IAA AFIR/LIFE Colloquium 2009

September 9th,2009

Coffee Break

Please make sure that you have your invitation to the Reception by the City of Munich with you tonight!

AFIR MUNICH LIFE 2009

Landeshauptstadt München

Einladung

The Lord Mayor
of the City of Munich,
Christian Ude

requests the honour of your presence
at a Civic Reception

on the occasion of the
IIIrd International LIFE Colloquium
and the XIXth International AFIR Colloquium

on Wednesday, September 09, 2009
at 19.00 - 21.00 hrs.

in the Old Town Hall Munich
(entrance in the driveway).

– This invitation serves as admission ticket for 1 person. –

Der Oberbürgermeister der
Landeshauptstadt München,
Christian Ude

lädt Sie ein zu einem Stehempfang

anlässlich des
3. Internationalen LIFE Kolloquiums
und des 19. Internationalen AFIR Kolloquiums

am Mittwoch, 09. September 2009,
von 19.00 bis 21.00 Uhr

in den Saal des Alten Rathauses
(Eingang in der Unterführung zum Tal).

– Diese Einladung gilt als Einlasskarte für 1 Person. –

Breakout Session Topic 3:

Life insurance and financial markets

9 September 2009

AFIR LIFE MUNICH 2009

Section Finance de l'AAI
IAA Finance Section
AFIR

Section Vie de l'AAI
IAA Life Section
LIFE

DAV
DEUTSCHE AKTUARVEREINIGUNG e.V.

Hedging against volatility, jumps and longevity risk in participating life insurance contracts – a Bayesian analysis

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University of Tampere, Finland

Overview

- Introduction
- The participating life insurance contract
- Finance model
- Mortality model
- Model estimation
- Pricing with regression method
- Fair bonus rate
- Minimum variance hedging
- Empirical results
- Conclusions

Introduction

- A full Bayesian analysis of valuation and hedging of a participating life insurance contract is presented.
- The contract embeds an American-style surrender option which is valued with regression method.
- Both financial and mortality risks are taken into account.
- The financial model allows the interest rate, volatility and jumps in the index process to be stochastic.
- As a stochastic mortality model we use a generalization of the Gompertz model.
- Hedging of the contract is performed with a single-instrument minimum variance hedge.
- The results are compared with a simpler competing model and delta-neutral hedge.

Introduction

- The duration of the contract is the most significant factor to produce large hedging errors.
- To make the different hedging strategies comparable, it is important to determine a fair bonus rate for each case studied.
- If the bonus rate is set at a high level, the contract is almost never reclaimed before the final expiration date.
- If the bonus rate is too low, early surrender is highly probable.
- A special method to solve a fair bonus rate is introduced.

Participating life insurance contract

The growth of the amount of savings in the insurance contract during a time interval of length $\delta = t_{i+1} - t_i$ is given by

$$\log\frac{A(t_{i+1})}{A(t_i)} = g\,\delta + b\max\left(0, \log\frac{X(t_{i+1})}{X(t_i)} - g\,\delta\right),$$

where $A(t_i)$: amount of savings at time t_i

$X(t_i) = \sum_{j=0}^{q} S(t_{i-j})/(q+1)$

$S(t_i)$: equity index of total return at time t_i

g : guarantee rate, b : bonus rate

Finance model

The following system of SDEs is assumed for the dynamics of stock index, variance and riskless short-term interest rate

$$\mathrm{d}\log S_t = \left(r_t - \frac{1}{2}V_t - \lambda\mu_J\right)\mathrm{d}t \;+\; \sqrt{V_t}\,\mathrm{d}Z_t^{(1)} + U_t\mathrm{d}q_t$$

$$\mathrm{d}V_t = (\alpha_1 + \beta_1 V_t)\mathrm{d}t + \sigma_V\sqrt{V_t}\,\mathrm{d}Z_t^{(2)}$$

$$\mathrm{d}r_t = (\alpha_2 + \beta_2 r_t)\mathrm{d}t + \sigma_r\sqrt{r_t}\,\mathrm{d}Z_t^{(3)}$$

where $\mu_J = \exp(a + \frac{1}{2}b^2) - 1$, and $Z_t^{(1)}$, $Z_t^{(2)}$ and $Z_t^{(3)}$ are three standard Brownian motions with correlation structure

$$\mathrm{Cor}\left(Z_t^{(1)}, Z_t^{(2)}, Z_t^{(3)}\right) = \begin{pmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{12} & 1 & \rho_{23} \\ \rho_{13} & \rho_{23} & 1 \end{pmatrix}$$

under a risk-neutral probability measure.

Mortality model

For the intensity of mortality, we will use a generalization of the Gompertz model of a form

$$\log(\mu_{ku}) = \beta_{00} + \beta_{01}u + \beta_{10}k + \beta_{11}ku + \epsilon_{ku}$$

where μ_{ku} is the death rate for age k and for cohort u set by the year of birth and

$$\epsilon_{ku} = \phi\epsilon_{k-1,u} + a_{ku}, \quad a_{ku} \sim \text{i.i.d. N}(0, \sigma_m^2)$$

Model estimation

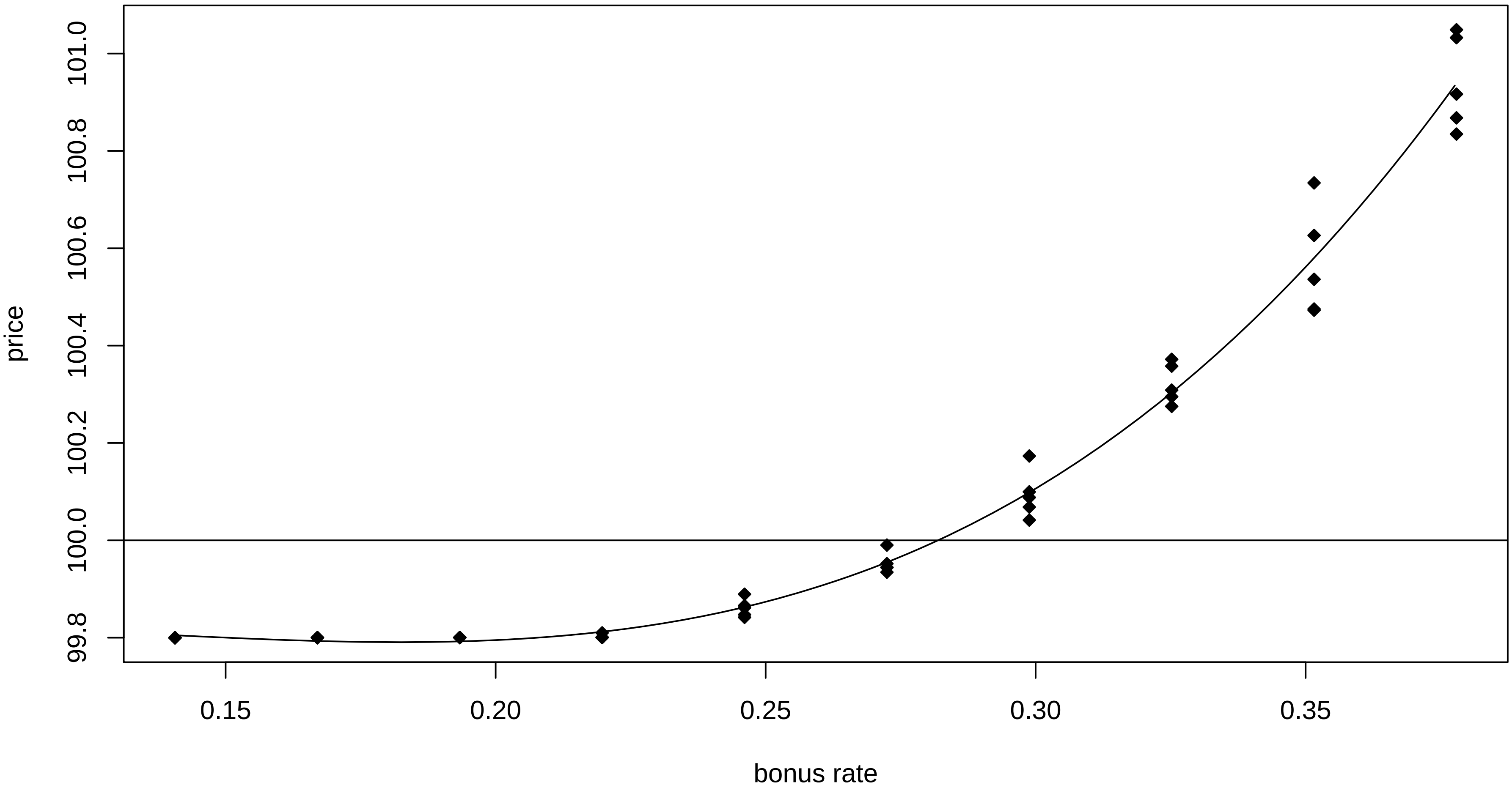
- Based on Bayesian methods and Euler discretization
- A single-component (or cyclic) Metropolis-Hastings algorithm is used
  - finance model parameters updated with Gibbs sampler
  - volatility and jump processes updated with Metropolis-Hastings algorithm
  - mortality model updated with Gibbs sampler except that the correlation parameter is updated with a Metropolis step

Pricing an American option

- Based on an optimal exercising strategy, such that the expected discounted payoff of an option is maximized.
- The discounted immediate exercise value and the corresponding discounted continuation value are compared.
- In regression method the continuation value is expressed as linear regression using some basis functions.
  - used variables: the current values of the index, interest rate and variance, moving average of the index and the first index value appearing in the moving average
  - as basis functions Laguerre polynomials are used

Fair bonus rate

- We determine a fair bonus rate so that the price of the contract will be equal to the initial savings. This gives the contract a transparent structure.
- Determining b is a inverse prediction problem.
- Option is valued for various values of b and repeated several times for each value.
- We need to estimate a regression model where option price estimates are regressed on bonus rates.
- Third degree polynomial curve is flexible enough for this purpose.
- After fitting the curve, we solve the bonus rate b for which the option price is equal to 100, which we assume to be the initial amount of savings.


price
101.0
100.8
100.6
100.4
100.2
100.0
99.8
0.15
0.20
0.25
0.30
0.35
bonus rate

Minimum variance hedging

- A single instrument hedge, which employs only the underlying stock
- The time t value of the replicating portfolio is $N_t^0 + N_t^S S_t$ where N_t^S denotes number of shares of the stock to be purchased and N_t^0 the residual cash position
- The time $t+\delta$ hedging error $H_{t+\delta} = N_t^S S_{t+\delta} + N_t^0 e^{r_t \delta} - C_{t+\delta}$
- When minimizing the variance the amount of stock in the portfolio is as follows

$$N_t^S = \Delta_t^{(S)} \frac{V_t}{(V_t + V_t^J)} + \Delta_t^{(V)} \frac{\rho_{12}\sigma_V V_t}{S_t(V_t + V_t^J)} + \Delta_t^{(r)} \frac{\rho_{13}\sigma_r \sqrt{V_t r_t}}{S_t(V_t + V_t^J)} + \frac{\lambda\left[\mathrm{E}_t\left(J_t C_t(S_t + J_t S_t, V_t, r_t)\right) - C_t(S_t, V_t, r_t)\mu_J\right]}{S_t(V_t + V_t^J)}$$

where V_t^J denotes the instantaneous variance of the jump component and $C_t(S_t, V_t, r_t)$ denotes the value of the contract

Competing model and delta-neutral hedging

When a delta-neutral hedge is constructed we use a simpler model described as

$$\mathrm{d}r_t = \kappa(\xi - r_t)\mathrm{d}t + \sigma r_t^{\gamma}\mathrm{d}Z_t^{(1)}$$

$$\mathrm{d}S_t = r_t S_t \mathrm{d}t + \nu S_t^{1-\alpha}\mathrm{d}Z_t^{(2)}$$

where $Z_t^{(1)}$ and $Z_t^{(2)}$ are two standard Brownian motions under a risk-neutral probability measure, correlated through $Z_t^{(2)} = \rho Z_t^{(1)} + \sqrt{1-\rho^2}Z_t^{(3)}$, where $Z_t^{(1)}$ and $Z_t^{(3)}$ are independent standard Brownian motions.

In the delta-neutral hedge corresponding to this model, the number of shares in the replication portfolio is given by

$$N_t^S = \frac{\partial C_t(S_t, r_t)}{\partial S_t} = \Delta_t^{(S)} \geq 0$$

which is computed using the estimated regression model.

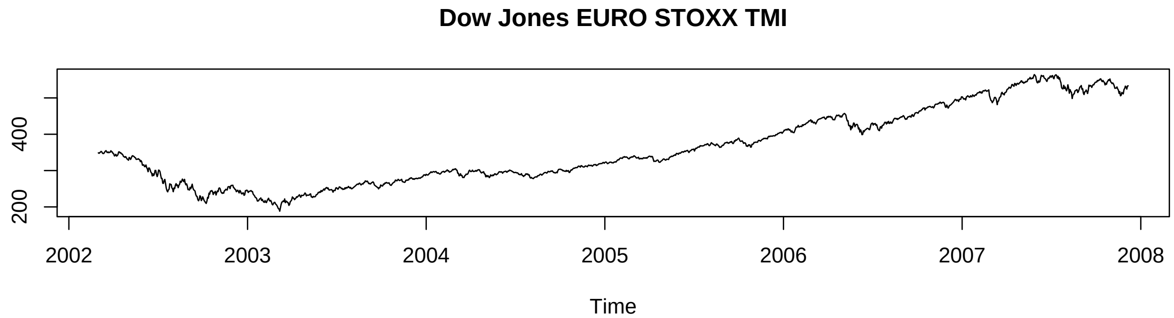
Comparison of the hedging schemes

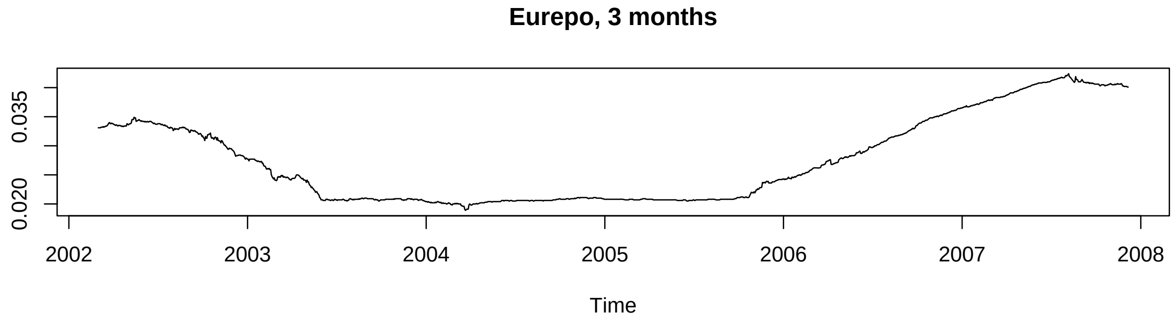
- The difference of the replication portfolio and the balance is computed for each simulation path at its estimated optimal stopping time.
- The used stopping time is the first time when the estimated continuation value is smaller than the immediate exercise value.
- The mean difference and mean squared error is computed over all the simulation paths.
- The procedure is repeated 50 times using different regression estimates and the results of the repetitions are pooled.

Finance data

We use the following data (4.12.2002 – 6.12.2007):

- Total return equity index of Dow Jones EURO STOXX Total Market Index
- 3 month Eurepo interest rate





Mortality data

- We use data provided by Human Mortality Database (see http:// www.mortality.org)
- In our work we use Finnish mortality data for females between ages 30 and 80. More specifically, we use cohort death rates for cohorts born between 1926 and 1961.

Empirical results – SVJ-SI model

- The mean difference (MD) of the replicating portfolio value and the balance is positive in the 3-year contracts.
- In the 10-year contracts the sign of the MD is sometimes positive and sometimes negative.
- The hedging MSEs are considerably larger in the 10-year contracts than in the 3-year contracts due to extremely heavy-tailed error distributions in the longer contracts.
- The updating frequency of the replicating portfolio has no systematic effect on the hedging error.

Empirical results – effect of mortality

- When pricing and hedging the contract, we use the worst-case scenario of mortality from the insurance company's viewpoint, i.e. simulated minimum death rates.
- There is only slight difference in the results between the case when mortality is not taken into account and the case when it is and the contract is started at the age of 60.
- When comparing the models without mortality with those with mortality and 80 years old clients, we note that for some reason the MD is clearly negative in the 10-year contracts.

Empirical results – hedging comparison

- In almost all cases the hedging MSE with CEV-SI model and delta-neutral hedging is slightly lower than that with SVJ-SI model and minimum variance hedging.
- When the real-world predictions are simulated from the SVJ-SI model and the hedging is based on the CEV-SI model, the MDs are substantially negative.

Conclusions

- In some cases the distribution of the hedging error has a negative mean.
  - can be easily shifted by reducing the bonus rate slightly
- Some rare paths of the financial series cause a heavy left tail in the hedging error distribution in long-term contracts.
  - might lead to crucial hedging errors and large losses to the insurance company

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