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COMPARATIVE STUDY OF MORTALITY FORECASTING MODELS

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MORTALITY FORECASTING METHODOLOGIES

- Biomedical Process-based
- Expert based
- Structural Modelling (Explanatory or Econometric)
- Decomposition and Disaggregation
- Trend Modelling (Extrapolation)

REFLECTIONS

- Extrapolation methods fail to account for future structural change.
- Expert opinion has been conservative e.g. choice of target, target date, interpolation path.
- Expert approach limited to aggregate measures.

REFLECTIONS (cont…)

- Structural models can allow for feedback, limiting factors, lagged variables. Theoretical advantages not matched by forecasting performance.

- Decomposition by cause of death has led to conservative forecasts.

- Criteria for choosing a method: central estimates, measures of uncertainty (note different elements: process error, parameter error, model error).

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ACTUARIAL JUSTIFICATION FOR EXTRAPOLATION METHODS

- Complexity and stability of historical trends

- Extrapolation may be the most reliable approach in terms of forecast accuracy.

- Note trade-off between model fit and forecast accuracy: in-sample errors may not be a good guide to forecast errors.

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ACTUARIAL JUSTIFICATION FOR EXTRAPOLATION METHODS (cont…)

- "…we cannot afford to be ashamed of extrapolating the observed regularities of the past" (Keyfitz, 1982)

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SIGNIFICANT RECENT DEVELOPMENTS

- LEE-CARTER MODEL 1992
- AGED-PERIOD-COHORT VERSION OF LC MODEL 2006 (RENSHAW & HABERMAN)
- BLAKE-CAIRNS-DOWD MODELS 2008

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LEE-CARTER MODEL

$$\mu_x(t) = \exp(\alpha_x + \beta_x \kappa_t + \varepsilon_{xt})$$

- Parameter constraints: $\sum_x \beta_x = 1$ and $\kappa_{t_1} = 0$
- Interpretation of parameters
- Methods of fitting: SVD,WLS, GLM formulation
- Diagnostics
- Smoothing of β_x
- Forecasting: based on time series models of κ_t

Also fitted to log $\left[\frac{q_x(t)}{1-q_x(t)}\right]$

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EXTENSIONS TO IMPROVE FIT

- Optimize choice of fitting period
- Add extra time factors

$$\exp(\alpha_x + \beta_x^{(1)}\kappa_t^{(1)} + \beta_x^{(2)}\kappa_t^{(2)})$$

- Allow for cohort effect

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APC VERSION OF LEE-CARTER

$$\mu_x(t) = \exp(\alpha_x + \beta_x^{(1)}\kappa_t + \beta_x^{(0)} i_z + \varepsilon_{xtz}) \text{ where } z = t - x$$

Note strong observed cohort effect for particular cohorts in UK (1925-45), US, France, Germany, Japan, Sweden.

Parameter constraints: $\sum_x \beta_x^{(1)} = 1,\ \sum_x \beta_x^{(0)} = 1,\ \kappa_{t_1} = 0\ \ \beta_x^{(1)} > 0.$

Also fitted to log $\left[\frac{q_x(t)}{1 - q_x(t)}\right]$.

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BLAKE-CAIRNS-DOWD MODELS

4 main models proposed for log $\left[\frac{q_x(t)}{1-q_x(t)}\right]$

M5 $\kappa_t^{(1)}+\beta_x^{(1)}\kappa_t^{(2)}$

M6 $\kappa_t^{(1)}+\beta_x^{(1)}\kappa_t^{(2)}+i_z$

M7 $\kappa_t^{(1)}+\beta_x^{(1)}\kappa_t^{(2)}+\beta_x^{(2)}\kappa_t^{(3)}+i_z$

M8 $\kappa_t^{(1)}+\beta_x^{(1)}\kappa_t^{(2)}+\beta_x^{(3)}i_z$

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BLAKE-CAIRNS-DOWD MODELS (cont…)

The following deterministic functions are specified:

$$\beta_x^{(1)} = (x - \bar{x}),\ \beta_x^{(2)} = \left\{ (x-\bar{x})^2 - \frac{1}{k}\sum_{i=x_1}^{x_k}(i-\bar{x})^2 \right\},\ \bar{x} = \frac{1}{k}\sum_{i=x_1}^{x_k} i,\ \beta_x^{(3)} = (x_c - x)$$

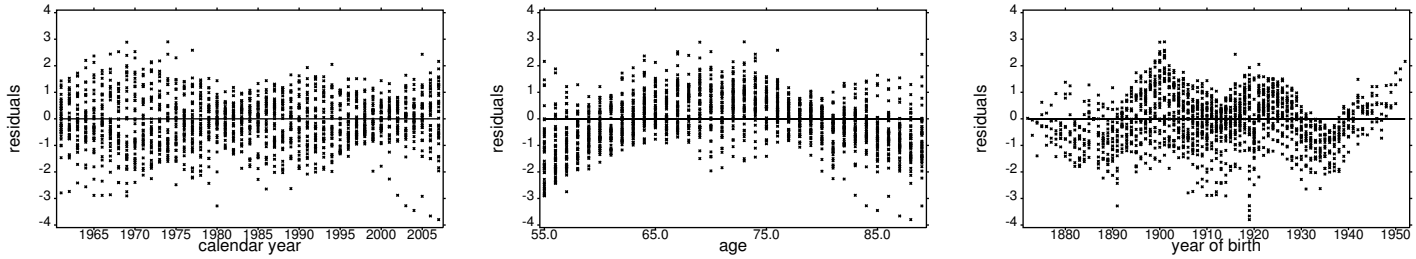
and x_c is a pre-determined constant.

CASE STUDY

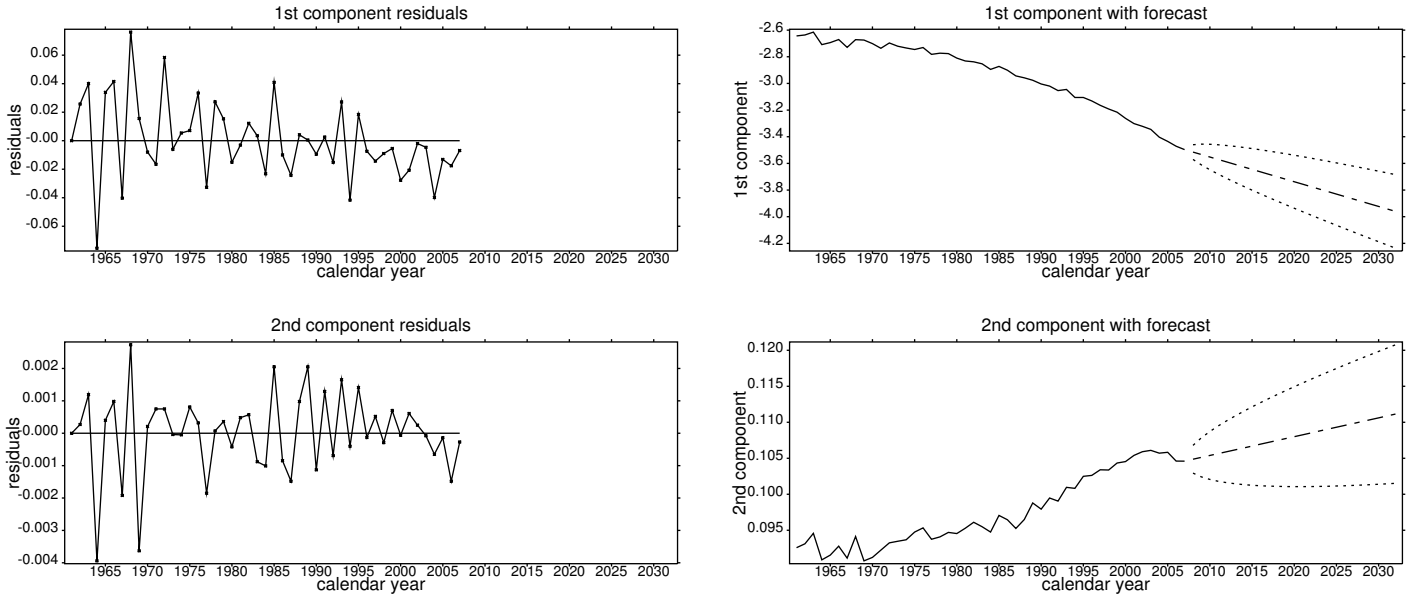
Models are fitted to England & Wales male mortality experience for 1961-2007, for ages 55-89.

We compare goodness of fit and predictions of key indices for individuals aged 60, 65, 70 and 75 for the most recent period using data for 1961-1997, 1961-1999,…, 1961-2007.

We also compare predictions for 2007 using data for 1961-2007, 1963-2007, …, 1971-2007.

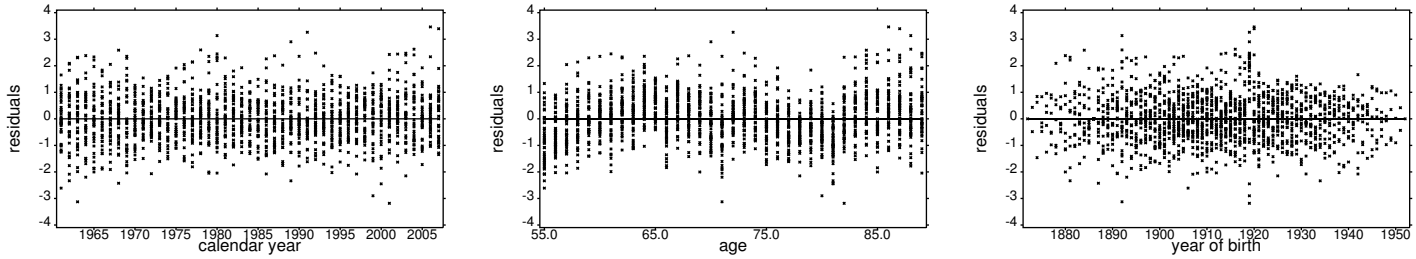


Model M5: deviance residual plots

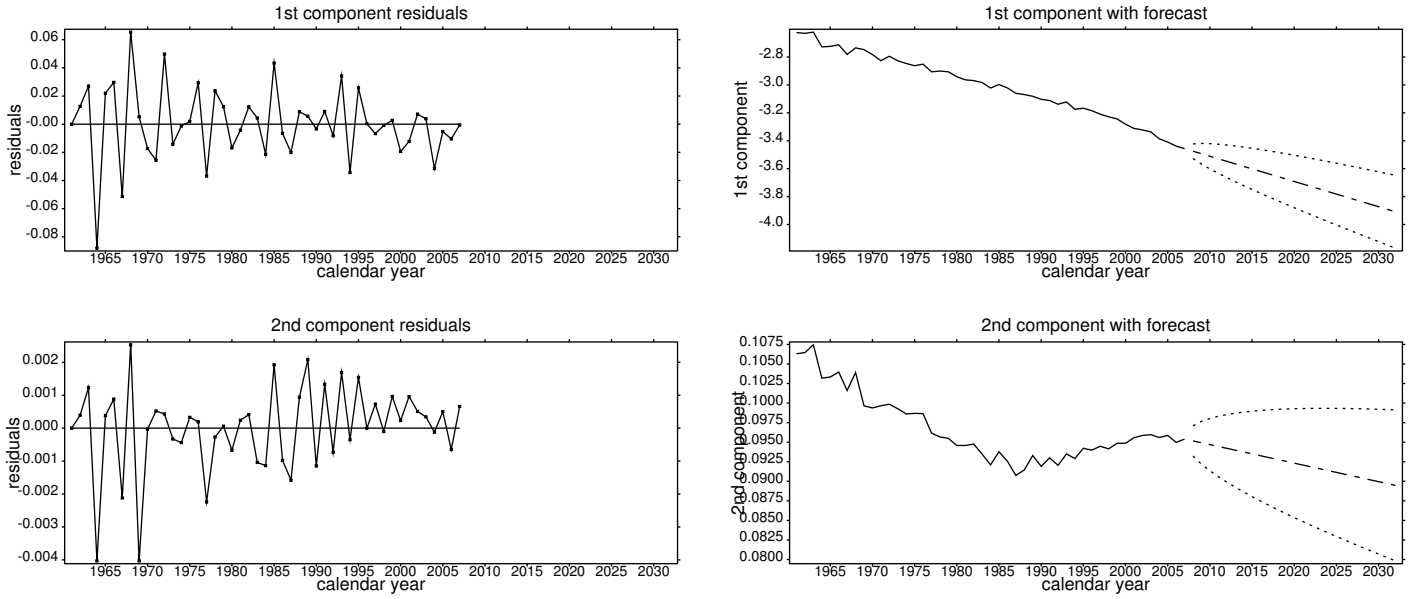


Model M5: period indices bi-variate random walk

Fig 1. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target q(x,t), predictor M5.

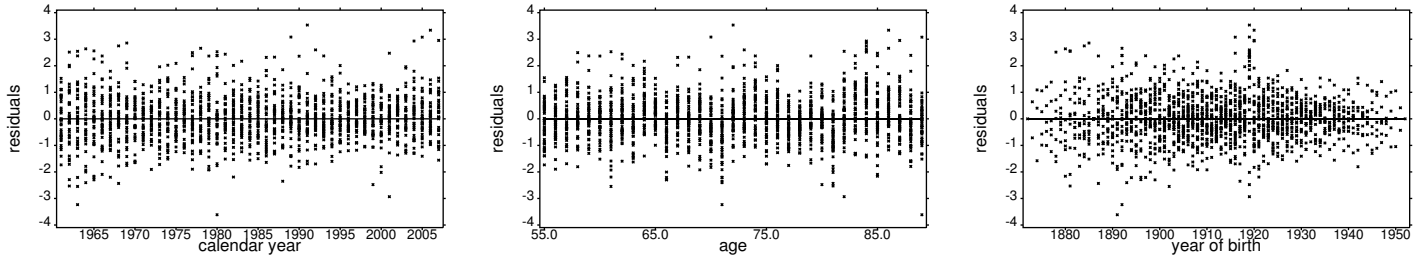


Model M6: deviance residual plots

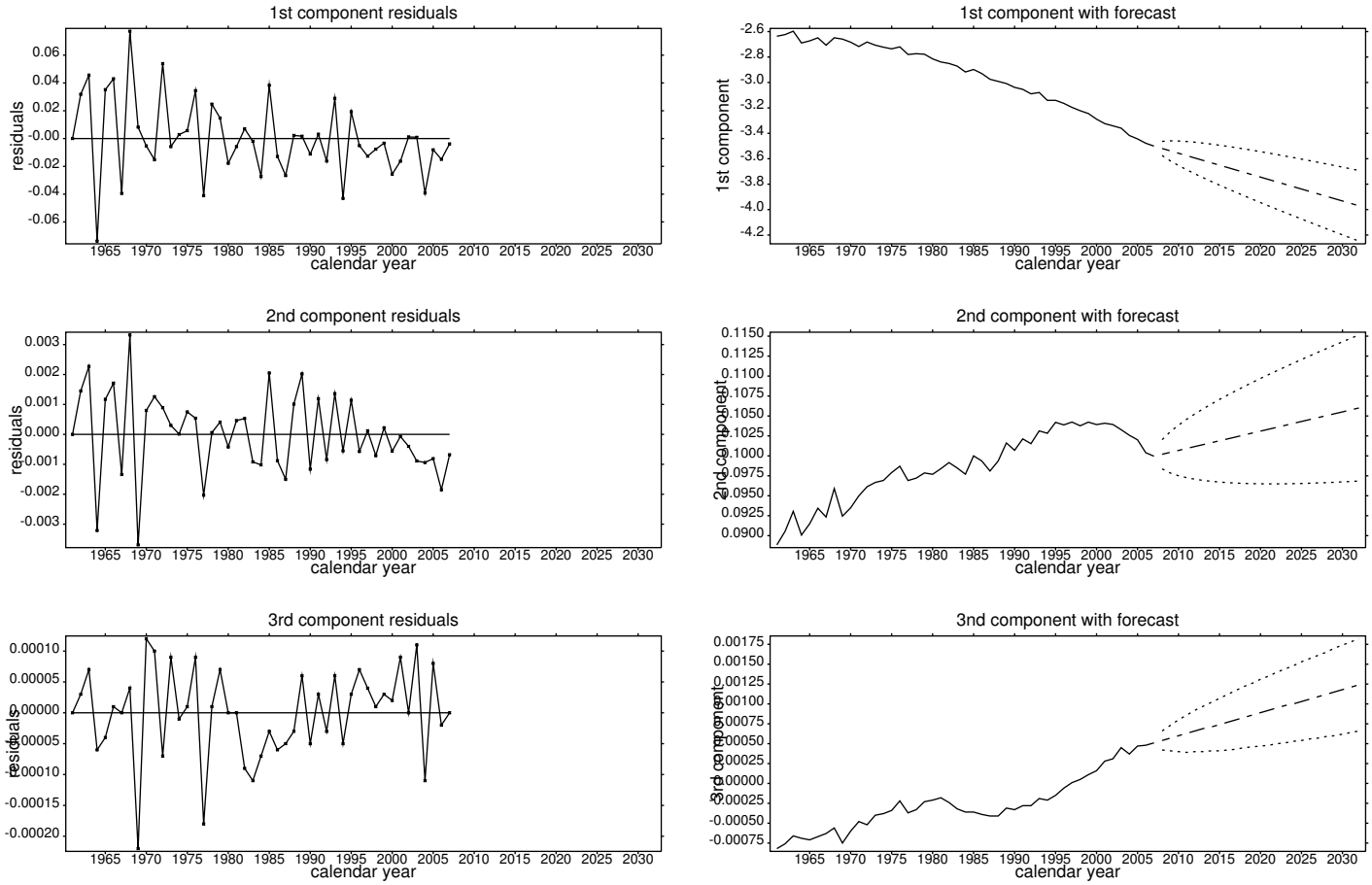


Model M6: period indices bi-variate random walk

Fig 2. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target q(x,t), predictor M6.

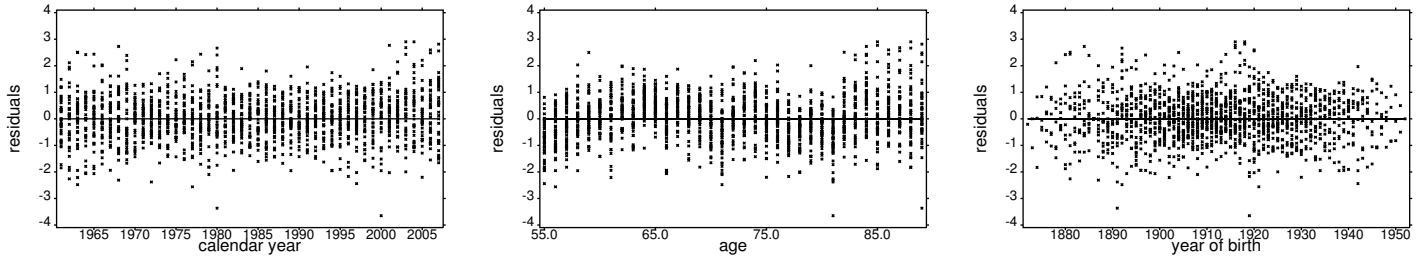


Model M7: deviance residual plots

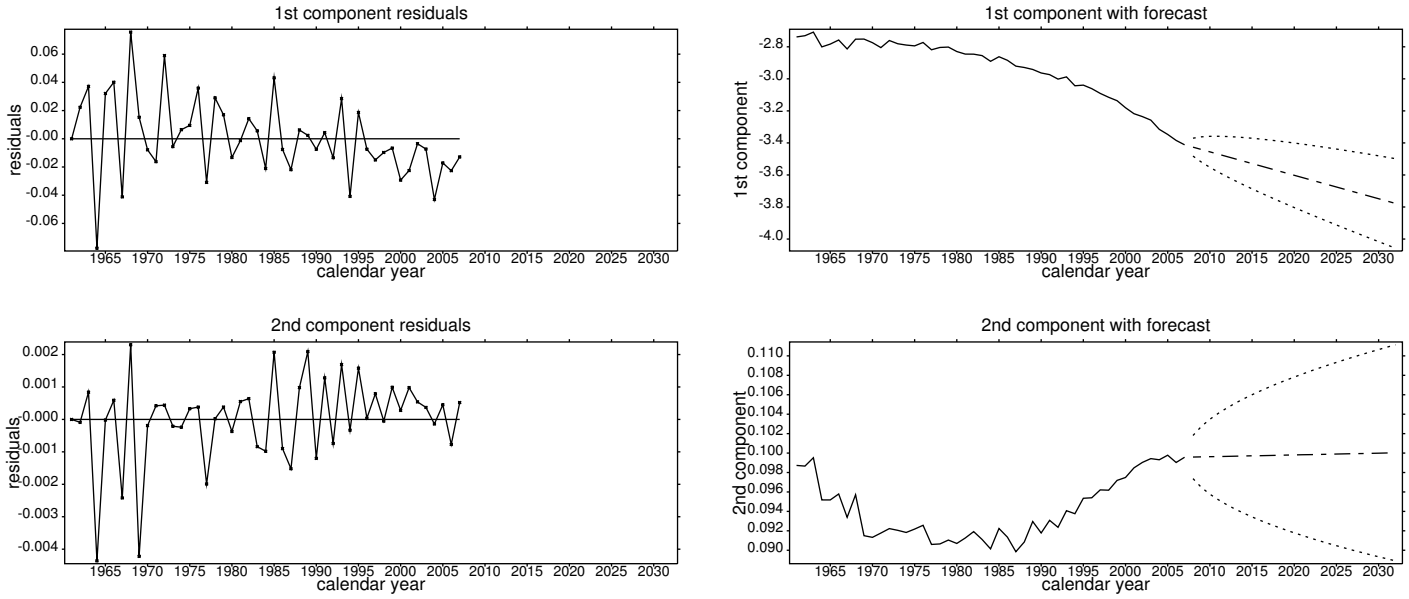


Model M7: period indices tri-variate random walk

Fig 3. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomila responses, log-odds link, target q(x,t), predictor M7.

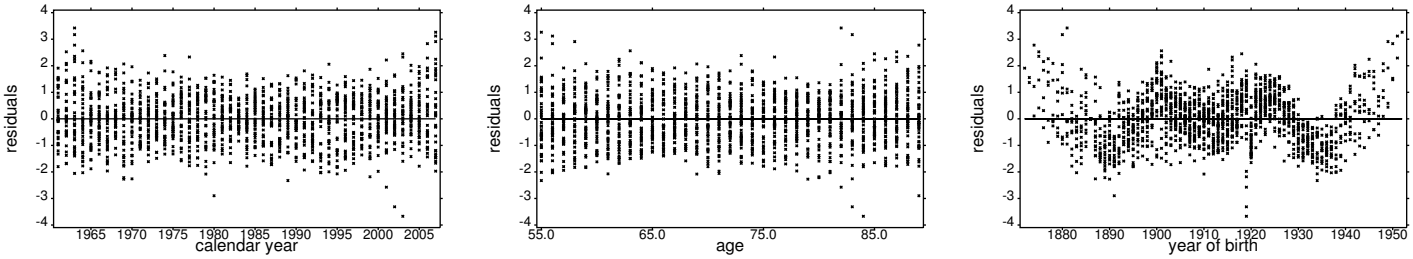


Model M8: deviance residual plots

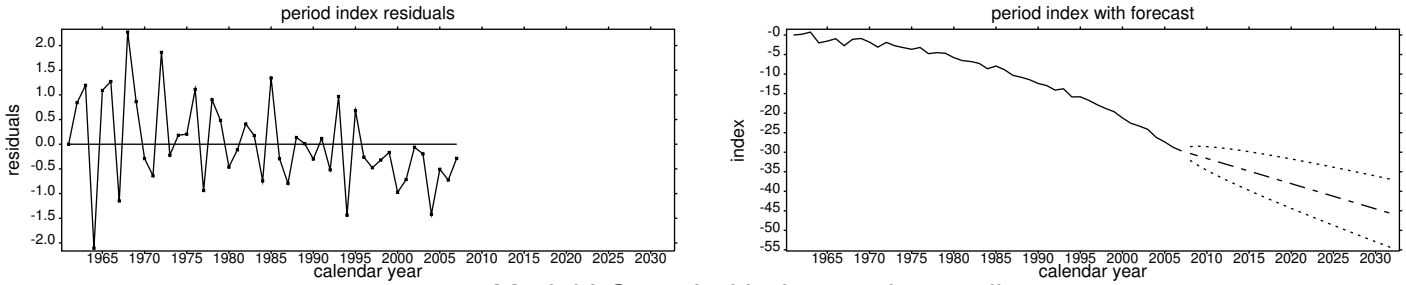


Model M8: period indices bi-variate random walk

Fig 4. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target q(x,t), predictor M8.

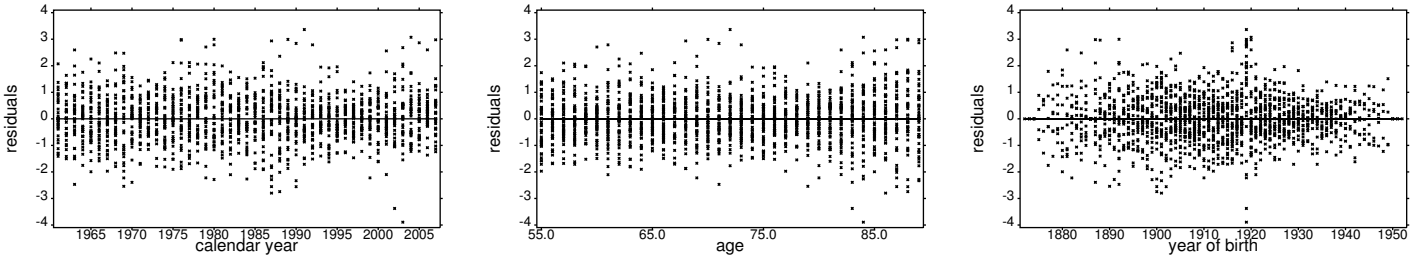


Model LC: deviance residual plots

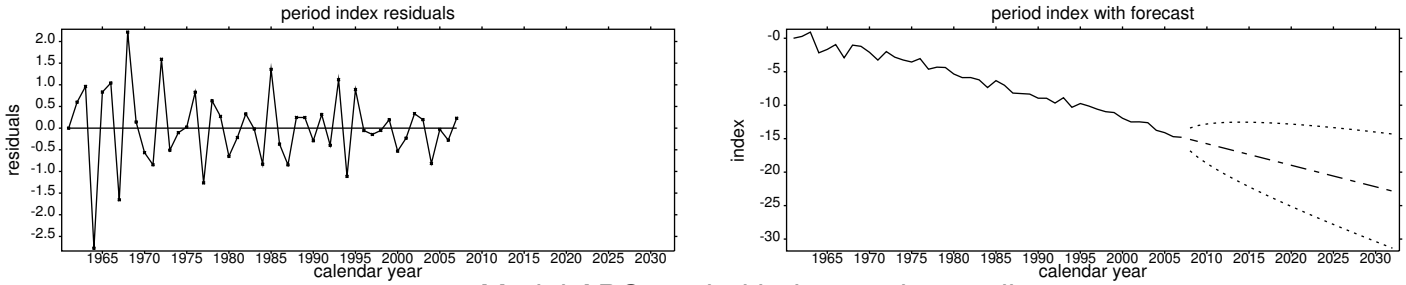


Model LC: period index random walk

Fig 5. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target q(x,t), predictor LC.

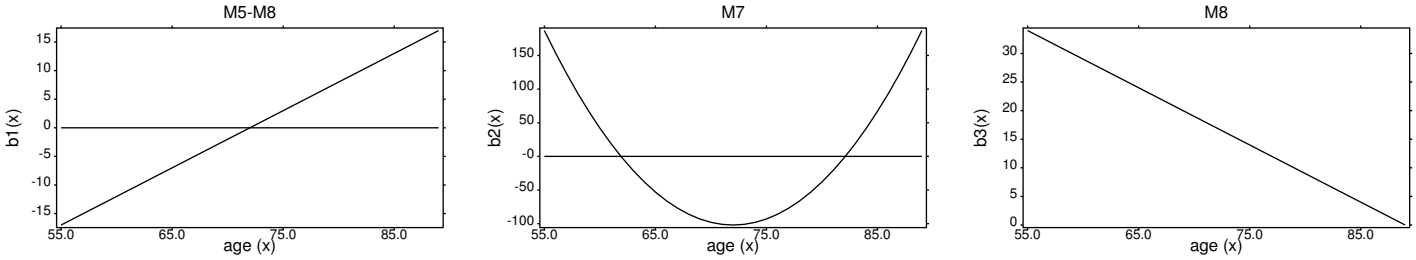


Model APC: deviance residual plots

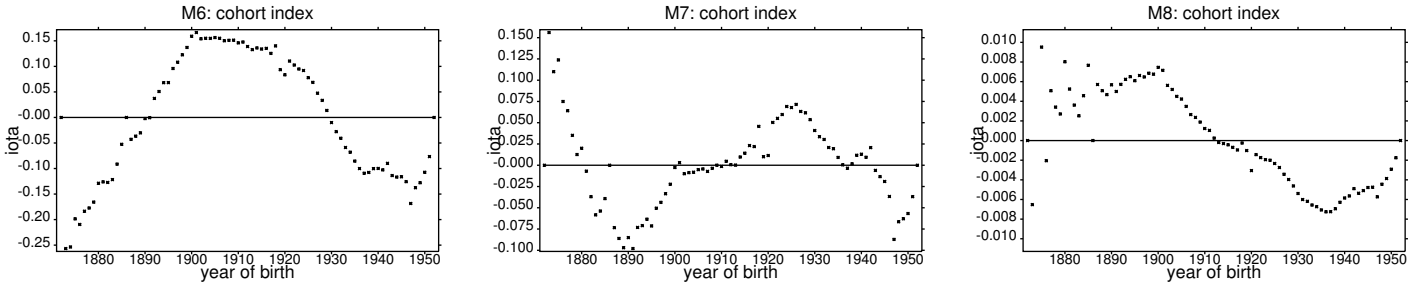


Model APC: period index random walk

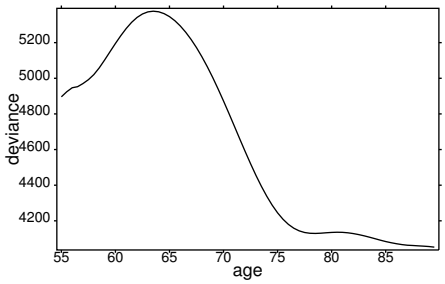
Fig 6. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target q(x,t), predictor APC.



Models M5-M8: age modulating functions

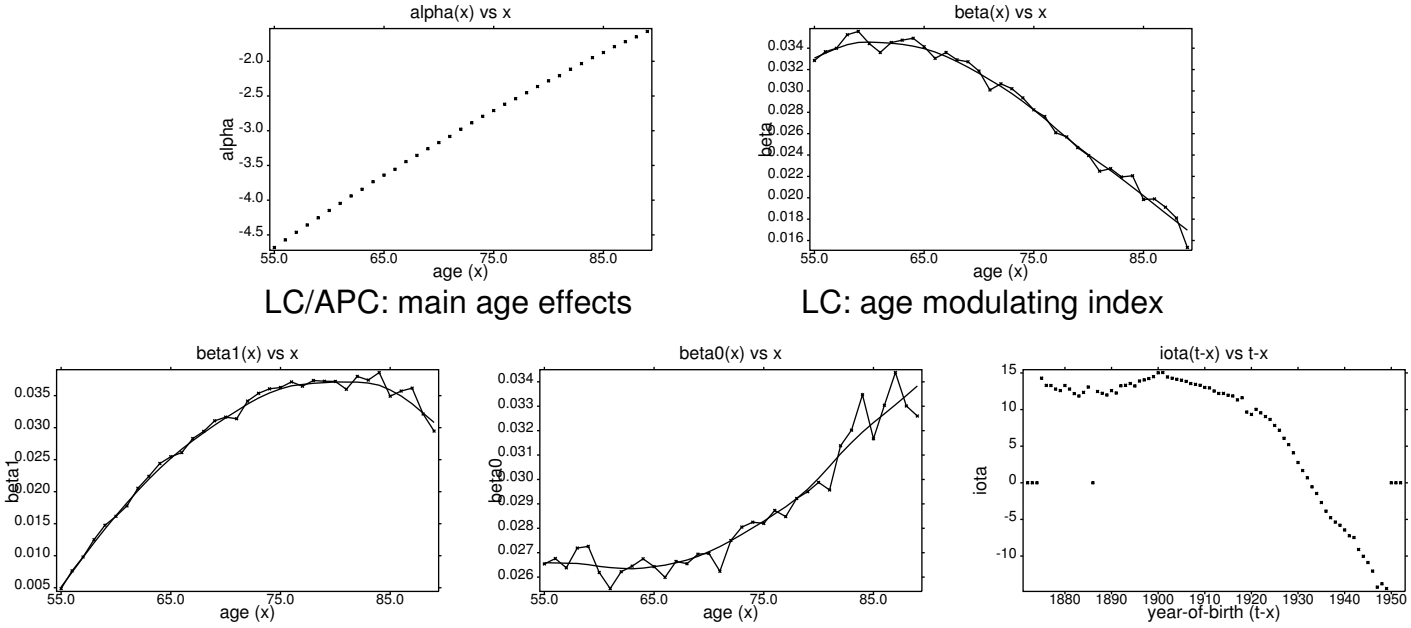


Models M6-M8: year-of-birth indices



Model M8: deviance profile

Fig 7. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target q(x,t), predictors M5-M8.



LC/APC: main age effects

LC: age modulating index

APC: age modulating indices; year-of-birth index

Fig 8. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target q(x,t), predictors LC, APC.

GOODNESS OF FIT

Note successful capture of

- period effects by all 6 models
- cohort effects by M6,M7,M8, APC
- age effects by LC,APC

GOODNESS OF FIT (cont…)

Note models M5-M8 do not include a main age effect term, α_x. Age effects are allowed for through the specified $\beta_x^{(1)}, \beta_x^{(2)}, \beta_x^{(3)}$ functions. The justification for this approach is that these models are designed for data sets with relatively short age ranges.

Not shown: supporting Q-Q residual plots, confirming normally distributed residuals.

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SUMMARY

Model	Age effects	Period effects	Cohort effects
M5	x	✓	x
M6	?	✓	✓
M7	?	✓	✓
M8	?	✓	✓
LC	✓	✓	x
APC	✓	✓	✓

MODEL DYNAMICS

- Characteristic features of random walk projections.
- Dominant primary period component exhibits a downward trend.
- Primary period component is linear for M6 and APC models, but shows curvature for other models.

MODEL DYNAMICS (cont…)

- For M5-M8, the forecast trend in secondary period component varies in direction across models. Also form of $\beta_x^{(1)}$ means that contribution switches direction at mid point of age range.

- For M7, form of $\beta_x^{(2)}$ means that tertiary period components changes direction twice.

PREDICTION INTERVALS

For cohort-based life expectancy and annuity values at ages 60, 65, 70, 75:

- calculated for the last year, using data for 1961-1997, 1961-1999, …, 1961-2007.
- calculated for 2007, using data for 1961-2007, 1963-2007, …., 1971-2007.

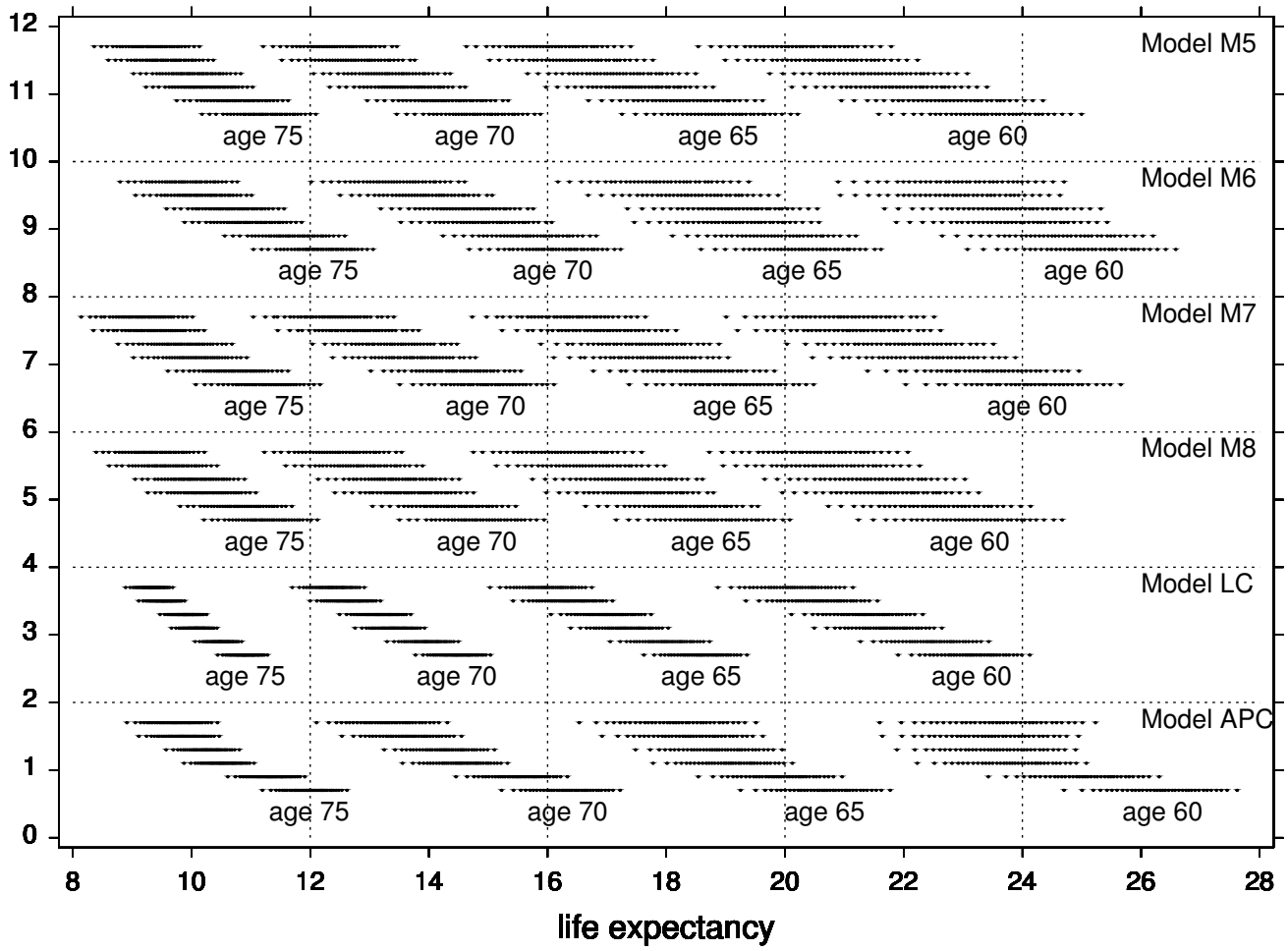

Fig 9. England & Wales male mortality experience, age range 55-89. Evolving biennial 1997(02)07 life expectancy prediction intervals: presented in decreasing sequence, for individuals aged 60(05)75, and each model M5-M8, LC, APC.

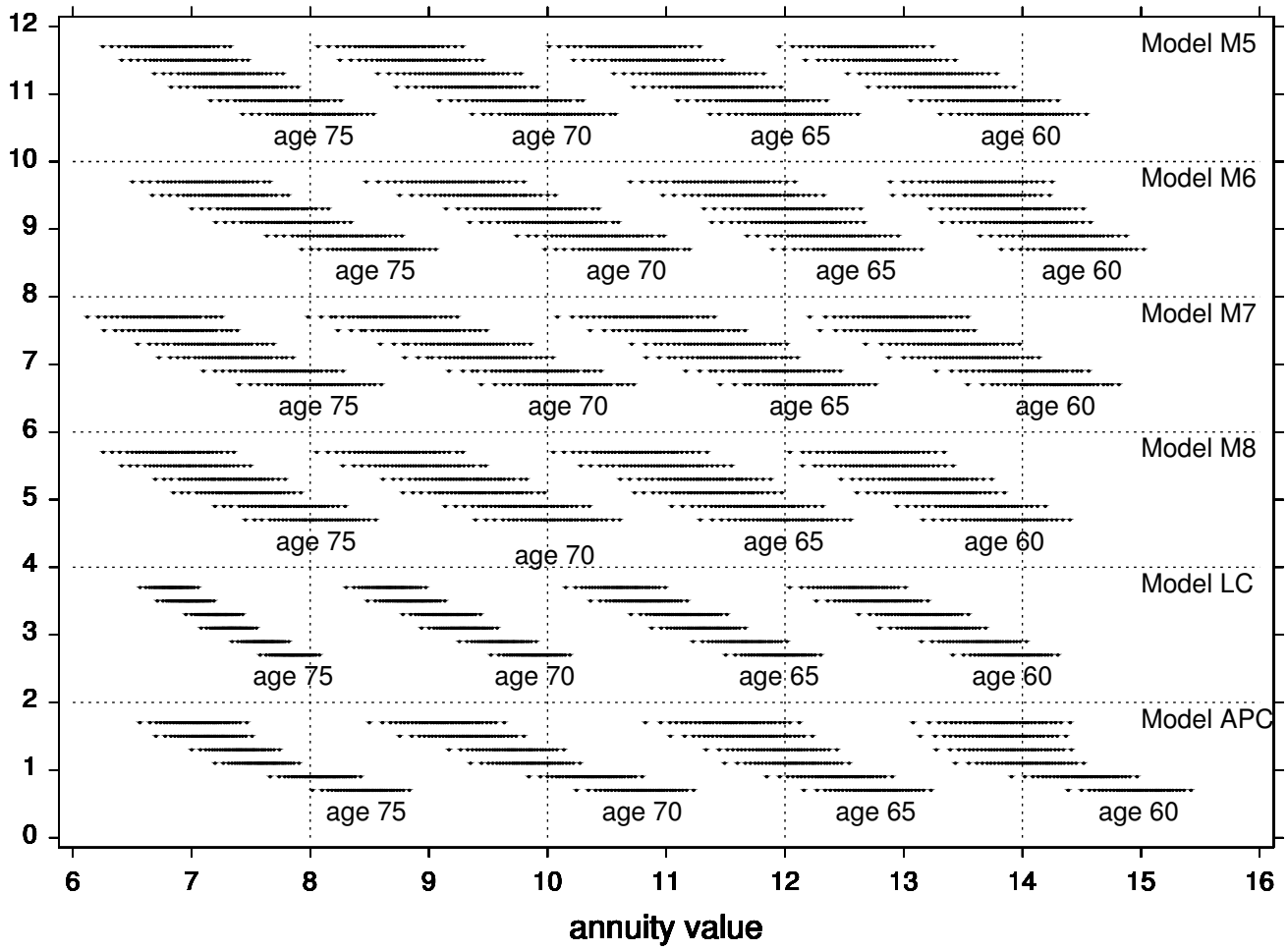

Fig 10. England & Wales male mortality experience, age range 55-89. Evolving biennial 1997(02)07 present value 4% immediate annuity prediction intervals: presented in decreasing sequence, for individuals aged 60(05)75, and each model M5-M8, LC, APC.

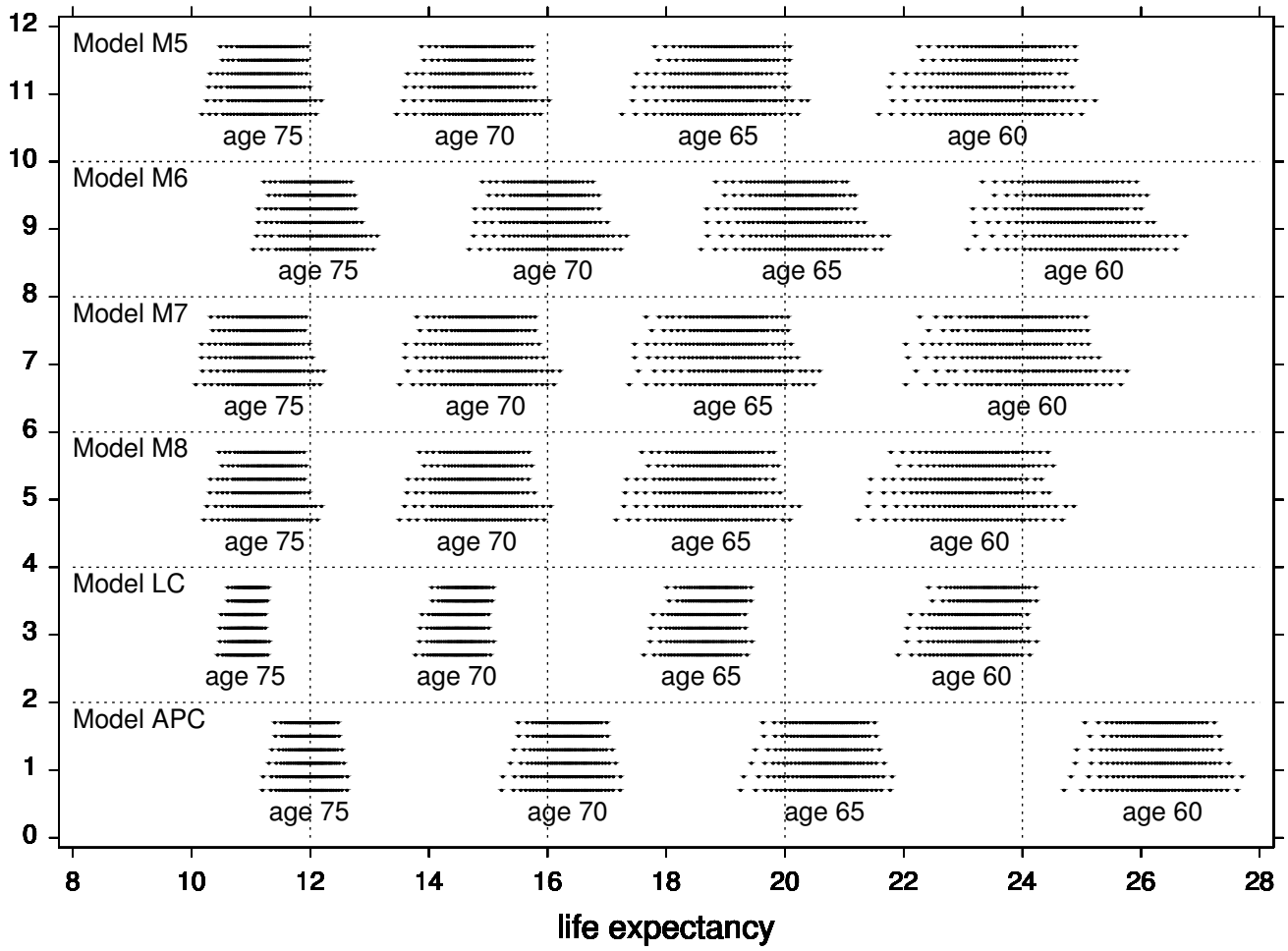

Fig 11. England & Wales male mortality experience, age range 55-89. 2007 life expectancy prediction intervals, subject to biennial front-end data deletion 1961(02)71: presented in asending sequence, for individuals aged 60(05)75, and each model M5-M8, LC, APC.

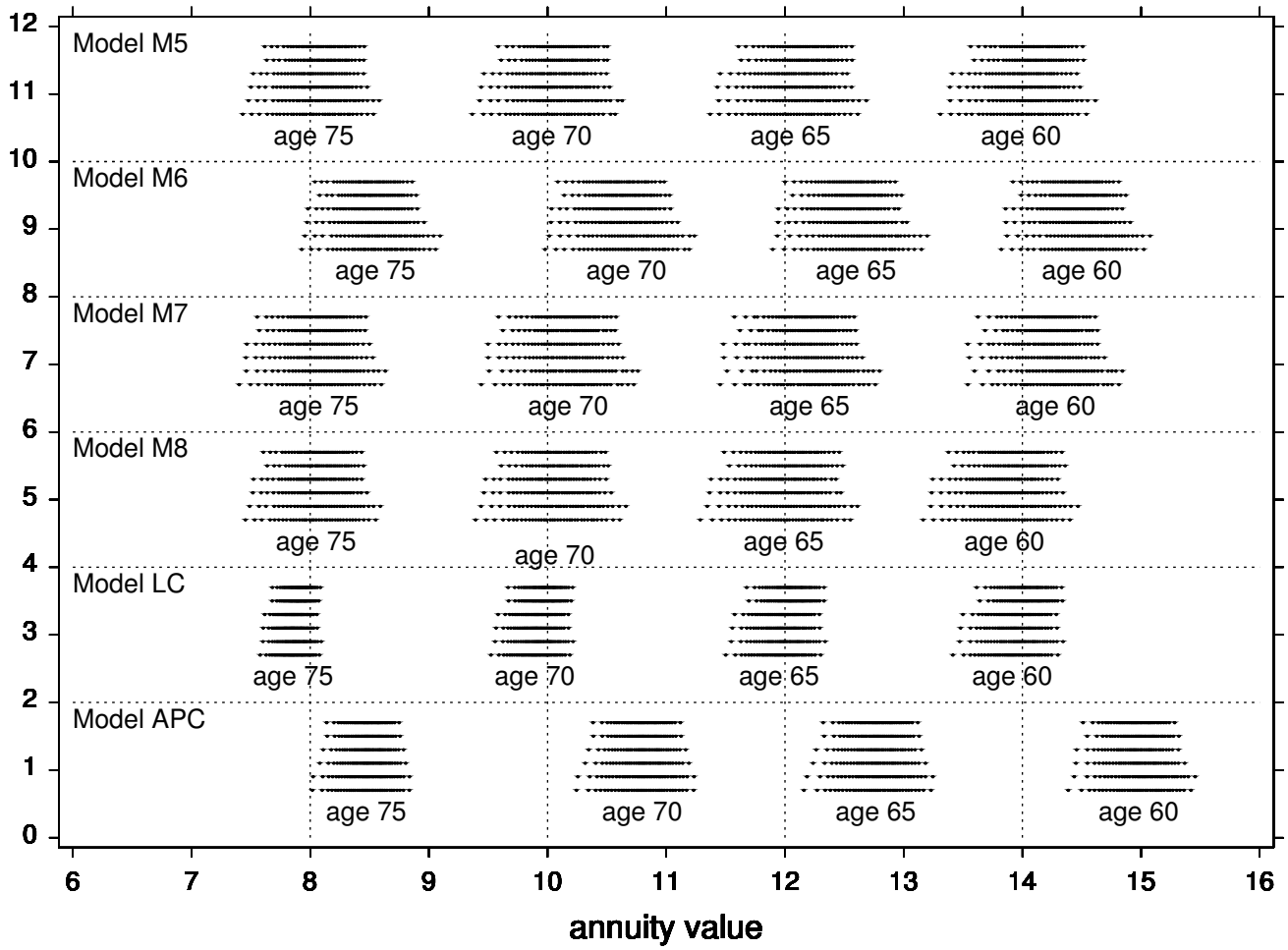

Fig 12. England & Wales male mortality experience, age range 55-89. 2007 present value 4% immediate annuity prediction intervals, subject to biennial front-end data deletion 1961(02)71: presented in ascending sequence, for individuals aged 60(05)75, and each model M5-M8, LC, APC.

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PREDICTIONS AND PREDICTION INTERVALS

Key features for predicting indices at end of data period are:

- Differences between LC and APC results indicates the impact of capturing the cohort effects.
- Similarity between M6 and APC, but failure of M7 and M8 to generate significant responses to modelling of cohort effects.
- Similarity between results for M5, M7, M8, and LC

PREDICTIONS AND PREDICTION INTERVALS (cont…)

Key features for predicting indices for 2007 are:

- Robustness of result across all models and ages.
- Increased values for APC relative to LC, as a result of capturing cohort effects.
- Approximate alignment of result for M5, M7 and M8 <u>even though</u> M7 and M8 include a cohort effects term.

REFLECTIONS

- APC model provides good fit; leads to stable estimates and forecasts: uses 1 time factor; converges slowly (without adjustment).

- CBD models (M6, M7, M8) provide good fit although cohort effect is less prominent; lead to stable estimates and forecasts; converge quickly; use multiple time factors; lack of α_x term means age range needs to be restricted.

OTHER POINTS TO NOTE

- Time series methods and their application to long forecasting periods: data requirements.
- Quality of data sources and appropriateness for particular applications: adverse selection and "basis risk".
- Model error – essential to investigate more than one modelling framework.
- Sources of uncertainty – process, parameter, model, judgement. Not all sources of uncertainty can be quantified.