IMPLEMENTING a PENSION PLAN ALONG WITH the AGE of the PLAN PARTICIPANT

by

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How to approach the issue of fairness?
The accrual function M(x)

$$M(x) = \begin{cases} 0, & x \prec a \\ \int_a^x m(t)dt, & a \leq x \prec r \\ 1, & x \geq r \end{cases}$$

It represents the fraction of the actuarial value of future pensions accrued as an actuarial liability at age x under the actuarial cost method.

What are the possible candidates for application to pension funding methods?

- The Power function
- The Truncated Exponential
- The Truncated Pareto

Why this choice?

The Uniform distribution is the special case of the Power function when p =1.

Under Uniform distribution, m(x), M(x) coincide with the benefit accrued under the Normal Cost and Actuarial Liability for the Projected Unit Credit.

THE MODEL

- DEFINED BENEFIT PENSION SCHEME
- INDIVIDUAL COST METHODS
- STATIONARY POPULATION, entry age α, retirement age r
- SALARY FUNCTION: $g(t) = s_{\alpha} * e^{\tau(x-a)}$
- ONLY RETIREMENT BENEFITS are ALLOWED
- INITIAL PENSIONS are a FIXED PERCENTAGE, b, of FINAL SALARY. THEY INCREASE by β(x) i.e. $\beta(x)=e^{\beta(x-r)}$

$$NC_x = \begin{cases} B_r * m(x) * (Dr / Dx) * \bar{a}_r^{(\delta-\beta)}, & \alpha < x < r \\ 0, & x > r \end{cases}$$

$$AL_x = \begin{cases} B_r * M(x) * (Dr / Dx) * \bar{a}_r^{(\delta-\beta)}, & \alpha < x < r \\ B_r * \bar{a}_x^{(\delta-\beta)} * e^{\beta*(x-r)}, & x > r \end{cases}$$

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Categorisation of m(x), Cooper & Hickman 1967

- If $m'(x) > 0$ the actuarial cost method associated with m(x) is an accelerating actuarial cost method.
- If $m'(x) < 0$ the actuarial cost method associated with m(x) is a decelerating actuarial cost method.
- Power function may be categorised as either a decelerating ($p<1$) or an accelerating cost method ($p>1$).

m(x) development under different distributions, at specific ages (α=30,r=65)

age	Pr, p=0.3	Pr, p=0.8	Pr, p=1	Pr, p=1.5	TEl σ=30	TEl σ=40	TEl σ=50	Preto k=0.3	Preto k=0.8	Preto k=1.5
35	0.033	0.034	0.029	0.016	0.041	0.038	0.036	0.040	0.044	0.050
40	0.021	0.029	0.029	0.023	0.035	0.033	0.033	0.033	0.034	0.035
45	0.016	0.027	0.029	0.028	0.029	0.029	0.029	0.029	0.028	0.026
50	0.013	0.026	0.029	0.032	0.025	0.026	0.027	0.025	0.023	0.020
55	0.011	0.024	0.029	0.036	0.021	0.023	0.024	0.022	0.019	0.016
60	0.010	0.024	0.029	0.040	0.018	0.020	0.022	0.020	0.017	0.013

Comparison between the traditional and the new defined cost methods in terms of Normal Cost and Actuarial Liability at age x

Actuarial Cost Method	Normal Cost, NC_x $a \le x \le r$	Actuarial Liability, AL_x $a \le x \le r$
Current Unit Credit	$\frac{s_x}{S_r} * \frac{D_r}{D_x} \ddot{a}_r^{(\delta-\beta)}$	$\frac{1}{S_r} * S_x * \frac{D_r}{D_x} * \ddot{a}_r^{(\delta-\beta)}$
Projected Unit Credit	$\frac{1}{(r-a)} * \frac{D_r}{D_x} \ddot{a}_r^{(\delta-\beta)}$	$\frac{x-a}{r-a} * \frac{D_r}{D_x} \ddot{a}_r^{(\delta-\beta)}$

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Actuarial Cost Method	Normal Cost, NC_x $a \le x \le r$	Actuarial Liability, AL_x $a \le x \le r$
Entry Age Normal	$\frac{s_x}{s_a * {}^s\ddot{a}_{a:\overline{r-a}\rvert}} \frac{D_x}{D_a} \frac{D_r}{D_x} \ddot{a}_r^{(\delta-\beta)}$	$\frac{{}^s\ddot{a}_{a:\overline{x-a}\rvert}}{{}^s\ddot{a}_{a:\overline{r-a}\rvert}} \frac{D_r}{D_x} \ddot{a}_r^{(\delta-\beta)}$
Power function	$p * \frac{(x-a)^{p-1}}{(r-a)^p} \frac{D_r}{D_x} \ddot{a}_r^{(\delta-\beta)}$	$\frac{(x-a)^p}{(r-a)^p} \frac{D_r}{D_x} \ddot{a}_r^{(\delta-\beta)}$

Comparison between the traditional and the new defined cost methods in terms of Normal Cost and Actuarial Liability at age x

Actuarial Cost Method	Normal Cost, NC_x $a \le x \le r$	Actuarial Liability, AL_x $a \le x \le r$
Truncated Exponential	$\frac{1}{\sigma} * \frac{1}{1-e^{\left(-\frac{r-a}{\sigma}\right)}} * e^{-\frac{x-a}{\sigma}} \frac{D_r}{D_x} \ddot{a}_r^{(\delta-\beta)}$	$\frac{1-e^{-\frac{x-a}{\sigma}}}{1-e^{-\frac{r-a}{\sigma}}} * \frac{D_r}{D_x} \ddot{a}_r^{(\delta-\beta)}$
Truncated Pareto	$\frac{\frac{k}{a} * \left(\frac{a}{x}\right)^{k+1}}{1-\left(\frac{a}{r}\right)^{k}} \frac{D_r}{D_x} \ddot{a}_r^{(\delta-\beta)}$	$\frac{1-\left(\frac{a}{x}\right)^{k}}{1-\left(\frac{a}{r}\right)^{k}} \frac{D_r}{D_x} \ddot{a}_r^{(\delta-\beta)}$

Normal Cost under the new and traditional cost methods

Age	CUC	PUC Unif	Pr p=1.5	TrE σ=30	TrE σ=40	TrE σ=50	Par k=0.3	Par k=0.8	EAN
35	0.04	0.06	0.04	0.09	0.08	0.08	0.09	0.10	0.08
40	0.06	0.08	0.07	0.10	0.09	0.09	0.09	0.10	0.09
45	0.09	0.11	0.10	0.11	0.11	0.11	0.11	0.10	0.11
50	0.14	0.14	0.16	0.12	0.13	0.13	0.12	0.11	0.13
55	0.22	0.18	0.23	0.13	0.15	0.15	0.14	0.12	0.15
60	0.34	0.24	0.34	0.15	0.17	0.19	0.17	0.14	0.17
64	0.50	0.32	0.47	0.17	0.20	0.22	0.20	0.16	0.19

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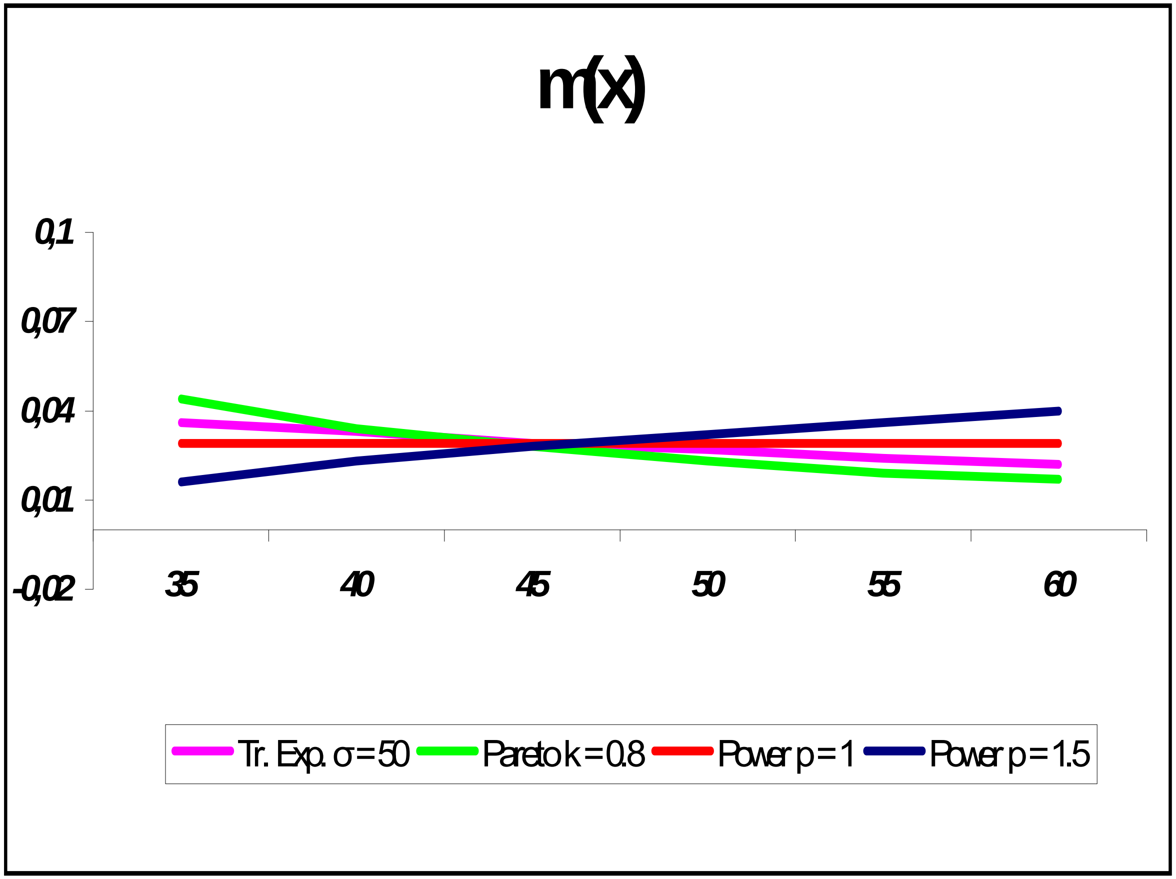

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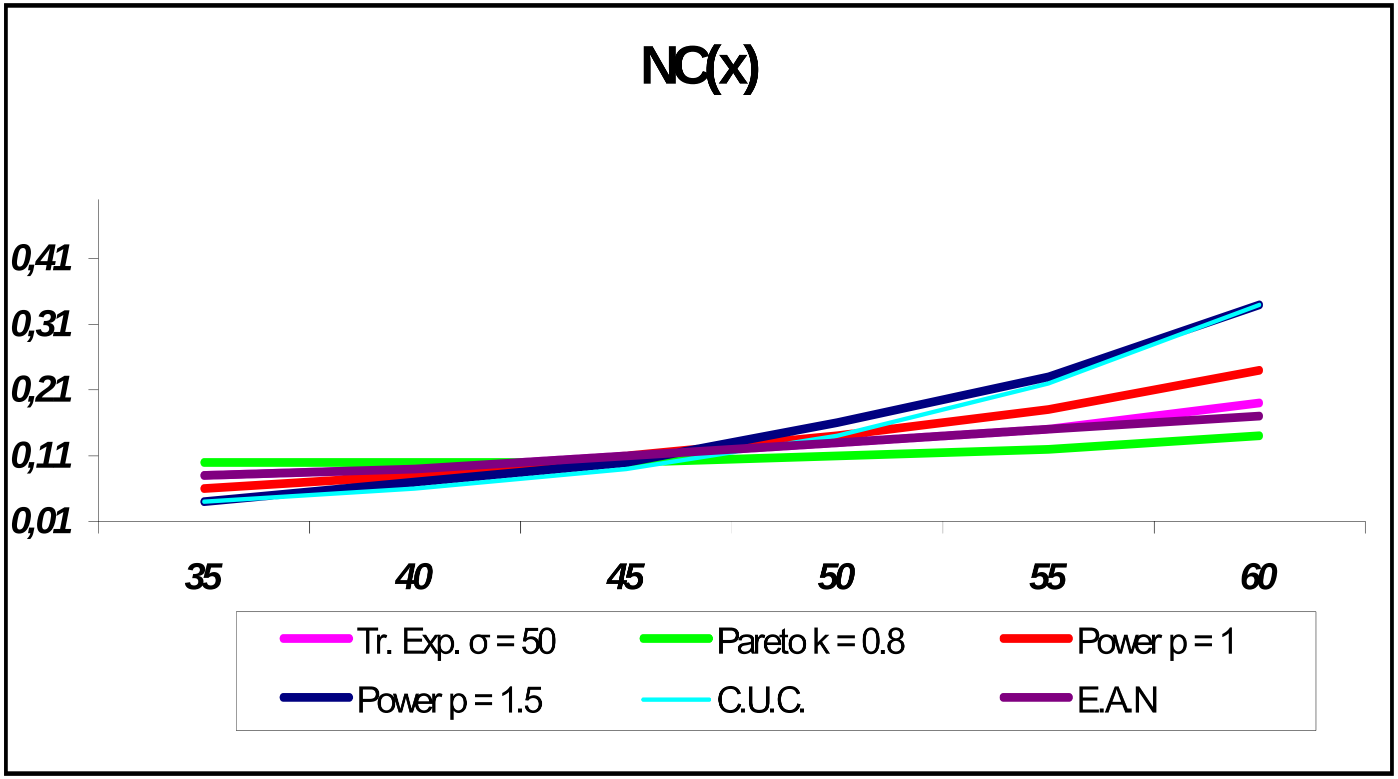
NC(x)
0,41
0,31
0,21
0,11
0,01
35
40
45
50
55
60
Tr. Exp. σ = 50
Pareto k = 0.8
Power p = 1
Power p = 1.5
C.U.C.
E.A.N

Actuarial Liability under the new and traditional cost methods

Age	CUC	PUC Unif	Pr p=1.5	TrE σ=30	TrE σ=40	TrE σ=50	Pr k=0.3	Pr k=0.8	EAN
35	0.19	0.31	0.12	0.49	0.44	0.42	0.48	0.55	0.43
40	0.54	0.81	0.43	1.17	1.08	1.02	1.13	1.27	1.06
45	1.13	1.58	1.03	2.10	1.98	1.90	2.04	2.21	1.96
50	2.14	2.75	2.08	3.40	3.25	3.15	3.30	3.50	3.24
55	3.84	4.54	3.84	5.22	5.07	4.97	5.11	5.30	5.08
60	6.74	7.35	6.80	7.87	7.75	7.68	7.77	7.91	7.77
64	10.6	10.78	10.63	10.93	10.90	10.88	10.90	10.94	10.91

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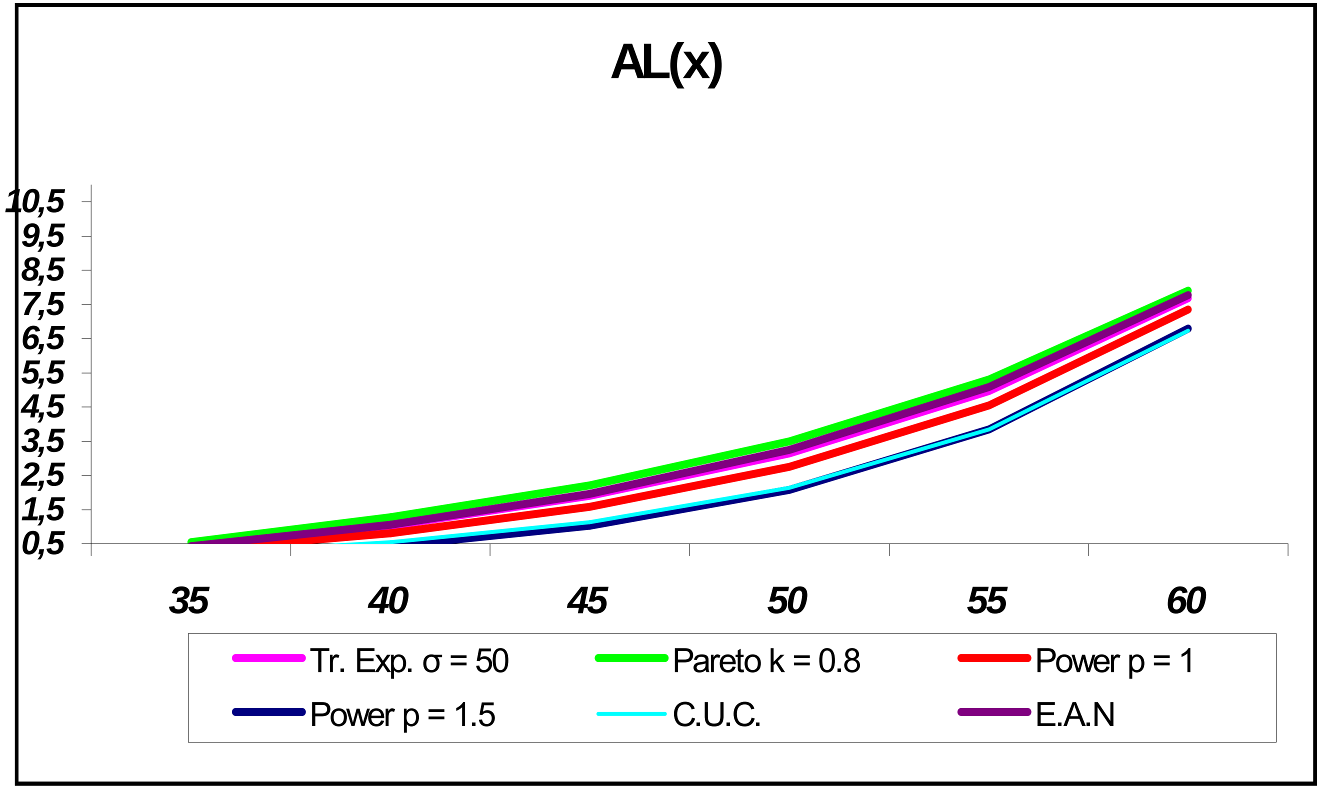
AL(x)
10,5
9,5
8,5
7,5
6,5
5,5
4,5
3,5
2,5
1,5
0,5
35
40
45
50
55
60
Tr. Exp. σ = 50
Pareto k = 0.8
Power p = 1
Power p = 1.5
C.U.C.
E.A.N

The New Cost Methods

- $NC(t) = \int_a^r h(t+r-x)*m(x) * e^{-\delta*(r-x)} * dx$

- $AL(t)= \int_a^r h(t+r-x)*M(x)*e^{-\delta*(r-x)}dx +$

$$+ \int_r^w h(t+r-x)*e^{\beta*(x-r)}dx$$

h(t): the density at time t of the amount of newly incurred age r pensions.

Proposition:

Bowers et al (1986): Consider two accrual functions $M_I(x)$, $M_{II}(x)$.

If $D(x) = M_I(x) - M_{II}(x)$ is such that $D'(\alpha) > 0$

and $D'(x) = 0$ has exactly one solution, $\alpha < x < r$,

then $AL_I(t) > AL_{II}(t)$.

Comparison of the new defined cost methods in terms of the Accrued Liability at time t

$^{CUC}AL(t) < {}^{PUC}AL(t) = AL(t)_{(Uniform)} < AL(t)_{(Truncated\ Exponential)}$

$^{CUC}AL(t) < {}^{PUC}AL(t) = AL(t)_{(Uniform)} < AL(t)_{(T\ Pareto,\ k<1,\ k<p/d)}$

$AL(t)_{(Power,\ p>1)} < {}^{PUC}AL(t) = AL(t)_{(Unif)} < {}^{EAN}AL(t)$

- The above conclusions are as those derived from the comparison in terms of the AL at age x.

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Concluding Comments

If the benefit is allocated in higher proportions as age increases, the Normal Cost values are very similar when they are calculated either under the Current Unit Credit method or using the Power function,p>1. On the other hand, if it is allocated in lower proportions as age increases, they are very similar under the Entry Age Normal the Truncated Exponential and the Truncated Pareto methods.

Among the different accrual functions, a lower Actuarial Liability may be expected from the accelerating cost methods than from the decelerating ones while the Normal Cost follows the opposite trend.

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- THANK YOU !