A stochastic investment model for asset and liability management

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Abstract

This paper describes a stochastic investment model, designed for use as a tool in the asset and liability management of UK pension funds. A full description of the model is given, including the equations and parameters. Rates of return are not modelled directly, but are transformed into forces of return. Modelling forces makes the relationships between the variables additive rather than multiplicative. This is the first fully published model to use earnings rather than dividends to generate price returns. Another feature of the model is that the equity return is divided into three components – dividend yield, earnings growth, and change in market rating. By modelling these components separately the model is able to capture one of the key features of the equity market, namely the high short term volatility (which derives from market sentiment) and the much lower long term volatility (which arises from economic fluctuations). The technique of decomposing the return should be applicable to other equity markets around the world.

Keywords: Stochastic Investment Modelling, Asset/Liability Modelling, Price Earnings Ratio, UK Equity, Pension Funds and Insurance Companies.

1 Introduction

This paper gives a description of a stochastic investment model that is primarily designed to be used in an integrated asset and liability model for UK pension funds. The main objective of the model is to give an understanding of the source and nature of the financial and economic risks to which pension funds are exposed, so enabling better management of these risks.

Asset and liability models are widely used by financial institutions to manage their assets and liabilities in order to achieve their business objectives. A stochastic asset and liability model consists of three integral parts; an economic and asset projection model; a liability projection model; and a decision-making model used to define an optimal strategy based on the projections of assets and liabilities.

This paper provides the stochastic model of the economic and asset projections for generating simulations of a range of economic and asset returns. Each simulation scenario represents a possible development of the economy and investment markets. The model structure and parameters determine the range of possible simulations. Within the asset models, the objective is to assess risk relative to an assumed central rate of return or yield, rather than to predict an absolute level of return or yield.

2 Stochastic Asset Modelling

In this section we describe the thought process behind the development of our model. We start in section 2.1 with a consideration of fundamentals. In section 2.2 we review aspects of fully published existing models that are of relevance to the construction of our model, contrasting the different approaches that have been used with our own approach.

2.1 The three fundamental inputs

There are three fundamental inputs to the design of any stochastic model: economic theory, investment practice, and historic data. The weight given to these inputs will depend partly on the purpose for which the model is being constructed, and partly on the personal opinions of the people who are building the model.

2.1.1 Economic theory

There are macro-economic theories about the relationships between key economic variables such as price and salary inflation. They suggest that price inflation tends to feed into salary inflation, but with some time lag. They also suggest that certain scenarios such as very high inflation, are unlikely to be sustained for any length of time, as the economy will either move into hyper-inflation (which itself is unsustainable) or the government will act to bring down inflation.

Financial economic theory often starts with the assumption that in a perfectly informed market of rational investors, there can be no persistent arbitrage opportunities. In assessing the usefulness of this theory, the modeller should take into account investment practice. The applicability of financial economic theory will vary over time, and between investment markets.

2.1.2 Investment practice

The practice of investment markets varies around the world. Some differences are due to external factors such as taxation and accounting rules, while others are cultural and historic. This is explained through the example of UK equity dividends.

In the UK equity market there has been a strong tradition of paying substantial dividends to shareholders. Until 1997, this practice was encouraged by the UK tax structure. The government received less tax if a company paid earnings out as dividends and then raised capital separately, than if it retained earnings to use for capital. However, there was also a strong cultural resistance to cuts in dividends. A company which cut its dividend was assumed to be in serious trouble. The combination of these two opposing influences (tax pressure for high dividends and cultural resistance to cuts in dividends) meant that the companies aimed to distribute the highest dividend that they thought they could sustain.

In consequence, the dividend payout effectively represented the company board's estimate of sustainable earnings, and this helped to make the dividend yield a good measure of value that could be used by investment analysts and in the construction of investment models.

Although the use in the UK of the dividend yield as a measure of value has been effective in the past, it was less appropriate in markets where dividends are less important. It is also doubtful whether the approach will continue to be as valid in the future even in the UK. The tax position has now changed. There is no longer any tax incentive to pay dividends. In addition, investment analysts have almost ceased to use dividends as a measure of company value, which suggests that the cultural resistance to dividend cuts may reduce.

2.1.3 Data

For mature markets like the UK there is a substantial amount of historic data available from which to build an asset model. The problem is the extent to which the past is a guide to the future. There have been important changes in capital markets, namely the relaxation of government controls and the improvement of market information. These changes are likely to make the future fundamentally different from the past. For example, for most of the twentieth century markets were heavily constrained by controls on foreign exchange, which restricted the free movement of capital. There have also been periods of control on dividends, and on profits. In the nineteenth century there was much less government control, but the information was very poor in most markets.

There is another change that may be relevant to the future development of markets: the introduction of index linked bonds. In the 1970s there was high inflation in many countries around the world, and index linked bonds hardly existed. The real return on fixed interest investments was substantially negative, but the only alternatives for investors were equities or property. We now have index linked bonds in many countries. These provide a much more suitable alternative for investors during any period of high inflation in the future. This might influence the return that could be obtained on fixed interest stocks. (The appendix shows the data sources we have used.)

2.2 Features of existing stochastic models

There are good surveys of existing models in Ziemba and Mulvey (1998) and Huber (1996). Possibly for commercial reasons, some models have not been fully described in the academic literature, although some general descriptions have been given (Mulvey and Thorlacius, 1998; Dempster and Thorlacius,1998). As the validity of these models has never been externally tested, we do not consider them further.

2.2.1 The Wilkie model and the cascade structure

Wilkie (1984) published the first comprehensive UK stochastic investment model and it has contributed immensely to the acceptance of stochastic techniques by the UK actuarial profession. The model was originally designed for assessment of maturity guarantees for unit linked life insurance policies. However, since its publication it has been used extensively in a range of other applications, such as life and general insurance and asset and liability modelling for pension funds. The original model has been updated and extended to include additional asset classes (Wilkie, 1995).

Some problems with Wilkie's model were raised in papers by Daykin and Hey (1990) Geoghegan *et al* (1992) and Huber (1995): Some of the parameters were unstable over time; there was significant cross-correlation between the out-of-sample residuals; the price inflation model did not have normal residuals; the model did not allow for irregular shocks or periods of high inflation; the probability of negative inflation was high; and the inflation model did not react adequately to changes in the mean rate.

The Wilkie model adopts a cascade structure with price inflation influencing salary inflation and other asset returns. The more complete 1995 version includes models of yields on cash, irredeemable government bonds, index linked long term government bonds; and equity dividend yield and growth, property yield and rental growth.

Other models with a similar cascade structure have been developed for Australia (Carter 1991), South Africa (Thomson 1994), Japan (Tanaka 1995) and Finland (Ranne 1998). A feature of these models is that the designer specifies the model structure to some extent and makes judgements about the variable linkages. The cascade structure only allows one way causality and requires the modeller to focus on the important links. This precludes feedback, for example, in the model price inflation influences salary inflation but not the reverse. These feedback effects are secondary in the long term.

2.2.2 Vector autoregression models

An alternative approach is to allow the data to "recognise" a structure between variables, to model the economic variables and to fit parameters to the structure. For example, vector autoregressions (VAR) have been used for this purpose (Harris 1997).

A criticism of these models is that they are commonly overfitted to the data on which they are based. When relative returns have different characteristics to the historic period on which the model was based, this could result in poorer models of returns. These models do not have a restrictive cascade structure, allowing each variable to be influenced by any other. We have found VAR techniques useful for indicating the links between variables.

2.2.3 Efficient market based models

A number of models are based on the theory that markets are either weakly efficient (ie prices in the market reflect all previous price information) or strongly efficient (ie prices in the market reflect all available information). Such an approach is generally appropriate for short term modelling, particularly where the model is to be used for pricing. For a long term model, the approach seems to have less value, as it does not take sufficient account of macro economic fundamentals. Smith (1996) and Dyson and Exley (1995) have produced UK based models on efficient market principles.

In particular, efficient market models usually seek to exclude arbitrage opportunities and often do not contain any mean reversion. However, mean reversion is a feature of some investment markets in the long term, chiefly because market participants ensure that it is. For example, if the yield on government bonds gets too high, then the government will act to reduce it. We have therefore used a mean reverting model for the yield on government bonds.

Our equity model includes aspects of market efficiency. We allow equity yields to drift rather than forcing them to revert to a particular value. The absence of reversion removes an important arbitrage opportunity.

2.2.4 Continuous and discrete time modelling

Continuous time modelling provides an alternative to the time series approach for modelling movements in asset returns and yields. Continuous models are widely used in derivative mathematics where the asset return or interest rate processes are modelled as stochastic differential equations.

Time series and differential equations both can be used to describe the processes in the economy and in the markets. Medvedev (1998) showed that the AR(1) is the discrete equivalent of the Ornstein-Uhleneck process, $dx = \mu(x,t)\, dt + \theta(x,t)\, dW(t)$ if the function μ is linear in x and t. He compared the two approaches and showed that higher order autoregressive and moving average processes cannot be obtained via a first order differential equation

We chose the time series approach because a range of estimation techniques are available both in single and multi-equation settings. A further advantage of the discrete approach is that time series are better known and therefore the models can be understood by a wider audience.

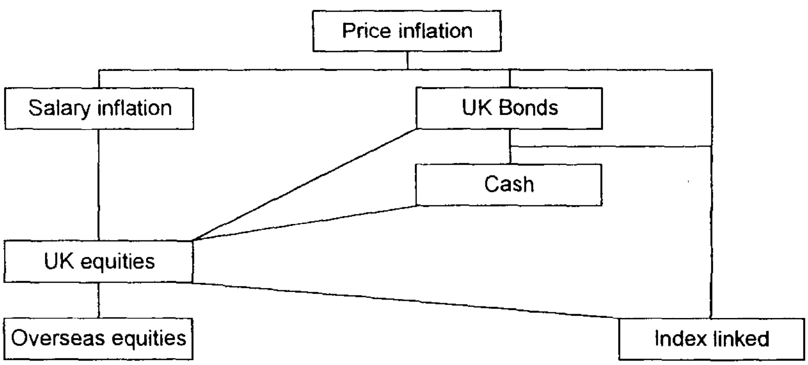
3 Construction and testing of our model

The process of constructing an investment model is inevitably cyclical. An initial model is constructed, and tested against the data, and the results of the tests are used to refine the design of the model. A number of key decisions are required in the modelling process. What basic structure should be used to order the modelled variables? What transformations should be made to the modelled variables? What equations should link the variables, and with what parameters?

3.1 Structure

We chose to adopt a cascade structure for the model, as we were interested in a long time horizon, and a purely data driven model would not take sufficient account of fundamental macro economic considerations. In common with other actuarial models, price inflation was selected as the main driver. This is due to the use of price inflation in assessing real returns and the belief that the links from price inflation to the other asset classes are stronger than the reverse effects. The subsequent development of cascade structure was largely based on statistical evidence, although we did take account of economic and investment considerations.

We used a number of statistical methods to determine the structure of the model including output from simple VAR models, the Granger causality test and the ADF unit root test. The diagram below shows the structure adopted, and the links between the models of each variable.



We model the force of yield on long term government fixed interest bonds and on cash, since these are the main fixed interest investments that are used by UK pension funds. This means that we are effectively modelling the short and long term ends of the yield curve but not the medium term. To create a more complete model of the government bond market it would also be necessary to model medium term bonds, but this refinement was not necessary for our purposes.

The index linked bond model is derived from the models for fixed interest government bonds and inflation. Actuarial and economic theory would suggest that index linked bond yields are more fundamental than fixed interest yields, and so should appear higher up in the cascade. However, practical investment experience indicates that the market actually works the other way round, primarily because the index linked market is so much smaller and less liquid than the fixed interest market.

The UK equity model is the most innovative part of our model, and is influenced by most of the other asset classes. We introduce the use of a force of change in yield both to derive the dividend yield and to generate the total return. This recognises that the return on equities depends on the relative yields at which the equities are bought and sold, rather than assuming that equities are always priced as the discounted value of their cashflow stream (Teeger and Yakoubov, 1998). We link salary rather than price inflation into equity earnings growth.

The overseas equity models are only linked to the UK equity return. This is reasonable for UK pension fund purposes as overseas equities are usually only compared with UK equities for asset allocation purposes. The structure involves specifying the correlations between overseas equity returns. These were derived from a combination of data and expectations of future experience

3.2 Transformation of variables; returns and yields

A feature of time series models is that they consist of linear equations and therefore impose a particular structure on the development of the modelled variable. We have applied transformations to the modelled variables in order to produce a (log) linear structure that is consistent with a time series modelling approach. We use forces of return to allow negative values of variables since using a log transformation would not allow negative values.

Throughout the model we use forces of yields and returns. Use of forces means that many key relationships become additive rather than multiplicative. For example if inf_t is the force of price inflation over the year t and the force of salary inflation over the year is sal_t then the force of real salary growth is equal to $sal_t - inf_t$. Similarly if the forces of price inflation over each quarter are $qinf_t$, $qinf_{t-1/4}$, $qinf_{t-1/2}$, $qinf_{t-3/4}$, then, $inf_t = qinf_t + qinf_{t-1/4} + qinf_{t-1/2} + qinf_{t-3/4}$.

Our UK equity model divides neatly into the sum of three (log) linear elements. In the UK equity model, one variable is needed in several forms (dividend yield). Here we have adopted the form that has the greater impact on equity total returns, and model the force of change in yield rather than the yield itself.

3.3 Derivation of equations and parameters

Having defined the broad structure of the model, we considered the nature of the variables to be modelled. We tested variables for co-integration and considered the relevance of mean reversion. We derived the equity model structure by considering how equity returns can be decomposed.

Statistical techniques were used both to choose the equations and to assess the validity of the model. The set of criteria which were used to choose and validate the model included the Akaike Information Criterion (AIC), log likelihood, R squared, normality and serial correlation of residuals, standard error of residuals and parameter stability over rolling subperiods.

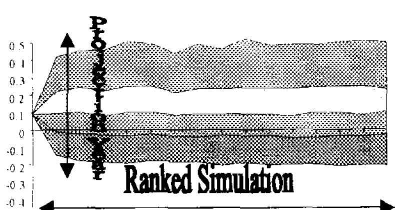
For some variables it appeared that the structure has changed over time and we have used appropriate statistical techniques such as intervention variables to handle this. (An example is the relationship between fixed interest yields and inflation.) Sections 4 to 10 explain the equations and parameters.

3.4 Model testing

In theory, it might be desirable to separate the processes of construction and testing, to ensure a completely independent test of the validity of the model. However, this could only be achieved by ignoring information in the construction of the model, and we prefer to use all available data available, especially for modelling index linked bonds. Since our model is designed for informing users about the shape of the asset return risks relative to central rates of return, rather than for predicting economic variables, we did not consider that the fit of forecasts to out-of-sample data would be an appropriate test of model validity.

We ran simulations to test the model output and the relationship between variables. We investigated the median, standard deviation, correlation, skewness, and kurtosis statistics using both projection year and ranked simulation approaches.

3.4.1 Single projection year statistics



The projection year approach is to look at the distribution of the simulations across each year of projection period. However, this looks at the statistics across the "funnel of doubt" which are not comparable with the historic observations used to derive the models, which are taken over a period of time.

3.4.2 Ranked simulation statistics

Each simulation can be looked at as a single realisation out of number of possibilities. We therefore prefer to calculate these statistics for each simulation, rank them, and then examine the resulting distribution, which we call the ranked simulation distribution. We calculate the "ranked-simulation" distributions of the median, standard deviation, correlation, skewness and kurtosis statistics.

Thus, for example, each simulation has an individual associated standard deviation calculated over the projection period. The ranked-simulation standard deviation refers to the distribution of these standard deviations. The median of the ranked-simulation standard deviation is comparable with a historic standard deviation eg over the last 30 years. The tails of the ranked-simulation standard deviation indicate the variability of the standard deviation statistic over different simulations. Section 9 shows tables and graphs of the statistics.

3.5 A brief comparison with the Wilkie model

Our model is similar to the Wilkie model, which has been the most widely used model in the UK. However, there are significant differences, which we believe make our model rather more robust. In this section we comment on some of the key differences between our model and the Wilkie model. This should assist those who are familiar with the Wilkie model.

UK Equities

- The Wilkie model had a link from UK equities into fixed interest whereas we have modelled a link from fixed interest into UK equities.
- Wilkie's model is based on dividends whereas our UK equity model is based on company earnings. While dividends may have been an appropriate base in the past, we believe that this is unlikely to be the case in future.
- Wilkie modelled the log of the yield whereas we have modelled the first difference in the log of the yield.
- Wilkie included only income and growth elements, whereas our model has three underlying elements.
- We link salary rather than price inflation to UK equity earnings growth.

Wilkie modelled fixed interest bonds as a lognormal real yield + a normal smoothed inflation. This allows only positive real yields on fixed interest but allows negative nominal yields. We believe it is preferable to allow negative bond real yields and model the force of bond yield to achieve this.

The Wilkie model only has one link into his index linked model, from long term government bonds. We also have links from price inflation and link the residuals with equity earnings growth residuals.

Wilkie used different transformations to model each variable; we have used consistent transformations throughout and modelled forces rather than rates of return. Our approach ensures that the key relationships are ones of addition and subtraction, rather than multiplication and division. This gives greater stability to the model.

4 Price inflation

Price inflation is a critical element in many actuarial calculations. In pension schemes, the liabilities are often linked to salaries, which may be expected to increase in excess of inflation, whilst pensions in the course of payment may be directly linked to price inflation. In general insurance, general price inflation is one of the drivers of claims inflation.

So far as the assets are concerned, inflation has an influence on investment returns. Economic theory suggests that investors should look for a real rate of return on their investments. This is also supported by investment practice, where actual and expected levels of inflation have a major influence on cash and bond yields.

In common with most other models, we have chosen to use the national index of consumer prices as our measure of inflation. Whilst the deficiencies in this index are important for short term economic modelling these are outweighed for our purposes by the long data series. The public prominence of the index means that it is used in pension indexation, indexed linked government bonds, and by employees in making claims for pay increases.

Inflation modelling is a difficult task for several reasons. The level of inflation depends on different factors such as political, economic and employment decisions as well as one-off shocks such as the oil crises in the middle of the 1970s and the beginning of the 1980s. In stochastic investment models, inflation is often described as an AR(1) process (Wilkie, 1986) or as a diffusion process in the continuous case. Some authors have introduced modifications to these two basic approaches by explicitly modelling the volatility structure (Wilkie, 1995) or by using transfer functions to take account of one-off events (Ranne, 1998; Clarkson, 1991). We follow Wilkie's approach and have chosen an ARCH (1) model.

We model force of price inflation, ie $inf_t = log\ (I_t^{inf} / I_{t-1}^{inf})$ where I_t^{inf} is the price index.

$$inf_t = \mu^{inf} + \alpha_1^{inf} (inf_{t-1} - \mu^{inf}) + \varepsilon_t^{inf}$$
$$\sigma_t^2 = \beta_1^{inf} + \beta_2^{inf} \varepsilon_{t-1}^{inf\,2}$$
$$\varepsilon_t^{inf} \sim N(0, \sigma_t^2)$$

In each section the symbol, μ, represents the mean value of the variable in the data period.

%	μ^{inf}	α_1^{inf}	β_1^{inf}	β_2^{inf}
Value	4.9	60	9.38 (=3.06^2)	72
Standard Error		7.6	2.86	20

We found three residuals (1940, 1975 and 1980) which fall outside the corridor *(-2 s.e. , 2 s.e.)*. The Jarque-Bera (JB) statistic, JB = 6.71, so the normality of residuals can be rejected at the 5% level but not at the 2.5% level. For all our models, the Ljung-Box Q statistics, measuring serial correlation in the residuals, indicates that the residuals are white noise at all lag levels at the 5% level.

We have tried to fit an asymmetrical ARCH to the inflation series, where good news ($\varepsilon_t<0$) and bad news ($\varepsilon_t>0$) have different effects on the conditional variance. However the fitted asymmetrical parameter was unstable over time and simulations showed excess positive asymmetry.

5 Salary inflation

In final salary pension schemes, the liabilities are directly linked to salaries. We use salary inflation as an input to the UK equity model, where it is used as a proxy for growth in the economy. Around 65% of UK GDP derives from employment income with about 15% from gross trading profits of companies (Central Statistics Office).

The central value should therefore be targeted at expectations of National Average Earnings. Where separate estimates of scheme specific salary inflation are required, adjustments relative to this earnings model should be made.

We model force of salary inflation, which is $sal_t = log\ (I_t^{sal} / I_{t-1}^{sal})$ where I_t^{sal} is a salary inflation index. The following model gives the best fit to data and has a simple structure.

$$sal_t = \mu^{sal} + \alpha_1^{sal}(sal_{t-1} - \mu^{sal}) + \alpha_2^{sal}(inf_t - \mu^{inf}) + \varepsilon_t^{sal}$$
$$\varepsilon_t^{sal} \sim N(0, \sigma^2)$$

%	μ^{sal}	α_1^{sal}	α_2^{sal}	σ
Value	6.6	33	54	2.32
Standard Error		7.8	7.0	

There are four residuals (1946, 1963 and 1977, 1978) which fall outside the corridor *(-2 s.e. , 2 s.e.)*. The residuals in 1977 and in 1978 cancel out on average, i.e. the average fitted salary inflation over these 2 years is very close to the average actual salary inflation. The Jarque-Bera statistic, JB = 8.52 and normality of residuals can be rejected at the 5% level but not at the 1% level.

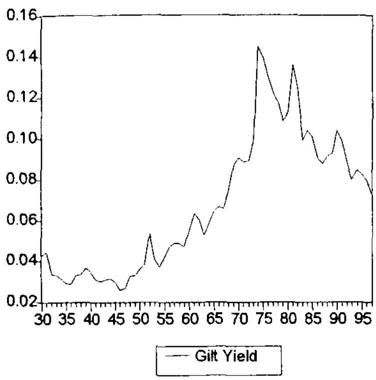
Wilkie's model has a similar salary inflation equation, but uses inf_{t-1} in place of sal_{t-1}. Wilkie has commented that in his model there is significant crosscorrelation between the residuals of the price inflation model and first order lagged residuals from the salary inflation model. We tested for crosscorrelation in our model and found that it is lower than in the Wilkie model and is below the 5% borderline of significance.

6 Fixed interest government bonds

In our model the long term government bond yield drives the other interest rate models. This is similar to other actuarial models (Wilkie,1995; Ranne, 1998) and differs from some financial economics models where short term rates drive the whole yield curve (Vasicek, 1977).

In both economic theory and investment practice, fixed interest government bonds are fundamental to valuing a number of asset classes. The size and liquidity of the market makes it autonomous and relatively independent of other UK markets.

We model the force of gross redemption yield (*gry*), which is $gilty_t = ln(1 + gry_t)$. We investigated the link between inflation and the yield on long bonds. Economic theory suggests that gross redemption yield = real yield +expected inflation + any risk premium.



UK data suggests that before 1960, investors did not envisage a great risk of inflation and considered long term British Government Bonds to be really "gilt-edged". After 1960, the fear of inflation grew and investors started to demand a real rate of return on their investments. Since 1960, the yield has moved with expectations of inflation, so that a rise in expected inflation will tend to lead to a rise in yields as investors demand higher returns for investing in nominal assets. In our modelling we therefore divided the period into the experience up to 1959 and from 1960 onwards.

Expected inflation is the average inflation over the life of the bond. An exponentially weighted average of inflation, $infwe_t = c*inf_t + (1-c)*infwe_{t-1}$, can be used as a proxy for the expected inflation. Using statistical techniques, the optimal parameter appears to be $c = 0.3$.

We fitted first order autoregressive models for the yield series over the two periods. The model had a weak inflation link over the first period, whilst the link was much stronger over the second period. The opposite relationship can be observed with the autoregressive coefficient. Over the whole period, using an intervention variable to separate the two periods, we have the following model:

$$gilty_t = \mu^{gilty} + \alpha_1^{gilty}(infwe_t - \mu^{inf}) + u_t^{gilty}$$
$$u_t^{gilty} = \alpha_2^{gilty} u_{t-1}^{gilty} + \varepsilon_t^{gilty}$$
$$\varepsilon_t^{gilty} \sim N(0, \sigma^2)$$

1931-1959 | 1960-1997

%	μ^{gilty}	α_1^{gilty}	α_2^{gilty}	σ	%	μ^{gilty}	α_1^{gilty}	α_2^{gilty}	σ
Value	3.6	8	99	4.7	Value	9.2	34	65	9.6
Standard Error		6.5	2.5		Standard Error		7.1	8.2	

Two residuals, 1974 and 1981, fall outside the corridor *(-2 s.e. , 2 s.e.)*. In both cases, the yield had risen more than our model predicted. The 1974 residual is perhaps due to the distortion of the large oil price inflation shock. The 1981 residual is likely to have been caused by a combination of the introduction of index linked gilts and the high interest rates used to maintain the tight monetary policy then in place.

If these two points are stripped out, the residuals become much more normal, with a Jarque-Bera statistic of JB = 2 (JB = 146 when the two points are included).

In period one, the inflation link parameter, α_1^{gilty}, has high standard error relative top the mean, which means that it can be assumed to be zero. This is consistent with the view that investors in that period were not particularly concerned about the inflation risk. The autoregressive parameter, α_2^{gilty}, is near to 100% so the model is a random walk. We chose to use the model from period 2 for simulation. However, in a period of long term stable inflation, it might be more appropriate to use model parameters adjusted slightly towards those estimated for period one.

7 Cash model

We modelled the short end of the yield curve (the 3-month rates). The cash model is linked to the government bond yield model, as in the Wilkie model. The expectations and liquidity preference theories of the yield curve suggest a link between the yields on cash and long term bonds. This would also be consistent with the idea of a risk premium concept between bonds and cash.

We examined the slope of the yield curve in terms of the difference between bond and cash yields and $(\mu^{gilty} - \mu^{cash})$ represents the average slope in the yield curve. The following model is appealing in its simplicity and goodness of fit:

$$cash_t = \mu^{cash} + gilty_t - \mu^{gilty} + u_t^{cash}$$
$$u_t^{cash} = \alpha_1^{cash} u_{t-1}^{cash} + \varepsilon_t^{cash}$$
$$\varepsilon_t^{cash} \sim N(0, \sigma^2)$$

%	μ^{cash}	α_1^{cash}	σ
Value	5.6	70	1.5
Standard Error		8.65	

There are three residuals (1980, 1981 and 1989) which fall outside the corridor *(-2 s.e. , 2 s.e.)*. These are times at which government policy was affecting the relationship between the short and long ends of the yield curve. The Jarque-Bera (JB) statistics, JB = 15.88 and normality of residuals can be rejected at the 5% level. This is to be expected since government actions are difficult to model.

8 Index linked government bonds

Index linked government bonds offer a risk-free real yield. Although they were first issued in 1981, the market was restricted in the first year and illiquid initially, which distorts the data from those years. In consequence, we used data from 1984 onwards.

A common assumption is that index linked bond yields should be fully explained by the yield on fixed interest bonds less expected inflation, suggesting that index linked yields should be barely affected by inflation (as they are designed to be inflation proof). However, the demand for index linked bonds may rise in times of uncertainty about the level of future inflation. This suggests that the index linked yield and the real component of the fixed interest yield are not the same, although they are related. Considering the smaller size and liquidity of the index linked market we decided to model the index linked yields using the fixed interest yields as an exogenous variable.

Other influences, which we have not explicitly included, may come from the world bond markets, because global bond yields are not affected by UK inflation. Exchange rate fluctuations or demand from institutions such as pension funds may have an effect over the shorter term.

We model the force of gross redemption yield, $ilg_t = ln(1 + ilgry_t)$, similar to our conventional bond model. We have modelled index linked bond yields using conventional bond yields and expected inflation. We found that using a proportion of expected inflation provides a better fit and that the data suggested that there was a need for a lagged value. The following model gives the best fit:

$$ilg_t = \mu^{ilg} + \alpha_1^{ilg}(gilty_t - \mu^{gilty} - \alpha_2^{ilg}(infwe_t - \mu^{inf})) + \alpha_3^{ilg}(ilg_{t-1} - \mu^{ilg}) + \varepsilon_t^{ilg}$$

$$\varepsilon_t^{ilg} \sim N(0, \sigma^2)$$

%	μ^{ilg}	α_1^{ilg}	α_2^{ilg}	α_3^{ilg}	σ
Value	3.8	41	49	31	0.22
Standard Error		4.4	4.9	6.4	

All residuals fall in the corridor *(-2 s.e. , 2 s.e.)*, only the 1992 is close to *2 s.e.* The Jarque-Bera statistic (JB = 0.42).

We investigated possible links between the index linked bond yields and equities, as investors may regard equities as also providing some protection against inflation. The dividend yield on equities is similar in size to the index linked yield suggesting that dividends are expected to grow at least in line with expected price inflation. We did not pursue an explicit link between index linked bond yields and equities because of the limited amount of data available to estimate the parameters of the model. However, we have correlated the index linked residuals with the residuals from the equity model in 9.4.4.

It is not clear at this stage whether the link should lie within the equity or the index linked model. Due to the shorter length of data available for the UK market, there may be a lower degree of confidence in the index linked model. For this reason the residual link has been placed within the index linked model.

9 UK equities

Our UK equity model divides the total equity return into three separate elements: force of dividend yield, force of earnings growth and force of change in earnings yield. The change in earnings yield is used to derive equity prices. The change in earnings yields is a market sentiment term. Its inclusion allows us to model the decay of equity risk over time.

9.1 Earnings based (not dividend based)

In the UK it has been common actuarial practice to use the dividend yield to derive equity prices, ie as a measure of value. However, earnings yields (or price/earnings (P/E) ratios) are widely accepted as a better indicator of equity valuation than dividend yields. Investment analysts rarely use dividend yields even as a secondary measure of value but often assess the value of shares using the P/E ratio as a starting point.

Of course equities can be modelled using any measure of value such as; dividend yield, earnings yield, cashflow yield, net asset value, etc. These measures of value correspond to investment styles used by fund managers to identify value. A paper by Fitzherbert (1998), published whilst our model was being finalised, discusses the possible use of a variety of accounting measures.

Any model of equity total return involves the modelling of dividend yields, to take account of the income portion of the return. However, the major advantage of analysing earnings (or in our case earnings yields) is that one can use changes in both earnings yields and dividend yields to explain changes in equity prices. We believe this should enhance the model fit to data and its stability over different time periods. Earnings growth as a time series is easier to model than dividend growth, as earnings growth is more reactive to changes in the economy whilst dividend growth is smoothed by company directors.

There are limits to the quality of earnings data since earnings are an accounting concept, which have undergone significant changes over the period under investigation. (Similar criticisms can be made of other series. For example, dividends have been affected by changes in corporate taxation and restrictions on dividend increases.) However, most of these changes are step changes, which have a one off effect. Since we model the "change in yield", each step change in yield only affects one term in the series and so is unlikely to introduce serious distortion into the model.

9.2 The three components of equity returns

We have ensured an additive relationship between the modelled variables by decomposing the total return into three separate elements: force of dividend yield, force of earnings growth and force of change in earnings yield. If we denote equities total return at time t as TOT_t then:

$$1+TOT_t=\frac{P_t+Y_t*P_t}{P_{t-1}}=\frac{P_t}{P_{t-1}}*(1+Y_t)$$

$$etot_t=\ln(1+TOT_t)=\ln(\frac{P_t}{P_{t-1}})+\ln(1+Y_t)=ep_t+ey_t$$

Where P_t is the price index
Y_t is dividend yield
$etot_t$ is the force of total return over year t,
ep_t is the force of price growth over year t,
ey_t is the force of dividend yield at time t

The price growth element of the above equation can be further decomposed.

$$\frac{P_t*ye_t}{P_{t-1}*ye_{t-1}}=\frac{E_t}{E_{t-1}}=(1+ge_t)\text{, i.e. }\frac{P_t}{P_{t-1}}=(1+ge_t)*\left(\frac{ye_t}{ye_{t-1}}\right)^{-1}$$

$$ep_t=\ln\left(\frac{P_t}{P_{t-1}}\right)=\ln(1+ge_t)-\ln\left(\frac{ye_t}{ye_{t-1}}\right)$$

Hence, $etot_t = ey_t + eeg_t - dlogeey_t$,

Where, E_t is the earnings index
ge_t is the annual effective rate of growth in earnings over the year
$eeg_t = ln(1+ge_t)$ is force of earnings growth over the year
ye_t is the earnings yield
$dlogeey_t = \log(ye_t/ye_{t-1}) = \log(1+(ye_t-ye_{t-1})/ye_{t-1}) \sim (ye_t-ye_{t-1})/ye_{t-1}$

The great part of the volatility in the force of price growth comes from the final term, which we call the change in rating. We can look at $(ye_t-ye_{t-1})/ye_{t-1}$ as the relative change in the yield and therefore the $dlogeey_t$ term can be seen as a force of relative change in yield.

9.3 Features of income, growth and change in rating

Descriptive Statistics 1931-1997

%	*ey*	+	*eeg*	*-dlogeey*	=	*etot*
Mean	4.86		4.92	1.20		11.10
Median	4.76		7.17	1.97		13.00
Std.Dev.	1.05		14.95	22.08		18.19
Skewness	81.24		-56.80	-111.46		-78.96

The table above shows that the most important terms contributing to the annual volatility in the force of equity returns are the change in rating term, *-dlogeey*, and then the eeg term. The equity return data suggests an overall negative skewness of 79%, which reduces to 35% if the 1974 crash is excluded.

9.3.1 Income

The dividend yield term, *ey*, has a very low standard deviation relative to the other terms. It therefore has a relatively small effect on the volatility of the annual equity return over the short term. The equity dividend yield is positively skewed. This positive skewness is due the exponential form of the relationship between the yield and change in rating term (which we model as having zero skewness).

9.3.2 Growth

The equity earnings growth term, *eeg*, is derived from the underlying growth in the economy and the structure of corporate debt. The skewness of *eeg* is a reflection of equity earnings falling faster in a recession than rising in a boom. The earnings growth has an increasing effect on annualised equity returns over longer periods.

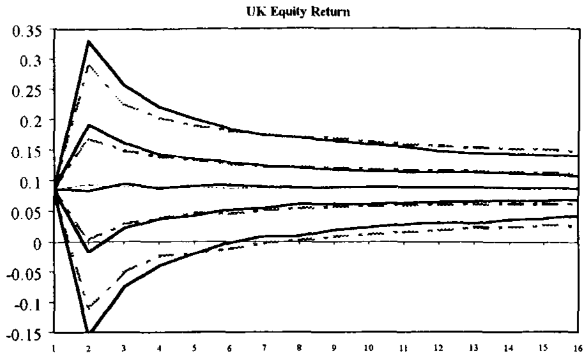
9.3.3 Change in rating

The change in rating is expressed here as *-dlogeey*. The negative median value of *-dlogeey* reflects a falling earnings yield over the period. The negative skewness of *-dlogeey* means that equity market falls (crashes) are steeper than rises. However, if the number for the 1974 crash is removed, the large negative skewness of *-dlogeey* reduces from 110% to –10%.

The change in rating is the major volatility effect in short term equity returns. The rating is a measure of the amount the market is prepared to pay for equities and it changes according to short term influences such as investor sentiment; supply and demand; valuation relative to other assets etc. The change in rating for equities is positively correlated with the change in rating for long term government bonds (change in gross redemption yield).

The data shows an improvement in rating over the period, with earnings yields falling from highs of 20% to present levels of around 5%. The question arises as to whether this trend should be projected to continue into the future. We concluded that there must be a limit to the re-rating that can occur. Accordingly within each individual scenario we allow sustainable changes in earnings yield to occur, but over the long term the mean annual change earnings yield is zero. If it were not zero, then in the long term earnings yields would move to either zero or infinity. Across all scenarios, the mean earnings yield over the short and long term is defined in the basis.

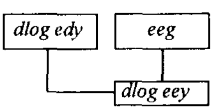
Since the change in rating is modelled as a stationary series we allow equity dividend and earnings yields to drift rather than forcing yields to revert to a particular level. This removes an arbitrage opportunity by not allowing equity rating to be predicted in advance. In the very long term, the absolute value of the yield can, in extreme individual projection years of simulations, reach relatively large or small numbers. However, this does not arise over the periods in which we are interested.



The advantage of splitting out the rating term is that it enables the model to encapsulate high short term volatility, without excessive long term volatility. If the model were constructed without such a term, it would show too small a volatility of annualised equity returns over shorter time periods and excessive volatility over longer periods.

The above graph plots annual equity return over a 15 year projection period and the dotted line shows the effect of excluding rating from an annualised UK equity return model. Whilst the change in rating is the major volatility effect in short term equity returns, over longer projection periods, the effect of the change in rating on the annualised return diminishes as 1/n, and the earnings growth term dominates.

9.4 The equity model



Our model is based on earnings and hence we need to model *eeg*, *dlogeey* and *ey*. Earnings growth (*eeg*) is based on the fundamental state of the economy, and is therefore modelled first. Change in earnings yield (*dlogeey*) depends on the change in dividend yield (*dlogedy*) and the earnings growth (*eeg*), using information from the other variables to better model *dlogeey*.

Some equity models are derived from a yield and a growth term, with the price being generated by dividing a growth index by the yield. Such a division of random variables can produce a distribution that is difficult to understand and is likely to contain extreme values.

We believe that modelling three homogeneous segments produces a better description of the shape of the volatility structure of equity returns and hence give the model a better fit to data than only using an income and a growth term. However, this approach does impose weaker restrictions on the absolute value of the yield over the very long term.

9.4.1 Earnings growth (eeg_t)

$$eeg_t = \mu^{eeg} + \alpha_1^{eeg}(sal_{t-1} - \mu^{sal}) + \alpha_2^{eeg}(cash_t - cash_{t-1}) + \alpha_3^{eeg}(dloggilt_t - dloggilt_{t-1} - dloggilt_{t-2} + dloggilt_{t-3}) + \varepsilon_t^{eeg}$$

$$\varepsilon_t^{eeg} \sim N(0,\sigma^2)$$

Where $dloggilt_t$ is the force of change in yield of long term bonds over year t and has an average value of zero as the bond yield is assumed to have a stable long term mean.

μ^{eeg} is the user defined mean equity earnings growth.

%	μ^{eeg}	α_1^{eeg}	α_2^{eeg}	α_3^{eeg}	σ
Value	4.7	86	142	22	11.7
Standard Error		17	85	5.5	

There are three residuals (1940, 1962 and 1981) which fall outside the corridor *(-2 s.e. , 2 s.e.)*. The first two distortion are data driven, 1940 relates to the second world war and 1962 is the point at which we link our two equity data series. Removing the 1962 distortion via an intervention variable reduces the Jarque-Bera (JB) statistic, JB = 3 (JB = 29 including 1962) and residuals are normal at the 5% level.

The major effect on *eeg* is salary inflation, the role of which is analogous to nominal GDP growth in some other models.

The $dloggilt_t$ term is worthy of some explanation. It represents the change in capital value over the year of a bond perpetuity with yield $gilt_t$ and $gilt_{t-1}$ at times t and $t-1$ respectively. We came to investigate the use of this function (change in bond rating) by considering the excess return or risk premium equation between equities and bond perpetuities.

excess return	= *total return on equities less total return on bonds,*		
	= *(ey – gilty)*	+ *eeg*	+ *dloggilt - dlogeey*
	= *income*	+ *growth term*	+ *relative changes in rating*

This suggested investigating the use of *dloggilt* rather than *gilty* (the force of bond yield) in modelling some of the equity variables. In fact, it was found that using *dloggilt* gave better adherence to data and parameter stability for the equity model, in much the same way we prefer to model change in earnings yields rather than the earnings yield itself.

One might expect *eeg* to be affected by relative changes in the bond yield to reflect a change in the cost of corporate borrowing (hence the relevance of the *dloggilt* term). Although the parameters were derived with reference to the data, the form of the dloggilt expression can be further explained. Since the signs on the coefficients of the *dloggilt* terms are equal and opposite, a change in bond yields has only a one off impact on *eeg*.

There is currently speculation that UK companies may change their capital structure, possibly choosing more long term debt rather than equity to finance future expansion. Such a change, if it occurs, may affect the size of the bond and cash links in this equation.

9.4.2 Change in dividend yield ($dlogedy_t$)

We have seen that a change in the *log(yield)* represents an important component of the force of price growth. This suggests modelling the change in the *log(yield)* and then using an appropriate starting value (allowing for any permanent changes in dividend paying policy) one can derive the absolute values. This approach also prevents the possibility of a negative dividend yield. [$y_t = y_{t-1} * e^{-dlogedy_t}$]

There are two possible methods of modelling dividend and earnings yield. The first approach is to model the change in earnings yield and then the dividend yield. The alternative is to model dividend yields first and then explain the change in earnings yields via the dividend yields. We found that the latter approach gave a better statistical fit to UK market data.

$$dlogedy_t = \alpha_1^{edy}\,(dloggilt_t + dloggilt_{t-1}) + \alpha_2^{edy}\,dlogedy_{t-1} + \varepsilon_t^{edy}$$
$$\varepsilon_t^{edy} \sim N(0,\sigma^2)$$

We set $\mu_{edy} = 0$ ie anticipating a zero average future change in rating.

%	μ^{edy}	α_1^{edy}	α_2^{edy}	σ
Value	-3.3	73	-41	15
Standard Error		11	15	

There are two residuals (1940, 1968) which fall outside the corridor *(-2 s.e. , 2 s.e.)*. Excluding the war period, when parts of the equity data series had to be interpolated, the Jarque-Bera (JB) statistic is JB = 2.4 (JB = 5.1 when we retain the distortion) and normality of residuals can be accepted at the 5% level.

The $(dloggilt_t + dloggilt_{t-1})$ term is equal to the change in bond rating over the two years from t-2 to t. The parameter α_2^{edy} implies an autocorrelation which suggests that market sentiment tends to overshoot. This effect has changed over the period and an intervention variable was used to model the more pronounced autocorrelation in the second half of the century. This autocorrelation will also act to reduce volatility over periods of greater than one year.

9.4.3 Change in log earnings yield ($dlogeey_t$)

For reasons similar to ones given in the dividend analysis section, we prefer to model the change in the log earnings yield rather than the absolute value.

$$dlogeey_t = \alpha_1^{eey}\,dlogedy_t + \alpha_2^{eey}\,(eeg_t - salw_{t-3} - (\mu^{eeg} - \mu^{sal})) + \varepsilon_t^{eey}$$
$$\varepsilon_t^{eey} \sim N(0,\sigma^2),\ salw_t = 0.12\,salw_t + 0.88\,salw_{t-1}$$

$salw_t$ is an exponentially weighted moving average of salary inflation.
We set $\mu_{eey} = 0$ ie anticipating a zero average future change in rating.

%	μ^{eey}	α_1^{eey}	α_2^{eey}	σ
Value	-2.0	93	73	5.3
Standard Error		3.4	5.2	

There is one residual (1993) falling outside the corridor *(-2 s.e. , 2 s.e.)*. The Jarque-Bera (JB) statistic, JB = 3 and normality of residuals is accepted at the 5% level.

Earnings yields and dividend yields are linked via the payout ratio which is reflected in a 91% observed correlation in historical data between the change in rating series for earnings and dividends and in our model by the 93% parameter in the above equation.

The above equation can be transformed to leave the force of change in payout ratio on the left:

$$-(dlogeey_t - dlogedy_t) = (1-\alpha_1^{eey})\,dlogedy_t - \alpha_2^{eey}\,(eeg_t - salw_{t-3} - (\mu^{eeg} - \mu^{sal})) - \varepsilon_t^{eey}$$

Since α_l^{cey} is close to one, this transformation shows that payout ratios fall when corporate earnings growth exceeds salary inflation. This is understandable since companies are reluctant to increase dividends on the basis of one good year of earnings.

9.4.4 Correlating Index Linked Gilt Residuals With Equities

As discussed in section 8, we found the relationship between the index linked gilts and equities yields has been consistent over the period for which index linked yields have been available. We therefore investigated the model residuals for evidence of correlation. The only significant correlation, –66%, was between the *ilg* (index linked yield) and *eeg* (equity earnings growth) model residuals. Since there is less index linked data the index linked model may be less stable than the other models. We therefore prefer to include the links in the error term of the index linked bond model, leaving the relationship between UK equities and fixed interest bonds unchanged. The revised error term is:

$\varepsilon^{ilg} = -0.00124\,\varepsilon^{eeg} + 0.00146\,z^{ilg}$
Where the z^{ilg} is a unit normal variable, independent from ε^{eeg}.

Although there was some evidence of the change in payout ratio, *dlogedy* - *dlogeey*, affecting the index linked yield, we do not feel that the small number of data points warrants another parameter and prefer to leave only the residual correlation link.

10 Overseas equities

10.1 Introduction and model structure

In September 1998, 19% of UK pension funds assets (WM, 1998) were invested in overseas equities, of which 51% was invested in Europe, 22% in North America, 13% in Japan, 9% in Pacific (Ex Japan), and 5% in others. Hence, overseas equities are the second most important asset class after UK equities for the average UK pension fund. The allocation to overseas equities is usually considered relative to the UK equity allocation. It is therefore important to investigate how overseas and UK equities are related as well as the relationship between how the different overseas markets.

We modelled sterling total return data rather than using local currency returns and a currency model. We believe the former approach is reasonable as currency modelling has a poor long term record and currency and stock markets are interrelated. We considered applying the UK equity model structure but considered this too complex for UK pension fund applications. We therefore decided that a total return model of the four overseas markets would be sufficient.

Whilst a vector autoregression model would have captured the historical data links between the different markets, the historic returns from these markets may be atypical and may not continue into the future.

We worked from the premise that if markets are efficient then the risk-adjusted expected returns from the four overseas markets and UK should be equal. We therefore decided to "estimate" standard deviations and correlations of the four markets combining historical data and our own expectations about future performance. These were used to fit correlated random noise around fixed means with the desired covariance characteristics built in.

The calculated standard deviations are very dependent on the chosen period. We examined monthly-calculated standard deviations of rolling 3-year returns, and derived gaps in standard deviations between countries. Finally we added the gaps to an assumed UK equity standard deviation of 20%.

uk	usa	eur	jap	pac
20%	25%	22%	35%	30%

Correlations of rolling 3-year returns are more difficult to predict than standard deviations and have shown great shifts over different periods. Therefore we combined our judgement with evidence from data to produce the following correlations between the UK and four overseas equity markets.

%	uk	usa	japan	europe	pacific
uk	100				
usa	85	100			
japan	20	40	100		
europe	85	70	20	100	
pacific	50	30	60	60	100

10.2 The Model

We aimed to build a model which forecasts the total returns from the overseas markets in a manner consistent with the UK equity market return. The central rates of returns are set equal to the UK equity return and the following equations specify deviations from this central rate:

$$usa\_dev = 21\%*\varepsilon_{uk\_dev} + 13\%*\varepsilon_{us}$$
$$jap\_dev = 7\%*\varepsilon_{uk\_dev} + 15\%*\varepsilon_{us} + 31\%*\varepsilon_{jap}$$
$$eur\_dev = 19\%*\varepsilon_{uk\_dev} + -1\%*\varepsilon_{us} + 1\%*\varepsilon_{jap} + 12\%*\varepsilon_{eur}$$
$$pac\_dev = 15\%*\varepsilon_{uk\_dev} + -7\%*\varepsilon_{us} + 21\%*\varepsilon_{jap} + 7\%*\varepsilon_{eur} + 12\%*\varepsilon_{pac}$$

Where ε_{us}, ε_{jap}, ε_{eur} , ε_{pac} are independent unit normal random variables and ε_{uk_dev} is the normalised UK equity model deviation from the central value and is derived from the UK equity model.

We chose the following overseas equity asset allocation to illustrate the features of the models: *world=50%*us + 25%*eur + 15%*jap + 10%*pac*

There are a number of advantages to this approach. The model is fairly simple and replicates the desired variance and correlation structures. It also allows for the use of individual judgement in addition to data considerations in choosing the parameters. The model is also robust to changing investment conditions.

The disadvantages include no attempt to autoregress on previous years' returns (a trade off for being able to specify a correlation structure). In addition, no growth or rating measures are used in its forecasting and there is no direct link to global or domestic yields on short or long bonds, (although UK market correlations provide some link). Finally a fixed correlation structure will artificially understate or overstate the diversification provided by overseas markets (i.e. the model may limit the correlations which will appear in extreme conditions).

11 Use of the model

11.1 Setting the basis

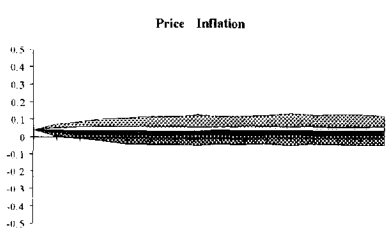
The models require a "basis" containing the long term central values of the projected variables. The existing model parameters are acceptable in their current form for use with a range of bases. We set out below an example basis, which is in the form of annual effective returns or yields (in accordance with most actuarial modelling) but the corresponding forces of interest ie ln(1+annual rate) are used for simulations.

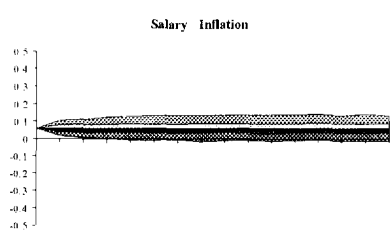
price inflation	4.0%	cash yield	5.0%	earnings yield	6.0%
salary inflation	6.0%	index linked bond yield	3.0%	earnings growth	5.8%
long bond yield	7.0%	dividend yield	3.0%	equity return	9.0%

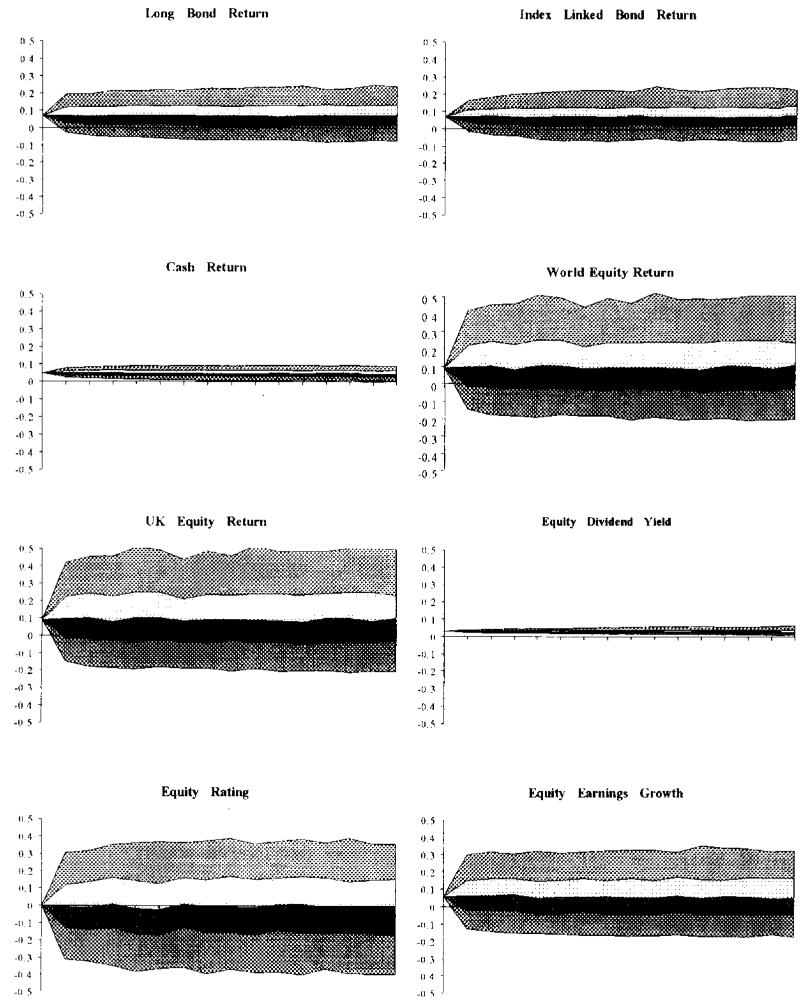
When an asset model is used in practice, the choice of the mean values for the economic variables is important. For example, the estimated historic mean value of price inflation is close to 5% but in the current economic conditions there is a focus on keeping inflation low (eg the 2.5% inflation target in the UK) and we chose 4% as a prudent estimate of the long term level. However, it should be noted that changes from the historical averages may require adjustments to other parameters. Changing the standard deviation after a large change in central value is the most obvious example. Ranne, 1998 showed a range of central values and standard deviations for several variables experienced by the twelve countries that were investigated. He concluded that there is a positive correlation between the two statistics.

11.2 Model output – "projection year" results

We investigated the ranges within which each modelled variable may lie. One of our objectives was to obtain plausible values from the model output.







The graphs give an indication of the ranges over a 15 year projection period, the middle line is the median, the lines below and above it – the lower and upper quartiles respectively, and the last two lines – the 5% and 95% percentiles.

11.3 Model output – "ranked simulation" distributions

As noted earlier in 3.4.2, we have calculated "ranked simulation" distributions of the mean, standard deviation, skewness and kurtosis of forces of yields and returns over the 15-year projection period. The table below gives the lower quartile (LQ), median (MED) and upper quartile (UQ) of the distributions. These numbers represent the ranges of experience for the projection period under investigation.

%	mean			standard deviation			skewness			kurtosis		
	LQ	MED	UQ	LQ	MED	UQ	LQ	MED	UQ	LQ	MED	UQ
prices	3.2	3.9	4.6	2.3	3.1	4.3	-59.2	-2.4	53.1	-39.4	39.4	147.9
salaries	5.0	5.9	6.6	2.5	3.0	3.7	-42.6	1.1	48.2	-64.0	-5.6	84.0
long bond	6.4	6.8	7.1	0.6	0.8	1.0	-42.5	-1.6	40.6	-86.2	-34.4	32.6
cash yield	4.2	4.9	5.6	1.5	1.8	2.2	-35.3	0.6	40.3	-86.5	-40.1	19.8
index-linked	2.8	3.0	3.1	0.3	0.4	0.4	-35.3	3.1	39.6	-90.7	-41.6	27.1
dividend yield	2.5	3.0	3.5	0.5	0.6	0.8	-1.9	33.1	74.9	-96.8	-50.1	29.6
equity rating	-2.3	-0.1	2.4	18.3	21.0	24.1	-41.7	-0.5	34.6	-57.4	-2.4	79.5
earnings growth	3.9	5.6	7.5	11.0	12.7	14.5	-40.8	1.6	41.7	-58.2	-3.9	80.4
UK equity return	7.2	8.7	10.7	15.2	17.6	20.2	-37.2	0.7	40.3	-60.1	-6.3	76.5
World equities	6.8	8.7	10.9	16.0	18.3	21.1	-37.3	0.1	40.4	-60.0	-7.5	77.7

11.4 Model output – ranked simulation correlation results

Asset and liability models are often used to choose optimal asset allocations (taking liabilities into account). The technique of the efficient frontier can be used to produce the optimal asset allocation for each return-risk profile. Correlation assumptions contribute significantly to determining the efficient frontier. We calculated the correlations over the projection period for each simulation and then calculated the correlation distributions. The first table gives the median correlation between the yields and the second, the median correlation between returns.

Yield correl %	prices	salaries	long bond	cash	index linked	dividend yield	equity rating
prices	100.0						
salaries	65.3	100.0					
long bond	51.2	41.8	100.0				
cash	22.8	18.8	48.2	100.0			
index linked	-22.3	-19.2	36.3	13.2	100.0		
dividend yield	24.2	22.0	54.7	24.3	13.7	100.0	
equity rating	30.4	24.3	28.6	17.2	-6.5	38.0	100.0

Return correl %	prices	salaries	long bond	cash	index linked	earn growth	UK equities	world
prices	100.0							
salaries	65.3	100.0						
long bond	-50.3	-29.0	100.0					
cash	22.8	18.8	-21.9	100.0				
index linked	65.8	42.1	13.6	3.8	100.0			
earnings growth	21.9	24.4	-34.4	14.5	29.3	100.0		
UK equity return	-19.6	-9.9	34.4	-9.1	7.3	7.4	100.0	
world return	-16.1	-8.2	28.5	-7.9	5.6	7.0	82.8	100.0

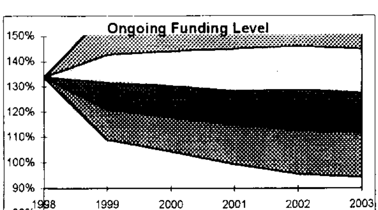
11.5 Model output – projection year correlations of annualised returns

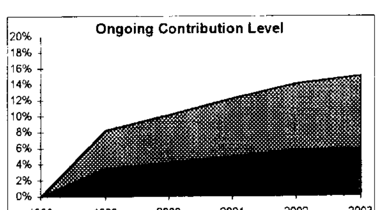
%		1	5	10	15	30
price	salary	38.9	77.5	89.5	91.7	99.5
	long bond	-34.0	-66.5	-82.1	-86.4	-98.6
	index linked	52.2	83.1	83.1	79.9	39.8
	cash	11.9	23.3	44.5	49.9	91.4
	uk equities	-0.6	-26.4	-13.9	9.5	76.4
	world equities	1.6	-4.4	-7.1	-5.2	14.9
salary	long bond	-10.8	-54.5	-75.3	-80.5	-97.9
	index linked	22.4	67.1	78.0	75.7	40.8
	cash	3.1	20.9	40.2	45.7	91.0
	uk equities	3.4	-13.7	-4.6	18.0	77.6
	world equities	6.8	-0.3	-4.6	-1.5	15.9
long bond	index linked	44.7	-44.4	-73.7	-77.4	-28.9
	cash	-39.5	-43.5	-54.7	-59.9	-90.2
	uk equities	28.9	32.0	12.3	-14.2	-74.5
	world equities	21.5	18.1	15.9	9.9	-14.0
index linked	cash	-19.0	15.3	36.8	44.0	41.5
	uk equities	25.8	-11.2	6.1	31.1	50.3
	world equities	22.0	1.1	-6.1	-2.4	12.6
cash	uk equities	-6.0	-7.4	-7.5	10.8	72.3
	world equities	-1.3	0.9	-3.7	-2.3	15.1
uk equities	world equities	77.6	51.1	45.1	38.6	30.6

11.6 An example asset and liability model

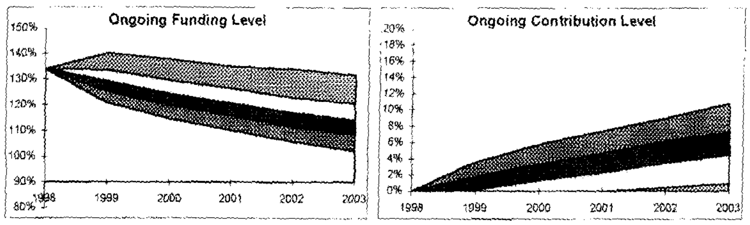
In this section we give a simple example of how the model is used in practice. We consider the case of a UK defined benefit pension scheme and investigate the effect of different asset allocation policies on the key variables of the funding level and the employer contribution rate. The scheme's objectives for these key variables are firstly to maintain an adequate funding level in all but the worst circumstances, and secondly to have as low an employer contribution as possible. Surplus and deficits are spread over the average future working lifetimes. We have considered the following asset allocations:

A - 100% UK equities



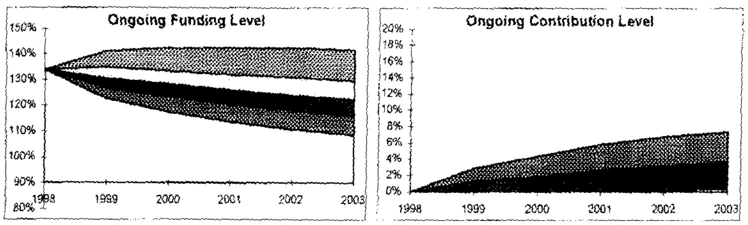


B – 100% government bonds

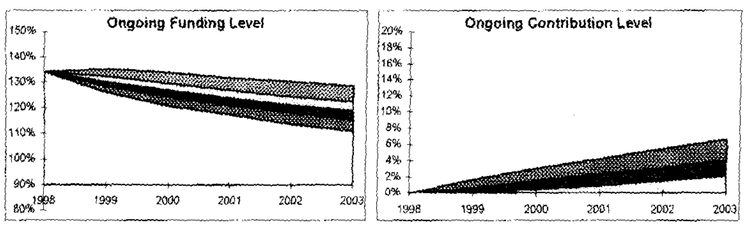


The 100% equity strategy results in an unacceptable risk of a below 100% funding level. This is due to the high volatility of equity returns. The 100% government bond strategy results in an unacceptable median decline in funding level, due to the lower return.

C – 50% UK equities and 50% government bonds.



D – 25% UK equities and 75% government bonds.



Whilst both mixed strategies remain above 100% funding level, they have very different risk characteristics. The 25% equity strategy has a lower short term risk of rising contributions, but it will result in steadily deteriorating funding and contribution levels. The 50% equity strategy has a greater risk of high contributions being required in the short term, but produces more sustainable funding and contribution levels.

The model enables the trustees (or company management) to make an informed decision on asset allocation. If the paramount consideration is company cash flow, then the 25% equity strategy may be appropriate. However, in most other circumstances, a 50% equity strategy would be more appropriate for the long term stability of the scheme.

12 Conclusions and further research

Our main aim was to produce an asset model that would be of use to investors with a time horizon of over five years. Such investors want to achieve high returns by investing in equities and will wish to analyse the extra risk that this strategy may entail.

The decay of equity risk over time is therefore of great importance. We have captured this decay feature by dividing the equity return into its three fundamental components: yield, growth and change in market rating. By modelling these components separately the model is able to capture the high short term volatility (which derives from market sentiment) and the significantly lower long term volatility (which arises from economic fluctuations).

Since the 1950s, the effect of earnings yields falling from 20% to 5% has made a substantial contribution to the total return on UK equities. This has caused much confusion when people have tried to calculate the retrospective excess return on equities over gilts. A sustainable prospective excess return might perhaps be better estimated by considering the historic equity return without the rating component.

The model was built primarily for asset and liability modelling in UK pension funds; however the model can be used for applications in life and longer term general insurance with some extensions being required, in particular a model of medium term government bonds.

There are a number of areas where we are continuing to develop the model, either by extension into other asset classes or by using alternative statistical techniques to capture more features of the investment markets. These are outlined below.

12.1 Extension into other asset classes

It would be relatively simple to extend the model to include medium term government bonds, which will be useful in insurance applications. Although the model could be used in its current form with medium term bonds yields taken to equal long term ones, this would effectively assume that the long end of yield curve is always flat.

There are formidable difficulties confronting anyone who tries to produce a stochastic model of the investment performance from property. There is not a great amount of data, and the data that is available requires great care in its use. Most of these problems derive from the fact that property is not easily marketable, and so market tests of value are much less frequent than with equities or bonds. For UK pension funds property is usually only a small proportion of the assets of the fund. Where a fund does hold a significant amount of property, the portfolio will be unique, and it is likely that general modelling of the whole property market will not be applicable. Our work on a property model is not concluded.

The model is UK based, and models other equity markets by reference to the UK. This has the considerable advantage of simplicity in practical work. However, the essential three component structure of our equity model should be applicable to other equity markets, and this would be a useful and interesting extension.

12.2 Alternative statistical techniques

Our model for price inflation is symmetrical around the mean. But with inflation now at quite low levels in the UK, it may be more appropriate to have an asymmetric distribution, which recognises that if mean inflation is low, say 3%, then the probability of inflation of 13% is greater than the probability of 7% deflation.

In price and salary inflation, as well as the cash models, the residuals are not very normal. Different approaches can be used, eg use of non-normal error terms, GARCH modelling and regime switching techniques. We intend to investigate the latter.

We are currently investigating modelling the changes in the variance of the change in rating. We have a variance model that reflects the low sustainability of volatile rating movements over annual periods.

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Appendix

For all series the June values on an annual basis were used to estimate the model parameters.

UK inflation data	1930-1947	Cost of Living Index
	1947-1997	Retail Prices Index
UK salary data	1930-1967	Basic Weekly Wage Rates Index
	1967-1997	Index of Average Earnings, All Employees
Fixed interest bond yield	1930-1962	2 1/2% Consols, Actuaries investment index
	1963-1977	2 1/2% Consols, FT-Actuaries Investment Index
	1977-1991	25 year FT Actuaries Index
	1991-1997	20 year FT Actuaries Index
Index linked bonds	1981-1985	FTA Index linked All Stocks Index, 5% inflation
	1986-1997	Over 5 years FTA Index linked Stocks, 5% inflation
Cash yields	1930-1969	Three Month Bank Bills
	1969-1979	Three month interbank rates
	1980-1997	Three month eurosterling deposit rates
UK equity prices and dividend yields	1930-1962	Actuaries Investment Price Index - Ordinary Shares
	1962-1997	FTSE All Share Index
Equity earnings yield	1930-1962	P/E ratio, Actuaries Investment Index
	1962-1992	P/E ratio, FTA 500-Share Index
	1993-1997	P/E ratio, FTSE All Share
Foreign equity data	1987-1998	FT/S&P A World Indices in Sterling terms