Investment strategies and risk management for participating life insurance contracts

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Introduction & Motivation

Risk management of guarantees

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- New supervisory framework for the (life) insurance industry:
  - IASB Insurance Project
  - EU Solvency II Review
- Basic concepts
  - Market consistent valuation of A & L
  - Target Capital
- Focus directed especially on
  - "Fair valuation": what is it? How to carry out the program?
  - Market modelling
  - Identification of embedded options: the default option & safety loading

Aims and objectives

- **Risk management** of financial risk induced by a participating contract with minimum guarantee
  - Consiglio et al. (2006): reference portfolio structuring by means of non linear programming using the Wilkie model
  - Bernard et al. (2006): investment strategies for the corresponding safety loading in a market set up á la Merton (1974)

Aims and objectives

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- **(Static) Asset allocation approach**
  - Risk minimization
  - Chosen "risk measure": volatility of the guaranteed benefit with respect to prespecified target

Aims and objectives

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  - Consiglio et al. (2006): reference portfolio structuring by means of non linear programming using the Wilkie model
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- **(Static) Asset allocation approach**
  - Risk minimization
  - Chosen “risk measure”: volatility of the guaranteed benefit with respect to prespecified target
- Analysis of
  - market consistent value of embedded options and safety loading
  - probability of default and Solvency II capital requirements
  - robustness of the approach

Agenda

1. The Participating contract
   - Design
   - Embedded options
   - Safety loading
2. The reference portfolio
3. The market model
4. Asset Allocation
5. Stress Testing

The Participating Contract

- Starting time: $t = 0$; maturity: $T = 20$ years
- Single premium: π_0
- Reference fund: $F(t)$
- Leverage coefficient: θ such that

$$\pi_0 = \theta F(0)$$

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- **Benefit at maturity**:
  - **Guaranteed component**: $\pi(T)$
  - **Discretionary component** (terminal bonus):

$$R(T) = (\theta F(T) - \pi(T))^+$$

- Terminal bonus rate: γ

The fair pricing condition

- Benefit paid **IF** company solvent at T

  $\Rightarrow$ overall liability:

  $$\pi(T) + \gamma R(T) - D(T)$$

  where

  $$D(T) = (\pi(T) - F(T))^{+}$$

  payoff of the "Default Option"

- No arbitrage condition:

  $$\pi_0 = V_{\pi}(0) + \gamma V_R(0) - V_D(0)$$

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- No arbitrage condition:

  $$\pi_0 = V_\pi(0) + \gamma V_R(0) - V_D(0)$$
  $$\leftrightarrow \pi_0 + V_D(0) = V_\pi(0) + \gamma V_R(0)$$

- Price of the Default Option: additional premium to gain "insurance" against possible default $\Rightarrow$ **Safety Loading**

  $$V_D(0) = \varphi \pi_0$$

The design of the guaranteed component

- $\pi(t) = \pi(t-1)(1 + r_\pi(t)) \qquad t = 1, 2, ..., T$

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- $r_\pi(t) = \max\left\{ r_G, \frac{\beta}{n}\left(\frac{F(t)}{F(t-1)} + ... + \frac{F(t-n+1)}{F(t-n)} - n\right)\right\}$
- r_G is the minimum guarantee
- $\beta \in (0,1)$ is the participation rate
- $n = min(t, \tau)$, where τ is the length of the smoothing period

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The design of the guaranteed component

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- r_G is the minimum guarantee
- $\beta \in (0, 1)$ is the participation rate
- $n = min(t, \tau)$, where τ is the length of the smoothing period
- $r_\pi(t) = r_G + \left(\frac{\beta}{n} \sum_{i=1}^{n} \frac{F(t_i)}{F(t_{i-1})} - (\beta + r_G) \right)^+$
- Sequence of **Asian call options** + risk free bond
  (No closed formulae for the price of the embedded options)

Market model

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- **Reference Portfolio**: equity & bonds

$$F(t) = \alpha F(t-1)\underbrace{\frac{S(t)}{S(t-1)}}_{equity} + (1-\alpha)F(t-1)\underbrace{\frac{P(t,T)}{P(t-1,T)}}_{bond}$$

- **Market Model** (simplified)
  - Equity $\rightarrow$ Geometric Brownian motion

    $$dS(t) = \mu S(t)\,dt + \sigma S(t)\,dW(t)$$

  - Interest rates $\rightarrow$ Hull and White model

    $$dr(t) = \kappa(a(t) - r(t))\,dt + v dZ(t)$$

  - Equity and interest rate are correlated

Asset allocation strategy

- How to fix α?
- Idea: stabilize the expected guaranteed benefit due at maturity, with respect to
  - prespecified **target**
  - prespecified **optimality criterion**

Asset allocation strategy

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- **Prespecified Target**

$$t \doteq \pi_0 \left(1 + r_G (1 + h)\right)^T$$

$h =$ "spread" representing the policyholder participation in the asset returns

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- **Optimality Criterion**: MINIMIZE volatility of guaranteed benefit with respect to the target

$$\min_\alpha \mathbb{E}\left[\left(\pi\left(T\right) - \mathrm{t}\right)^2\right]^{1/2}$$

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- **Optimality Criterion**: MINIMIZE volatility of guaranteed benefit with respect to the target

$$\min_\alpha \mathbb{E}\left[\left(\pi\left(T\right) - t\right)^2\right]^{1/2} = \min_\alpha \left[\mathbb{V}ar\left[\pi\left(T\right)\right] + \left(\mathbb{E}\left[\pi\left(T\right)\right] - t\right)^2\right]^{1/2}$$

Implementation

- Monte Carlo simulation

1. Solve numerically the given optimization problem $\longrightarrow \alpha^*$
2. Obtain the corresponding no-arbitrage prices of the embedded options $\longrightarrow V_\pi(0), V_R(0), V_D(0)$
3. Derive the "fair" terminal bonus rate $\longrightarrow \gamma$
   $\gamma = (\pi_0 + V_D(0) - V_\pi(0)) / V_R(0)$
4. Calculate the safety loading $\longrightarrow \varphi$
   $\varphi = V_D(0)/\pi_0$
5. Compute the corresponding solvency indices

Results: Case 1

- Parameter set

Market model			
Equity	$\mu = 10\%$ p.a.	$\sigma = 20\%$ p.a.	
Interest rate	$r_0 = 4.5\%$	$v = 0.2942\%$	$\kappa = 0.9866\%$
	$\lambda = -0.015$	$\rho = -0.2$	
Policy design			
$\beta = 70\%$	$\theta = 90\%$	$r_G = 4\%$	**h=70%**
$F(0) = 100$	$\tau = 3$ years	$T = 20$ years	

- Prices

β	α^*	$\mathbf{V}_\pi(0)$	$\mathbf{V}_R(0)$	$\mathbf{V}_D(0)$	γ %	φ %
0.3	1	68.6183	35.8058	13.1643	96.48	14.63
0.4	0.8924	75.1433	29.0018	12.4418	94.13	13.82
0.5	0.6563	76.6864	22.1740	7.0943	92.04	7.88
0.6	0.4835	78.4642	16.9290	3.6285	89.57	4.03
0.7	**0.3448**	**80.7334**	**12.3448**	**1.5056**	**87.26**	**1.67**
0.8	0.2222	84.0793	7.7263	0.4614	82.60	0.51
0.9	0.1114	89.6243	2.7147	0.2836	24.28	0.32

- $\varphi(\alpha = 100\%; \beta = 70\%) = 53.79\%$

Solvency indices

- $A =$ insurer total assets available s.t.

$$A(0) = F(0) + V_D(0)$$

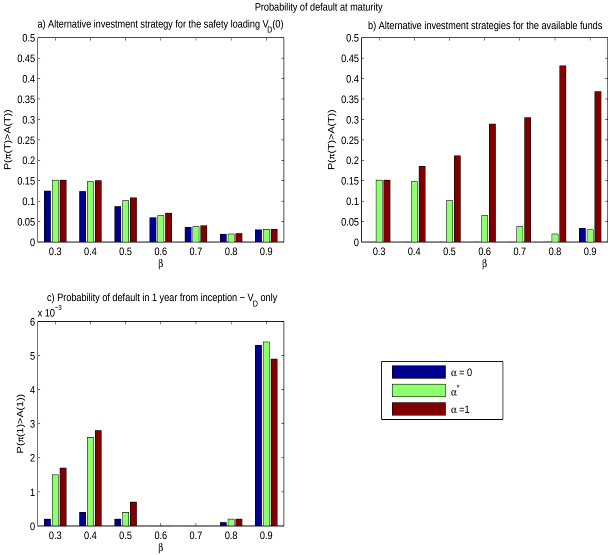
- Risk measures
  - $\mathbb{P}(\pi(T) > A(T)) \qquad \mathbb{P}(\pi(1) > A(1))$
  - $TVaR$ of the solvency index

    $$s(t) = \frac{\widetilde{RBC}(t+1) - RBC(t)}{A(t)}$$

  - $RBC(t) = \underbrace{A(t)}_{\mathbb{P}} - \underbrace{V_\pi(t) - \gamma V_R(t)}_{\hat{\mathbb{P}}}$

"Risk Bearing Capital" (FOPI 2006)

Probability of Default



Comments

- Base case: $\beta = 70\%$; $\varphi = 1.67\%$

	α	Probability of Default @ T	Probability of Default @ t = 1
(F, φ)	(α*, α*)	1.98%	0.02%
	(α*, 100%)	2.08%	0.02%
A	α*	1.98%	-
	100%	43.11%	-

Comments

- Base case: $\beta = 70\%$; $\varphi = 1.67\%$

	α	Probability of Default @ T	Probability of Default @ t = 1	Expected severity (TVaR) @ t = 1 AAA	Expected severity (TVaR) @ t = 1 BBB
(F, φ)	(α^*, α^*)	1.98%	0.02%	8%	7%
	(α^*, 100%)	2.08%	0.02%	9%	7.5%
A	α^*	1.98%	-	-	-
	100%	43.11%	-	-	-

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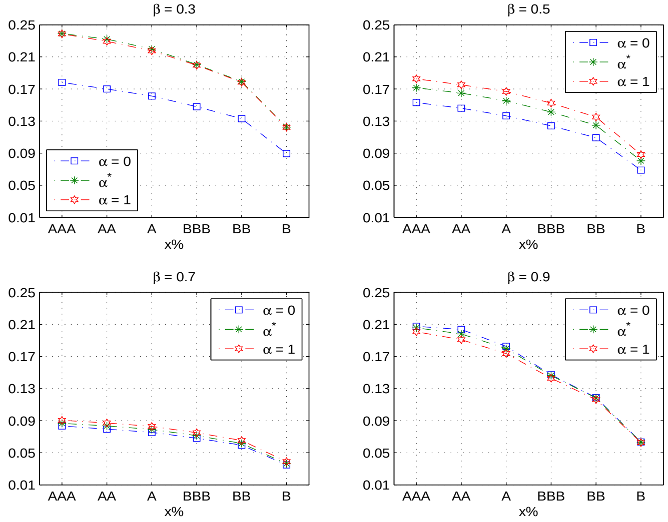
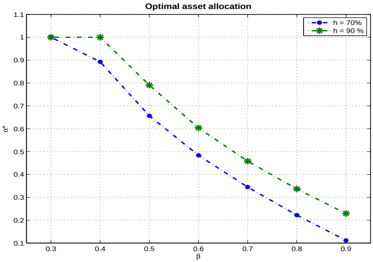
	(α*, 100%)	2.08%	0.02%	9%	7.5%
A	α*	1.98%	-	-	-
	100%	43.11%	-	-	-

- $(\beta = 50\%; \varphi = 7.88\%; \alpha^*) = (0.04\%; 15\%; 12\%)$
- $(\beta = 90\%; \varphi = 0.32\%; \alpha^*) = (0.54\%; 20\%; 14\%)$

Risk Bearing Capital



Increasing the target: $h = 90\%$



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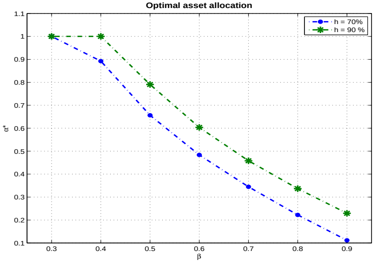

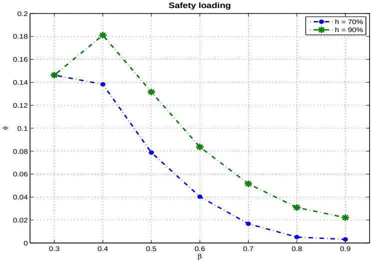

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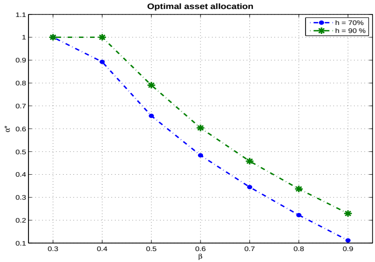

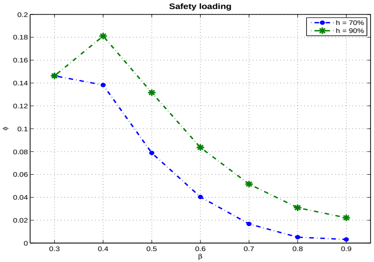

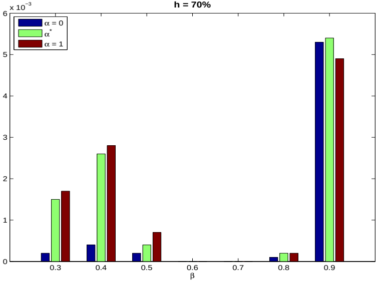

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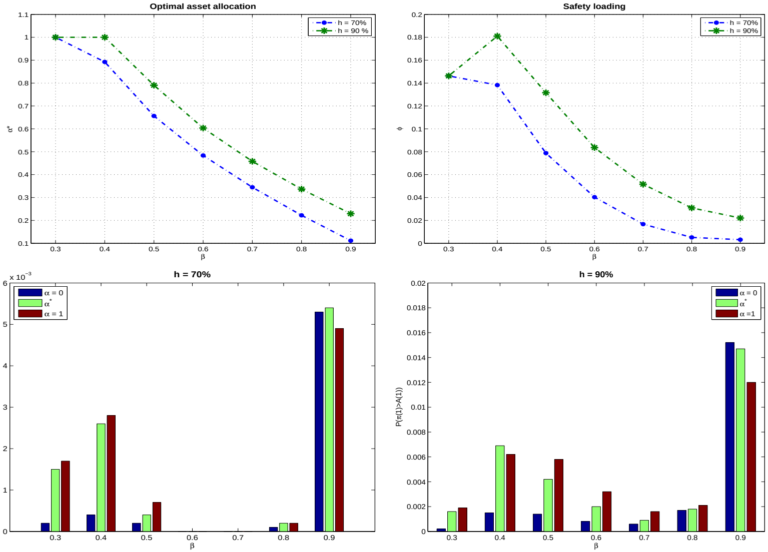

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Stress testing

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- Aim: to assess the robustness of the proposed asset allocation
- Adverse (extreme) movement at $t = 1$ year
  - equity volatility
  - interest rates
- $\Delta\sigma = +10\%$, i.e. $\sigma_{ST} = 30\%$ at $t = 1$
- $\Delta r = -1\%$, i.e. $r_{ST}(1) = r(1) - 1\%$
- F invested optimally; alternative strategies for the Safety Loading

Probability of Default

- Base case: $\beta = 70\%$, $\mathbb{P}(\pi(1) > A(1)) = 0.02\%$

Investment strategy		$\mathbb{P}(\pi(1) > A(1))$ %					
		a) $\Delta\sigma = +10\%$			b) $\Delta r(1) = -1\%$		
β	α^*	$\alpha = 0$	α^*	$\alpha = 1$	$\alpha = 0$	α^*	$\alpha = 1$
0.3	1	0.42	1.21	1.17	0.80	1.74	1.75
0.5	0.6563	1.44	1.90	2.44	1.69	2.22	2.76
0.7	**0.3448**	0.17	**0.19**	0.25	3.67	**3.87**	4.17
0.9	0.1114	1.25	1.20	1.19	99.92	99.92	99.90

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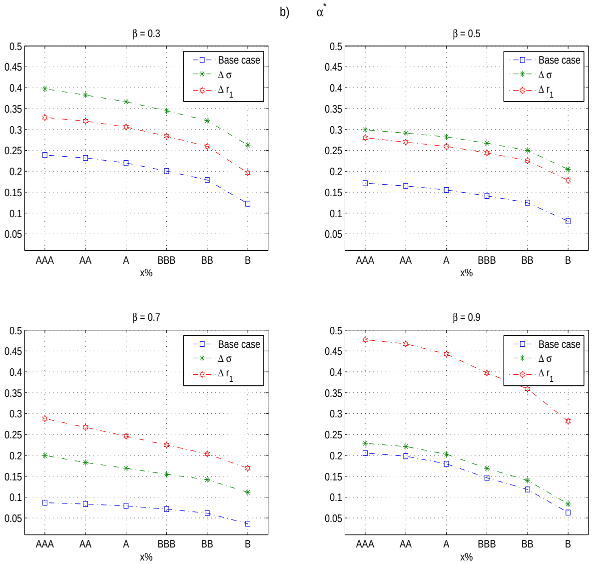
0.7	**0.3448**	0.17	**0.19**	0.25	3.67	**3.87**	4.17
0.9	0.1114	1.25	1.20	1.19	99.92	99.92	99.90

	Expected severity			
	$\Delta\sigma$		$\Delta r(1)$	
β	**AAA**	**BBB**	**AAA**	**BBB**
0.5	30%	27%	29%	24%
0.7	20%	15%	29%	22%
0.9	23%	17%	48%	40%

Risk Bearing Capital



Conclusions

- Asset allocation strategy aimed at risk management of participating life insurance
  - minimum guarantee
  - reversionary bonus
  - terminal bonus
- Impact on capital requirements
- Results consistent with regulatory requirements imposed by Solvency II regime
- Optimal value of the design parameter β
- Results are robust under stress testing
- Work in progress: further investigation of approach robustness via scenario generation