AFIR LIFE MUNICH 2009

# Building blocks for a mortality index in an international context


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# Outline

- Longevity risk:
  - Identification
  - Assessment
  - Management
- Securitization → Mortality Indexes
- Stochastic forecasts of life expectancy
  - Univariate ARIMA
  - Multivariate VAR → Vector Error Correction

# Identification

- "Mortality risk" is divided into:
  - Process risk: random fluctuations due to the stochastic nature of mortality
  - Catastrophe risk: unexpected shocks of mortality
  - Uncertainty risk: random deviations due to
    - choice of the projection model (model risk)
    - estimated parameters of the model (param.risk)

    → **Longevity Risk** refers to old age mortality

# Assessment

- The impact of the LR is measured via:
  - Discounted cash flows
  - Random annual cash flows
  - Random number of annuitants alive in t
  - Loss function
- Combined with different indexes:
  - Expected value
  - Variance
  - Quantile

# Management

- Portfolio strategies of loss control
  - Loss prevention
  - Loss severity

- Portfolio strategies of loss financing
  - Natural hedging
  - Risk transfer → Securitization
  - Risk retention

# Securitization

- LR is transferred to capital markets via mortality-linked securities:
  - Longevity Bonds (EIB/BNP in 2004)
  - Survivor Swaps (Swiss Re in 2007)
  - Mortality Futures
  - Mortality Forwards (JP Morgan in 2007)
  - Mortality Options

# Securitization

- LR is transferred to capital markets via mortality-linked securities:
  - Longevity Bonds (EIB/BNP in 2004)
  - Failed due to:
    - Presence of basis risk
    - Lack of transparency
    - Capital strain

# Mortality indexes

- Recently proposed:
  - Longevity Index by Credit Suisse (2005)
  - LifeMetrics Indices by JP Morgan (2007)
  - Xpect-Indices by Deutsche Börse (2008)
- Our proposal:
  - Life expectancy at birth
  - Publicly available mortality data (HMD)
  - Multivariate analysis between countries

# Stochastic models to forecast mortality

- Univariate ARIMA models
  - De Beer and Alders (1999)
  - Keilman, Pham and Hetland (2001)
  - Alders Keilman and Cruijsen (2007)

- Multivariate VAR models
- Multivariate VEC models

→ Economics

# ARIMA models

- Based on the theory of stationary stochastic processes. ARIMA(p, d, q):

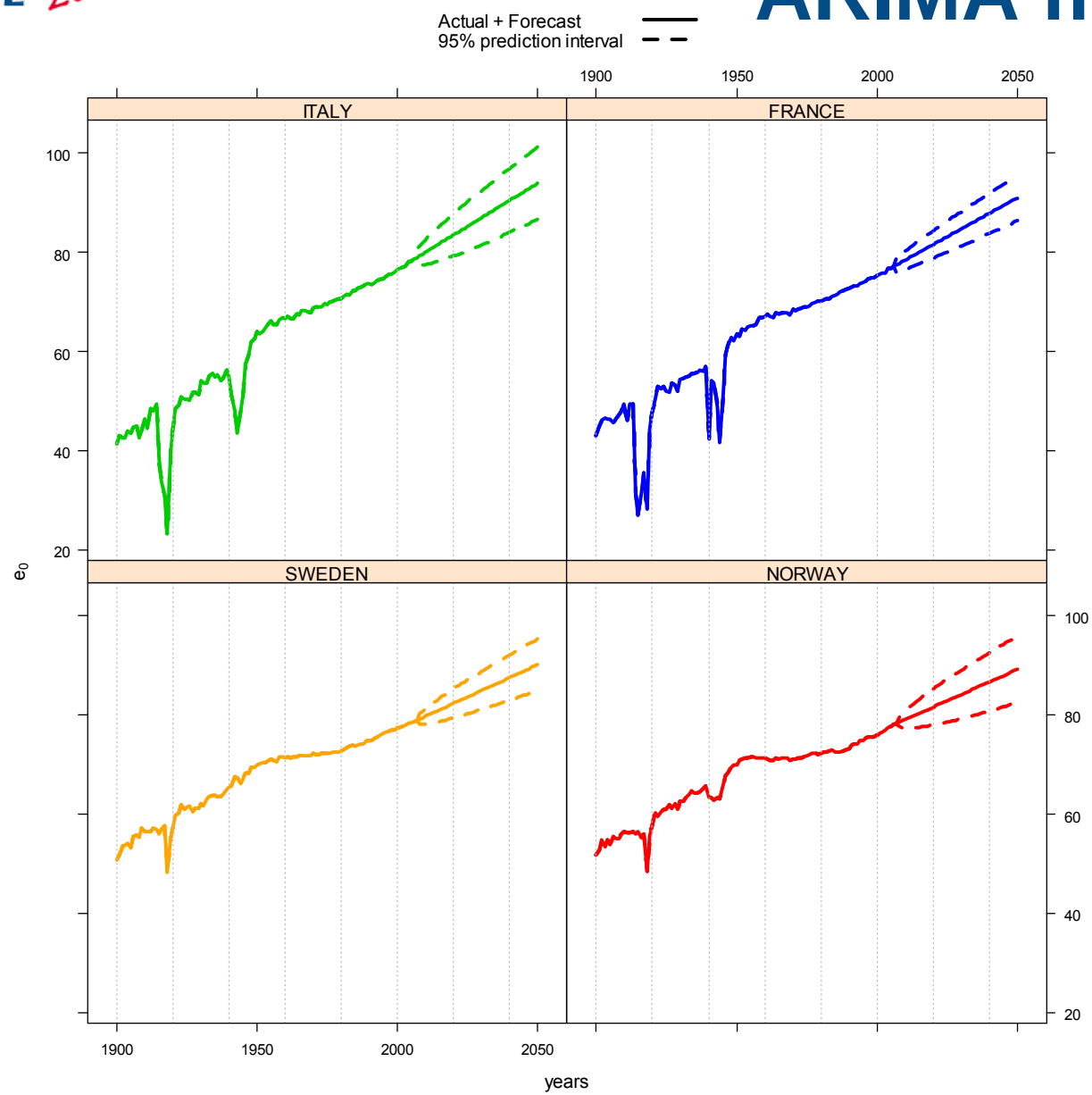
$$\nabla^d Y_t = \delta + \phi_1 \nabla^d Y_{t-1} + \ldots + \phi_p \nabla^d Y_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + \ldots + \theta_q \epsilon_{t-q}$$

- Box and Jenkins' iterative model selection strategies:
  - Model identification
  - Model estimation
  - Diagnostic checking of model adequacy

# ARIMA models

| | Male | | | |
|---|---|---|---|---|
| | Italy | France | Sweden | Norway |
| ARIMA(p,d,q) | (0,1,1) | (0,1,1) | (0,1,1) | (2,1,0) |
| $\delta$ | 0.3479 | 0.3127 | 0.2601 | 0.3086 |
| s.e. | (0.0542) | (0.0335) | (0.0385) | (0.0508) |
| $\phi_1$ | - | - | - | -0.2188 |
| s.e. | - | - | - | (0.0964) |
| $\phi_2$ | - | - | - | 0.2670 |
| s.e. | - | - | - | (0.1028) |
| $\theta_1$ | -0.3092 | -0.4979 | -0.2871 | - |
| s.e. | (0.0886) | (0.0829) | (0.0901) | - |

# ARIMA models



# VAR models

- Variables are explained by their own past values and the past values of all the other variables in the system.

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$$Y_t = \mu + A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + \varepsilon_t$$

  – Model identification
  – Model estimation
  – Diagnostic checking of model adequacy

# VAR(2) model

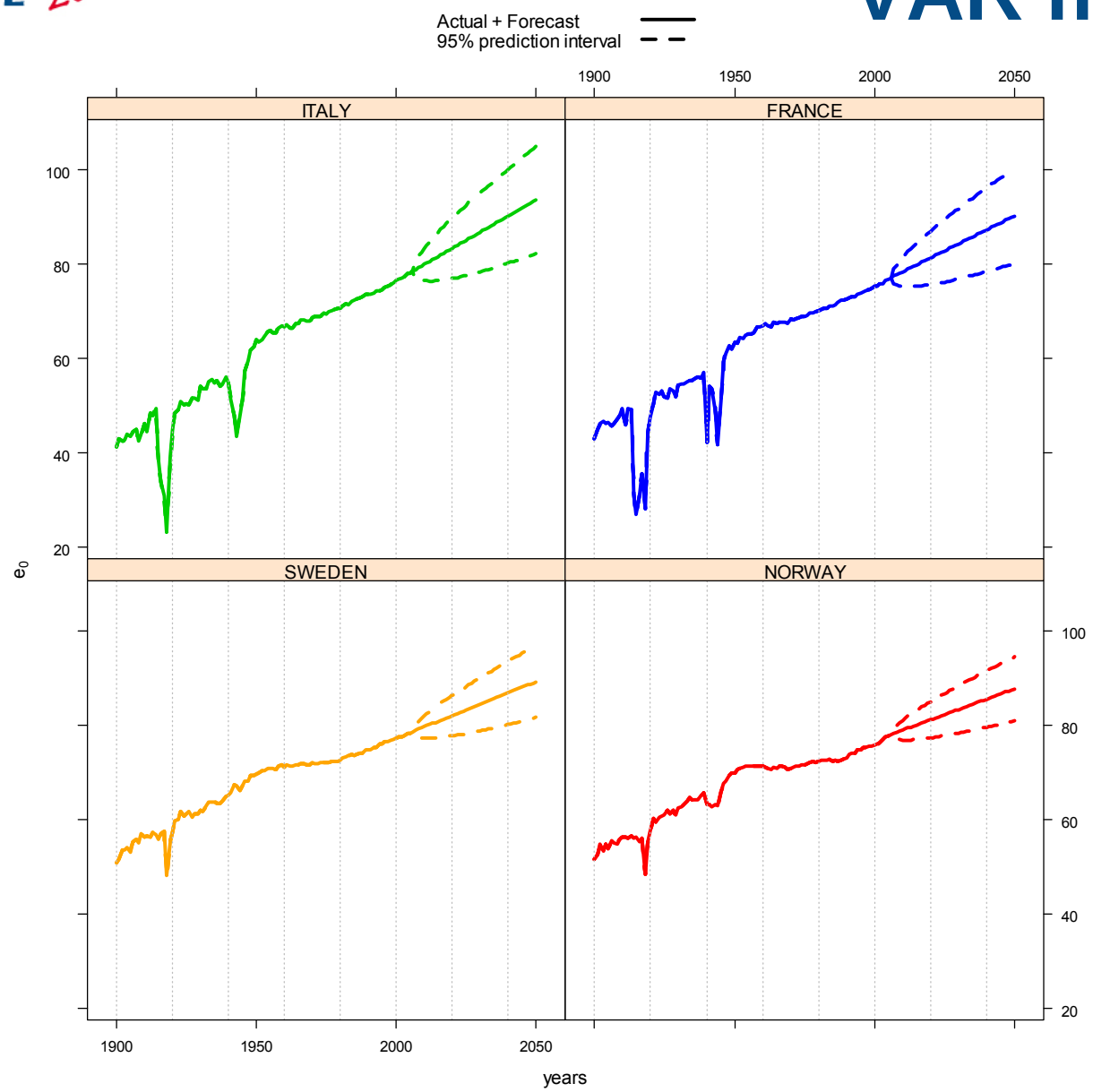
$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \varepsilon_t$$

$$\Delta e_0^{Ita}(t) \quad \Delta e_0^{Fra}(t) \quad \Delta e_0^{Swe}(t) \quad \Delta e_0^{Nor}(t)$$

$$a_1^{Ita} \quad a_1^{Fra} \quad a_1^{Swe} \quad a_1^{Nor}$$

$$\Delta e_0^{Ita}(t-1) \quad \Delta e_0^{Fra}(t-1) \quad \Delta e_0^{Swe}(t-1) \quad \Delta e_0^{Nor}(t-1)$$

# VAR models


Actual + Forecast
95% prediction interval
ITALY
FRANCE
SWEDEN
NORWAY
$e_0$
years
1900
1950
2000
2050
20
40
60
80
100

# Cointegration

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- "If each element of a vector of time series $Y_t$ achieves stationarity after differencing, but a linear combination $Z_t = \beta' Y_t$ is already stationary, the time series are said to be cointegrated"

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- "If each element of a vector of time series $Y_t$ achieves stationarity after differencing, but a linear combination $Z_t = \beta' Y_t$ is already stationary, the time series are said to be cointegrated"
- Grangers' representation theorem: "For each cointegrated system exists a VEC representation; if exists a VEC representation and the series are integrated, then they are also cointegrated"

# VEC models

- A VAR in a VEC representation:

$$\Delta Y_t = \Gamma_1 \Delta Y_{t-1} + \ldots + \Gamma_{p-1} \Delta Y_{t-p+1} - \Pi Y_{t-1} + \varepsilon_t$$

where $\Gamma_i = -(I - A_{i+1} - \ldots - A_p)$, $\Pi = -(I - A_1 - \ldots - A_p)$

- Example of a VAR(1): $Y_t = A Y_{t-1} + \varepsilon_t$

$\Delta Y_t = \Pi Y_{t-1} + \varepsilon_t$ where $\Pi = A - I$

# VEC models

- Studying the rank of $\Pi$ we obtain information on the number of cointegrating relationship:

$$0 \leq r \leq n$$

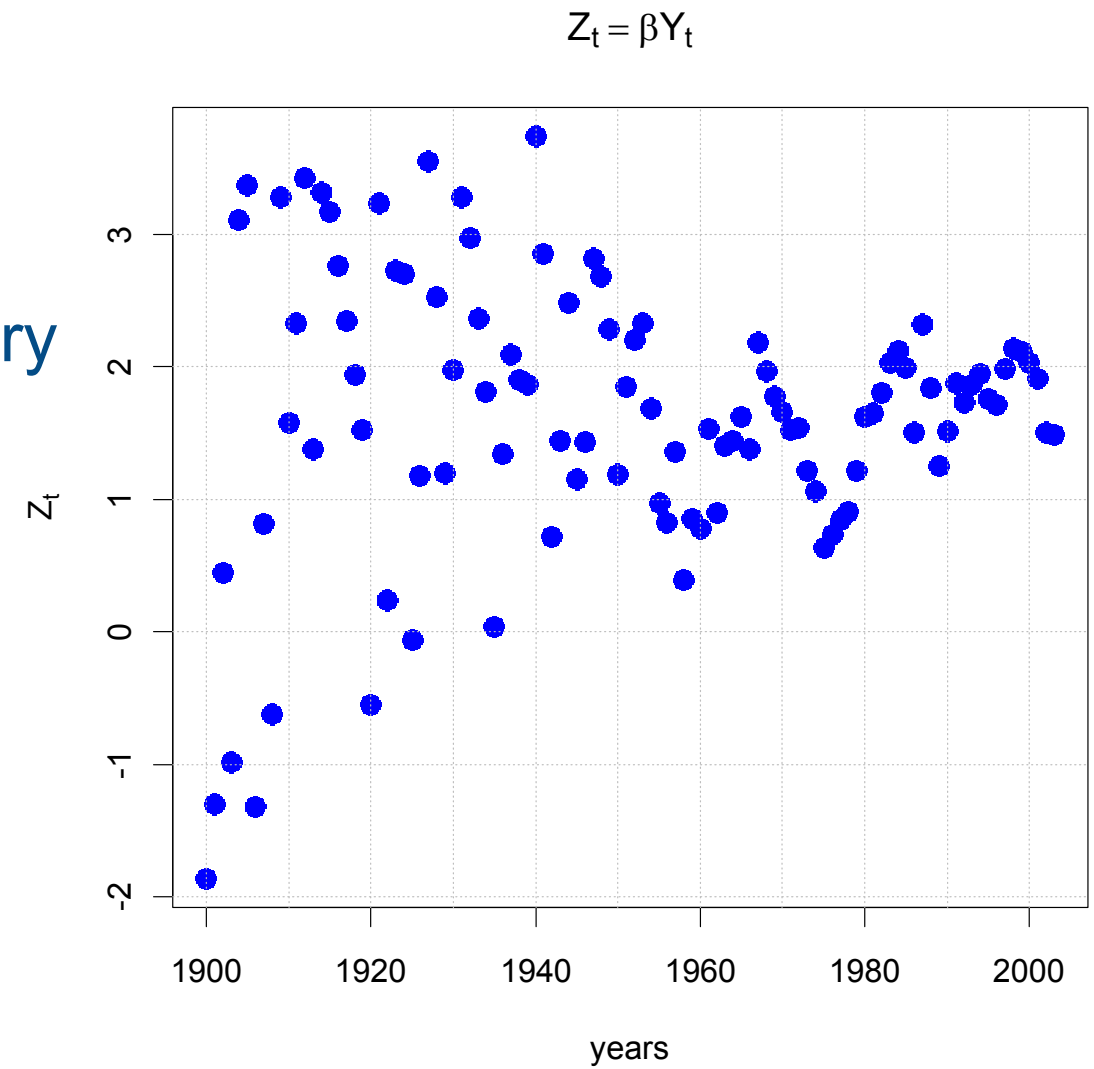
- $\Pi = \alpha' \beta$ is decomposed into the loading matrix α and the cointegrating matrix β

$$\Delta Y_t = \Gamma_1 \Delta Y_{t-1} + \ldots + \Gamma_{p-1} \Delta Y_{t-p+1} - \alpha Z_{t-1} + \varepsilon_t$$

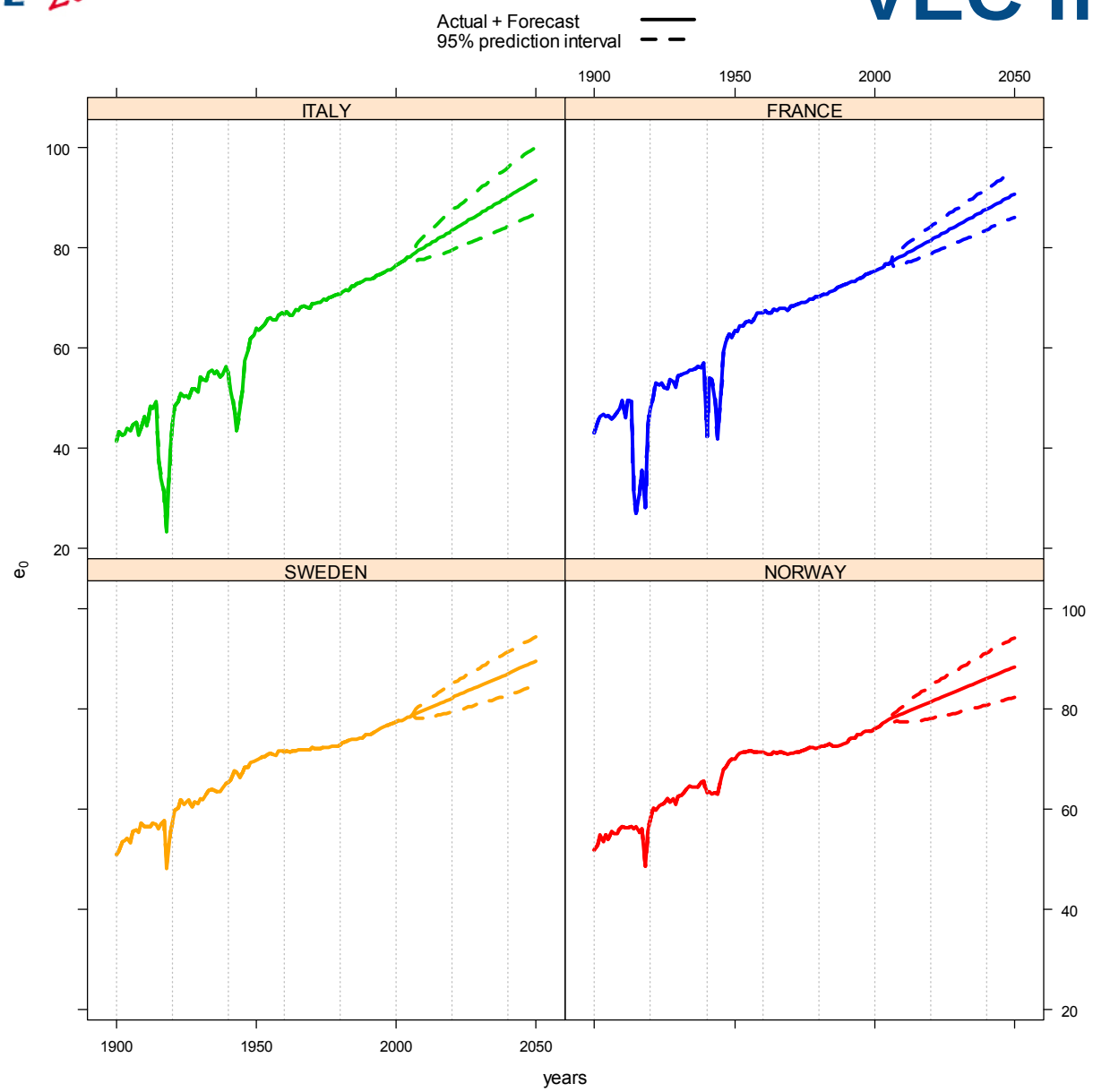
- VEC is a VAR model on the first differences plus a vector of cointegrating residuals

# VEC models

- Lag-order: 2
- Rank($\Pi$)=1
- $Z_t = \beta' Y_t$ stationary



# VEC models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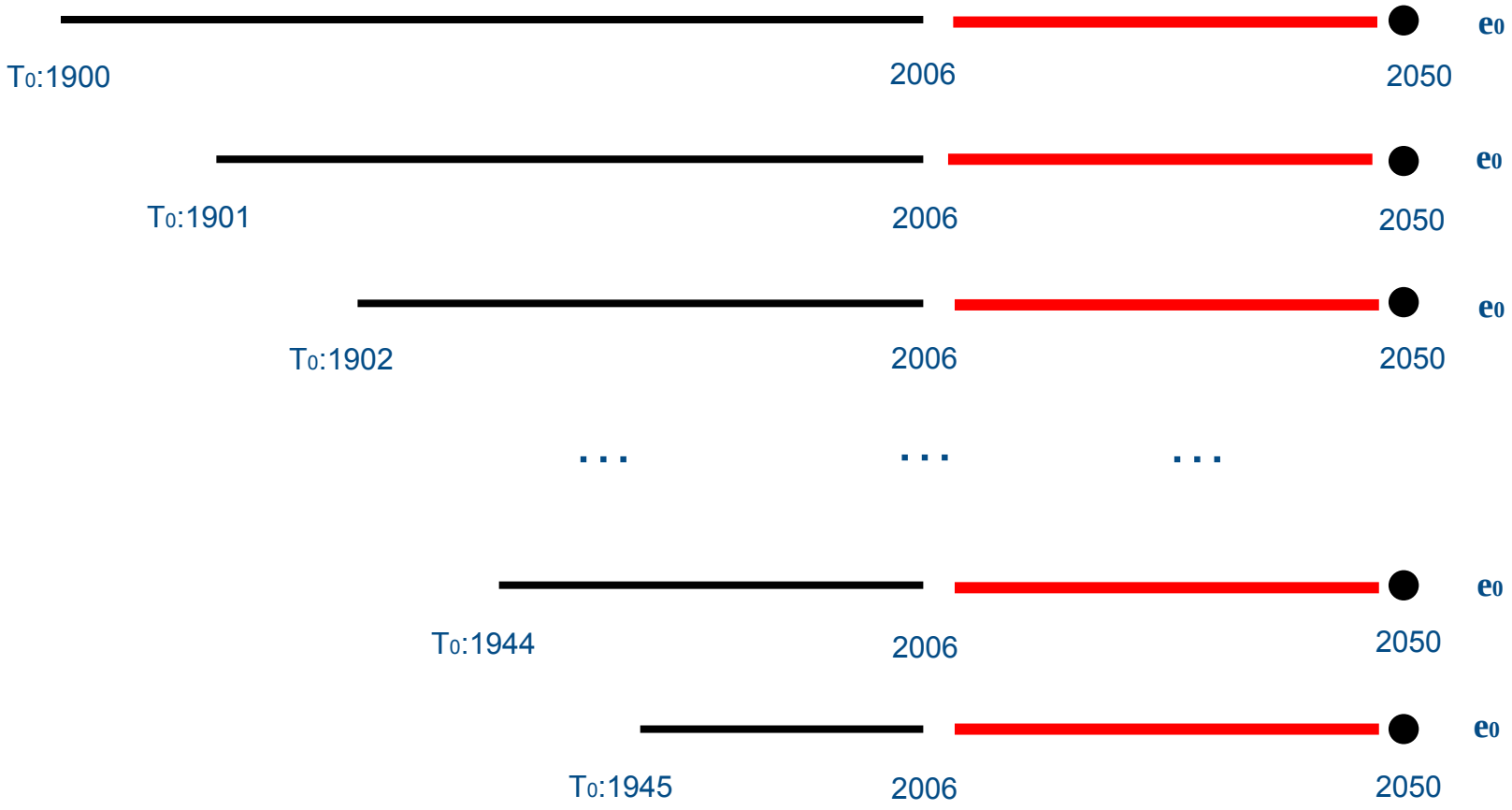
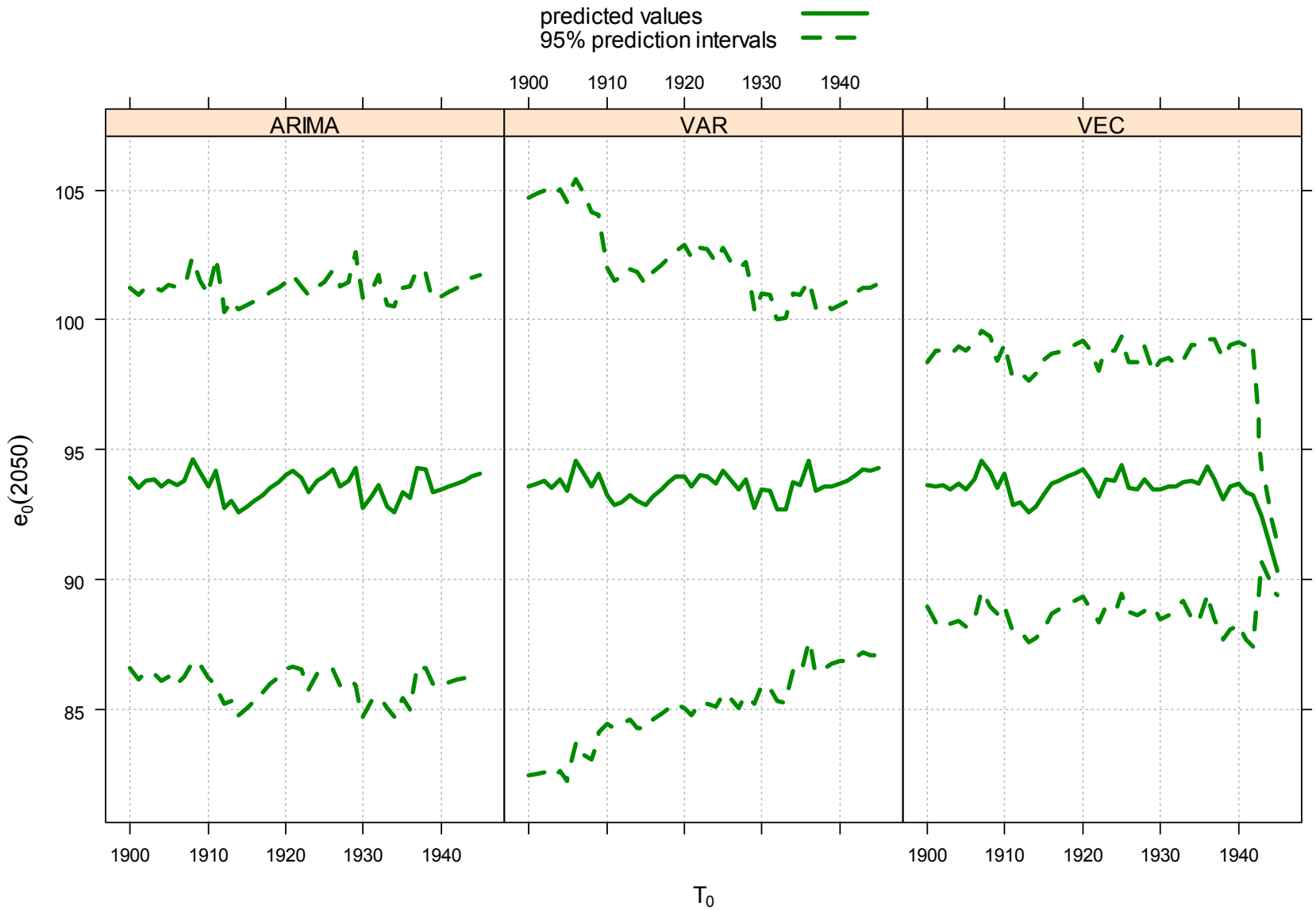
# Comparison of results: 2050

| | Italy | France | Sweden | Norway |
|---|---|---|---|---|
| ARIMA | | | | |
| $e_0$ | 93.91 | 90.93 | 87.47 | 86.06 |
| 95% CI | (86.59–101.22) | ( 86.35–95.52) | (82.21–92.75) | (80.33–91.79) |
| VAR | | | | |
| $e_0$ | 93.66 | 90.29 | 89.34 | 87.79 |
| 95% CI | (82.32–105.01) | (80.21–100.37) | (81.71–96.96) | (81.03–94.57) |
| VEC | | | | |
| $e_0$ | 93.18 | 90.11 | 86.80 | 88.22 |
| 95% CI | (88.03–98.33) | (85.46–94.76) | (80.59–93.01) | (82.31–94.13) |

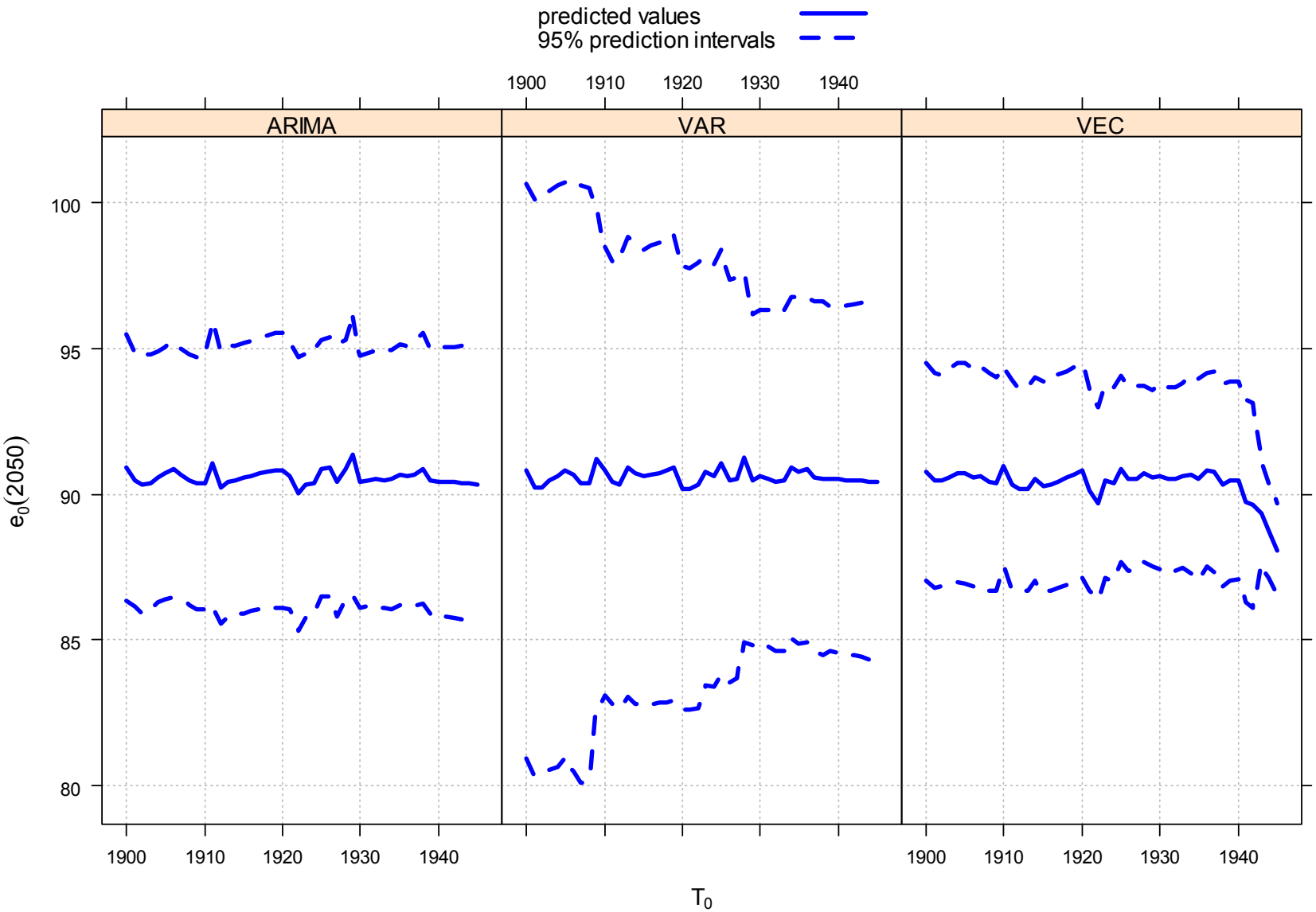
# Changing time-window



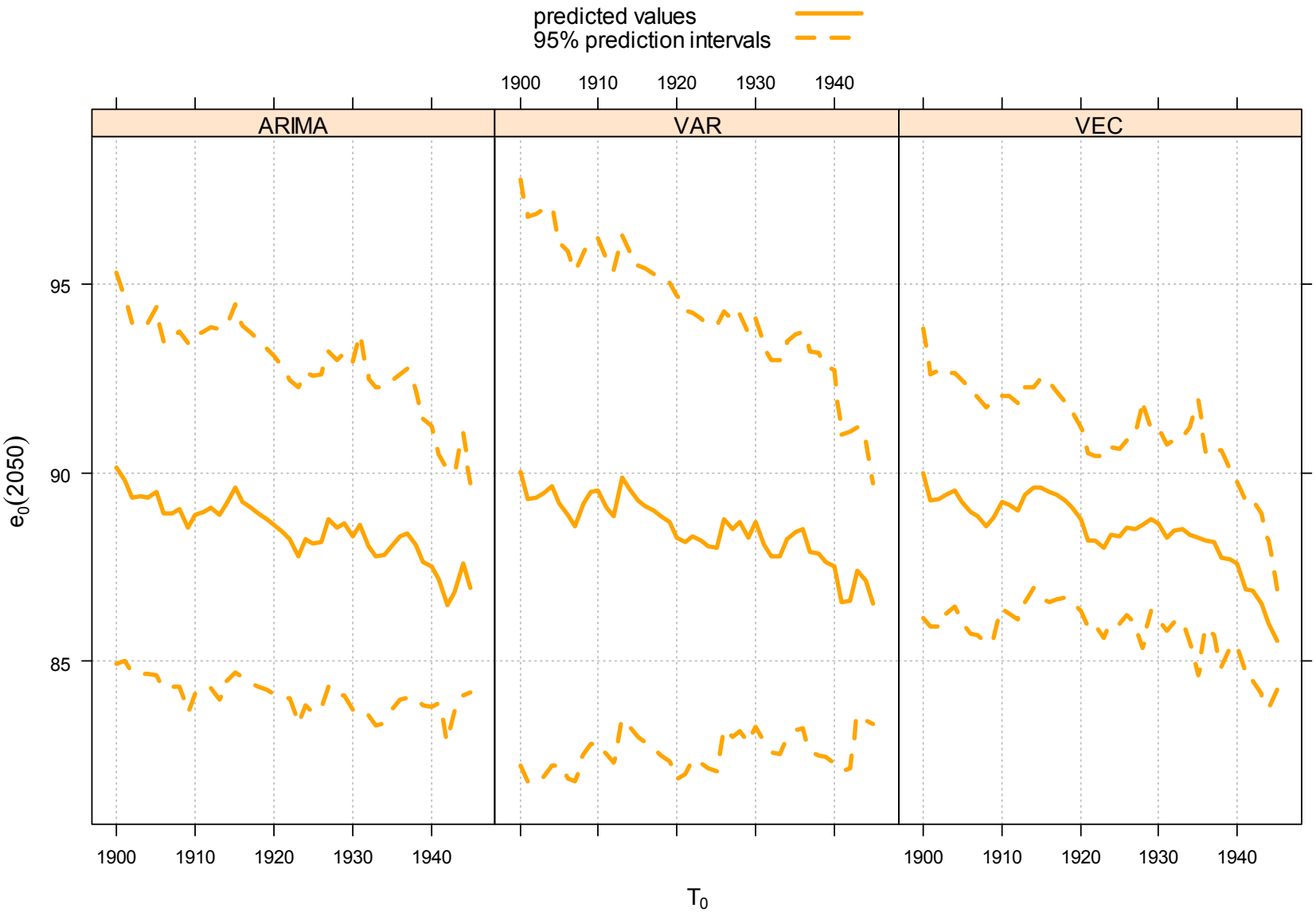
# Changing time-window: ITALY



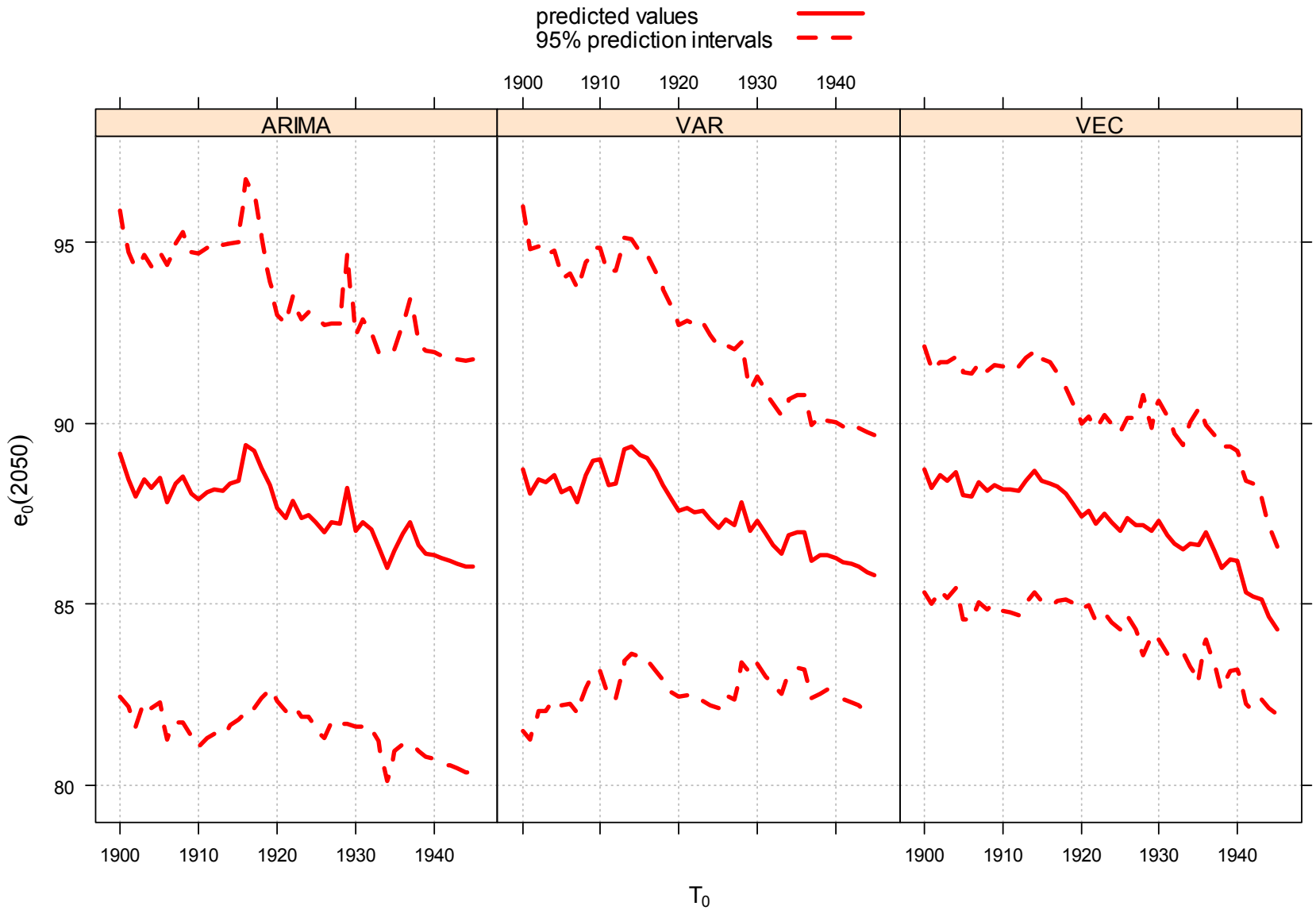
# Changing time-window: FRANCE



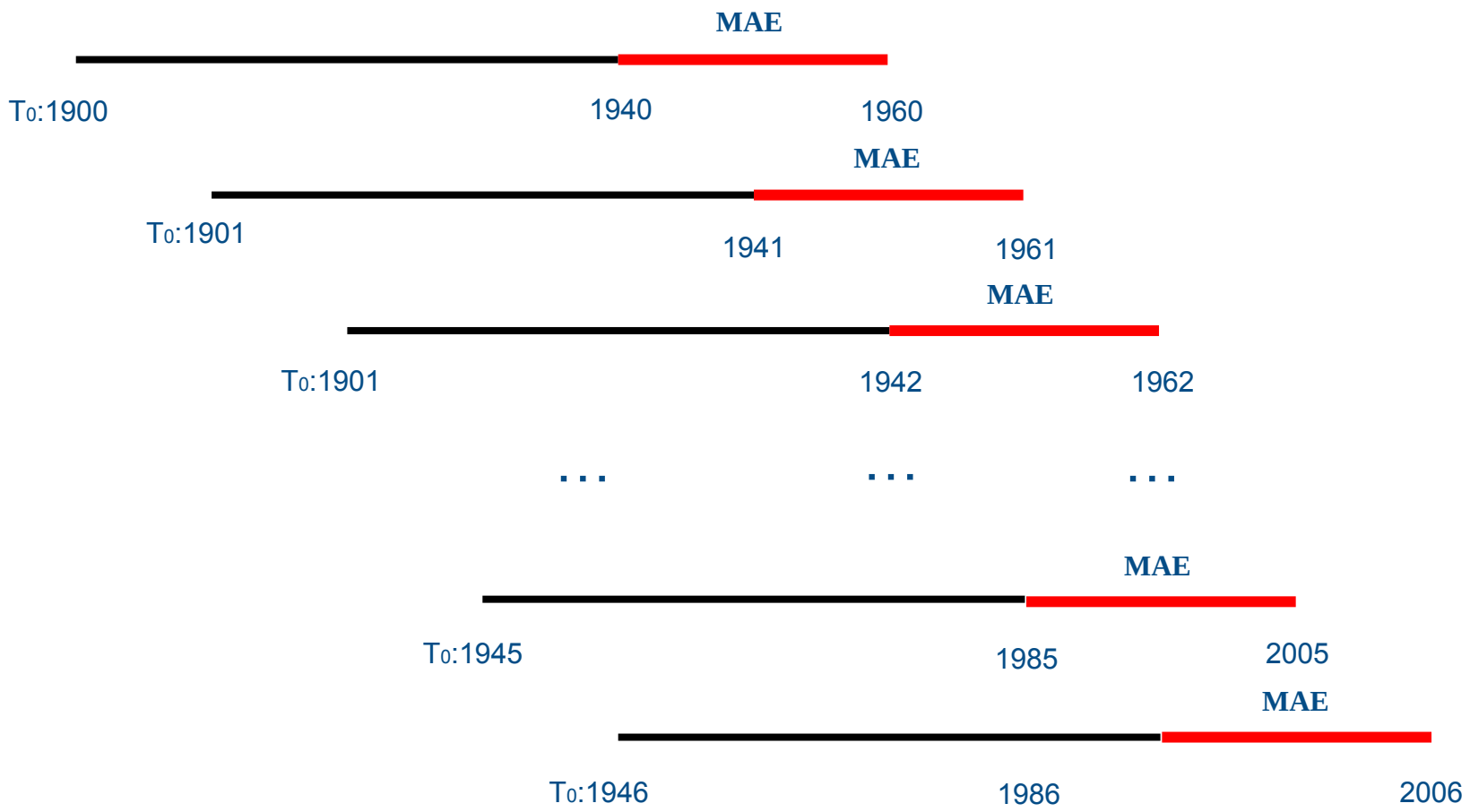
# Changing time-window: SWEDEN



# Changing time-window: NORWAY



# Changing the in-sample period



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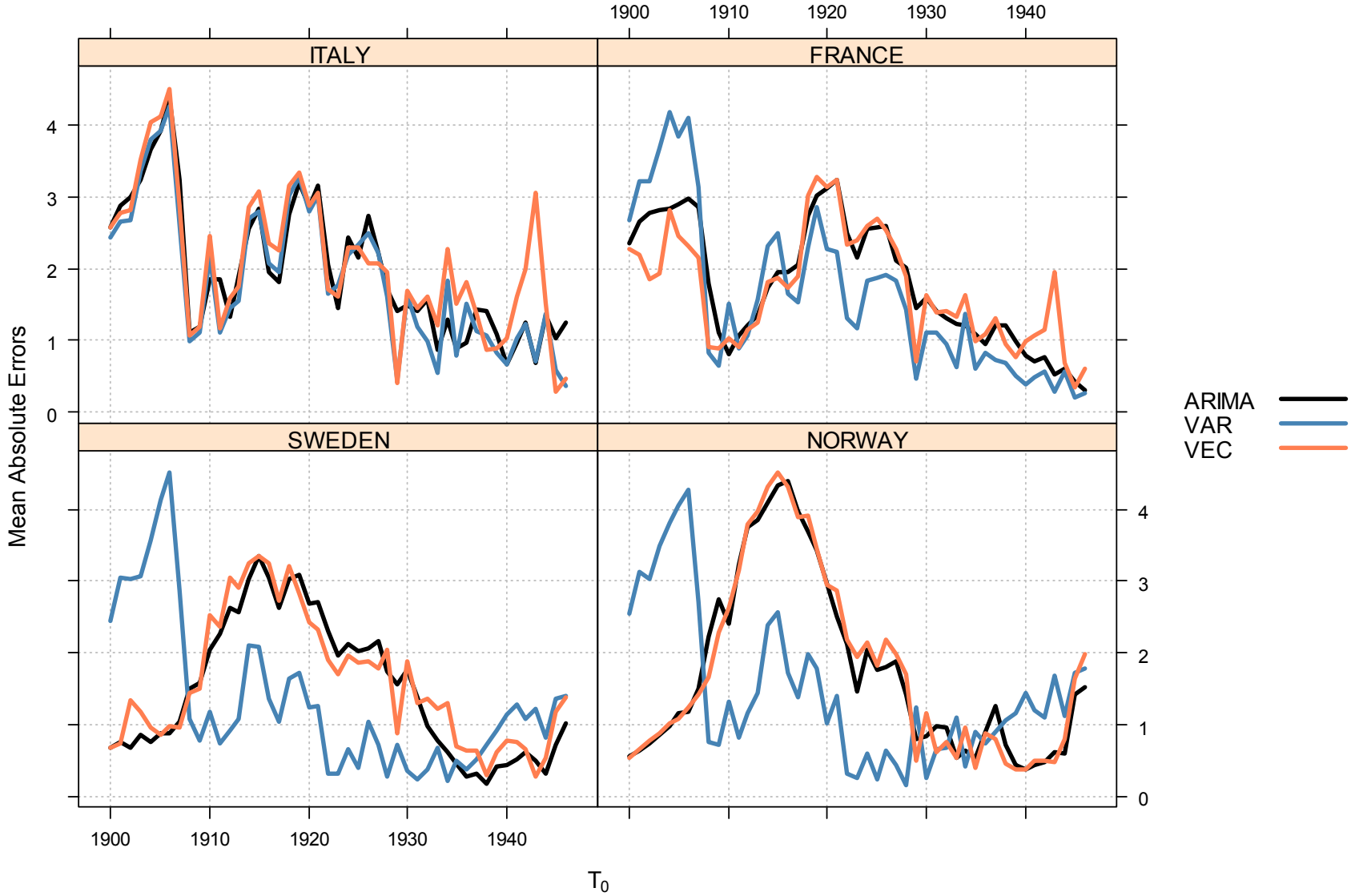

# Conclusions

- Our models:
  - Multivariate framework
    - VEC more coherent, VAR performs better forecasts
- Questionable the use of $e_0$:
  - Not strictly representing the annuitants' mortality
    - Basis risk present for the insurance company

however:

  - A well known and understood measure
  - Easily available and constantly updated
    - Transparency guaranteed to investors

# Thanks for your attention!

# Comments
# or
# questions?

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# VAR models

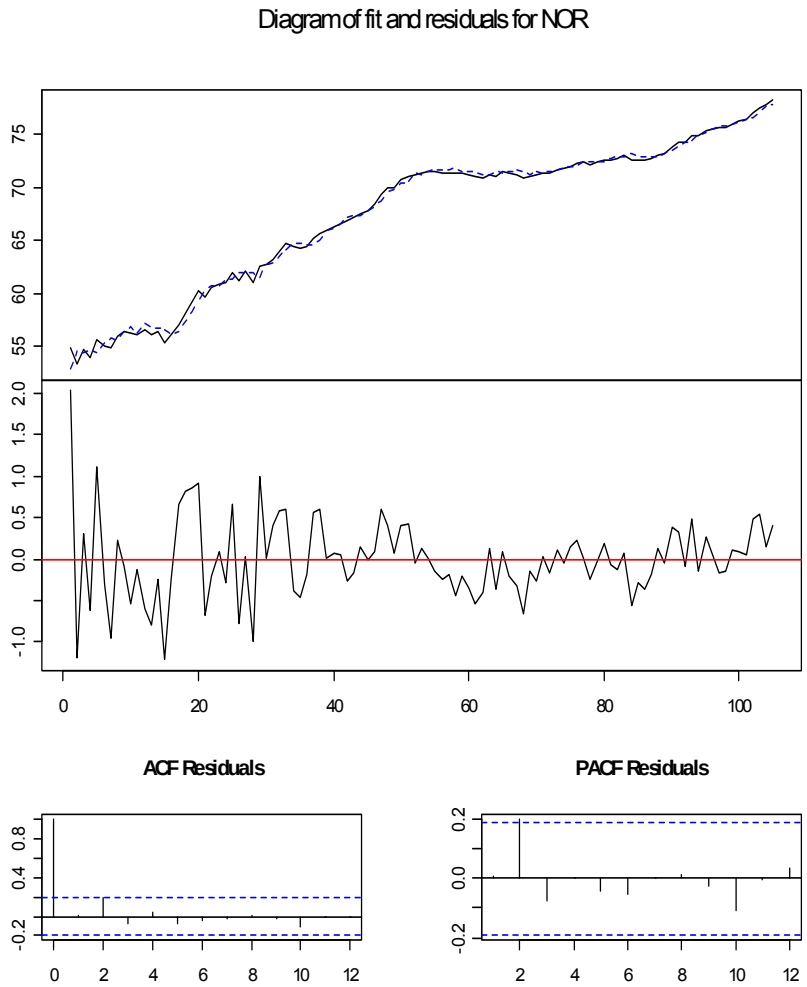
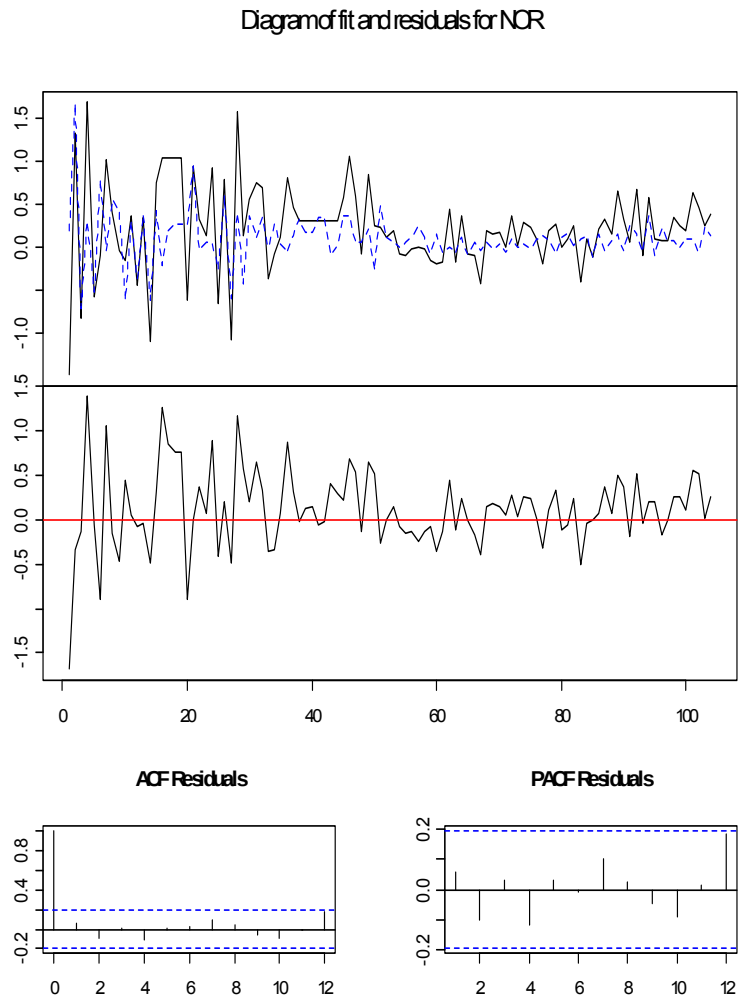


Diagram of fit and residuals for NCR
ACF Residuals
PACF Residuals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