Asset and Liability Management by CADES,
a manager of public debt

Name	Eric Ralaimiadana
Department & affiliation	Asset and Liability Management, Caisse d'Amortissement de la Dette Sociale
Mailing Address	CADES 15, rue Marsollier 75002 PARIS (France)
e-mail address(es)	eric.ralaimiadana@cades.fr, eric.ralaimiadana@ensae.org
Phone number	331 55 78 58 19 / 331 55 78 58 00
Fax number	331 55 78 58 02

Abstract

The method chosen by CADES to steer the process of paying down the social security debt it has assumed is related to our particular asset and liability management policy. The economy is ruled by three factors, the dynamics of which govern the principal classes of negotiable debt instrument and our asset, which is the CRDS (*Contribution au Remboursement de la Dette Sociale*) and, partly, CSG (*Cotisation Sociale Généralisée*) joint taxes revenue, generated via a levy on nearly all forms and sources of income in France.

Risk is defined as the probability that we will not achieve an acceptable performance level in terms of debt repayment capacity, while our aversion to risk is reflected in the convexity of the relationship between performance and the redemption horizon.

We build projections of our balance sheet through the implementation of its components dynamics. As we formalize the amortizing process, we introduce the notion of amortizing capacity at the core of the expression which is the object of our optimization problem.

Then we exhibit different subsets of portfolios, subject to a pre-defined rule of re-balancing, in a two-axis space representing performance and risk respectively. Based on this representation, we define a direction of performance and risk optimization, thereby managing to provide guidance concerning the matter of optimizing the debt portfolio structure.

We finally describe the decision support tools which we have rolled out based on the results of the modeling, and which we have designed to provide guidance for debt allocation optimization.

Keywords : refinancing, amortizing capacity, redemption horizon, direction of performance and risk optimization, risk threshold.

Introduction

The role of CADES (*Caisse d'Amortissement de la Dette Sociale*) is to reimburse the accumulated deficits of the French social security or health insurance system. To this end initially, a single and exclusive resource has been allocated to CADES by law—the CRDS (*Contribution au Remboursement de la Dette Sociale*), to which a portion of the CSG (*Cotisation Sociale Généralisée*) was added by 2008 year-end. To respond to the following question—How can this debt be optimally repaid ?—we use our asset and liability management model.[1]

The management of debt offers some striking analogies with the management of assets. For example, a company that insures a given fleet of risks receives premiums and constitutes a portfolio of invested assets. It builds this portfolio to maximize return so that it can meet its liabilities toward the policyholders, cover its own operating costs and generate a profit margin. Similarly, CADES receives tax proceeds which are used to cover debt service and redemption costs.

Besides, while the taxable base fluctuates with the value of its components, the taxable base and rate are defined by law, and have not changed but once since the tax was first levied. Accordingly, we manage what might be considered a defined contribution plan, with the notion of contribution corresponding to tax inflows. Conversely, our liabilities are made up of the outflows by which we amortize the debt.

While the modeling of our asset and liability management is largely inspired by the theoretical tools used in asset management, articles on the management of debt *stricto sensu* are rare, since this kind of analysis is primarily conducted by organizations that are responsible for "sensitive" debt, i.e., within a government's public finance administration, a public service agency or a very large corporation.

The research work that our modeling strongly resembles is that of Brennan and Xia (2002)[1]. The authors defined the optimal investment strategy within a universe that did not contain any instruments generating an inflation-indexed return, made up of a savings account, a risky asset, and nominal fixed coupon bonds. They demonstrated that, for an agent with a finite investment horizon T, in the presence of an unanticipated inflation component not hedged by a market instrument, the optimal portfolio is the sum of two portfolios : one providing the return most strongly correlated to that of an indexed bond with a maturity of T; the other being the minimum variance portfolio as intended by Markowitz (1959)[2], combining the risky asset and the savings account.

Their findings revealed a strong sensitivity to agent risk aversion: the higher it is, (i) the higher the allocation to a portfolio replicating the indexed security, and (ii) the more the maturity of the nominal bond diminishes.

[1] We would like to thank Jean-François Boulier for having encouraged us to publish this article. We would also like to thank the anonymous arbitrator(s) of Banque et Marchés for their work and their extremely useful comments, as well as Patrice Ract Madoux and Christophe Frankel for their insightful remarks.

In their study, they cite work done by Campbell and Viceira (1999)[3], whose thinking our own closely mirrors. The latter used a numerical method to solve the optimization of the strategy of an investor with no horizon limitation, operating in the same investment universe as Brennan and Xia, using a so-called myopic strategy, i.e., with constant proportions. By exhibiting an optimal solution in an analytic form, Brennan and Xia underscore the loss of value generated by the myopic strategy, as well as the sensitivity of the results to two characteristics of the model: the investment horizon and the mean return parameters of variate diffusion processes.

A former article in Banque et Marchés (2004)[4] assesses methods for managing pension funds. We detected several points of contact with our own approach on the level of modeling the processes followed by the variates under study. In particular, that of Cairns (1998)[5] introduces a stochastic retirement flow into the modeling of a defined benefits fund. A retirement entitlement or pension is a fixed percentage of an individual's wage, which the author models using a diffusion process that is not correlated with market noise. In addition, the analyses developed in articles by Svensson and Werner (1993)[6], as well as by Koo (1998)[7], resonate directly with our own reflection. Their authors examine the optimality of the portfolio and consumption in the case of an agent with a stochastic wage. They introduce a source of non-duplicable risk via a negotiable instrument, thereby placing the problem within a framework of market incompleteness.

The rest of the article is structured as follows. We briefly review the regulatory framework that governs the functioning of CADES. Then, we describe our representation of the balance sheet in simple components, resulting in an economy regulated by three variates, the nominal short-term rate, the rate of inflation, and the rate of volume growth in the joint tax. Having written the diffusion equations followed by their processes, we explain our optimization problem and its resolution. We then display the decision support tools we have rolled out based on the results of the modeling. The last part of our paper is devoted to a critical review of the model and the changes envisioned, ending with a few conclusion on debt management informed by asset and liability management.

2. Review

Four texts mark the history of CADES:

- The seminal text is the French ordinance dated January 24, 1996, which defines the mission of CADES (i.e., extinguish the French social security debt) and sets its life span (i.e., until January 31, 2009). Debt outstanding totals 21 billion euros, plus annual payments to the French government of 1.9 billion euros, over a period of twelve years.
- The Social Security Financing Act (SSFA) of December 19, 1997 for year 1998 extends the remit of CADES for an additional five years, and transfers an additional 13.3 billion euros of debt.
- The Health Insurance Act of August 13, 2004 transfers 35 billion euros worth of deficit accumulated through 2004 to CADES, to which is added estimated debt of up to but not more than 15 billion euros. The Act also strikes all reference to a defined date on which CADES ceases to exist and holds that all new deficits from the social security system must be financed by a new resource. In addition, the Parliament asks CADES to report twice a year on its most probable final year of operation.

- The 2009 SSFA of December 17, 2008 transfers an additional 26.9 billion euros of debt, and allocates an additional resource of 0.2% from CSG (Cotisation Sociale Généralisée), which is roughly equivalent to a 0.19% levy from CRDS.

In what follows, we will illustrate the way CADES has tackled its asset and liability management issues in a changing environment due to regulatory amendments.

3. Methodology

3.1. Balance-sheet modeling

The CADES balance sheet can be broken down into four major items. The real estate holdings (assets) that were inherited when CADES was formed having been disposed of in full, its assets today consist of a receivable on the nationwide tax levied on nearly all sources of income (the CRDS), to which was added a portion of the CSG – with a quite similar taxable base. Its liability is financing debt, and it has no shareholders' equity.

3.1.1. Asset

The taxable base for the CRDS is earned income from work (67%), replacement income (21%), income earned from assets and investments (10%), gaming proceeds and the proceeds from the sale of precious metals (2%). Similarly, the breakdown of the income base for the CSG is work income (67%), replacement income (20%), income earned from assets and investments (12%), gaming proceeds and the proceeds from the sale of precious metals (1%).

To the extent that this joint tax is levied globally on all forms of income, a very straightforward way of modeling our revenue is to use gross available income, the national accounting aggregate, as a proxy for the taxable base. Another option would be to model separately transfer and wage income, asset and investment income, and assimilate growth in the remainder to a random walk. We opted for the most straightforward solution, noting that the taxable base has undergone numerous changes as well as various specific exemptions, and there is no reason to believe this will end : any advantage to be gained through more refined modeling by income category should be put into proper perspective considering the fluctuations due to changes in scope.

The next question, then, is to model the rate at which these income inflows grow, using a constant tax rate (0.69%). If we focus on the three most significant sources of revenue in the taxable base, we see that wages and old age income have experienced quasi-constant volume growth over the 1979-2001 period. Over the same period, investment income has undergone volume growth that can be assimilated to a trend growth plus a white noise.

A fairly simple modeling of our assets is based on diffusion equations for two processes, the real rate of growth in our tax revenue and the rate of inflation. They follow Ornstein-Uhlenbeck processes. The value growth of our asset is calculated through the composition product of the latter.

At time t, we note A_t the value of the asset, k_t its value growth rate, g_t its real growth rate and i_t the rate of inflation.

The dynamic of A_t is described by the following diffusion equation

$$dA_t = A_t k_t dt$$

And its rate of value growth is modeled by

$$e^{k_t dt} = e^{(g_t + i_t)dt}$$

The diffusion equations followed by these rate processes will be developed further on.

3.1.2. Net debt dynamic

Net debt varies in the following manner: at the end of each year, we note the balance between CRDS revenue received and outflows, either interest paid on outstanding debt, or additional new debt. If this balance is positive, we reduce the debt through buy-backs. If it is negative, then we must increase the level of our borrowings. By "financing balance," we mean the change in net debt after employment of this balance, and we note it S_t. In what follows, it will also be referred to as "amortization capacity".

To elucidate the net debt dynamic, we adopt the following notations :

let,

- $\hat{L}_t$ the value of the debt payable year t,
- L_{t-1}^t the value in current euros in t, of the inherited debt for year t – 1 before payments falling due ,
- L_t^* debt at end of year t before reallocation
- L_t the value in current euros in t, of net outstanding

The net debt dynamic is written in simple fashion

$$L_t = L_{t-1} - S_t$$

Net debt observed for year t, before payables arriving at maturity, is the sum of debt payable in t and debt at end of year t before reallocation, which is written as:

$$L_{t-1}^t = \hat{L}_t + L_t^*$$

We can show that net debt at end of year t after reallocation is written

$$L_t = L_t^* - S_t^{net} \qquad (1)$$

where S_t^{net} designates the balance net of financing, i.e. the balance S_t that is modified by the effects of time and market fluctuations on the inherited debt from year t – 1, as well as payables arriving at maturity.

Proof

Indeed, the net balance reads

$$S_t^{net} = S_t - \hat{L}_t - \left(L_{t-1} - L_{t-1}^t\right) \tag{2}$$

The financing balance is derived by calculating D_t*, the amount available at time t, expression in which* V_t *designates an eventual new inflow of debt and* c_t *designates operating costs*

$$D_t = A_{t-4} \exp^{\int_{t-1}^{t} k_u du} - V_t - c_t$$

If the value of debt was reduced owing to market fluctuation, then the financing balance is increased, and vice versa. Accordingly, we add to D_t*, the opposite of this change in value, i.e.* $L_{t-1} - L_{t-1}^t$*, to derive* S_t

$$S_t = D_t + \left(L_{t-1} - L_{t-1}^t\right) \tag{3}$$

The net balance is then written as

$$S_t^{net} = D_t - \hat{L}_t \tag{4}$$

so that net debt dynamic can be written as

$$L_t = L_{t-1} - \left(D_t + L_{t-1} - L_{t-1}^t\right)$$

$$L_t = \hat{L}_t + L_t^* - D_t$$

which, in accordance with (4), gives back equation (1)

■

The net balance S_t^{net} is given by (4). Depending on its sign, it represents either a financing capacity or a borrowing need. It constitutes the total of buy-backs or taps (at the initial proportions) of existing bonds at their price, measured in t.

3.1.3. Liability

Our liabilities are almost exclusively limited to our debt portfolio. CADES does not have shareholders'equity.

Debt is classified according to three types: fixed-rate instruments, bonds pegged to French inflation (in France, inflation excluding tobacco), and floating rate instruments, including medium term notes. The factors which rule liabilities are nominal interest rates and the rate of inflation.

We specify a Vasicek[8] model for the yield curve. It offers the dual advantage of integrating a mechanism of return to the mean and of allowing us to rebuild the entire curve from the short-term interest rate alone. It reflects a characteristic that has been demonstrated by econometric studies, i.e., the fact that changes in the short-term interest rate alone explain about 80% of all yield curve movements.

The variates that rule liabilities are finally, the short-term interest rate and the rate of inflation.

3.2. Processes assumed by relevant variates

The nominal short-term interest rate process is described by the following SDE (*Stochastic Differential Equation*)

$$dr(t) = a(b - r(t))dt + \sigma_r dW_r(t)$$

The formulation of the zero-coupon rate for the [t,T] period is

$$R(T-t,r) = R_\infty - \frac{1}{a(T-t)}\left\{(R_\infty - r)\left(1 - e^{-a(T-t)}\right) - \tfrac{\sigma_r^2}{4a^2}\left(1 - e^{-a(T-t)}\right)^2\right\}$$

The inflation rate process is described by an SDE that is identical to that of the short-term rate

$$di(t) = c(d - i(t))dt + \sigma_i dW_i(t)$$

While the growth rate of the CRDS in volume terms is described by the following diffusion equation

$$dg(t) = (m - g(t))dt + \sigma_g dW_g(t)$$

The three sources of risk, the Brownian motions W_r, W_i, W_g, are linked by their instantaneous cross-correlations $\rho_{r,i}$, $\rho_{g,i}$, $\rho_{g,r}$, respectively.

One has to transform this vector of Brownian motions into a new vector of uncorrelated Brownian motions. This is obtained by transforming the covariance matrix of the initial vector into a triangular matrix. In a 2-dimension case, applying the copula theory to the 2-dimension vector of the first Brownian motions W_r and W_i for instance, would yield the same transformation.

The dynamics of both the short-term interest rate and the inflation rate processes can be re-written as

$$dr(t) = a(b - r(t))dt + \sigma_r dW_r(t)$$

$$di(t) = c(d - i(t))dt + \rho_{r,i}\sigma_i dW_r(t) + \sqrt{1 - \rho_{r,i}^2}\,\sigma_i dZ_i(t)$$

where W_r, Z_i are uncorrelated Brownian motions.

The dynamic of the CRDS growth rate in volume terms is re-written using the same technique, but will not be shown here because of the length of the SDE.

4. The Optimization Problem

4.1. Formalization

The financing balances that are accumulated year after year will determine our capacity to amortize the debt. By iterating the dynamic equation of the net debt, we see that the change in the latter between years 0 and t is equivalent to the accumulated financing balance for each period. Optimizing the amortization of the debt can be written, as a first approach, as the maximization of the expected aggregate of annual financing balances.

The optimization program is written as follows

$$\max \mathrm{E}\left[\sum_{l=0}^{t} S_l\right]$$

The solution (or solutions) of the optimization program consists (or consist) of portfolio weightings. The result depends in particular on the re-balancing rule that is adopted. We have opted for the rule of reallocating each portfolio by maintaining the initial proportions, which is tantamount to seeking target portfolio structures that are maintained constant throughout the term of the mandate. We will note $(\varpi_{k,m})$ the vector of the debt weightings for each class, flagged by the index k, and for each maturity, flagged by the index m.

Indeed, we are trying to determine one (or more) structure(s) which, over the long term, enables us to achieve our objective—i.e., that of paying down the debt at the lowest possible cost to the taxpayer. In the context of a debt management strategy to be carried over a long period, and which we want to be as transparent as possible, choosing the constant proportion rule allows us to present (for example, to the ministries with supervisory power over CADES) optimal debt structures that in essence do not fluctuate along with future events.

Since the debt is transcribed in a portfolio of zero-coupon bonds, all refinancing or buy-back transactions are carried out at going market rates, which, like S_t and S_t^{net}, are measurable in t.

We will note $E_{k,m}(t)$ the final value of a given outflow paid on the k–th class of debt with maturity m, and $B_{t,m}$ the price at time t of a zero-coupon bond of maturity m.

When the net financing balance is allocated, the current value of the debt becomes

$$L_t = L_t^* - S_t^{net}$$

$$L_t = \sum_{k=1}^{K} \sum_{m=1}^{M-t} \left[B_{t,m} E_{k,m} - \varpi_{k,m} S_t^{net}\right]^+$$

where, for a given term X, X+ designates the positive part of X.

Indeed, if the portion of the balance allocated to the amortization of outstanding debt $E_{k,m}(t)$ exceeds the market value of the latter calculated year t, this outstanding will be redeemed in full, and the remainder will be added to the remaining available balance.

The debt amortization mechanism entails that, the year in which the net debt crosses the null value, the financing balance is positive and exceeds the current value of the debt observed and recorded at the end of the preceding year. This allows us to represent our optimization program under a dual form.

4.2. Dual form of the optimization program

We will briefly leave behind the particular case of CADES and take a look at the "stylized" case of an indebted corporation, that is ordered by its shareholder to pay off its borrowings, by allocating all of its operating revenues to repayment. This corporation's guarantor of last resort is the State.

Let's suppose that the corporation has made a commitment to the financial community to a probable date of full repayment H, and that its last borrowing falls due on this date. There are two possible outcomes on date H :

- either the corporation has correctly estimated the full debt reimbursement date and will have it repaid in full or even earlier, which translates as

$$\mathrm{E}(S_H) \geq \mathrm{E}(L_{H-1})$$

- or the corporation has underestimated the full debt reimbursement date, in which case

$$\mathrm{E}(S_H) < \mathrm{E}(L_{H-1})$$

In the event of the second outcome, the corporation runs the risk of seeing its credit rating downgraded, and of finishing repayment on a date that is later than the estimated date of full repayment.

Let's look more closely at outcome number two: underestimating the amortization period means that the corporation's financing requirement is $(L_{H-1} - S_H)$, which in turn requires either re-borrowing on less appealing credit and liquidity terms or turning to the guarantor of last resort to "absorb" the financing requirement.

The envisioned consequences of this outcome do not exist for CADES. Indeed, the agency enjoys the implicit backing of the State, and its revenues are levied on and taken from the national income. However, risk analysis via the reimbursement horizon is valid, and allows us to express the probability α that the reimbursement target will not be met, as a function of the risk quantile H (α), in the following manner

$$\alpha = \mathrm{P}\left(S_{H(\alpha)} - L_{H(\alpha)-1} < 0\right)$$

Accordingly, we can write our optimization problem as the minimization of a risk, in the form

$$\min \mathrm{P}\left(L_{H(\alpha)-1} - S_{H(\alpha)} > 0\right)$$

We do not know the analytic form of the probability density of the variate $X_t = S_t - L_{t-1}^t$, conditional upon $(\Im_t)$ filtration - filtration engendered by brownian motions $W_r(t)$, $W_i(t)$, $W_g(t)$. The presence of time increment t – 1 within the expression of X_t shows the « path-dependency » of X_t, and that of its probability density. The expression of the latter might not be trivial.

Nevertheless, we can simulate drawings into the conditional probability distribution of $S_t \mid \Im_t$. This is what is done in the course of our resolution process.

4.3. Estimator of the expected amortization capacity

The conditional probability density of X_t thus depends on the level that has been reached by this variate on date t. Starting from an initial level of debt L_0 at the beginning of year 0, X_t depends on the level reached by the aggregate balances between years 0 and t, noted $S^c(0,t)$, i.e., of the event

$$\left\{ S^c(0,t) = \sum_{l=0}^{t} S_l = x \right\}$$

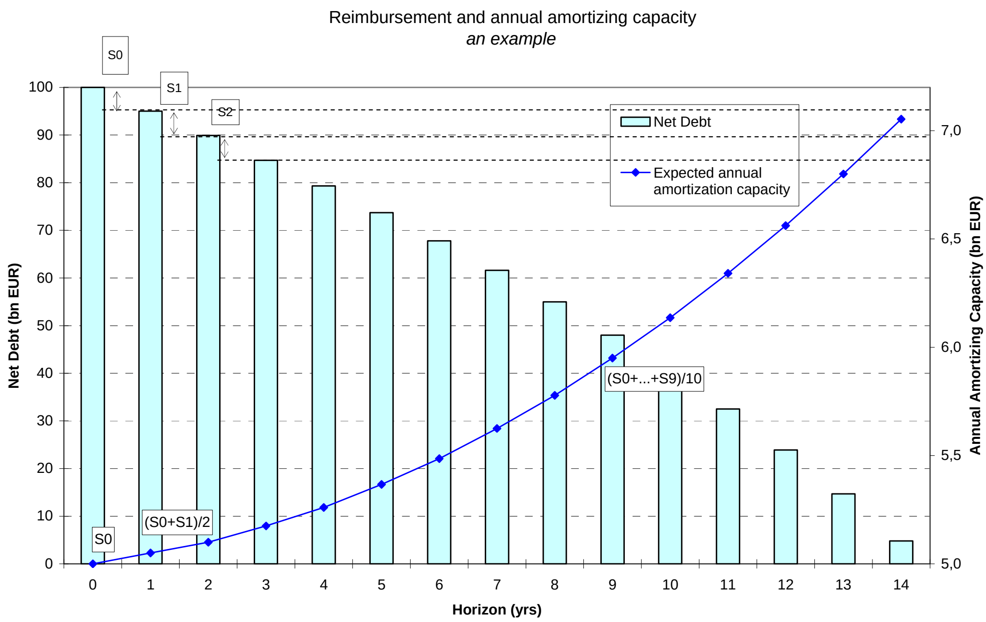
Our risk of failure, defined in the preceding paragraph as the risk of not achieving reimbursement by horizon H(α), grows with the decrease in x. The lower the aggregate balances, the more difficult it will be to reimburse before due date.

In like manner, we can use the same reasoning, considering the average annual balance (or the average annual amortization capacity) instead of the accumulated or aggregate balances for the period between years 0 and t, that we note $\bar{S}$ and calculate as follows

$$\bar{S} = \frac{1}{t} S^c(0,t)$$

As a matter of fact, the $\bar{S}$ statistic is a monotonic~~ous~~ function, strictly increasing, of the accumulated financing balances over the period extending from years 0 and t, scaled by a factor equal to $\frac{1}{t}$. For a given initial debt, the lower the average annual amortization capacity, the farther the reimbursement horizon will be. $\bar{S}$ is our estimator of the expected annual amortization capacity.

In the example that follows, of an initial debt of 100 billion euros reimbursed over 15 years, we illustrate the relationship between the level reached by $\bar{S}$, as observed at the end of the period, and the reimbursement horizon.



4.4. Risk aversion

At the same time, the statistic $\bar{S}$ is a decreasing function of horizon t, with a convex shape. The convexity of this function evidences CADES's aversion to risk. Indeed, we can write that, below some threshold of the annual amortization capacity achieved over the period, the lower the average annual amortization capacity the greater the increase in the amortizing period, for a deterioration in the capacity of the same magnitude.

Accordingly, there is a region of risky values that we wish to avoid, both for the accumulated balances between 0 and t and, likewise, for the average annual amortization capacity. The occurrence of such values, at the risk level α (%) at time t, entails the exploding trend of the reimbursement horizon.

4.5. The risk as seen from the perspective of the investor/taxpayer

To shed additional light on the risks we run, we look at the problem from the perspective of an investor who holds CADES debt and is a taxpayer a tthe same time.

Let us posit ourselves in the year H, our stated probable reimbursement horizon. Let us further suppose that the investor/taxpayer in question was holding a CADES bond reimbursed at horizon H, that CADES was obliged to issue a new bond to repay the bondholders, and that the investor reinvested in the new issue. The wealth of this investor, including taxes levied, will be reduced.

Indeed, assuming that the decision to postpone the date of final reimbursement does not have any impact on the credit spread, the investor will earn the same interest when the reinvestment is made, which is the yield on a CADES security. But he will be liable for an additional tax levy over a longer period due to the gap between the horizon estimated on date 0 and that which will in fact come to pass.

The analysis may be made from another angle : the Social Security Financing Act that was passed in August 2004 requires that all deficits be offset by additional tax revenue so as to maintain unchanged the date on which reimbursement is completed. Instead of bearing an extension of the contribution, the investor would be taxed “up front” for the resources needed to meet the financing needs of CADES on date H.

His profit/loss profile is that of a put selling position, the loss growing with the magnitude of the error of estimation committed with respect to the reimbursement horizon.

This analysis sheds light on the importance of the level of probability α . The more averse we are to the risk of making a mistake on the reimbursement horizon, and the consecutive one of having to resort to levying additional tax, the more we will require that α be small.

4.6. Optimality criterion

The selection of an optimality criterion is one of the pillars supporting any optimization program. Our variate under study, the expected annual financing balance, constitutes a gross performance measure. It is inadequate in that it evacuates from the criterion the impact of the risk, whose importance we have measured, and does not take into consideration CADES’ aversion to risk. Accordingly, we need to use a risk-adjusted performance measurement as our criterion.

At a time before the regulatory amendments aforementioned, when the reimbursement horizon was fixed, we were focused on the net financial position measured at this horizon. We used to compute the distribution of this variable, such as simulated at this fixed horizon. The risk-adjusted performance measurement we used was a Sharpe ratio, calculated over this distribution. The intermediate values reached by the variate under study was not the matter.

Within the new context that has emerged since the initial end date assigned to CADES was withdrawn – i.e., the initial end date set forth in the decree of December 19, 1997 for the social security financing act for 1998—we stop the dynamic of the annual financing balance process in the period where the net financial position becomes positive. For a given debt structure, we calculate the expected annual balance for each drawing, thereby computing its simulated distribution.

Next, we need to adjust this gross performance for risk, so that we can get an idea of the quality of the distribution. Indeed, an effective strategy must both optimize the expected annual balance and reduce the previously defined risk, to which we are averse.

We use the following criterion

$$\mathrm{E}(S_t) - k\sigma(S_t)$$

This criterion rewards strategies that optimize the expected annual balance and penalizes those that allow a large dispersion. According to the strong law of large numbers, if we postulate that the distribution of the expected annual balance is Gaussian, 5% of the distribution stands outside the radius interval $1.96\sigma(S_t)$ centered in $\mathrm{E}(S_t)$. We use this value for k, which is typically rounded off to 2.

4.7. Solving method

Using target portfolios, we bring up the debt portfolio structures that turn out to be optimal under the vast majority of scenarios, assuming that a given portfolio is reallocated systematically according to the weights of its initial structure with each refinancing or buy-back transaction. By doing this, we follow the method that was popularized in particular by Black and Perold (1992) [9], called CPPI (*Constant Proportion Portfolio Insurance*).

We use the Monte Carlo method to simulate scenarios and numerically construct the distribution of the results for each debt structure under the aforementioned method of reallocation. For this, we discretize the asset and liability dynamics following the Euler diagram, re-balancing each portfolio with its starting proportions.

Our optimization process entails, first of all, choosing the acceptable level of risk based on the simulated results profiles for the various eligible portfolios, including the current debt portfolio. The variate presented above, i.e. the annual amortization capacity, is a "good" test statistic. It allows us to build the following decision rule:

- if the annual amortization capacity is greater than the fractile T_α, we accept the strategy,
- if the annual amortization capacity is less than the fractile T_α, we reject the strategy.

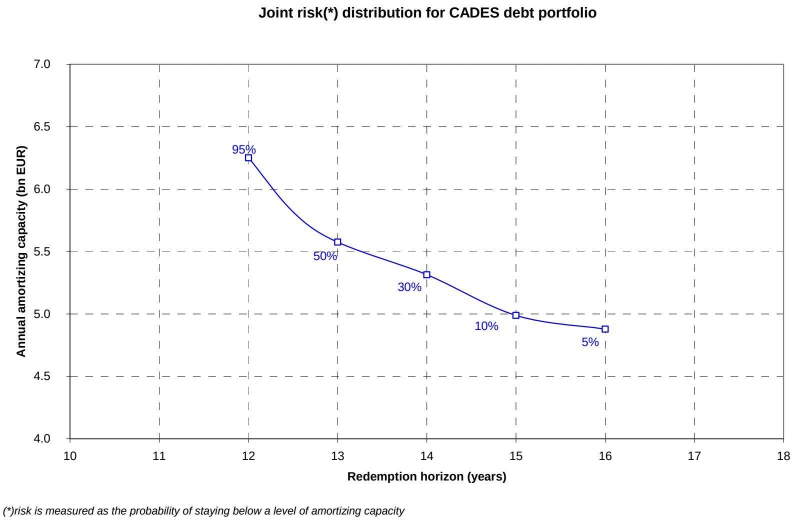
The probability level α (%) represents the risk of wrongly deciding that the strategy under consideration is admissible.

Accordingly, we will decide that a debt strategy with an expected amortization capacity that is sufficient to achieve full amortization of the debt before horizon H_α, with the risk α of making an error ; this risk is known as the « first-order » risk.

If such an event comes to realization, the consequence of actually underestimating the amortization duration is an additional tax levied on the taxpayer. The convexity of the risk profile accentuates the gravity of this consequence, because the lower the level of the capacity, the greater the magnitude of the amortization period increase, for the same decrease in amortization capacity.

Conversely, if the amortization period is overestimated, the consequence is less serious for the taxpayer, since the tax levied on the latter will be less than initially planned.

Below, we represent the joint distribution of redemption horizon and amortizing capacity for CADES debt portfolio. The distribution of either variate can be divided into strata based on the cumulative frequency. The strata are thus delimited by the levels of cumulative probability: 0%; 5%; 10%, etc. This cumulative probability is precisely the notion of risk that was defined above.

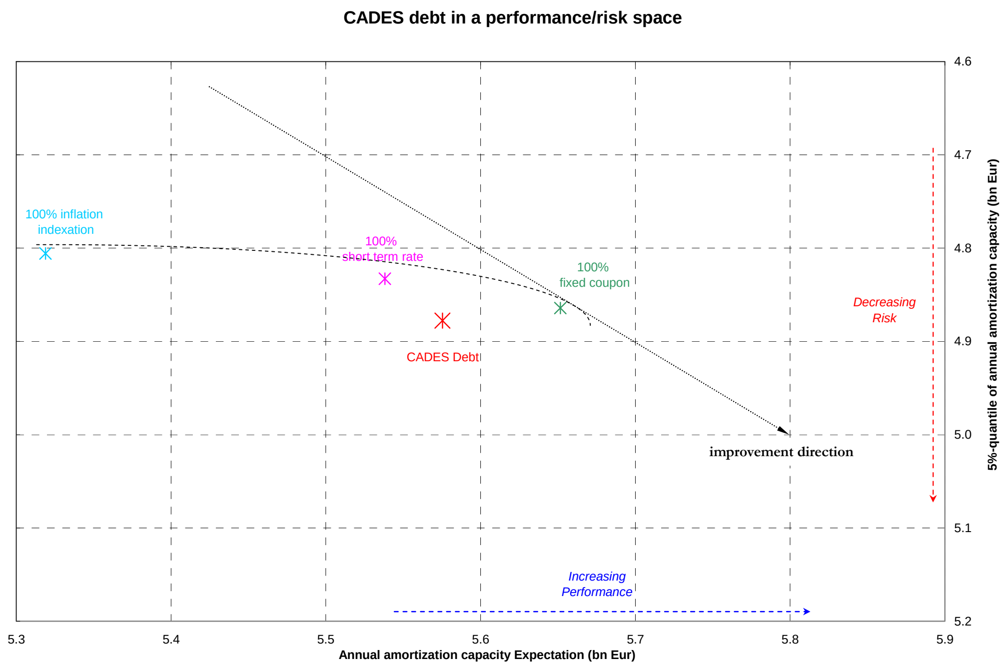


The graph shows, for some calculation date in 2008, that CADES debt portfolio had 1 chance over 2 of fulfilling its mandate in 13 years at most, and 1 chance over 20 of succeeding in 16 years at best.

5. Decision support tools

5.1. Improvement and no-arbitrage axis

Let our portfolio be represented in a two-axis space, as in the graph hereunder, where the X-axis bears performance, measured by annual amortizing capacity expectation, whereas risk, measured by the 5%-quantile of amortization capacity distribution - or CaaR(5%) -, is carried by the Y-axis. A move along the X-axis toward high values means a performance increase, and a move along the Y-axis toward high values means a risk decrease. Therefore, a move along the 2nd bisectional diagonal toward high values on both axes embodies the improvement direction. Conversely, the opposite and 1st diagonal stands for the no-arbitrage direction, for any move along this direction means that an improvement on one axis is annihilated by a step back on the other axis.



5.2. Performance and risk measurements

The performance and risk measurements that we have opted to use are both customary and critical for determining an optimum. Looking at the graph in the preceding paragraph, when aiming for an optimum portfolio one would move along the improvement axis, toward the lowest part of the graph, and furthest to the right.

However, this is not necessarily the portfolio that maximizes the optimality criterion we presented in the previous section. First, the fractile CAaR(α %) belongs to the "simulated" distribution of the annual amortization capacity. This may present distortions compared with a Gaussian distribution. Furthermore, the proposed criterion brings into play the standard deviation, square root of the second moment of the distribution. We know that it does not account for such distortions.

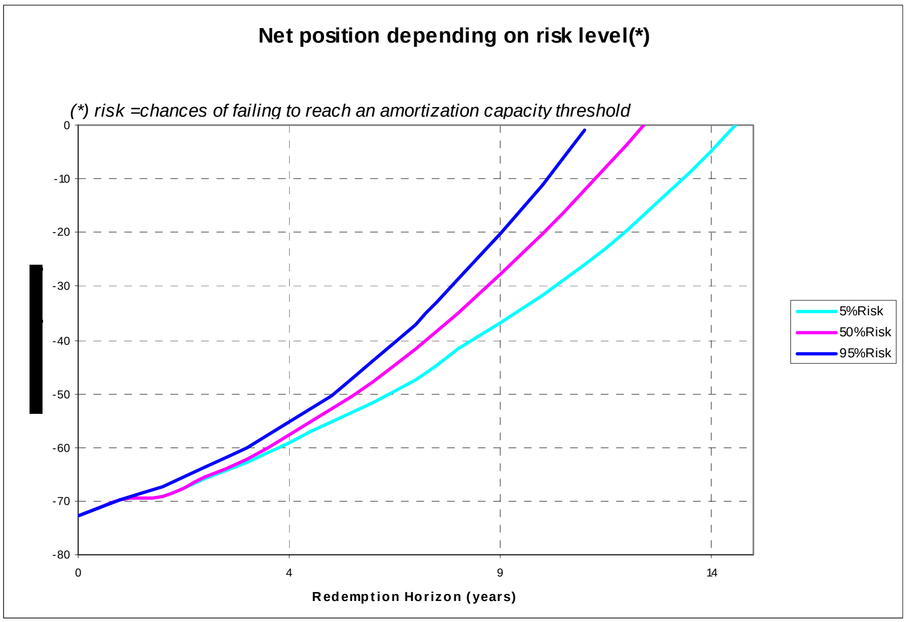
At the same time, the optimality criterion presented offers the advantage of constituting a scale of measurement that will allow us to rank the various portfolios.

5.3. Analysis of risk

5.3.1. Amortization profiles and dominance

We perform risk analysis at the portfolio level. Indeed, each portfolio can be characterized by amortization profiles that are comparable for an equal level of risk. An amortization profile is simply the amortization path of the outstanding debt for a given portfolio structure, corresponding to the quantile of annual amortization capacity of risk α (%).

Below, we represent the amortization profiles of the current debt portfolio as of some calculation date in 2008, at different levels of risk.



A profile will be acknowledged as better than another for the same level of risk, if it crosses the X-axis corresponding to the null value of net debt sooner. The conventional conditions of stochastic dominance of the first and second order will be translated as follows:

- a portfolio dominates another, in the sense of « first order dominance », if its profiles are better at every level of risk,
- a portfolio dominates another, in the sense of « second order dominance », if its profiles are better at risk levels that are lower than or equal to 50 %.

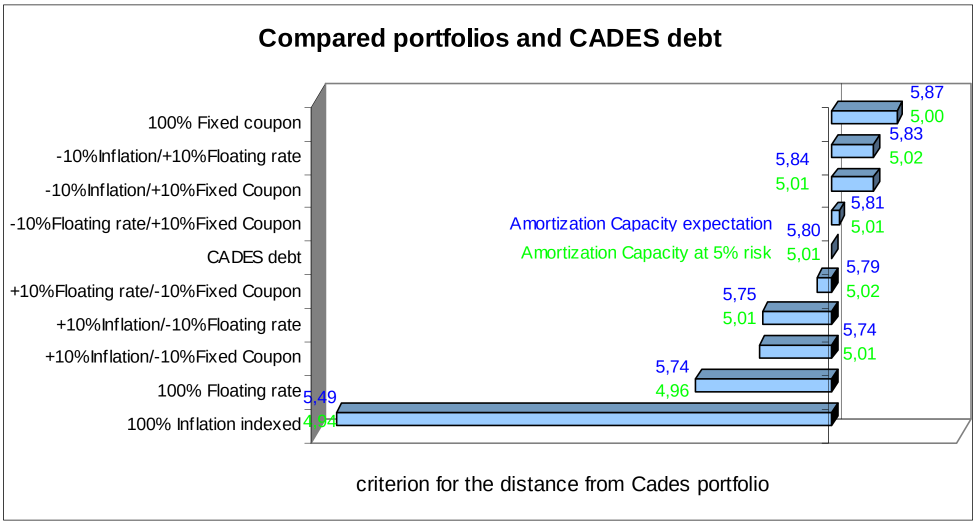
5.4. **Performance analysis**

5.4.1. Static analysis

We have built two sub-sets of comparison portfolios : one subset, named “extreme portfolios”, contains extensions of the actual CADES portfolio, constructed by setting at 100% one of its debt class, and the remaining debt classes at null ; the other subset is labelled “variational portfolios”, in which each one is built by stretching the actual portfolio, adding 10% of one debt class portion of the outstanding, and substracting 10% from another debt class.

Representing all these portfolios on one graph proves troublesome, for they may be scattered over the graph. So we build a plain criterion by adding up performance and risk indicators. We use this straighforward measure to rank the whole set of portfolios. It may be seen as another representation of our set of portfolios along the "improvement" direction.

The graph shown hereunder displays the whole set of portfolios ranked from top to bottom according to this criterion.



5.4.2. Dynamic analysis

When one compares the current debt from one date of calculation to another, it is useful to separate three effects.

The debt portfolio is composed of securities that are subject to market variations. The parameters of the model, calibrated upon market prices and yields, change along with them. Accordingly, the simulations are subject to this effect from one evaluation time to the other.

Next, it is necessary to separate the effect of a change in the nominal outstanding, from that of achieved transactions, i.e. a change of structure, which must be measured on a constant outstanding debt basis. This means we need to assess (i) the current debt portfolio with the new outstanding under the structure that prevailed before the transactions were completed—i.e., as if we had maintained the debt structure that prevailed on the date of the previous assessment, and (ii) this same current debt, but after completion of the transactions.

The gap in performance measurements between two assessment dates, for the former debt on a constant structure basis, with the same outstanding, represents the effects of time, market fluctuations and, where applicable, new data (realized inflation index, realized CRDS an CSG joint tax inflows).

For the same evaluation date, the gap between the measurements made while keeping constant the former structure, with different nominal outstanding, represents the effect of the change in the nominal outstanding.

Finally, for the same evaluation date, the gap in measurements made on the debt with the new nominal outstanding, between the old and the new structure, represents the effect of transactions.

The table hereafter shows the breakdown of the performance measure for CADES debt portfolio between october and december 2008, into the 3 components stated before :

- effect of market fluctuation and environment changes
- effect of outstanding amount and life-to-maturity variation
- effect of debt structure changes

	october 2008	december 2008	december 2008	december 2008
Effect		market change	oustanding change and time-decay	debt structure change
Outstanding (bn EUR)	69,8	69,8	69.7	69.7
performance	5.58	5.70	5.68	5.80

6. Asset and liability management, and the model

6.1. Modeling assumptions and parameter values

The fixed coupon debt class should benefit from positive or cyclical inflation, i.e., positively correlated to growth as well as to nominal rates. The results of the model will depend, at a first order, on the $(g - \tau_{reel})$ spread, with τ_{reel} designating the deflated nominal rate.

More generally, if the assumptions of the model yield the generation of deflated rates ruling a given debt class, such that the spread described hereabove is positive for the greatest part of the borrowing phase, this debt class will be one of the best candidates, particularly in terms of the expected reimbursement capacity.

The long-term trends, the eventual backward forces and, above all, the volatilities of factors, are therefore crucial for performance and risk measurement, as well as for the results of the optimization.

At a second order, the results will depend on correlations between risk factors.

6.2. Reallocation rules and optimality of resulting portfolios

Whether we use a criterion like the utility of terminal wealth or one like the utility of inter-temporal wealth, the resolution of the optimization cannot be different: otherwise, the agency would have two solutions to achieve a single objective, one being necessarily less optimal than the other. The same by-absurd reasoning can be used if we compare our method of optimization, which may be described as empirical, and the analytic resolution method of the program formalized in section 4, with the help of optimal control tools.

The refinancing/re-investment strategy adopted (of the CPPI type) leads us to integrate into the optimization program an a priori constraint. Such a constraint may be acceptable if it corresponds to a rule governing the operating process of the agency, or if it models its rationality in the future. For example, in the case of life insurance companies, rules for harvesting capital gains are integrated into the reinvestment strategy.

In our case, this strategy apparently leads to a behavior that is not always optimal.

6.3. Model risk: some responses

Unavoidably, the results are the product of the model's fundamental assumptions, and of the initial conditions.

To protect ourselves from model risk, we have recourse to the following actions:

- performing simulations of the model on contrasting market configurations to the extent permitted by the data,
- running regular simulations,
- implementing sensitivity tests to shocks on one or the other of our fundamental assumptions, in order to control the response of the model,
- developing alternative models

6.4. Contemplated changes

We have built other models, in particular a Vectorial Autoregressive Model within a purely econometric framework. We have also developed a quantitative model for the indexed debt, thanks to the work done by I. Toder (2004)[12], based primarily on the article written by Jarrow and Yildirim (2002)[10].

This model, written under the risk neutral probability measure, is still to be integrated within the modeling of our economy, which is under the real probability measure.

Finally, we still need to solve dynamically the program defined in section 4 in order to find an optimal portfolio using optimal stochastic control techniques.

6.5. Reflections on debt management

As the schematic representation of our balance sheet demonstrates, we are mainly « asset sensitive ». It is possible for us to model our assets in a relatively straightforward way, over a sufficiently long horizon. This exercise is far more difficult for a number of other institutions and companies, either with respect to modeling a portion of their balance sheet, or because of their weaker ability to make long projections due to a fairly short business cycle (2-5 years).

We can make full use of our asset and liability management capability to monitor debt allocation over a long time frame. If we compare ourselves to institutional investors such as life insurance companies that sell annuities, our balance sheet is in some respects a mirror image of theirs. Indeed, they invest in assets offering a return that is at least adequate to cover their policyholder retirement annuity liabilities, which are strongly indexed to wages. Whereas we issue inflation-linked bonds, among other instruments, and our asset grows roughly in line with wages. Our respective allocation processes seek to optimize a similar objective, of an opposite sign.

7. Conclusion

The nominal rate equal to our debt cost is related to its sustainable nature. The debt is sustainable as long as the total amount is at most equal to the sum of the discounted value of expected revenues. If the excess of debt nominal rate over the expected revenues rate of increase, exceeds an equilibrium value, the debt is no longer sustainable. Accordingly, gaining control over the expected future costs and the variability of the amortization is at the heart of debt management.

To fulfill a request from the Parliament, it is up to this agency to estimate the date on which the debt will be fully reimbursed. This is one of the results that we assess with measured cautiousness, and report monthly to the Board of directors.

As we saw in sub-section 4.7, the risk of underestimation in econometrics can be seen as a « first-order » risk, and is far more serious than the risk of overestimation. If this risk were to occur, it would lead to an additional cost to which CADES is, by definition, averse.

REFERENCES

[1] M.J.Brennan, Y.Xia (Juin 2002), « Dynamic asset allocation under inflation », *Journal of Finance*

[2] Harry Markowitz (1959), « Portfolio selection : efficient diversification of investment », *New York, Wiley*

[3] John Y.Campbell, Luis M.Viceira (1999), « Who should buy long term bonds », *Working Paper, Harvard University*

[4] M.Grasselli (Oct-Nov 2004), « Recent methods of pension funds management », *Banque et Marchés*

[5] A.J.G.Cairns (1998), « Some notes on the dynamics and optimal control of stochastic pension fund models in continuous time », *preprint*

[6] L.E.O.Svensson, I.M.Werner (1993), « Non-traded assets in incomplete markets », *European Economic Review*

[7] H.K.Koo (1998), « Consumption and portfolio selection with labour income : a continuous time approach », *Mathematical Finance*

[8] 0.Vasicek (1977), « An equilibrium characterization of the term structure », *Journal of Financial Economics*

[9] F.Black, A.Perold (1992), « Theory of Constant Proportions Portfolio Insurance », *Journal of Economic Dynamics and Control*

[10] R.Jarrow, Y.Yildirim (2002), « Pricing treasury inflation protected securities and related derivatives using an HJM model », *Working Paper, Cornell University*

[11] Robert C.Merton (1971), « Optimal consumption and portfolio rules in a continuous-time modem », *Journal of Economic Theory*

[12] I.Toder (Sept 2004), « Modelization of inflation-linked debt », *Memorandum of DEA « Probabilités et Finances » Pierre et Marie Curie PARIS VI University, CADES*