Investment strategies and risk management for participating life insurance contracts

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Cass Business School

AFIR Colloquium
Munich, September 2009
Introduction & Motivation

- New supervisory framework for the (life) insurance industry:
  - IASB Insurance Project
  - EU Solvency II Review

- Basic concepts
  - Market consistent valuation of A & L
  - Target Capital

- Focus directed especially on
  - “Fair valuation”: what is it? How to carry out the program?
  - Market modelling
  - Identification of embedded options: the default option & safety loading
Aims and objectives

- **Risk management** of financial risk induced by a participating contract with minimum guarantee
  - Consiglio et al. (2006): reference portfolio structuring by means of non linear programming using the Wilkie model
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- **(Static) Asset allocation approach**
  - Risk minimization
  - Chosen “risk measure”: volatility of the guaranteed benefit with respect to prespecified target
Aims and objectives

- **Risk management** of financial risk induced by a participating contract with minimum guarantee
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- **(Static) Asset allocation approach**
  - Risk minimization
  - Chosen “risk measure”: volatility of the guaranteed benefit with respect to prespecified target

- Analysis of
  - market consistent value of embedded options and safety loading
  - probability of default and Solvency II capital requirements
  - robustness of the approach
Introduction

Agenda

1. The Participating contract
   - Design
   - Embedded options
   - Safety loading

2. The reference portfolio

3. The market model

4. Asset Allocation

5. Stress Testing

Conclusion
The Participating Contract

- Starting time: \( t = 0 \); maturity: \( T = 20 \) years
- Single premium: \( \pi_0 \)
- Reference fund: \( F(t) \)
- Leverage coefficient: \( \theta \) such that
  \[
  \pi_0 = \theta F(0)
  \]
The Participating Contract

- Starting time: $t = 0$; maturity: $T = 20$ years
- Single premium: $\pi_0$
- Reference fund: $F(t)$
- Leverage coefficient: $\theta$ such that $\pi_0 = \theta F(0)$

**Benefit at maturity:**

- **Guaranteed component:** $\pi(T)$
- **Discretionary component** (terminal bonus): $R(T) = (\theta F(T) - \pi(T))^+$

- Terminal bonus rate: $\gamma$
The fair pricing condition

- Benefit paid **IF** company solvent at $T$
  $\Rightarrow$ overall liability:
  $$\pi(T) + \gamma R(T) - D(T)$$
  where
  $$D(T) = (\pi(T) - F(T))^+$$
  payoff of the “Default Option”

- No arbitrage condition:
  $$\pi_0 = V_\pi(0) + \gamma V_R(0) - V_D(0)$$
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- No arbitrage condition:
  $$\pi_0 = V_\pi(0) + \gamma V_R(0) - V_D(0)$$
  $\leftrightarrow \pi_0 + V_D(0) = V_\pi(0) + \gamma V_R(0)$

- Price of the **Default Option**: additional premium to gain “insurance” against possible default $\Rightarrow$ **Safety Loading**
  $$V_D(0) = \varphi \pi_0$$
The design of the guaranteed component

\[ \pi(t) = \pi(t - 1)(1 + r_\pi(t)) \quad t = 1, 2, \ldots, T \]
The design of the guaranteed component

- \( \pi(t) = \pi(t-1)(1 + r_\pi(t)) \quad t = 1, 2, ..., T \)
- \( r_\pi(t) = \max \left\{ r_G, \frac{\beta}{n} \left( \frac{F(t)}{F(t-1)} + ... + \frac{F(t-n+1)}{F(t-n)} - n \right) \right\} \)
- \( r_G \) is the minimum guarantee
- \( \beta \in (0, 1) \) is the participation rate
- \( n = \min(t, \tau) \), where \( \tau \) is the length of the smoothing period
The design of the guaranteed component

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\[
r\pi(t) = r_G + \left( \frac{\beta}{n} \sum_{i=1}^{n} \frac{F(t_i)}{F(t_{i-1})} - (\beta + r_G) \right)^+
\]

- Sequence of Asian call options + risk free bond
  (No closed formulae for the price of the embedded options)
Market model

- **Reference Portfolio**: equity & bonds

  \[ F(t) = \alpha F(t-1) \frac{S(t)}{S(t-1)} + (1 - \alpha)F(t-1) \left( \frac{P(t, T)}{P(t-1, T)} \right) \]

- **Market Model** (simplified)
  - Equity → Geometric Brownian motion
    \[ dS(t) = \mu S(t) dt + \sigma S(t) dW(t) \]
  - Interest rates → Hull and White model
    \[ dr(t) = \kappa (a(t) - r(t)) dt + \nu dZ(t) \]
  - Equity and interest rate are correlated
Asset allocation strategy

- How to fix $\alpha$?
- Idea: stabilize the expected guaranteed benefit due at maturity, with respect to
  - prespecified target
  - prespecified optimality criterion
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**Prespecified Target**

$$t \doteq \pi_0 (1 + r_G (1 + h))^T$$

$h =$ “spread” representing the policyholder participation in the asset returns
Asset allocation strategy

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- **Prespecified Target**

  $$t = \pi_0 (1 + r_G (1 + h))^T$$

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- **Optimality Criterion**: MINIMIZE volatility of guaranteed benefit with respect to the target

  $$\min_\alpha \mathbb{E} \left[ (\pi(T) - t)^2 \right]^{1/2}$$
Asset allocation strategy

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  **Prespecified Target**
  
  \[
  t = \pi_0 (1 + r_G (1 + h))^T
  \]
  
  $h$ = “spread” representing the policyholder participation in the asset returns

  **Optimality Criterion**: MINIMIZE volatility of guaranteed benefit with respect to the target

  \[
  \min_{\alpha} \mathbb{E} \left[ (\pi(T) - t)^2 \right]^{1/2} = \min_{\alpha} \left[ \text{Var}[\pi(T)] + (\mathbb{E}[\pi(T)] - t)^2 \right]^{1/2}
  \]
Implementation

- Monte Carlo simulation

1. Solve numerically the given optimization problem → $\alpha^*$
2. Obtain the corresponding no-arbitrage prices of the embedded options → $V_\pi(0)$, $V_R(0)$, $V_D(0)$
3. Derive the “fair” terminal bonus rate → $\gamma$
   $$\gamma = (\pi_0 + V_D(0) - V_\pi(0)) / V_R(0)$$
4. Calculate the safety loading → $\varphi$
   $$\varphi = V_D(0) / \pi_0$$
5. Compute the corresponding solvency indices
Results: Case 1

Parameter set

<table>
<thead>
<tr>
<th>Market model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
</tr>
<tr>
<td>Interest rate</td>
</tr>
<tr>
<td>$\lambda = -0.015$</td>
</tr>
</tbody>
</table>

Policy design

<table>
<thead>
<tr>
<th>$\beta = 70%$</th>
<th>$\theta = 90%$</th>
<th>$r_G = 4%$</th>
<th>$h = 70%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(0) = 100$</td>
<td>$\tau = 3$ years</td>
<td>$T = 20$ years</td>
<td></td>
</tr>
</tbody>
</table>

Prices

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\alpha^*$</th>
<th>$V_\pi(0)$</th>
<th>$V_R(0)$</th>
<th>$V_D(0)$</th>
<th>$\gamma%$</th>
<th>$\varphi%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>1</td>
<td>68.6183</td>
<td>35.8058</td>
<td>13.1643</td>
<td>96.48</td>
<td>14.63</td>
</tr>
<tr>
<td>0.4</td>
<td>0.8924</td>
<td>75.1433</td>
<td>29.0018</td>
<td>12.4418</td>
<td>94.13</td>
<td>13.82</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6563</td>
<td>76.6864</td>
<td>22.1740</td>
<td>7.0943</td>
<td>92.04</td>
<td>7.88</td>
</tr>
<tr>
<td>0.6</td>
<td>0.4835</td>
<td>78.4642</td>
<td>16.9290</td>
<td>3.6285</td>
<td>89.57</td>
<td>4.03</td>
</tr>
<tr>
<td>0.7</td>
<td>0.3448</td>
<td>80.7334</td>
<td>12.3448</td>
<td>1.5056</td>
<td>87.26</td>
<td>1.67</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2222</td>
<td>84.0793</td>
<td>7.7263</td>
<td>0.4614</td>
<td>82.60</td>
<td>0.51</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1114</td>
<td>89.6243</td>
<td>2.7147</td>
<td>0.2836</td>
<td>24.28</td>
<td>0.32</td>
</tr>
</tbody>
</table>

$\varphi(\alpha = 100\%; \beta = 70\%) = 53.79\%$
Solvency indices

- \( A = \) insurer total assets available s.t.
  \[ A(0) = F(0) + V_D(0) \]

- Risk measures
  - \( \mathbb{P}(\pi(T) > A(T)) \) \( \mathbb{P}(\pi(1) > A(1)) \)
  - \( TVaR \) of the solvency index
    \[ s(t) = \frac{\text{RBC}(t+1) - \text{RBC}(t)}{A(t)} \]
  - \( \text{RBC}(t) = A(t) - V_{\pi}(t) - \gamma V_{R}(t) \)

  “Risk Bearing Capital” (FOPI 2006)
Probability of Default

- Alternative investment strategy for the safety loading $V_D(0)$
- Alternative investment strategies for the available funds
- Probability of default in 1 year from inception – $V_D$ only
Base case: \( \beta = 70\% \); \( \varphi = 1.67\% \)

<table>
<thead>
<tr>
<th></th>
<th>Probability of Default</th>
<th>@ T</th>
<th>@ t = 1</th>
</tr>
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<tbody>
<tr>
<td>(F, ( \varphi ))</td>
<td>(( \alpha^* ), ( \alpha^* ))</td>
<td>1.98%</td>
<td>0.02%</td>
</tr>
<tr>
<td></td>
<td>(( \alpha^* ), 100%)</td>
<td>2.08%</td>
<td>0.02%</td>
</tr>
<tr>
<td>A</td>
<td>( \alpha^* )</td>
<td>1.98%</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>100%</td>
<td>43.11%</td>
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**Base case:** $\beta = 70\%; \, \varphi = 1.67\%$

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<td>@ $T$</td>
<td>@ $t = 1$</td>
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<tr>
<td>$(F, \varphi)$</td>
<td>$(\alpha^<em>, \alpha^</em>)$</td>
<td>1.98%  \hspace{0.5cm} 0.02%</td>
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<tr>
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<td>$(\alpha^*, 100%)$</td>
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**Comments**

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- $(\beta = 50\%; \varphi = 7.88\%; \alpha^*) = (0.04\%; 15\%; 12\%)$
- $(\beta = 90\%; \varphi = 0.32\%; \alpha^*) = (0.54\%; 20\%; 14\%)$
Risk Bearing Capital

Introduction

Agenda

The Contract

Market

Asset Allocation

Strategy

Implementation

Results 1

Results 2

Results 3

Comments

Results 4

Results 5

Stress Testing

Conclusion

15/20
Increasing the target: $h = 90\%$
Increasing the target: $h = 90\%$
Increasing the target: $h = 90\%$
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Stress testing

- Aim: to assess the robustness of the proposed asset allocation
- Adverse (extreme) movement at $t = 1$ year
  - equity volatility
  - interest rates
- $\Delta \sigma = +10\%$, i.e. $\sigma_{ST} = 30\%$ at $t = 1$
- $\Delta r = -1\%$, i.e. $r_{ST}(1) = r(1) - 1\%$
- $F$ invested optimally; alternative strategies for the Safety Loading
Probability of Default

- **Base case:** $\beta = 70\%$, $P(\pi(1) > A(1)) = 0.02\%$

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<th>$P(\pi(1) &gt; A(1))$ %</th>
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<td>a) $\Delta \sigma = +10%$</td>
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<tr>
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**Expected severity**

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<th>$\beta$</th>
<th>$\Delta \sigma$</th>
<th>$\Delta r(1)$</th>
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<tbody>
<tr>
<td>AAA</td>
<td>BBB</td>
<td>AAA</td>
</tr>
<tr>
<td>0.5</td>
<td>30%</td>
<td>27%</td>
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<td>20%</td>
<td>15%</td>
</tr>
<tr>
<td>0.9</td>
<td>23%</td>
<td>17%</td>
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**Risk Management of Guarantees**

Laura Ballotta  
FIN @ CASS

**Introduction**

**Agenda**
- The Contract
- Market
- Asset Allocation
- Stress Testing
- Results 1
- Results 2
- Conclusion

**Risk Bearing Capital**

![Graphs showing risk bearing capital for different levels of β (β = 0.3, β = 0.5, β = 0.7, β = 0.9).](image)

**Equation:**

\[ \beta = 0.3 \]

\[ \beta = 0.5 \]

\[ \beta = 0.7 \]

\[ \beta = 0.9 \]

**Symbols:**
- Base case
- \( \Delta \sigma \)
- \( \Delta r_1 \)

**Case Studies:**
- **Base case:**
  - \( \beta = 0.3 \)
  - \( \beta = 0.5 \)
  - \( \beta = 0.7 \)
  - \( \beta = 0.9 \)

**Notes:**
- Alpha (*) values for various risk levels.
Conclusions

- Asset allocation strategy aimed at risk management of participating life insurance
  - minimum guarantee
  - reversionary bonus
  - terminal bonus

- Impact on capital requirements

- Results consistent with regulatory requirements imposed by Solvency II regime

- Optimal value of the design parameter $\beta$

- Results are robust under stress testing

- Work in progress: further investigation of approach robustness via scenario generation