

Asset and Liability Management by CADES,

a manager of public debt

Executive summary

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CADES background

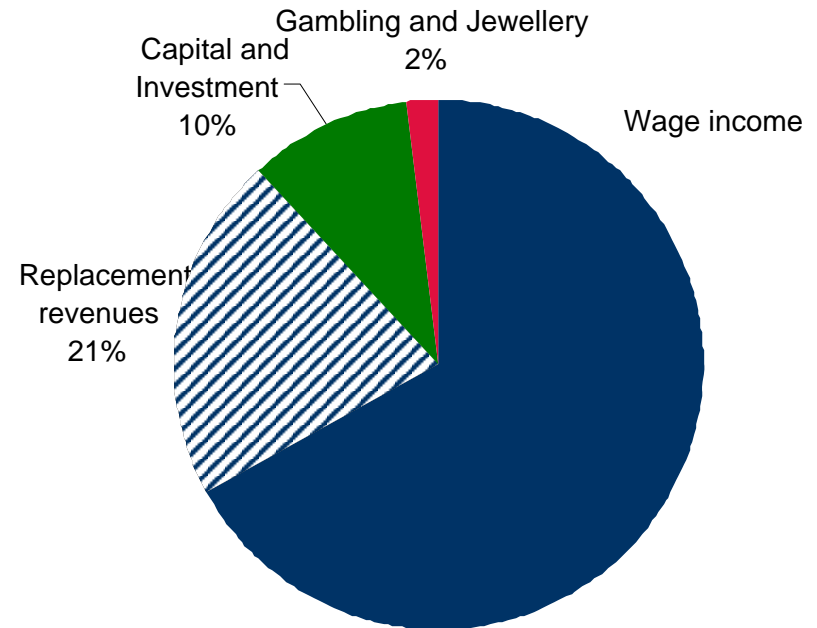
- A mission : CADES is the defeasance vehicle of social security cumulative deficits
- A unique resource : a tax levied on the nation overall revenues
- Debt outstanding has been extended twice since its birth in jan. 1996
- Its life span definition has moved from an adjustable end date with a fixed tax rate, to a fixed end date with an adjustable tax rate

Research work survey

- Few public or government debt managers have made their studies publicly available. Sweden, Denmark, Canada, and United Kingdom are among the rare who can be cited.
- Two main streams :
 - Macroeconomic modelling : the factors which determine the financing requirement, debt cost of servicing, tax income
 - Stochastic modelling of the drivers of debt financing costs, including yield curve models, and debt strategy simulation
- Generally the approach is pragmatic, and often combines the use of econometric and stochastic techniques
- Some of the research studies have tackled the global problem of optimizing both tax policy and debt strategy.
- In our case, the basics are grounded on the theory of portfolio investment, optimal strategy of the investor with either finite or unlimited horizon.
- a strong analogy can be found with retirement pension funds management.

Balance-sheet modelling : asset

- A unique asset: a receivable on a nationwide tax, the CRDS, levied on overall revenues, mainly on earned income.
- Taxable base is assumed to grow at the pace of national output (France's PIB)
- With A_t the asset value, k_t its growth rate, is modelled as the sum of growth rate in volume and inflation rate, in continuous time



the asset dynamic equation is : $dA_t = A_t k_t dt$

Balance-sheet modelling : liability

- The disposable is computed by a simple resources/expenses balance.
- From there, the net balance is attained by subtracting payables.
- Furthermore, as the value of each instrument outflows are rate dependent, a financing requirement will be met with fewer debt increase in a cheaper rates environment.

- The net debt dynamic defined as,
$$L_t = L_{t-1} - CA_t$$
- with CA_t the amortization capacity at year-end t
- , also reads
$$L_t = L_t^* - N_t$$
- with L_t^* the debt value before reallocation
- and N_t the net balance

Economy modelling

- 3 factors rule Cades economy : real growth, interest rates, inflation
- We assume their processes follow Ornstein-Uhlenbeck diffusion equations, parameterized by pullback force coefficient, long term mean, and standard deviation.
- The set of risk sources is a 3-d vector of correlated brownian motions, which one has to transform into an uncorrelated vector(*).
- For instance, the SDEs of the short-term interest rate and the inflation rate are re-written as

$$dr(t) = a(b - r(t))dt + \sigma_r dW_r(t)$$

$$di(t) = c(d - i(t))dt + \rho_{r,i} \sigma_i dW_r(t) + \sqrt{1 - \rho_{r,i}^2} \sigma_i dZ_i(t)$$

- where dW_r and dZ_i are uncorrelated brownian motion instantaneous changes.

(*) by applying the copula theory to the brownian changes, or the Cholesky transformation

Parameterization/calibration

- The O-U diffusions of the short-term rate and inflation rate are calibrated owing to market negotiable instrument prices.
- We adopt Vasicek specification for the nominal yield curve, and inflation rates curve.
- We take estimations
 - for correlation coefficients
 - as inputs for real growth

Net debt dynamic

- Our goal is to maximize amortization capacity at any time t , in order to have net debt reach the null value at the best pace.

- Therefore the objective function can be written as,

$$\text{Min } L_t^+ = \left[\sum_k \sum_m (B(t,m)E_k(t,m) - \alpha(k,m)N_t) \right]^+$$

- where $E_t(k,m)$ is the outflow at maturity m for the k -th class of debt, as of time t ,
- $B(t,m)$ the time t value of a risk-free zero-coupon bond maturing on year m ,
- $\alpha(k,m)$ the allocation coefficient for maturity m and k -th class of debt,
- and for any variate X , $X^+ = \text{Sup}(X,0)$

- The amortization capacity is cumulated until net debt value is null, in other terms debt is extinguished at time s when we reach

$$L_s^+ = 0$$

Dual form and risk definition

- Indeed an error in the estimation of debt redemption horizon H seen from time 0 bears some consequences.
- CADES would have to refinance the amount

$$L_{H-1} - CA_H$$

- Therefore, CADES faces the risk of an additional borrowing need, either met by its life extension, or by an extra tax raise.
- Let α designate this risk, the optimization problem can be written as :
- $$\text{Min } \alpha = P[L_{H(\alpha)-1} - CA_{H(\alpha)} > 0]$$
- The time dependancy of this risk reveals our risk aversion. For a given initial debt, the lower the amortization capacity, then for a same capacity decrease, the greater the reimbursement period lengthening.
- Seen from the tax payer standing point, this looks like a put selling position, for his loss is growing with the magnitude of the estimation error.

Solving method

- The adopted re-balancing rule is the CPPI(*) method.
- We do not solve numerically the optimization problem.
- Rather, we simulate a reasonable number of portfolio structures
 - specific structures called « extreme » portfolios
 - structures built by allowing to switch 10% of a portfolio between two debt classes arbitrage, keeping the third allocation equal to the current value in the actual debt portfolio
- In the past, we used to build two main sets of portfolios, one with no inflation indexed instruments, and the other with no short-term rate notes.
- Its is worth looking at the changes through time, at one year interval.

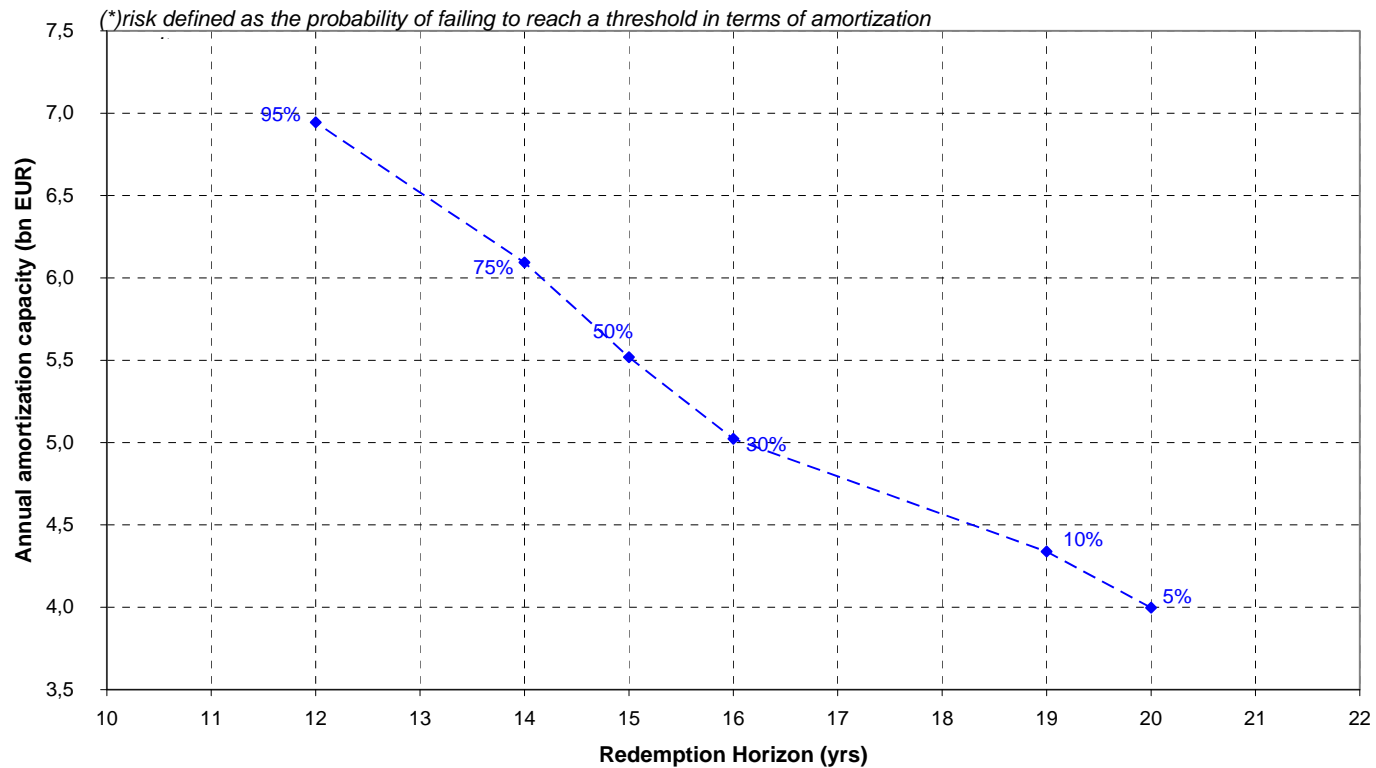
(*) Constant Proportion Portfolio Insurance

Results : Debt risk profile

- When drawing the « joint » distribution of amortization capacity and redemption horizon, we see a representation of our debt risk profile. Changing our debt structure would result in a change of debt profile.
- The curvature of the debt profile and the convexity phenomenon brings into light our risk aversion.

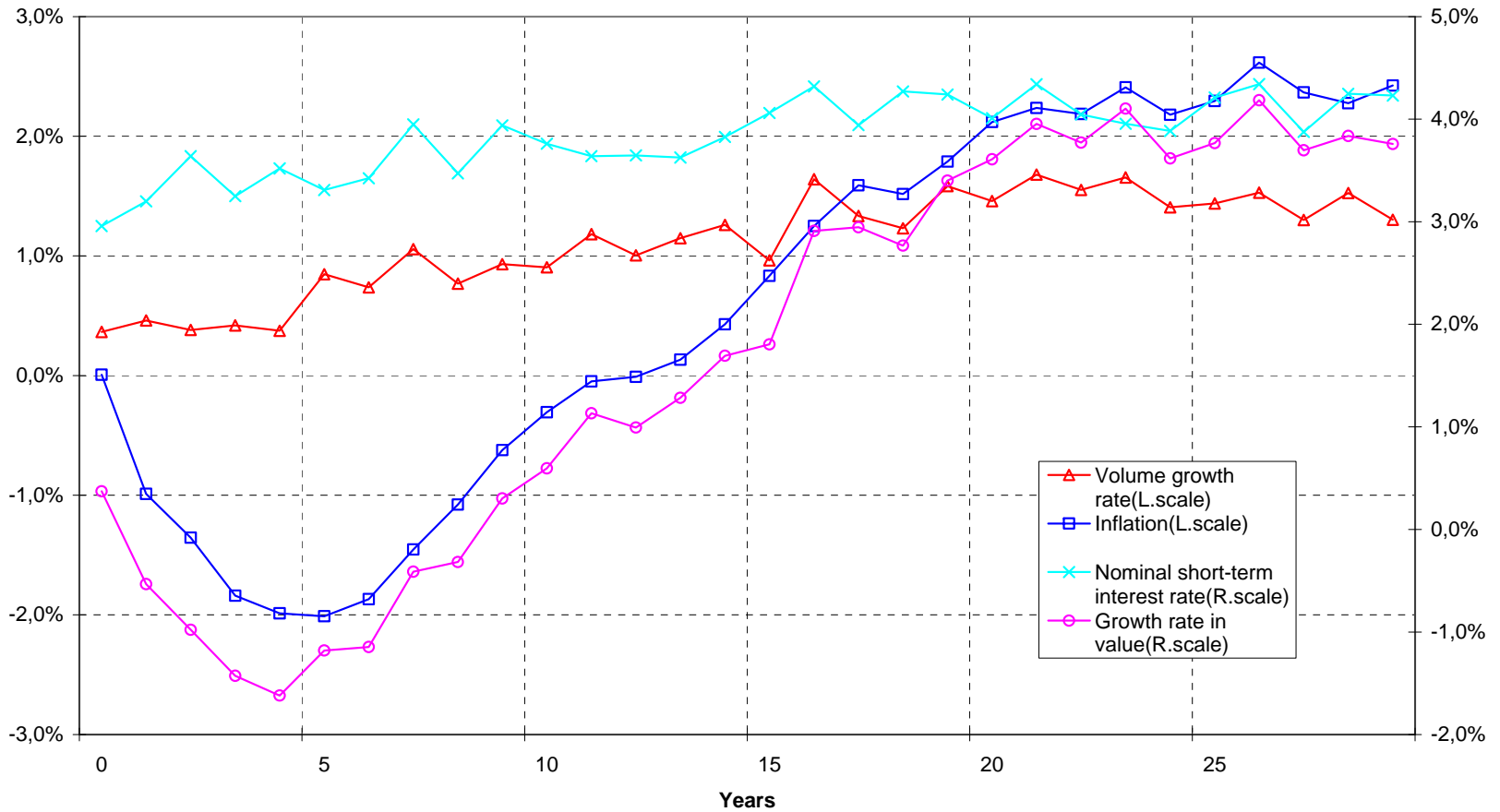
Results on the current portfolio

Debt Risk profile (*)



Results on the current portfolio : the risk region

Risk factors median path in the 5% risk region



Stochastic dominance

■ First order

- When for any level of risk, one portfolio quantiles are above the other's

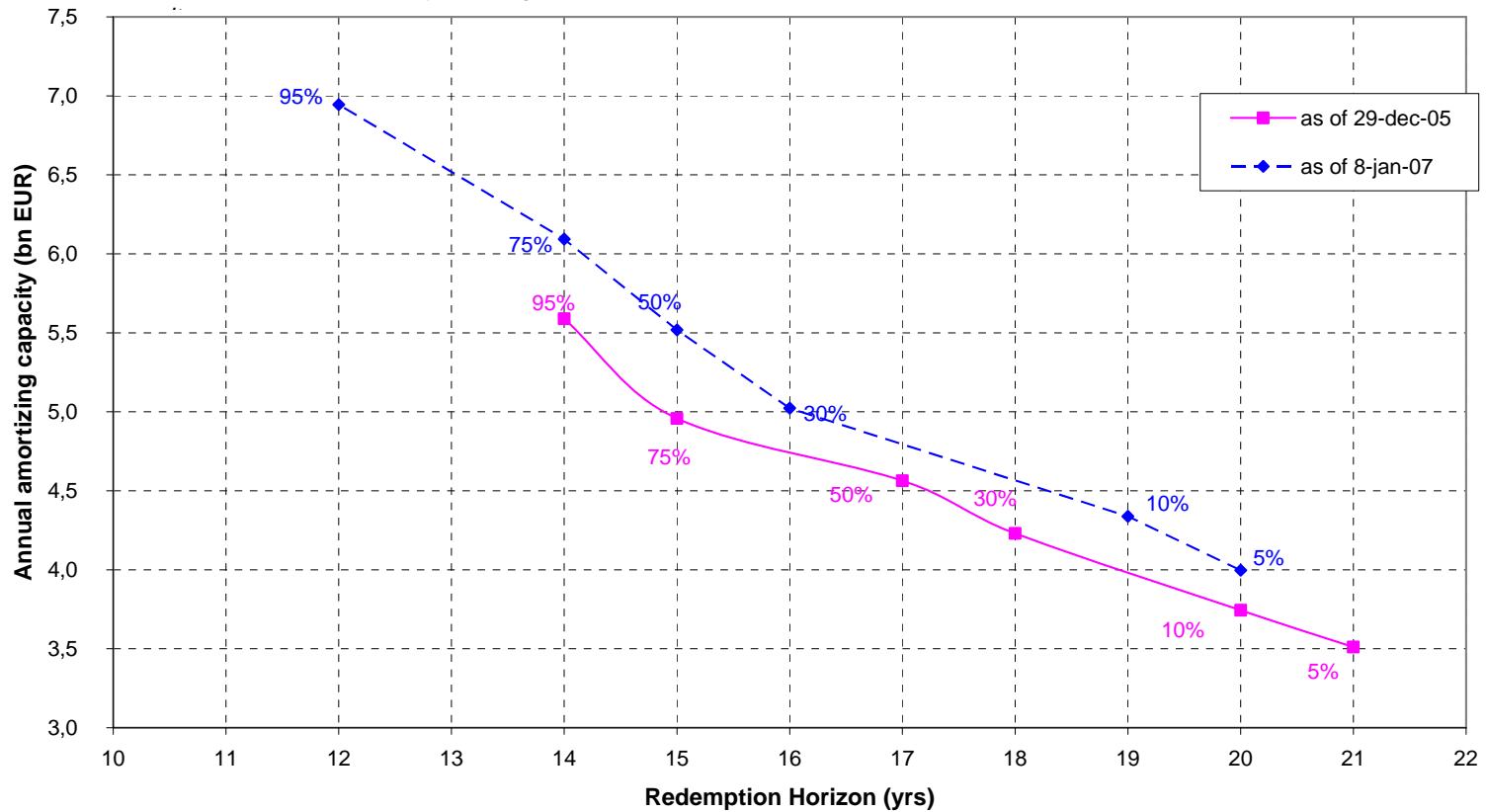
■ Second order

- When for all risk levels lower than or equal to 50%, one portfolio quantiles are above the other's

Stochastic dominance

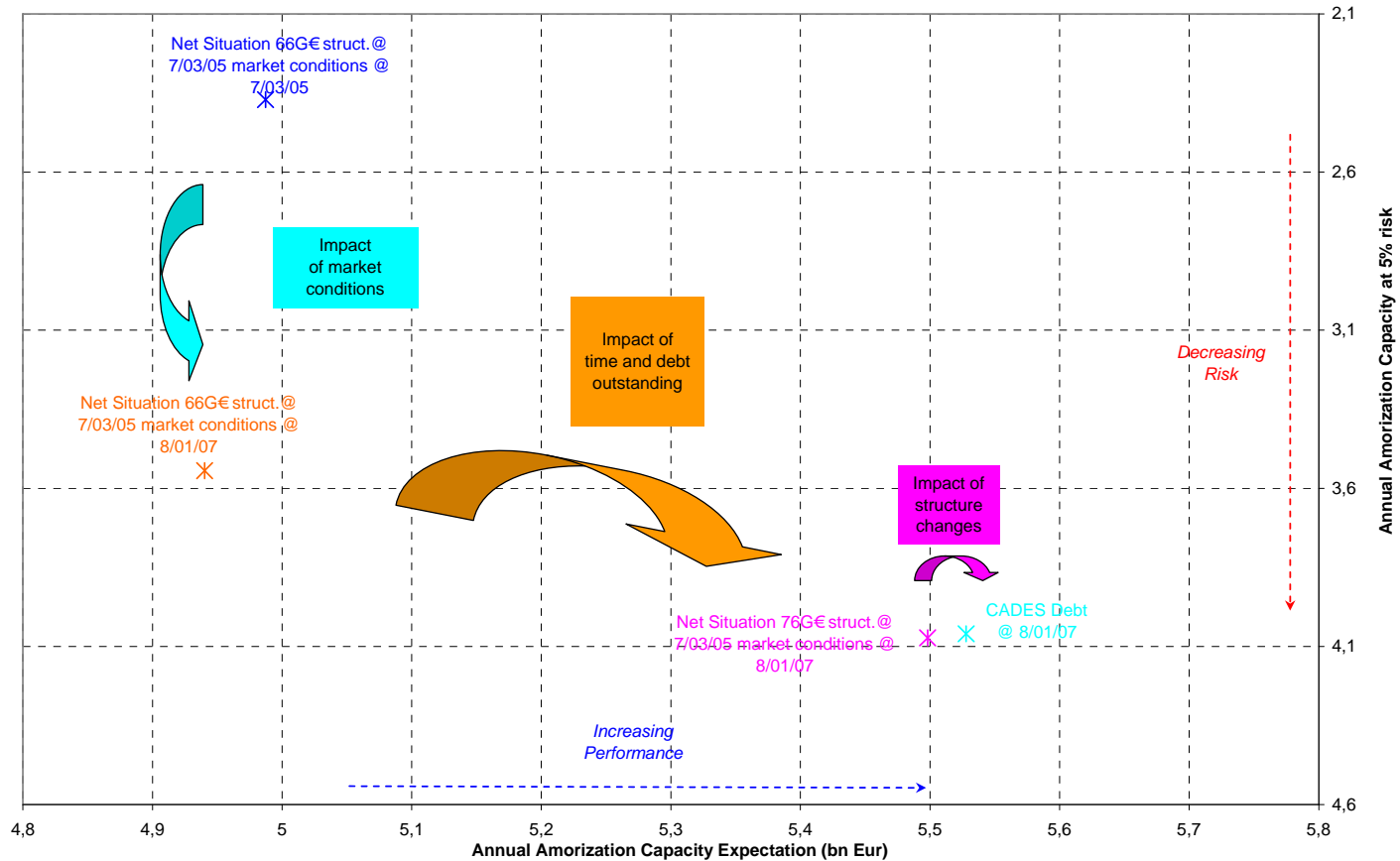
Debt Risk profile (*)

(*)risk defined as the probability of failing to reach a threshold in terms of amortization



Results on the current portfolio : dynamic analysis

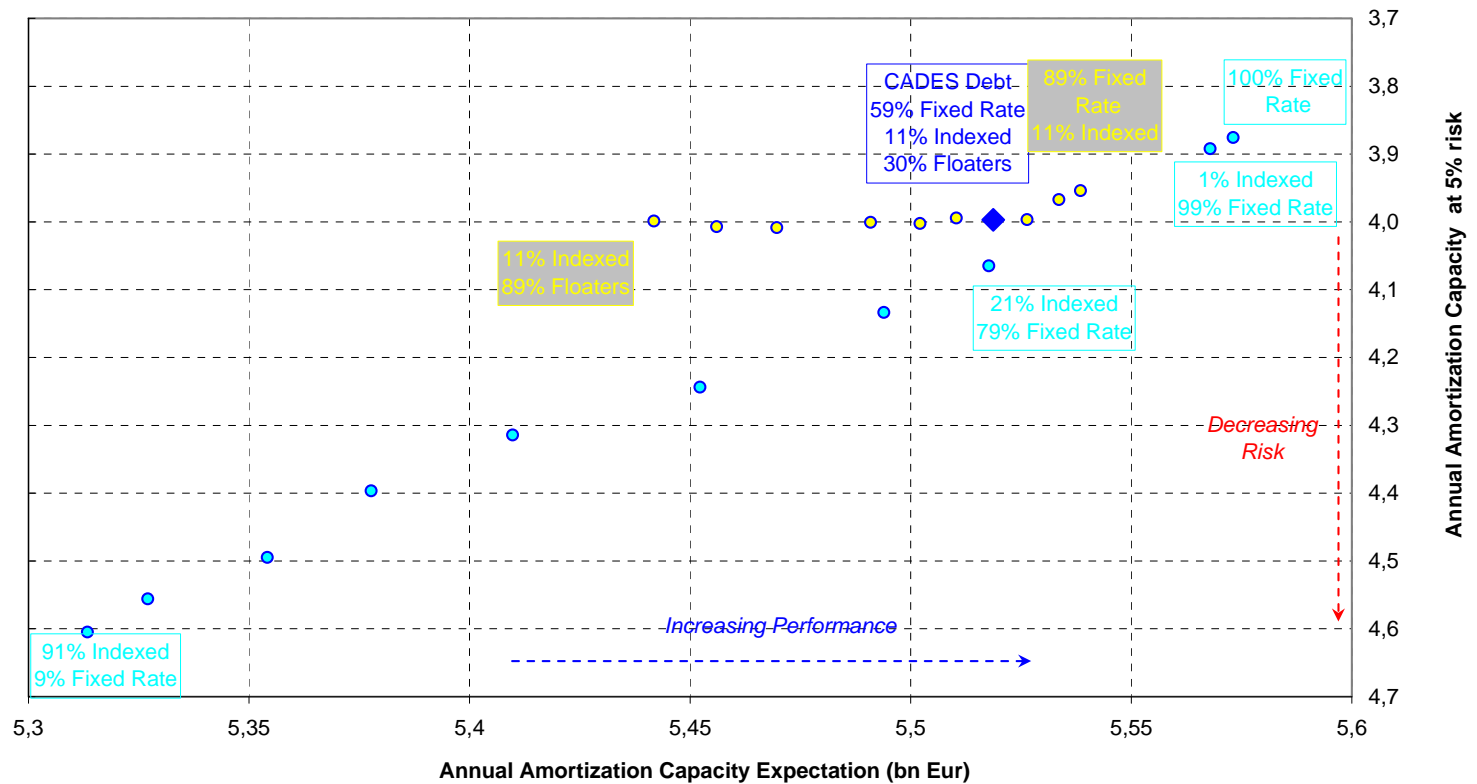
Actual debt analysis from 7-march-05 to 8-jan-07
focus on 3 separate impacts



Portfolio analysis :

Portfolio universe in a performance / risk framework

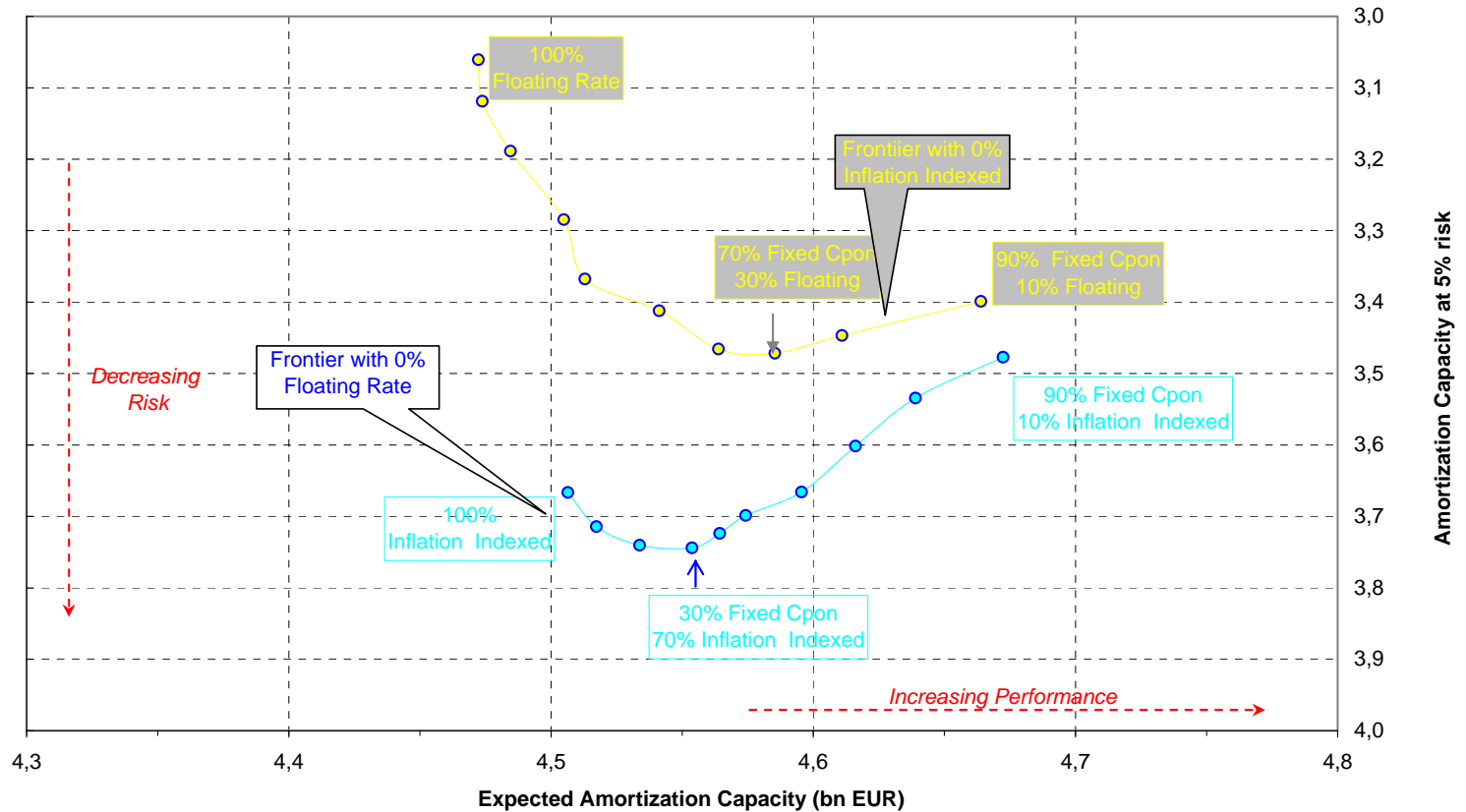
Efficiency Frontiers as of 8-01-07
in the Expected Amortization Capacity / CAaR 5% plan



Portfolio analysis :

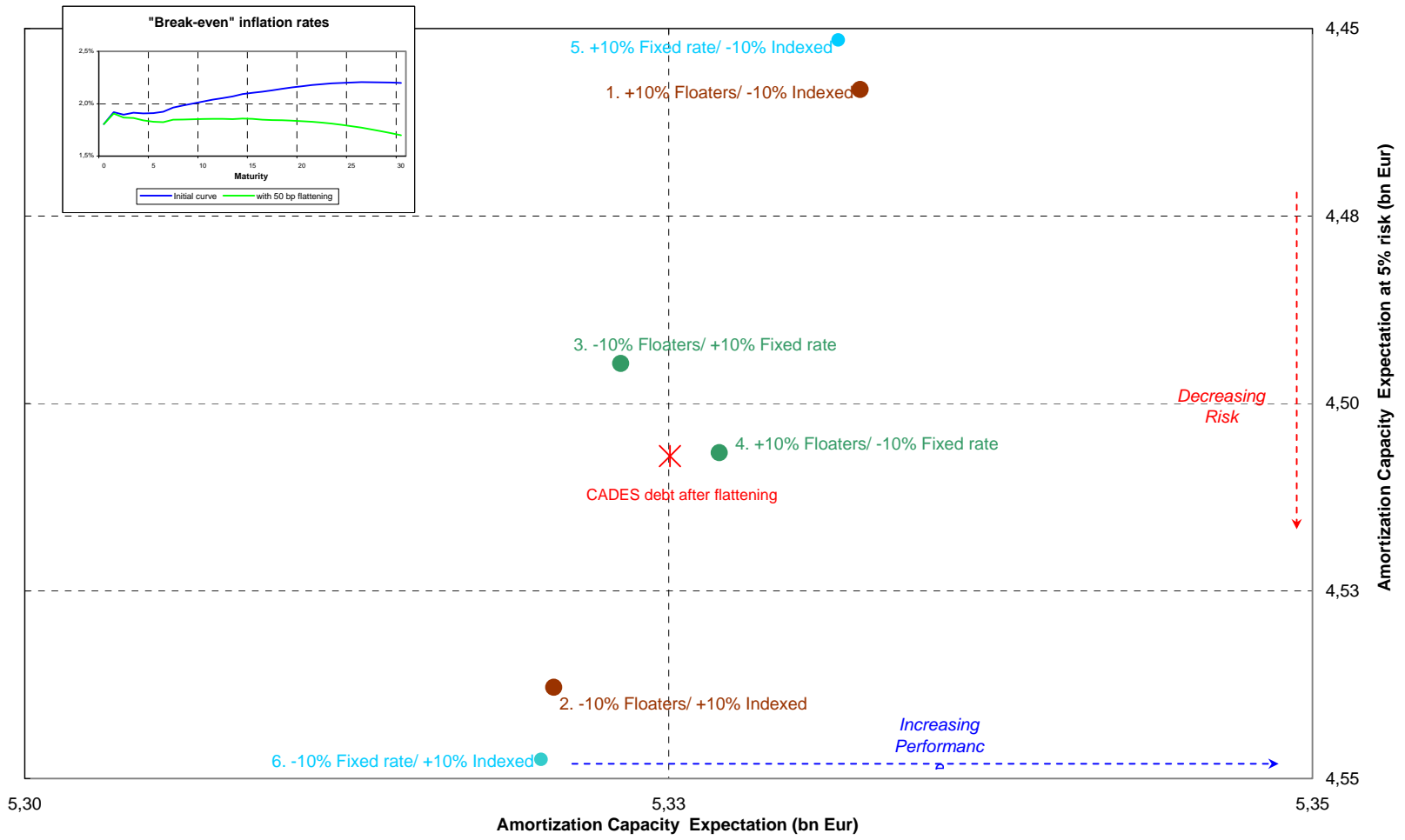
Portfolio universe in a performance / risk framework

Efficiency frontiers as of 28-dec-05
in the Expected Amortization Capacity / CAaR 5% plan



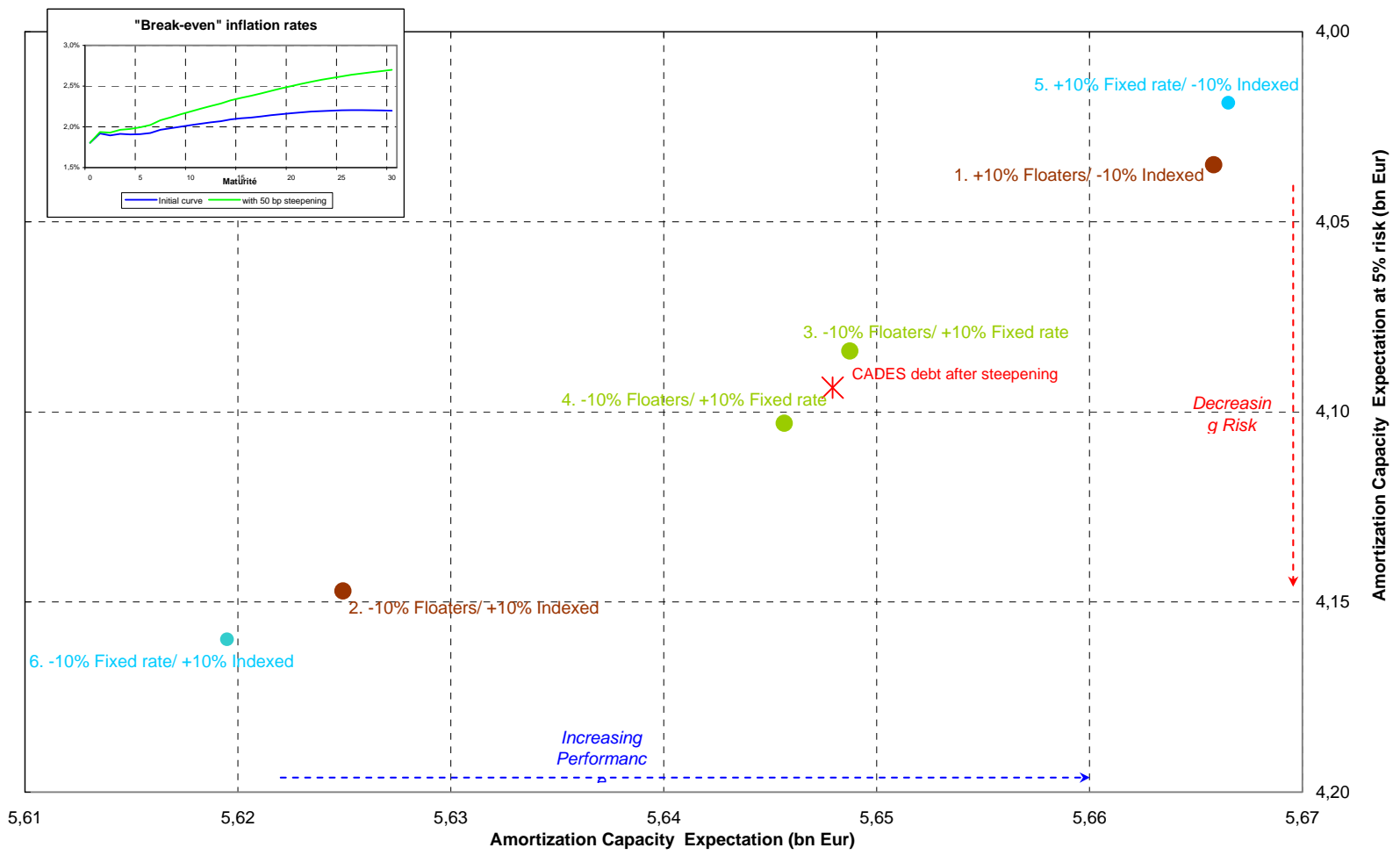
Model sensitivity – model risk

Sensitivity to a 50 bp flattening of break-even inflation rates curve



Model sensitivity – model risk

Sensitivity to a 50 bp steepening of break-even inflation rates curve



Conclusion

« being forewarned is being forearmed »

- A tool which allows for « enlightened » portfolio allocation recommendations, focused on the trade-off between performance and risk
- Anticipation, back-tracking of former strategies, and back-testing are also allowed