.000 times with annual errors for each path
- After 20 years, calculate the reserves with the EMT for each path
- Further simulate the AMT and compare the realized capital requirement with the reserves based on the EMT
- Optimal weighting (h_1, h_2) can be determined by minimizing the MSE between reserves and realized capital requirement

Combined AMT/EMT Model

EMT component - comparison



Calibration of the EMT components - comparison

- Unique solution: $(h_1 = 3{,}6,\ h_2 = 1{,}4)$
- Estimated present value of portfolio vs. realized present value

EMT estimation method	MSE	Root MSE
Optimal weighting	0.3216	0.5671
Optimal weighting (+0.5)	0.3261	0.5710
Optimal weighting (-0.5)	0.3259	0.5708
Regression last 5 years	1.026	1.0131
Regression last 10 years	0.3608	0.6007
Regression last 20 years	0.3794	0.6160

- The risk of a false estimation of the reserves based on future mortality can be minimized with the optimal weighting EMT approach

Combined AMT / EMT Model

EMT component

Calibration of the EMT components - comparison

- Practical implication:
  - Underestimation of reserves is critical
  - EMT approach has a crucial impact on the capital adequacy of reserves

EMT estimation method	>5% underestimation	>10% underestimation
Optimal weighting	3.6%	0.4%
Regression last 5 years	13.8%	1.5%

  - Use optimal weighting EMT approach instead of a linear regression on the last 5 years
    - The probability of underestimating the required reserves by more than 5% can be reduced from 13.8% to 3.6%
    - The probability of underestimating the required reserves by more than 10% can be reduced from 1.5% to 0.4%

Conclusion

- Two trends need to be distinguished and modeled
  - The actual mortality trend (AMT) is the prevailing, unobservable mortality trend
  - The estimated mortality trend (EMT) is the estimate of the AMT
- The trend to consider depends on the question in view
- The AMT is modeled as a continuous and piecewise linear trend with random changes in the trend's slope
  - The random walk with drift underestimates the longevity risk systematically
  - Based on the AMT model we can estimate an appropriate time period for the estimation of a deterministic trend
- Choice of EMT approach is crucial in many practical situations
  - A weighted regression approach seems reasonable
  - Optimal regression weights can be determined in a practical setting

Literature

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Appendix

Comparison with other AMT Models

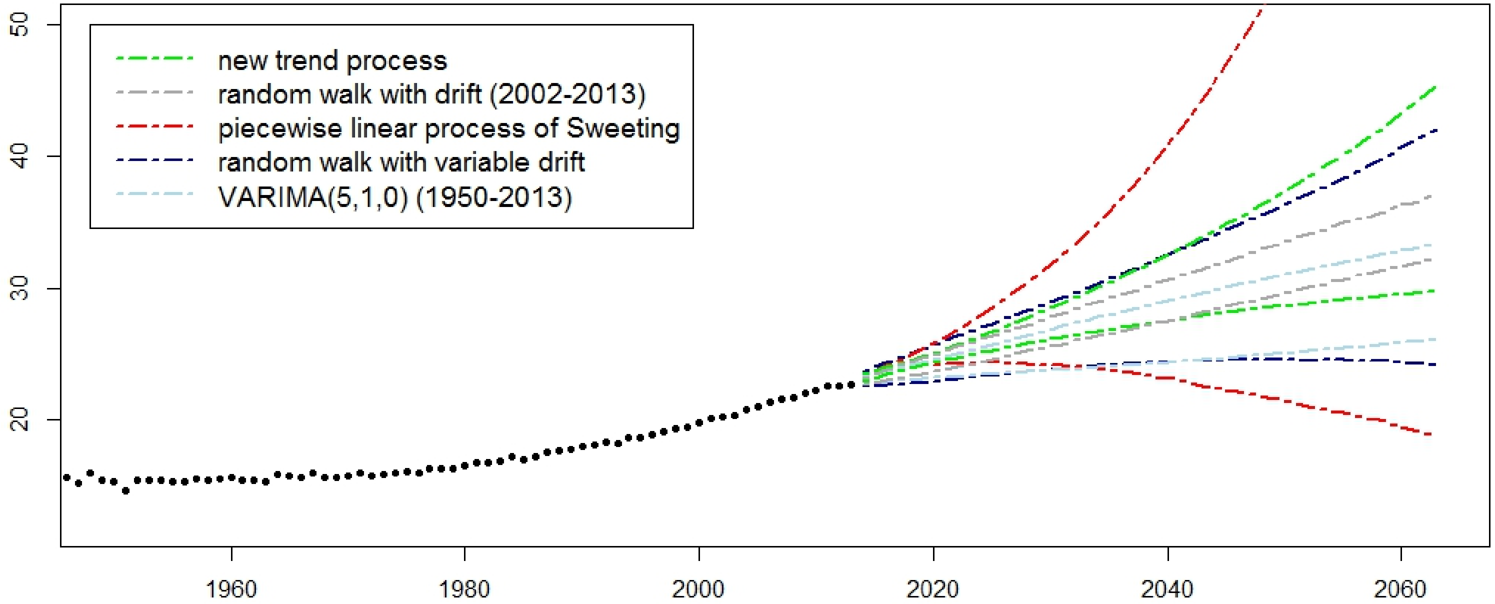
See Börger and Schupp (2015)

- RWD:
  - Bivariate random walk with one constant drift
  - Preselection of data history; here: data since last breakpoint
- Sweeting (2011):
  - Identification of trend model with Chow-test
  - Magnitude of changes normally distributed with mean 0
- Chan et al. (2014):
  - VARIMA process
  - Extrapolation of trends and errors
- Hunt and Blake (2014)
  - Random walk with variable drift
  - With parameter uncertainty

Appendix

Comparison with other AMT Models

- Remaining period life expectancy for a 60-year old (5th and 95th percentiles) by different approaches.



- Comparable medians but extreme differences in the percentiles
- Confidence bounds for RWD, VARIMA seem too narrow; Sweeting's approach produces unrealistically large bounds
- Trend process produces plausible confidence bounds
  - Possible continuation of latest improvements

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