

# Fair valuation of a non-statutory participating pension contract in the presence of economic cycles

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## Abstract

Market-consistent valuation (known as the fair valuation) of liabilities is an important part of analyzing a non-statutory participating pension contract. Liabilities can be valued in the Black-Scholes framework, however, the Black-Scholes approach does not capture the long-term behavior of financial markets well. One way to improve the long-term behavior is to model stochastic economic cycles in the form of an embedded Markov process, which modulates parameters of a geometric Brownian motion underlying the Black-Scholes approach. In this study, we consider the valuation of a participating pension contract in the Markov modulated Black-Scholes framework by employing the Esscher transform. Using this approach, we analyze how various parameters influence the fair value of a participating pension contract, and which sets of parameters are fair for such a contract.

*Key words:* Pension; Fair valuation; Participating contracts; Stochastic economic cycles; Markov-modulated geometric Brownian motion

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## 1 Introduction

Traditional valuation of insurance liabilities is being replaced by the so-called *fair valuation* of liabilities (for a recent discussion see, e.g., Jørgensen, 2004). An important reason for the change is the need for a more detailed valuation of previously neglected elements in insurance contracts, such as interest rate guarantees. Another factor in the shift is the IAS/IFRS framework for accounting that requires the use of fair valuation-like methods for corporate liabilities (IASB, 2004).

Option valuation models, e.g. of Black and Scholes (1973), capture many aspects of market's valuation process well, which suggests that such a model provides a good starting point for fair valuation of liabilities associated with insurance contracts (IASB, 2004). Recently, a large body of work has applied the option valuation methods to the valuation of life insurance contracts, e.g. Grosen and Jørgensen (2000), Ballotta (2005), Bacinello (2001, 2003), Kling et al. (2007), Siu (2005), Bernard et al. (2005). While Black-Scholes type valuation (Black and Scholes, 1973) is believed to be reasonably accurate for short time periods, it does not yield a good estimate of long-term behavior of the equity market (Bakshi et al., 2000) nor does it capture shocks in the equity value (Sornette and Johansen, 2001). In contrast to options on equity, maturity of a typical non-statutory pension product is tens of years in future. Consequently, it is not obvious that option valuation methods with static parameters are suitable for the valuation of pension contracts.

The impact of economic cycles is of fundamental importance to the estimation of liabilities associated with insurance contracts (Daykin et al., 1994). In particular, the phase of the economic cycle influences the volatility and mean of asset returns. Nevertheless, it has often been assumed that interest rates as well as volatilities are known with certainty in advance and are constant over an extended period (e.g., Black and Scholes, 1973). Most of the previous articles on fair valuation of insurance contract liabilities consider the Black-Scholes economy. Only a few authors (e.g., Ballotta, 2005; Siu, 2005) have considered other methods.

It has been shown that regime-switching can significantly improve modeling of financial time-series data (Hamilton, 1989; Hardy, 2003; Elliott et al., 2005), and that it may also be used in forecasting turning points in the economy (Koskinen and Öller, 2004). One of the attractive features of regime-switching is that it provides an intuitive description of the state of the economy as an embedded discrete-valued process influencing parameters of the asset return. Regime-switching is likely the most parsimonious method to improve the standard geometric random walk in the description of equity indices (Hardy, 2003). Hence, a regime-switching framework provides a natural extension of

the Black-Scholes framework, in particular when coupled with the Esscher transform (Gerber and Shiu, 1994; Esscher, 1932).

In this study, we examine the valuation of a long-term pension contract in a Markov-modulated Black-Scholes framework. In this method, the investment model incorporates changes in the state of the economy (regime-switching), which may improve estimation of fair value of a pension contract with long duration. Using this model, we assess the impact of economic cycles on a non-statutory participating pension contract.

## 2 The contract

### 2.1 Participating pension contract

In the following, we briefly describe a non-statutory participating pension contract, fair valuation of which is the main topic of this study. In a participating pension contract (also known as a with-profits contract), the pension paid from the contract depends on the performance of the assets associated with the contract. The pension contract consists of two time periods: an investing period lasting for  $T_I$  years, and a  $T_P$  year period during which the pension is paid. Initially, a policyholder invests for  $T_I$  years, after which he or she receives a pension for  $T_P$  years (Fig. 1). The total duration of the contract is  $T = T_I + T_P$ . The policyholder only receives the pension when alive and in the event of policyholder's death, a death compensation is paid.

We will denote value of the savings associated with the contract (payoff value) by  $P_t$ . The contract guarantees that the payoff value  $P$  increases by at least an interest rate  $r_G$  each year (annual interest). In addition, the payoff value  $P$  receives bonuses during both the investment years and during years when pension is paid.

There are two kinds of cash-flows associated with a pension contract: premiums paid by the policyholder and pensions received by the policyholder. We assume that premiums are predefined at the inception of the contract, which however is not the case for pensions. Pension  $c_t$  paid on year  $t$  (after the investment years) is an annuity based on the payoff value  $P_t$  of the contract and on the predefined discount rate  $r_A \in \mathbb{R}$ ,

$$c_t = \begin{cases} P_t D_{x+t} / (N_{x+t} - N_{x+T}), & \text{when } t \in [T_I, T), \\ P_T, & \text{when } t = T, \end{cases} \quad (1)$$

and it is recomputed yearly. Above we have defined  $D_x = e^{-\int_0^x \mu_t dx - \log(1+r_A)x}$

and  $N_x = \int_x^\infty D_t dt$  to incorporate mortality  $\mu_x$  of an  $x$ -years old person (see, e.g., Bowers et al., 1997). Mortality is modeled as the Gompertz mortality with parameters used in the Finnish statutory TEL pension system.

A pension contract may include a surrender option, which allows the contract to be terminated at any time. In this case, the contract is called an 'American' contract, otherwise the contract is 'European'. An American contract allows the surrender option to be exercised at any time, which is not necessarily the case in a typical non-statutory pension policy. For simplicity, we assume that there are no penalties for an early termination of the contract, that is, the entire payoff value is always paid to the policyholder when the contract is terminated.

## 2.2 Assets

An integral part of the valuation of a participating pension contract is forecasting the value of assets. We assume that the market consists of the participating contract  $P$  assumed to be tradable and of two other investment opportunities: an equity  $A$  and a riskless bond  $L$ . In probability space  $(\Omega, \mathbb{P}, F)$ , the time evolution of  $A$  and  $L$  we assume to be determined by

$$\begin{cases} L_t &= L_0 e^{\int_0^t r_{\alpha(t)} dt} \\ A_t &= A_0 e^{\int_0^t \gamma_{\alpha(t)} dt + \int_0^t \sigma_{\alpha(t)} dW_t}. \end{cases} \quad (2)$$

The equity  $A$  follows geometric Brownian motion modulated by regime-switching. The regime-switching is implemented by embedding a continuous-time, finite-state Markov process  $\alpha : \mathbb{R}_+ \rightarrow I_N$ ,  $I_N = \{1, \dots, N\}$ , with intensity matrix  $\Lambda$ . The parameters of the model are: interest rate  $r_\alpha$  of the riskless bond  $L$ ; the average return  $\mu_\alpha$  and the volatility  $\sigma_\alpha$  of the geometric Brownian motion part of  $A$ , each depending on the state of economy  $\alpha$ . For convenience, we denote  $\gamma_\alpha = \mu_\alpha - \frac{1}{2}\sigma_\alpha^2$  in each state  $\alpha \in I_N$ .

We assume that assets associated with the participating contract are invested in the equity market. The assets associated with the contract are denoted by  $A_t$ . The policy has an initial accrued pension value  $P_0$ , which may or may not coincide with the initial investment by the policyholder to the contract. In general, values of  $A_t$  and  $P_t$  do not coincide, even if they initially do. This will be necessary, as the contract will typically involve income guarantees that have to be compensated by dividing only a portion of the profits from the investments to the payoff value  $P_t$ .

A participating pension contract has a bonus policy, which determines an annual interest that payoff value  $P_t$  receives. The value of the annual interest depends on the evolution of the investment in the market. In particular, we shall inspect here a contract in which the annual interest depends only on (1) how long the contract has been in effect; (2) on the current payoff value  $P_t$  of the policy; (3) on the value  $A$  of the assets at present and at the granting of the previous interest. In a more mathematical notation, the evolution of the payoff value  $P_t$  is then given by

$$P_t = \begin{cases} P_{v-1}, & \text{when } t \in [v-1, v) \\ B_v(A_v, A_{v-1}, P_{v-1}) + C_v, & \text{at } t = v, \end{cases} \quad (3)$$

where  $C_v$  stands for a net cash-flow consisting of the net effect of premiums and pension payments at time  $v$ .

The bonus function  $B_v$  in equation (3) is determined at the time of signing of the contract, but it may depend on year  $v$ , and it is a continuous function of its parameters. Each  $B_v$  depends only on the present value of the assets,  $A_v$ , the value of the assets a year before,  $A_{v-1}$ , and the previous payoff value,  $P_{v-1}$ . The difference  $A_t - P_t$  determines how much excess assets the policy has at a time  $t$ , and we shall call it the bonus reserve—the bonus reserve can then also be negative.

The net cash flow  $C_t$  includes both possible continuing payments of the policyholder into the policy, as well as the pension payments out of the policy after retirement. Since pension is an annuity depending on the payoff value  $P_t$ , cash-flow  $C_t$  also depends on the payoff value  $P_t$ . In the following we assume that  $C_t$  depends only on the same parameters as the bonus function  $B$ . As is the case for the bonus function  $B$ , more complex dependencies on the state of the market could also be considered with the price of adding new variables to the fair value function, and thus making its analysis and simulation correspondingly more complicated.

The most common example of this kind of bonus policy is a contract which has a guaranteed minimum annual interest  $r_G$  (Grosen and Jørgensen, 2000), but which grants a higher interest  $R$  in a year when the investments have been doing well. Assuming the bonus policy is kept fixed, these policies can be described by setting all the functions  $B_v$  equal. Contracts with a bonus policy of this type have already been considered in the literature before, for instance, see Grosen and Jørgensen(2000), Jensen et al. (2001), Bacinello (2001), and Tanskanen and Lukkarinen (2003). Grosen and Jørgensen (2003) show that this kind of bonus policy approximates reasonably well bonused of actual

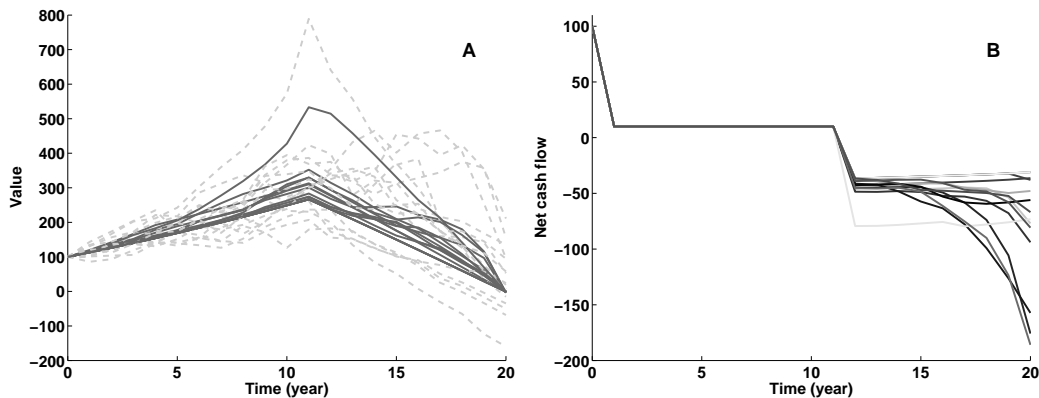


Fig. 1: A schematic figure showing (A) 15 sample paths of assets A (solid light gray lines) and savings P (solid dark gray lines); and (B) 15 sample paths of cash-flows associated with a pension contract. For the first 10 years, the policyholder invests (100 units initial investment, 10 units per annum in this case) into the policy. After 10 years, the policyholder will receive a pension size of which may increase during the pension years. Annual pension is computed according to equation (4).

policies of the Danish life insurance companies. For simplicity, we employ the bonus function

$$B(A_v, A_{v-1}, P_{v-1}) = [1 + \max\{r_G, \alpha(A_{v-1}/P_{v-1} - \gamma)\}] \quad (4)$$

used in Grosen and Jørgensen (2000). In equation (4), parameter  $\alpha$  can be interpreted as a distribution ratio and  $\gamma$  as a target bonus reserve ratio. It is also worth emphasizing that riskless interest rate  $r$  is a continuous interest rate, whereas  $r_G$  is an annual interest, and thus  $e^{r_G} - 1$  would be an annual interest comparable to  $r_G$ .

Fig. 1A shows 15 sample paths of value  $A$  of assets (dashed light grey line) and payoff value  $P$  (solid black line) associated with the contract. Asset value  $A$  varies significantly more than the payoff value  $P$ , which has a limited downside. An example of cash flows associated with a contract of this kind is shown in Fig. 1B, where initial investment is 100 units and annual premium 10 units during the investment period (years 1-10). During years 11-20, an annual pension is paid and it varies stochastically depending on the savings  $P$ .

### 3 Valuation of participating contracts

#### 3.1 Fair value of a contract

Our main goal is to estimate a fair, market-based valuation for liability  $V_t$  of a pension contract. Liability  $V_t$  measures the fair value of the policy defined as

the nominal present value of the policy with respect to a martingale measure  $Q$  at time  $t$ , taking into account the expected future cash flows, that is, premiums and pension payments. In particular,  $V_0$  then represents the fair value of the contract at the time of its signing. Fair value of the pension contract studied here depends only on processes  $A$  and  $P$  during their history, that is  $V_t = V_t(A(\cdot), P(\cdot))$ , hence we must estimate

$$E^Q[V_t(A(\cdot), P(\cdot))|F_t]. \quad (5)$$

The payoff value  $P(t)$  describes how much the policy is worth to the policyholder at time  $t$  if the policyholder exercises the surrender option. While  $P(t)$  is known definitely at time  $t$ , liability  $V_t$  is an estimate that is computed as an expected present value of the future cash flows. For bonus policy (4), it suffices to consider filtration that only depends on  $(A_t, A_{[t]}, P_t)$  (for details, see Tanskanen and Lukkarinen, 2007).

The fair value of the contract at maturity is equal to the final cash flow, that is, to the payoff value  $V_T = P(T)$ . Its fair value at any other time is determined by a measure  $Q$  of the market  $(L, A, P)$  such that the discounted value process  $e^{-\int_t^{t'} r_s ds} A_{t'}$  is a martingale for any  $t < t'$ . Given such a measure, and using the natural filtration  $\{F_t\}$ , the related fair value of a European contract is for  $t \leq T$  given by the random variable

$$V_t^E = E^Q\left[\sum_{k, t_k > t} e^{-\int_t^{t_k} r_s ds} C_{t_k} | F_t\right]. \quad (6)$$

where  $E^Q$  refers to an expectation value with respect to the measure  $Q$  and  $C_{t_k}$  to cash flow at time  $t_k$ ,  $k \in \mathbb{N}$ .

Let us then consider the evolution of the fair value during a given year  $v$ , that is, on the time-interval  $v - 1 < t < v$ ,  $v$ . Since during the time-interval both  $A_{[t]}$  and  $P_t$  remain constant, for any given  $z', p$  there is a function  $f_\alpha(t, x)$  such that  $f_{\alpha(t)}(t, Z_t) = E[V_t | G_t; Z_{[t]} = z', P_{[t]} = p]$ . From the known forms of the generators of the process  $Z_t$  and the embedded Markov chain, we can then derive (Buffington and Elliott, 2002) an equation satisfied by the vector-valued function  $f$ : for all  $\alpha \in I_N$

$$\begin{aligned} \frac{\partial}{\partial t} f_\alpha(t, x) + \frac{\sigma_\alpha^2}{2} \frac{\partial^2}{\partial x^2} f_\alpha(t, x) + \tilde{\gamma}_\alpha \frac{\partial}{\partial x} f_\alpha(t, x) - (r_\alpha + \tilde{\Lambda}_\alpha) f_\alpha(t, x) \\ + \sum_{\alpha'=1; \alpha' \neq \alpha}^N \Lambda_{\alpha' \alpha} f_{\alpha'}(t, x) = 0, \end{aligned} \quad (7)$$

where  $\tilde{\Lambda}_\alpha = \sum_{\alpha'=1; \alpha' \neq \alpha}^N \Lambda_{\alpha \alpha'}$  is the total transition rate from the state  $\alpha$ , and  $\tilde{\gamma}_\alpha = r_\alpha - \frac{1}{2} \sigma_\alpha^2$ . Equation (7) reduces to the Black-Scholes equation (in logarithmic variables) in the limit of a single state.

### 3.2 The 'no-jump' condition

The payoff value  $P$  changes discretely in time. In an analogous manner to the valuation of options on equity paying discrete dividends (Björk, 1998), we must derive a "no-jump" condition that applies during the discrete step to keep arbitrage possibilities away. This will be accomplished in the following.

Assume that  $P$  is updated at  $v$ . Time  $\epsilon$  before time  $t$ , the state of the economy  $\alpha$  is  $a$ . Since  $\alpha$  is a continuous-time Markov chain, the probability that  $\alpha$  changes on the interval  $(t - \epsilon, t + \epsilon)$  vanishes as  $\epsilon$  approaches 0. Hence, we can apply the no-jump argument of Tanskanen and Lukkarinen (2003) and conclude that the absence of arbitrage requires that value of liability  $V$  does not change at  $t = v$ .

An integral part of valuation of a pension contract is taking mortality into account. Given the probability  $\hat{p}_{t+1,t}^x$  of an  $x$  years old person to survive from year  $t$  until  $t + 1$ , the no-jump condition obtains the form

$$\lim_{t \rightarrow v^-} V = \hat{p}_{v,v-1}^x (F_v(z' + \ln(1 + e^{-z'} \tilde{c}_v), \tilde{B}_v(z', z, p) + \tilde{c}_v) - \tilde{c}_v) + \hat{q}_{v,v-1}^x y_{v-1} p A_0, \quad (8)$$

where  $\hat{q}_{d,t}^x = 1 - \hat{p}_{d,t}^x$ , and  $y_t$  is the mortality compensation assumed to be  $y_t = 1.0$  during years  $t \leq T_I$  and  $y_t = 0.8$  for  $t \in (T_I, T]$ . We make the additional assumption that mortality is taken into account only at full years. This has a natural interpretation that deaths are reported to the insurance company at full years, and the compensation is paid immediately after such a report.

## 4 Numerical results

How does the frequency of downturns influence the estimated liability associated with a pension contract? How does guaranteed interest influence it? In the following, we systematically analyze the fair value of a pension contract with a 2-state Markov modulated Black-Scholes model that incorporates the impact of stochastic economic cycles with a model of fair value of a pension contract. We compare the results to those obtained from a one-state model corresponding to valuation of the contract in an unmodulated Black-Scholes framework. We also analyze "fair sets" of parameters in such a model. For the numerical computations, we employ the fully implicit finite-difference method on a lattice (for details, see Tanskanen and Lukkarinen, 2007).

According to Hardy (2003), a 2-state Markov-switching geometric Brownian motion time-series model provides a good approximation to SP500 total return

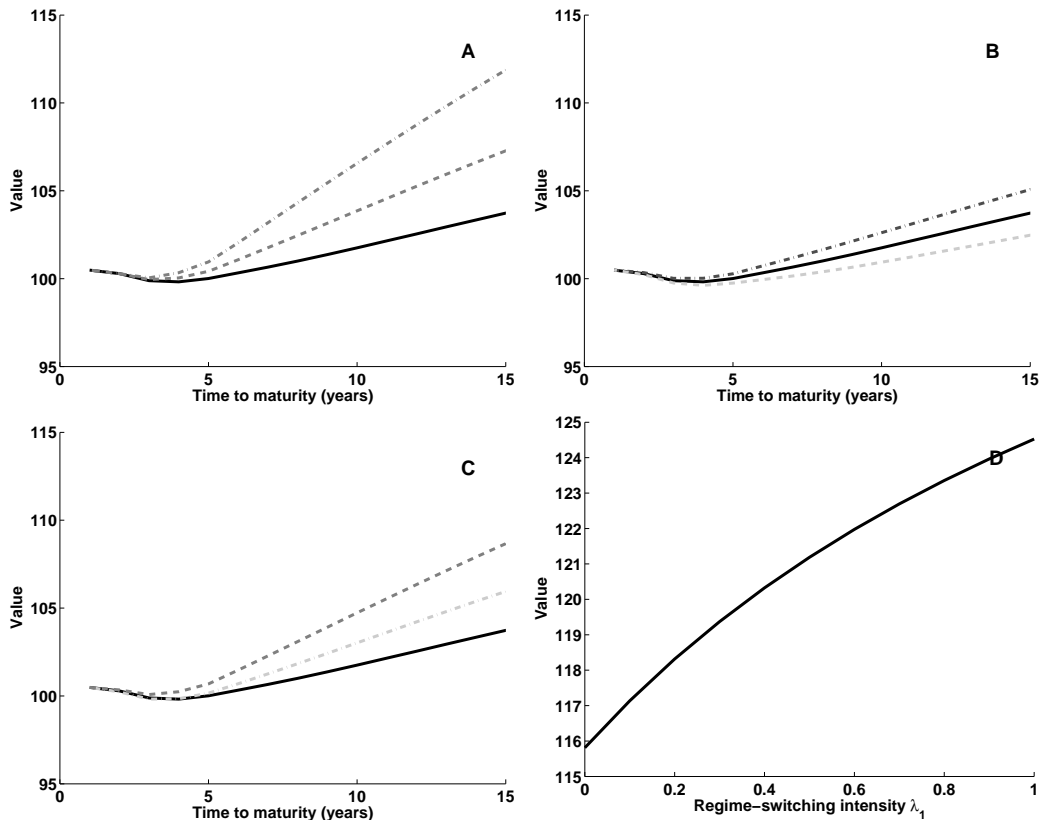


Fig. 2: Economic cycles. Fair value of a pension contract ( $\alpha = 0.3$ ;  $\gamma = 0.1$ ) in which the initial investment is 100 units, as a function of time to maturity. Estimating how the addition of a state impact the fair value: (A) Comparison of fair value estimated using a 1-state model (black solid line) with 11.6% volatility, a 2-state model (grey dashed line) with 11.6% state 1 and 22.5% state 2 volatility, and a 2-state model (grey dash-dotted line) with 11.6% state 1 and 33.4% state 2 volatility; (B) Comparison of fair value estimated using a 1-state model (black solid line) with  $r = 3.0\%$ , a 2-state model (dark grey dash-dotted line) with  $r_1 = 3.0\%$  and  $r_2 = 2.0\%$ , and a 2-state model (light grey dashed line) with  $r_1 = 3.0\%$  and  $r_2 = 4.0\%$ . In each case  $\sigma = 11.6\%$ ; (C) Comparison of fair value estimated using a 1-state model (black solid line) with  $r = 3.0\%$  and  $\sigma = 11.6\%$ , a 2-state model (dark grey dashed line) with  $r_1 = 3.0\%$ ,  $r_2 = 2.0\%$  and  $\sigma_1 = 11.6\%$ ,  $\sigma_2 = 22.5\%$ , and a 2-state model (light grey dash-dotted line) with  $r_1 = 3.0\%$ ,  $r_2 = 4.0\%$  and  $\sigma_1 = 11.6\%$ ,  $\sigma_2 = 22.5\%$ ; (D) Dependence of fair value (solid black line) on regime-switching intensity  $\lambda_1$  from state 1 to state 2, when  $\lambda_2 = 2.0$  per year ( $r_1 = 3.0\%$ ;  $r_2 = 3.0\%$ ;  $\sigma_1 = 11.6\%$ ;  $\sigma_2 = 22.5\%$ ).

index during years 1954-1999. Using the maximum likelihood method (Hardy, 2003), we estimate that in state 1 annual volatility of SP500 log returns during 1954-2006 is 11.6% and average dwell time is 26 months, while the high-volatility state 2 has 22.5% annual volatility and the average dwell time is 6 months. Hence, intensity parameters for the model are  $\lambda_1 = 12/26$  and  $\lambda_2 = 12/6$ , and volatilities  $\sigma_1 = 11.6\%$  and  $\sigma_2 = 22.5\%$ .

First, we consider how the presence of stochastic economic cycles influences

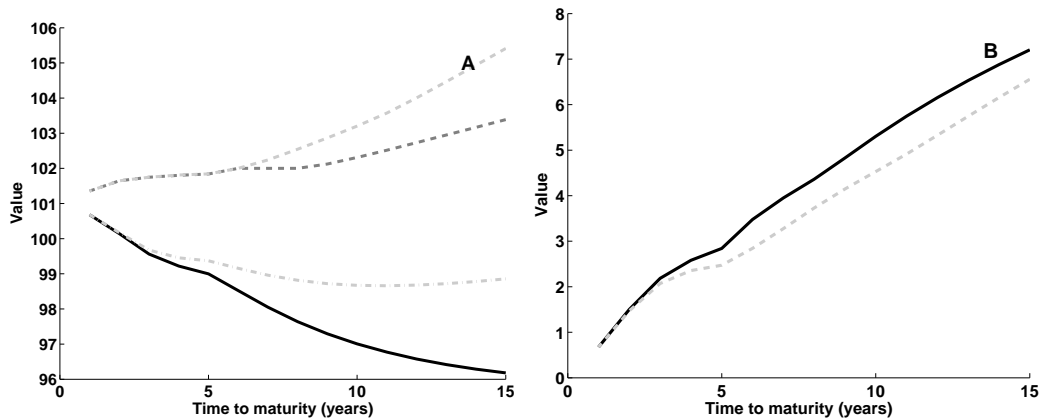


Fig. 3: Fair value of surrender option in a contract defined by  $(\alpha = 0.3; \gamma = 0.1; \sigma_1 = 11.6\%; \sigma_2 = 2.25\%; r_1 = 5.0\%; r_2 = 5.0\%; r_G = 2.0\%)$ : (A) Fair value of a European (black solid line) and American (dark grey dashed line) 1-state model, a European (light grey dash-dotted line) and American 2-state model (light grey dash-dotted line); (B) Fair value of a surrender option in a 1-state model (black solid line), and in a 2-state model with regime-switches (light grey dashed line).

fair value of a pension contract by employing a 2-state model. The presence of a high-volatility state increases the fair value of a participating pension contract (Fig. 2A). Similarly, the presence of a state with a low riskless interest rate will increase the fair value (Fig. 2B), while the presence of a state with a high riskless interest rate will lower the fair value (Fig. 2B). The combined influence of regime-dependent riskless interest rate and volatility have similar impacts, however, different choices may have influence in opposite directions (Fig. 2C). The frequency of regime-switches influences the fair value. When a regime-switch increases the fair value, frequent regime-switches will increase the fair value even further (Fig. 2D).

An American contract gives the policyholder a right to withdraw the investment at any time. Fair value of an American pension contract always exceeds that of a European contract (Fig. 3A). Fair value of the surrender options in a 1-state model is slightly higher than in a 2-state model (Fig. 3B). This is a result of a higher volatility in the 2-state model, which reduces fair value of the surrender option.

A fair set of parameters is defined as the set of parameters that all yield the contract the same fair value that corresponds to the premium paid by the policyholder. For a contract with only a single premium (the initial investment), a fair set of parameters gives the contract the fair value that equals the initial investment. Fig. 4A shows how variation in parameter  $r_G$  influences the choice of bonus distribution ratio  $\alpha$ , when pair  $(\alpha, r_G) \in F$  and set  $F \subset \mathbb{R}^2$  is a fair set for the contract as the initial investment by the policyholder. The presence of a high volatility state reduces the fair  $\alpha$  (Fig. 2A). This is an intuitive

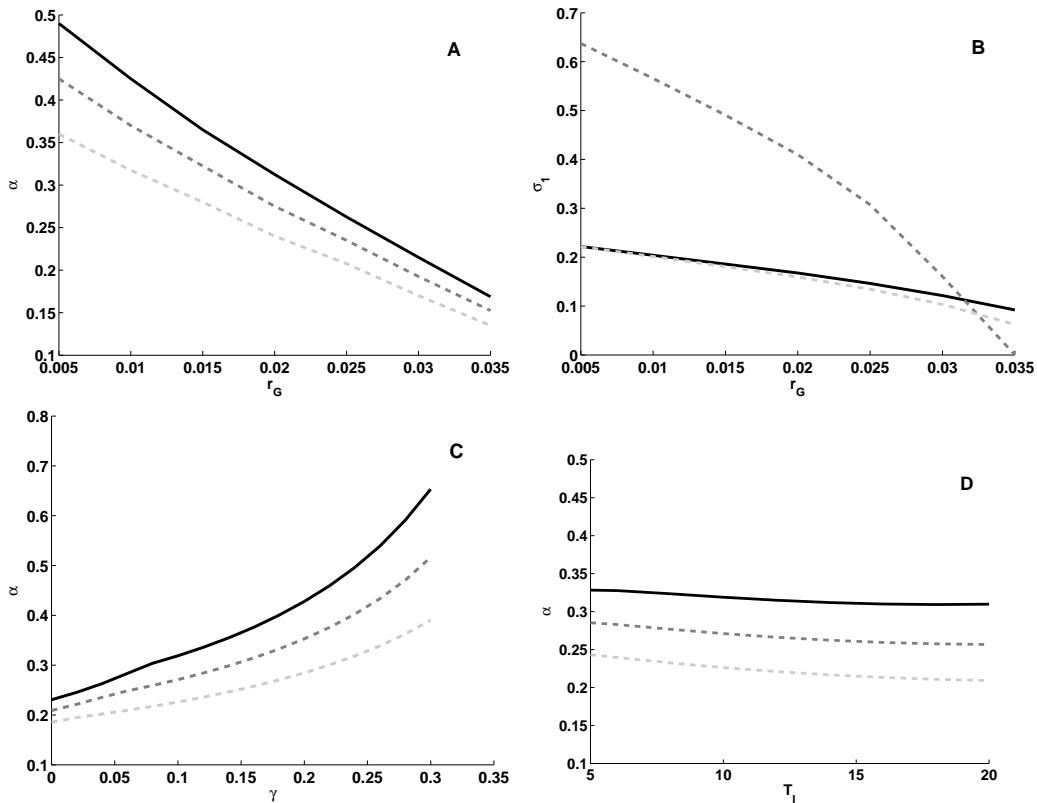


Fig. 4: The fair set of parameters. (A) Dependence of fair bonus distribution ratio  $\alpha$  on guaranteed interest  $r_G$  in a 1-state model with 11.6% volatility (black solid line), a 2-state model with 11.6% state 1 and 22.5% state 2 volatility (dark grey dashed line), and a 2-state model with 11.6% state 1 and 33.4% state 2 volatility (light grey dashed line); (B) Volatility  $\sigma$  implied by the requirement that the contract ( $\alpha = 0.3, \gamma = 0.1$ ) is fair in a 1-state model (solid black line), in a 2-state model with 22.5% state 2 volatility (light grey dashed line), and in a 2-state model with 11.6% state 1 volatility (dark grey dashed line); (C) Dependence of fair  $\alpha$  on the target bonus reserve ratio  $\gamma$  in a 1-state model (solid black line), in a 2-state model with 11.6% state 1 and 22.5% state 2 volatility (dark grey dashed line), and in a 2-state model with 11.6% state 1 and 33.4% state 2 volatility (light grey dashed line); and (D) The dependence of  $\alpha$  on contract duration  $T$  in a 1-state model (solid black line), in a 2-state model with 11.6% state 1 and 22.5% state 2 volatility (dark grey dashed line), and in a 2-state model with 11.6% state 1 and 33.4% state 2 volatility (light grey dashed line), given  $r_G = 3.0\%$  and  $\gamma = 0.1$ .

result, since higher volatility increases the value of a contract.

Given parameters  $\alpha, \gamma$  and  $r_G$ , volatility  $\sigma$  can be chosen so that the contract is fair. Fig. 4B shows that in a 1-state model, higher  $r_G$  implies a lower volatility. In a 2-state model with a 11.6% state 1 volatility, the implied state 2 volatility decreases in a fair contract dramatically from 65% at  $r_G = 0.5\%$  to 0% at  $r_G = 3.5\%$ . In a 2-state model with a 22.5% state 2 volatility, a fair contract has volatility in state 1 that is slightly lower than in the 1-state model.

Fig. 4C shows that the fair target bonus reserve ratio  $\gamma$  depends on  $\alpha$ : a higher  $\gamma$  requires a higher  $\alpha$  for the contract to be fair. Since a 2-state model has a higher volatility than a 1-state model, a fair bonus distribution ratio  $\alpha$  is lower in a 2-state model than in a 1-state model. Fig. 4D shows that fair  $\alpha$  slightly decreases with maturity of a contract.

## 5 Discussion

Markov process driven regime-switching often significantly improves models of financial time series (Hamilton, 1989; Hardy, 2003). Motivated by this, we have in this article considered the fair valuation of a participating non-statutory pension contract in the Markov-modulated Black-Scholes framework. The major improvement of the developed model over the Black-Scholes framework is that long duration liabilities are estimated in a more realistic way, that is, the influence of the state of the economy is taken into account.

A realistic description of pension liabilities with long duration requires consideration of multiple states of the economy, as is done in the models presented in this article. The method presented in this manuscript provides the minimal set of tools to estimate fair value of participating pension contracts in the presence of stochastic economic cycles. Our results suggest that while fair valuation is a reasonable approach for the valuation of insurance liabilities, its application to valuation of long duration liabilities requires carefully chosen parameters and appropriate methods.

## Acknowledgments

The work of AT was supported by the Jenny ja Antti Wihurin foundation. AT wishes to thank prof. Luis H.R. Alvarez, Asko Tanskanen and Jussi Pelkonen for helpful comments.

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