# Longevity risk and hedge effects in portfolios of life insurance products with investment risk

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# Introduction

# Measuring longevity risk



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- Future survival probabilities are stochastic → uncertainty in payments of life insurance products;
- Longevity risk is often quantified by distributional characteristics of the discounted cash flows, assuming a constant and deterministic interest rate, $r$.
- Problem: interpretation?
  - In real world there is investment risk;
  - Level of future payments is uncertain → cannot (fully) hedge against investment risk;
  - When return is as least $r$, this risk measure is a conservative one.
- We use an asset-liability approach to quantify longevity risk.

# Longevity risk



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- Focus on the effect of systematic longevity risk in a portfolio of life insurance products:
  - Effect of gender composition;
  - Effect of survivor annuities;
  - Effect of death benefits.
- Effect of investment risk when there exists longevity risk:
  - Effect of longevity risk on investment risk;
  - Effect of investment risk on hedge potential.
- Hedge effects of survival swaps:
  - Effect of survivor annuities;
  - Effect of basis risk.

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# Model

# Evolution of assets



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- Risk measure: $\mathbb{P}\left(A_T < 0\right)$.
- Evolution of the assets:
  $R_{s+1}$ return, $\widetilde{L}_s$ liability payment

$$A_{s+1} = \left(A_s - \widetilde{L}_s\right) \cdot \left(1 + R_{s+1}\right), \quad \text{for } s = 0, \cdots, T.$$

- Terminal asset value:

$$A_T = A_0 \cdot \prod_{s=1}^{T} \left(1 + R_s\right) - \sum_{s=1}^{T} \widetilde{L}_s \cdot \prod_{\tau=s+1}^{T} \left(1 + R_\tau\right).$$

- Capital requirement: required initial asset value in excess of the best estimate

$$A_0 = (1 + c) \cdot BEL.$$

# Risk measure



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- Investment portfolio is decomposed in:
  - Best estimate portfolio, with return $r_s^{be}$;
  - Buffer portfolio, with return $r_s^{bu}$.
- Probability of ruin:

$$\mathbb{P}\left(A_T < 0\right) = \mathbb{P}\left(L > (1+c)\cdot BEL\right),$$

$$L \equiv BEL + \sum_{s=1}^{T}\left(\frac{\widetilde{L}_t - \mathbb{E}\left[\widetilde{L}_s\right]\cdot P^{(s)}\cdot\prod_{\tau=1}^{s}\left(1+r_\tau^{be}\right)}{\prod_{\tau=1}^{s}\left(1+r_\tau^{bu}\right)}\right).$$

- Effect of longevity risk measured by the capital requirement in excess of best estimate:

$$c = \frac{\mathbb{Q}_{1-\epsilon}\left(L\right)}{BEL} - 1.$$

# Sources of risk



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Focus on non-hedgeable risks: Best estimate portfolio invested in bonds.
We decompose $L$ into four components:

i) Best estimate of the liabilities;
Deterministic, no uncertainty.

ii) Pure longevity risk component;
Uncertainty in $L$ given expected returns.

iii) Pure investment risk component;
Uncertainty in $L$ given expected cash flows.
*Hedgeable risk: generally set equal to zero.*

iv) Interaction investment and longevity risk component;
Uncertainty in $L$ which is not captured by the other components.
*Additional uncertainty!*

# Model



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- Life insurance products:
  - i) Single life annuity;
  - ii) Survivor annuity;
  - iii) Death benefit.
- Uncertainty in forecasting models includes: Process risk, parameter risk, and model risk.
- Longevity risk:
  - Variants of Lee-Carter (1992)-model;
  - Variants of Cairns-Blake-Dowd (2006)-model;
  - P-Splines model (Currie, Durbin, and Eilers; 2004).
- Investment risk:
  - Bond prices: Vasicek model;
  - Stock prices: Brownian motion with drift.

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# Effect portfolio mix

# Effect interest rate risk



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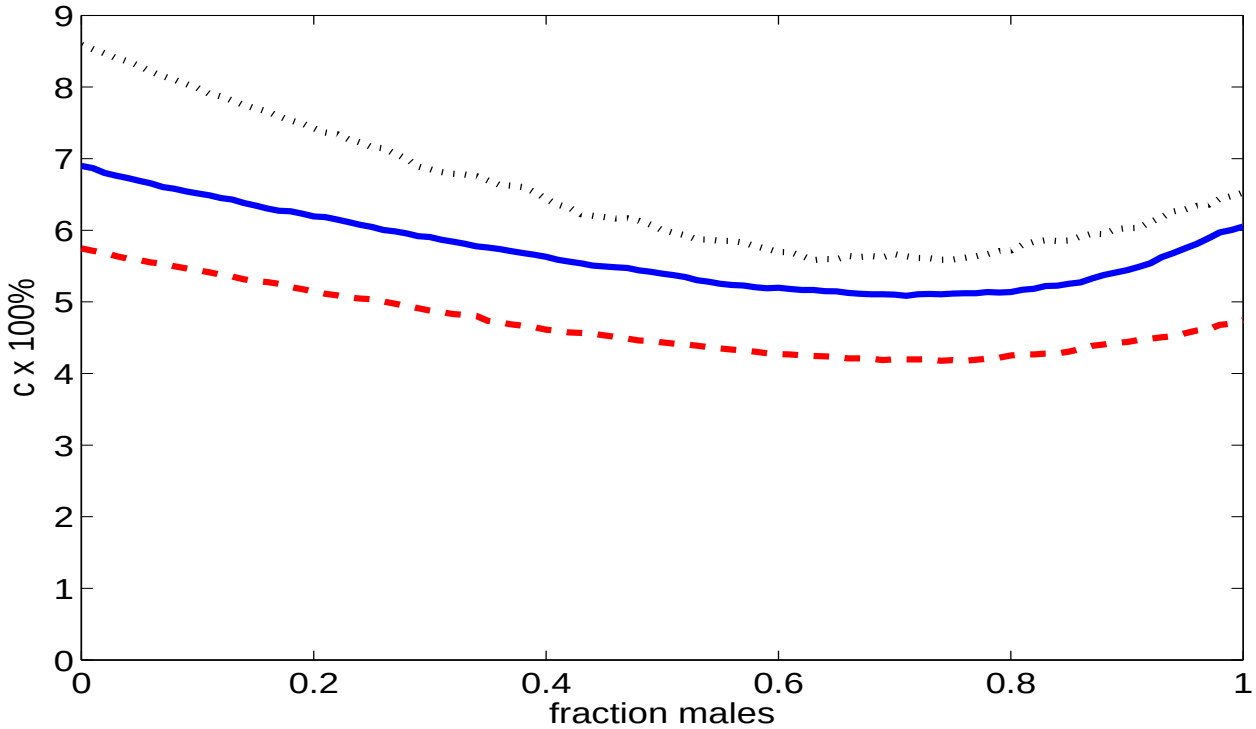
- Buffer requirement depends on investment strategy and liability portfolio of insurer.
- Buffer portfolio invested in one-year bonds, best estimate portfolio:
  - Only one-year bonds;
  - Portfolio of bonds eliminating pure investment risk.
- Buffer requirements:

| Product | $c^{1y,1y}$ | $c^{elh,1y}$ | $c^{LO}$ |
|---|---|---|---|
| Male single life annuity | 27.6% | 6.1% | 4.4% |
| Female single life annuity | 33.0% | 6.9% | 4.9% |

- We observe:
  - Investment risk might be large;
  - Liability only approach might underestimate longevity risk.

# Effect gender mix






Solid curve: 100% bonds; dashed curve: 1/3 equity, 2/3 bonds; dotted curve: 100% equity.

- Longevity risk is higher for females than males;
- Investment risk significantly affects required buffer;
- Higher duration typically increases impact of investment risk.

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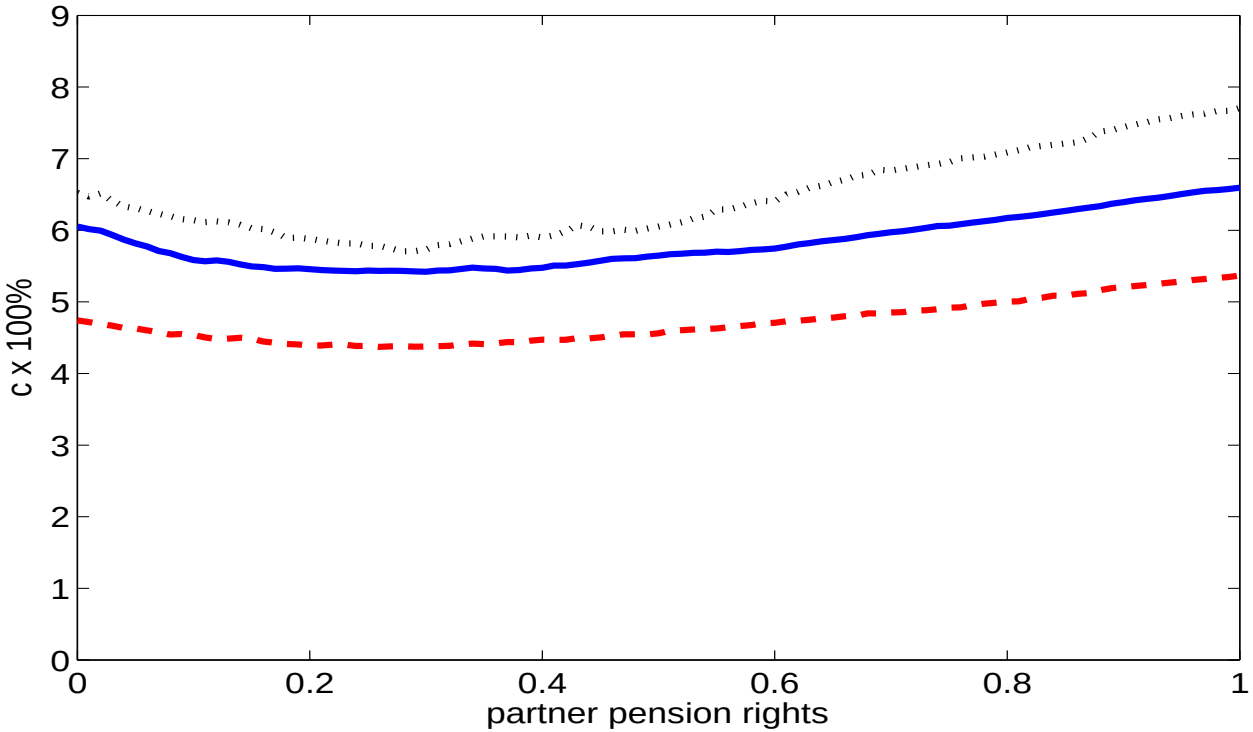
# Effect product mix






Solid curve: 100% bonds; dashed curve: 1/3 equity, 2/3 bonds; dotted curve: 100% equity.

- Survivor annuities can reduce longevity risk;
- Effect of hedge potential depends significantly on investment risk.

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# Effect death benefits I



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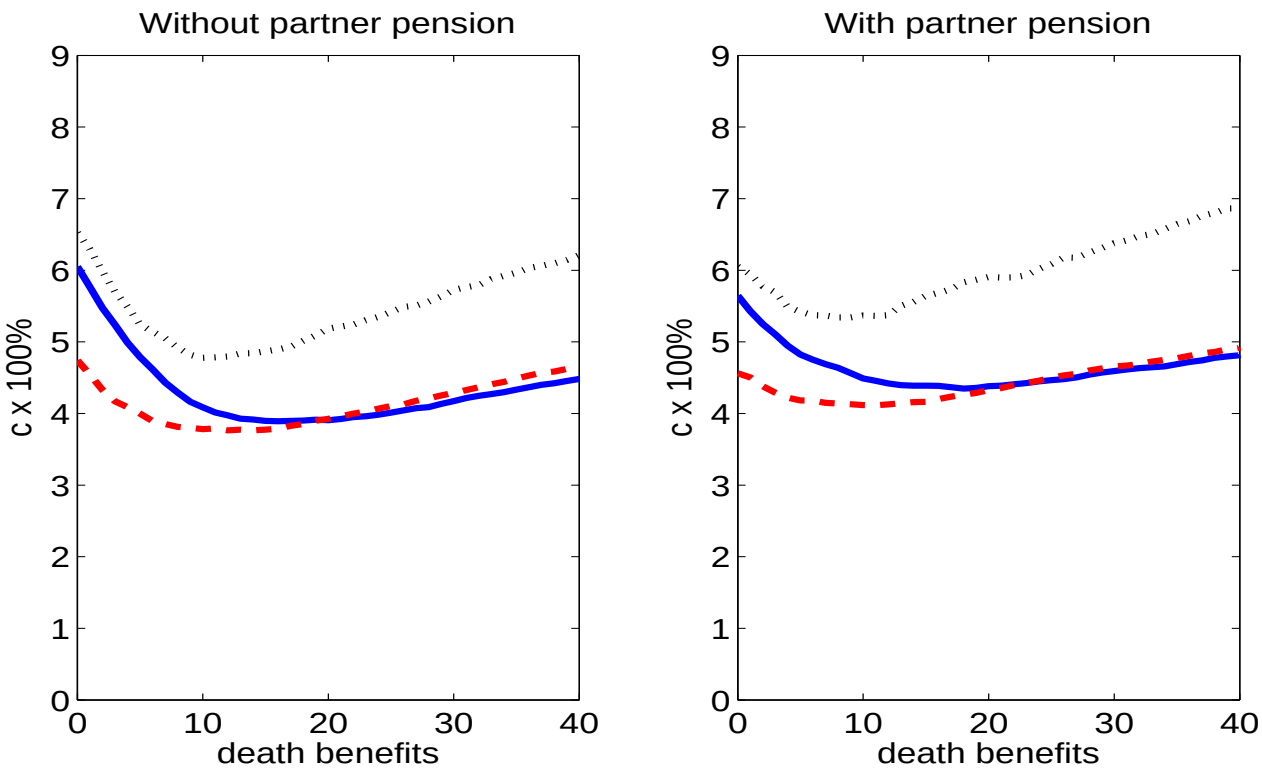
- Investment risk affects hedge potential.
- Death benefit consists of a single payment at the moment the insured dies.
  $\longrightarrow$ effect of living longer = postponing payment.
- Hedge effects of death benefits are due to discounting effects!
- When there is no investment risk:
  Consider an single life annuity with a yearly payment of 1 and a death benefit with a single payment $\delta$.
  For $\delta = \frac{1+r}{r}$, the level of the aggregate payments of the portfolio is independent of the number of survivors.

# Effect death benefits II






Solid curve: 100% bonds; dashed curve: 1/3 equity, 2/3 bonds; dotted curve: 100% equity.

- As expected: death benefits can reduce longevity risk;
- Survivor annuities reduce hedge effects of death benefits;
- Hedge effects of death benefits are significantly influenced by investment risk.

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# Effect survivor swaps

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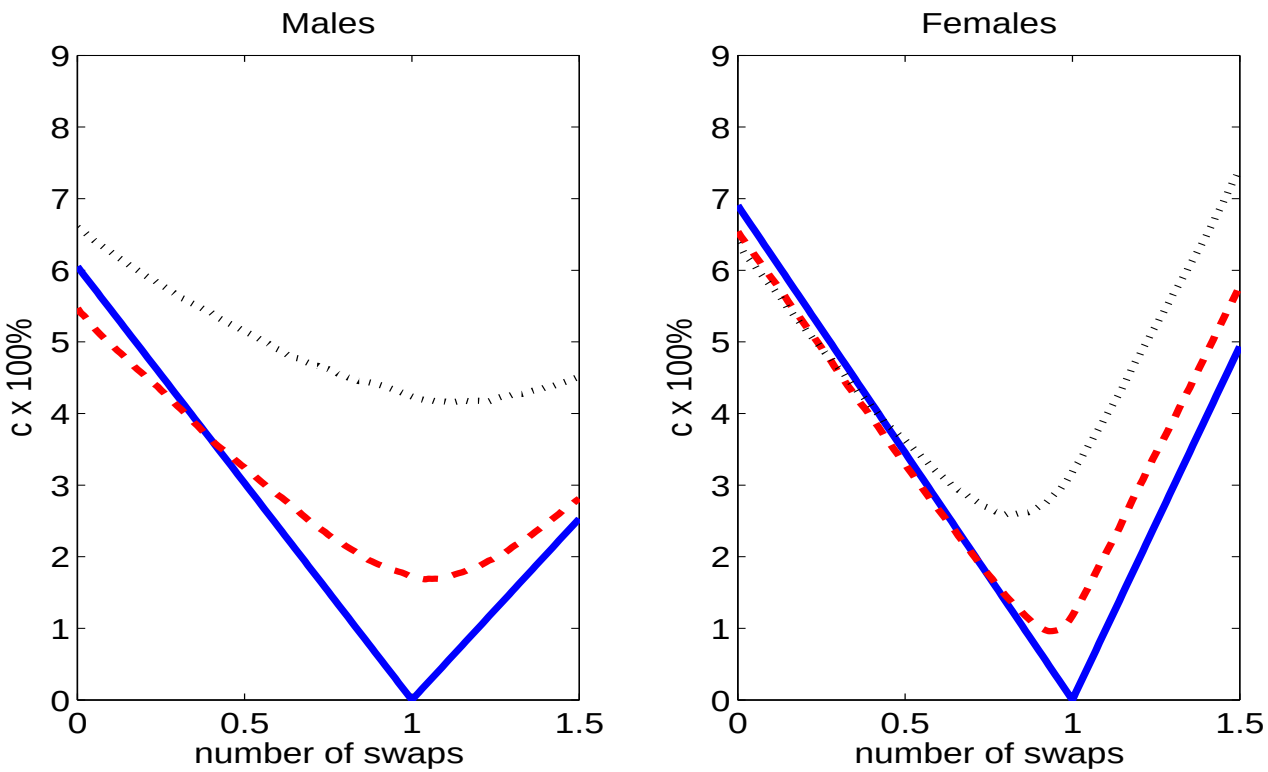
- Mortality linked assets can reduce longevity risk.
- An often proposed product is a survivor swap or longevity bond.
- The payments of a survivor swap in year $s$ are given by:

$$SS(s, ref) = S(s, ref) - K(s, ref).$$

- Without basis risk: survivor swaps can provide a perfect hedge for single life annuities;
- How well do they work:
  - In a portfolio of life insurance products?
  - When there is basis risk?

# No basis risk






Solid curve: no survivor annuities; dashed curve: 35% survivor annuities; dotted curve: 70% survivor annuities.

- Survivor annuities can significantly affect hedge potential of survival swaps;
- Maximum number of swaps depends on product and gender mix.

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# Basis risk



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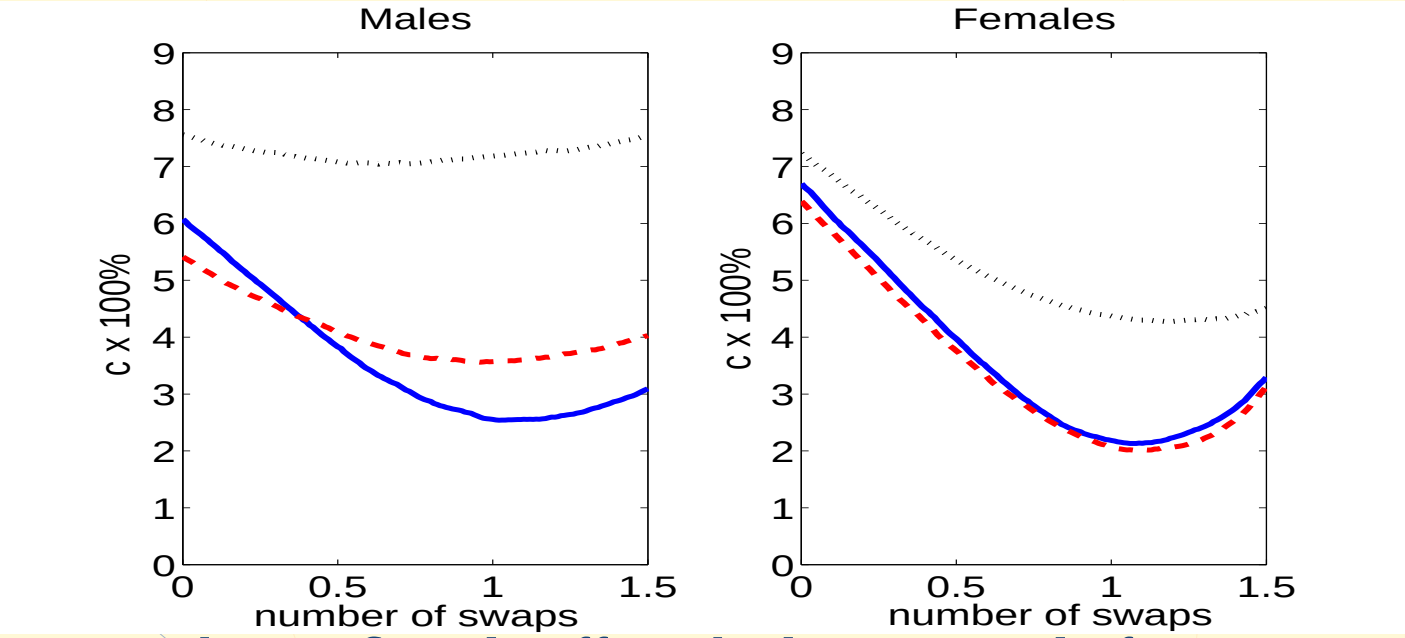
- Mortality rates of insureds and general population differ.
- Denuit (2008) uses a Cox-type relational model:

$$\log(\mu_{x,t}^{(h)}) = \alpha^{(h)} + \beta^{(h)} \cdot \log(\mu_{x,t}^{(g)}),$$

with $\alpha^{(m)} = -1.54, \alpha^{(f)} = -1.02, \beta^{(m)} = 0.82, \beta^{(f)} = 0.91.$




- Basis risk significantly affects hedge potential of survival swaps.

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- Liability only approach might underestimate reserve requirements;
- Product and gender mix influence required buffer;
- Investment risk affects hedge potential:
  - Gender mix and survivor annuities:
    A higher duration typically leads to a stronger influence of investment risk.
  - Death benefits:
    More investment risk leads to a weaker hedge potential of death benefits.
- Basis risk might significantly reduce the hedge potential of survivor swaps.