AFIR LIFE MUNICH 2009

Internal Models

Section Finance de l'AAI
IAA Finance Section
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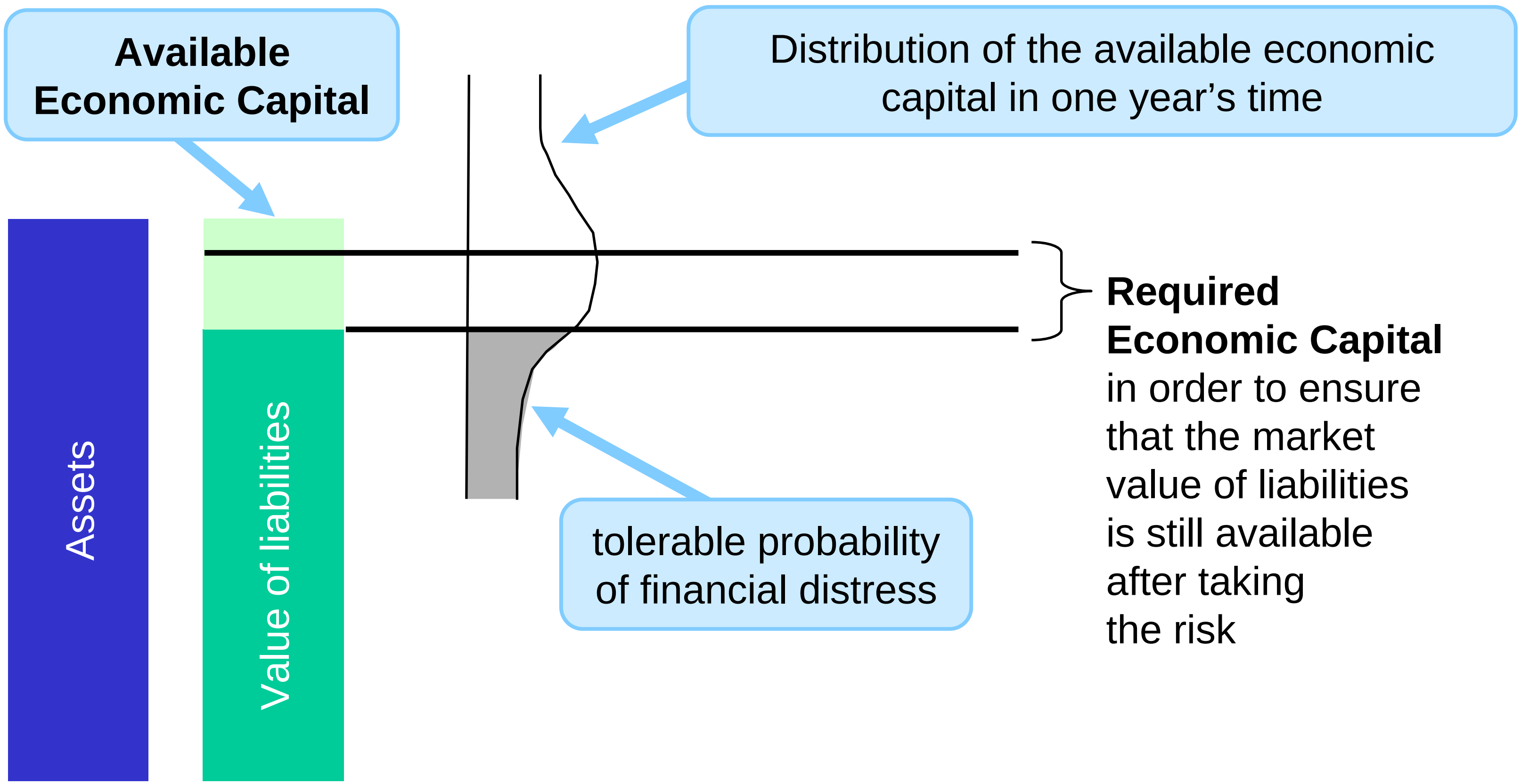
Section Vie de l'AAI
IAA Life Section
LIFE

DAV
DEUTSCHE AKTUARVEREINIGUNG e.V.

Agenda

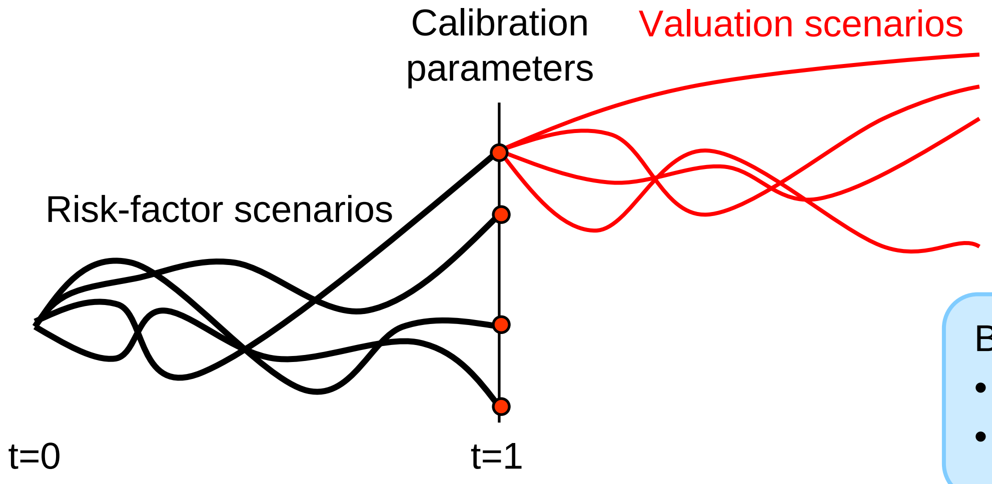
- The concept – required capital
- Typical issues
- The calibration-dilemma
- Typical approaches for internal models
- Linearisation approaches
- The Delta-Gamma-approach
- The numerical error in a full stochastic approach
- Using the Delta Gamma approximation as control variate

Required Economic Capital is the capital you need to have ensured that in most situations policyholder obligations can be fulfilled



Conceptually a full internal economic capital model is simple…

- Create a sufficiently large number of scenarios for all relevant risk factors
- Revalue the assets and liabilities for each risk factor scenario at the end of the first year
  - including all options and guarantees
    - which in most cases requires a stochastic valuation and thus leads to a nested stochastic calculation



But in most cases this approach is
- Technically not feasible
- Not necessary

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Typical issues

- We focus on actuarial issues here, not on the related issues, like data availability
- There are many risk factors:
  - Which distribution do I assume?
  - How do I **calibrate** these distributions?
  - How do I reflect extreme events properly?
  - How do I reflect the dependency structure of the risk factors?
- What are the numerical errors involved in a nested stochastic approach?
- How do I determine the net-value at time 1 for each risk-factor scenario?
- How do I reflect fungibility and intra-group dependency?

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Calibration: Does history tell us anything about the future?

The observed history is not a sufficient basis for projecting the future…

…because

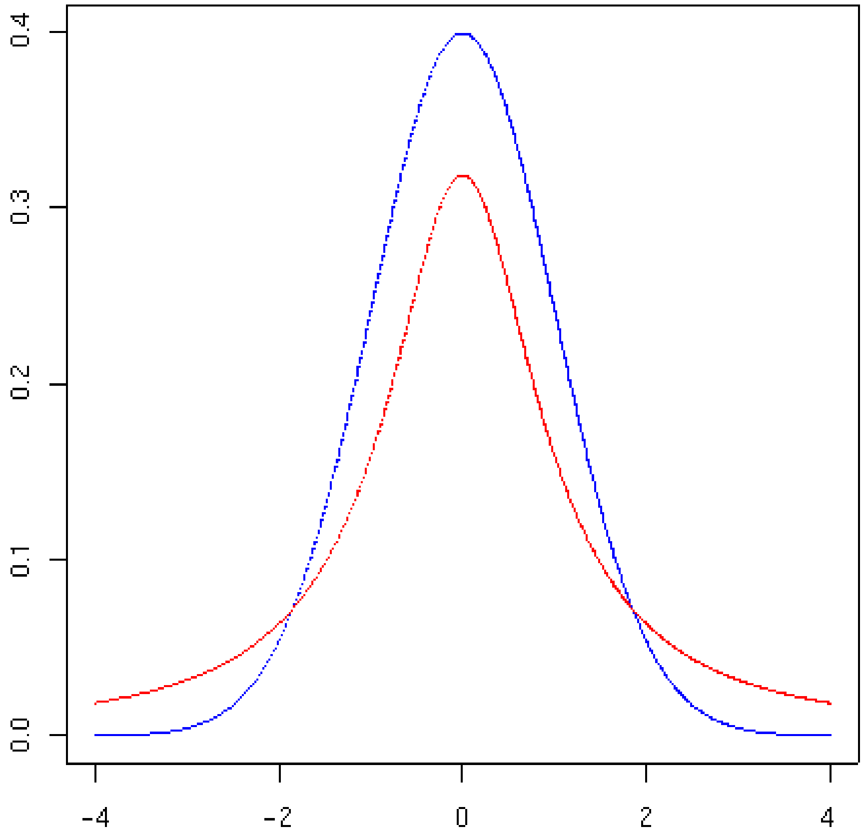
- time-series are too short to derive robust input for extreme events
- there will be new risks and developments, not yet observed

One solution:

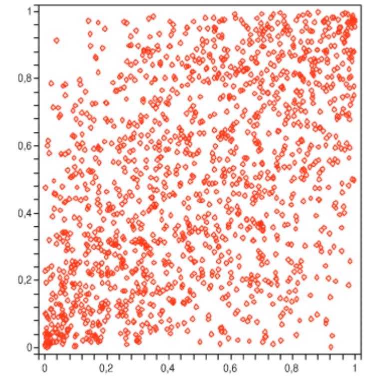
Actuarial judgement-based extreme-scenarios as basis for a calibration

of e.g. copulae

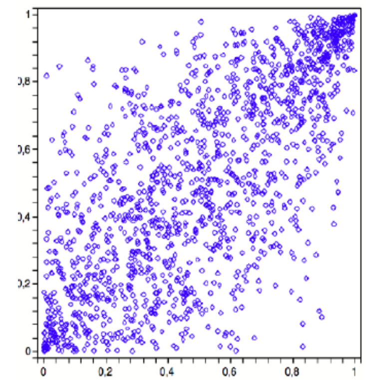
Two major issues: „fat tails“ and dependency structure



Normal (blue) vs t-Student (red)



Gaussian



Gumbel-copula

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Approaches for internal models observed so far

Risk-factors

- Multivariate normal – analytic
- Multivariate normal – stochastic scenarios
- Bootstrapping / historical resampling
- Extreme scenarios
- Copulae

Valuation

- Stress tests and linearity assumption
- Replication portfolio approach
- Full stochastic approach
- Roll-up-approach

The standard approach

Risk-factors

Multivariate normal – Analytic

Valuation

Stress tests and linearity assumption

- Assumes all risk factors are multivariate normal and
- Exposure is linear in risk factors
- Resulting net value distribution is normal again with known volatility
- All necessary statistics can be derived easily, but
- Too simplistic

A pragmatic approach

Risk-factors

- Multivariate normal – stochastic scenarios
- Extreme scenarios

Valuation

- Replication portfolio approach

- Use Cholesky-decomposition of correlation matrix and normal distribution assumption to produce stochastic scenarios
- Add extreme scenarios
- Determine net value at t=1 for each risk factor scenario using replication portfolios
- Captures non-linearity adequately
- Extreme scenarios can reflect tail-dependency and fat tails up to a certain degree
- Allows for non-trivial management actions
- Allows to model group diversification and fungibility

Cholesky-decomposition

- Let S be the covariance matrix of the multivariate distribution considered
- Determine a matrix such that $C*C^T = S$
  - If S is positive semi-definite (which we should expect) then Cholesky factorisation creates a lower triangular matrix such that $C*C^T = S$
- If Z is a vector of independent standard normal random variables then C*Z is a vector of normal random variables with covariance matrix S

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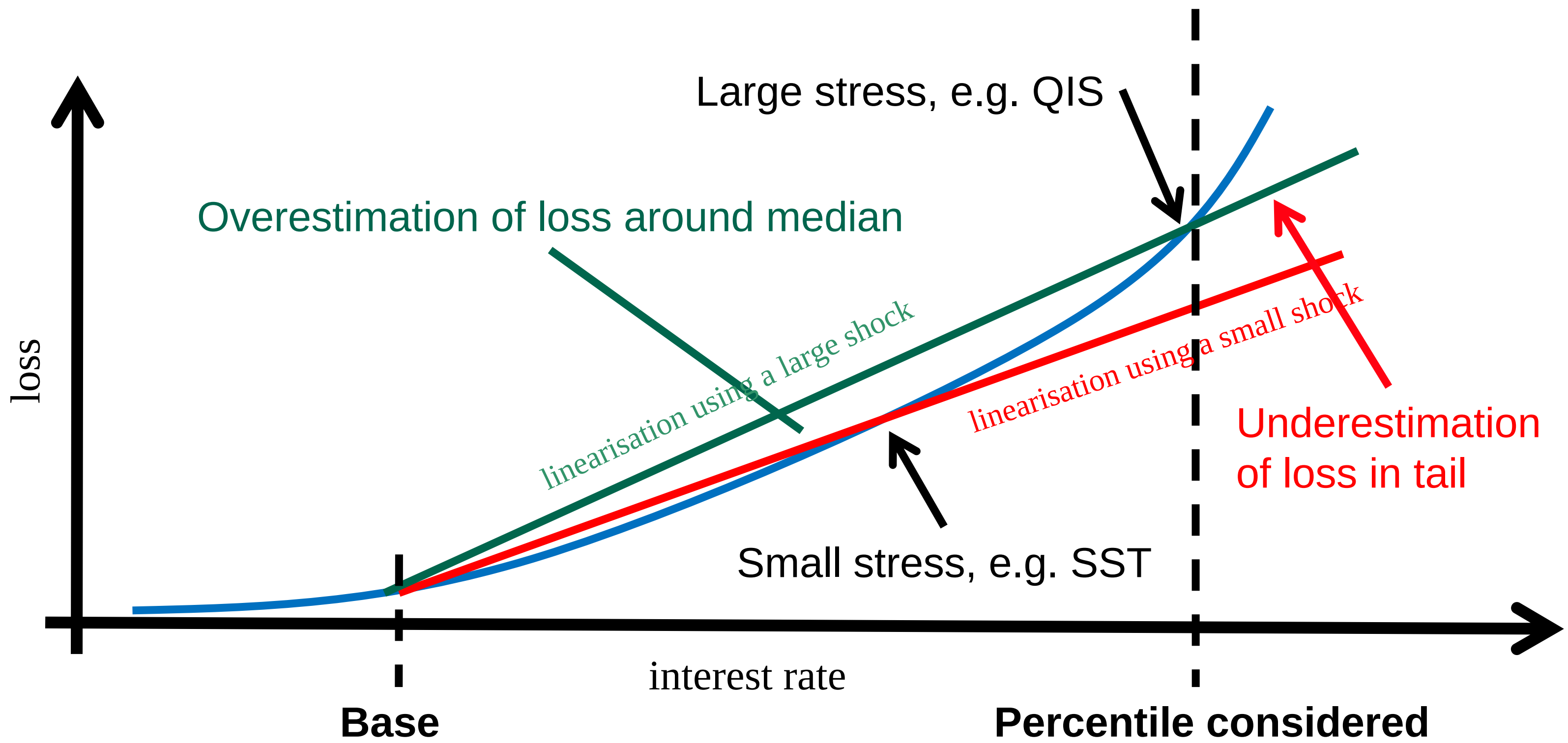
Non-linearity is an issue

An example is interest rate convexity: loss = $(1+i)^n \neq 1+i \cdot n$



Linear approaches can show all kind of behaviour

Example

- Loss function given by discounting: $100 \cdot (1+i)^{-20}$
- i fluctuates normally around 2% with volatility 0.8%
- Example A: exposure to this one risk factor only
- Example B: exposure based on the sum of the exposure of 6 similar but independent risk factors

VaR in % of correct VaR	Linearisation using small shock (1% – SST)	Linearisation using large shock (99.5% percentile: 2.06%)
One risk factor	**91%**	102%
6 risk factor exposures added	102%	**115%**

- This is by no means astonishing, looking at the graph

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Delta-Gamma helps

Idea: Determine two stresses for each risk factor and fit quadratic function to the three known values

- Typically an up-stress and a down-stress is used
- No cross-risk-factor stresses: “diagonal” approach

Example

- As above

VaR in % of correct VaR	Quadratic approximation using small shock (1% - SST)
One risk factor	97.76%
6 risk factor exposure added	98.75%

- But who knows whether in other situations the approximation works as well?

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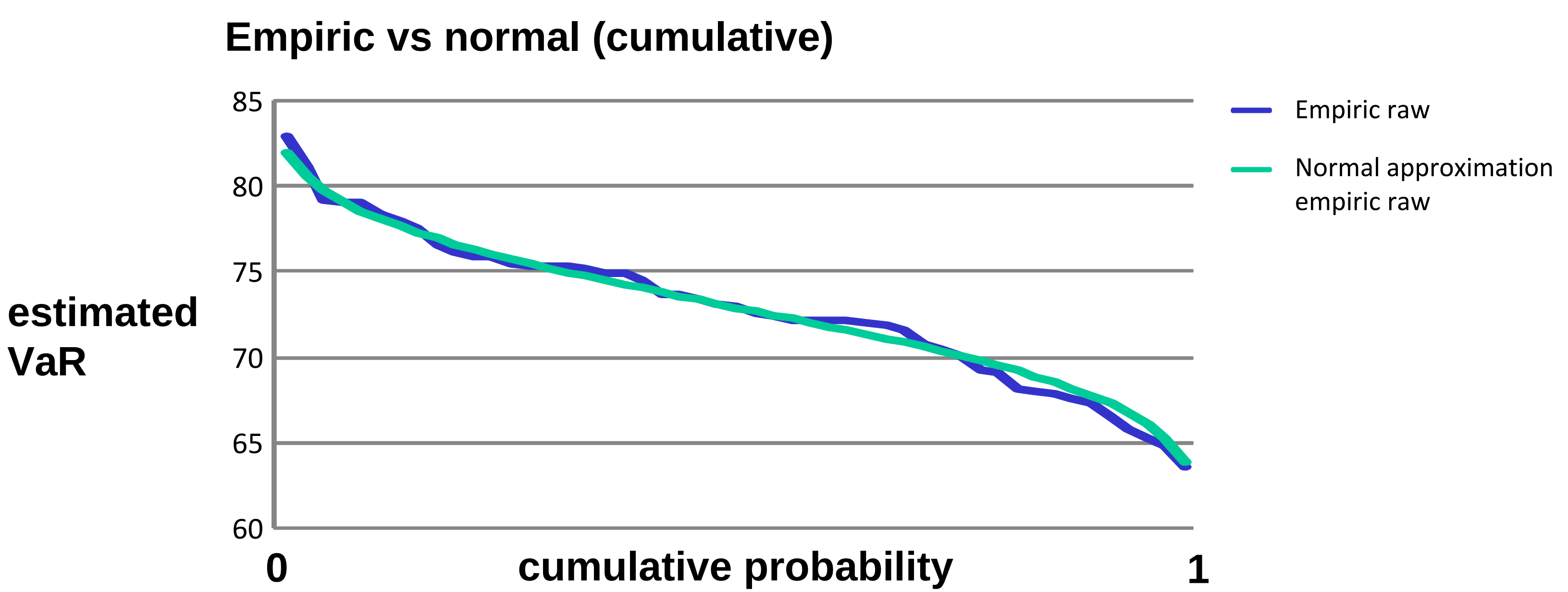
A full stochastic approach – even with accurate valuation – shows considerable numerical error

- Example as above – one risk factor
- 50 Batches of 1'000 simulations
- Distribution of empiric VaR
- Volatility is 6% of the VaR



Estimating the error in a stochastic approach

- In a nested stochastic approach we have two sources for numerical error
  - the error in determining the value for each risk-factor scenario
    - in fact we determine VaR(Accurate+valuation error) instead of VaR(Accurate)
  - and the numerical error in determining (e.g.) the 99.5% VaR
    - for this error there exists a nice non-parametric estimation
    - and a parametric formula

A non-parametric formula for VaR-estimation error

The probability that the correct VaR is higher than the m-th result L_m is $1-cum\_binom(m-1,n,\alpha)$, where $cum\_binom(m,n,\alpha)$ is the cumulative binomial distribution for m, n and α.

And the probability that the correct VaR is lower than the p-th result L_p is $cum\_binom(p-1,n,\alpha)$.

Here $cum\_binom(p-1,n,\alpha)$ is the cumulative binomial distribution for p-1 successes, n experiments and success-probability α.

α	n	Confidence-intervall (both sides)	p	m	P(VaR< L_p)	P(VaR> L_m)
0.995	100	0.95	98	101	0.014103	0
0.995	500	0.95	494	501	0.013944	0
0.995	1000	0.95	990	1000	0.013469	0.006654
0.995	10000	0.95	9936	9964	0.023315	0.023495

Read: the probability that the 9936-smallest scenario is larger than the 99.5% VaR when considering 10‘000 scenarios is smaller than 2.3%

A parametric formula for VaR-estimation error

- For a given quantile α define k:=int(α*n)+1
- The k-th result $\overline{L}_k$ is an estimation of the VaR
- The error $L_k - VaR$ has approximately variance $\frac{\alpha \cdot (1-\alpha)}{n \cdot f(VaR)^2}$ around 0
- f is the density of the distribution function of L

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Enhancing the full stochastic approach with the DeltaGamma-function as control-variate substantially reduces numerical error

We use that:

$$P(L > VaR) = E(I_{L>VaR}) = E(I_{L>VaR} - I_{DeltaGamma>VaR'} + I_{DeltaGamma>VaR'}) =$$
$$E(I_{L>VaR} - I_{DeltaGamma>VaR'}) + E(I_{DeltaGamma>VaR'})$$

where $E(I_{DeltaGamma>VaR'})$ **can be determined with a high degree of accuracy and**

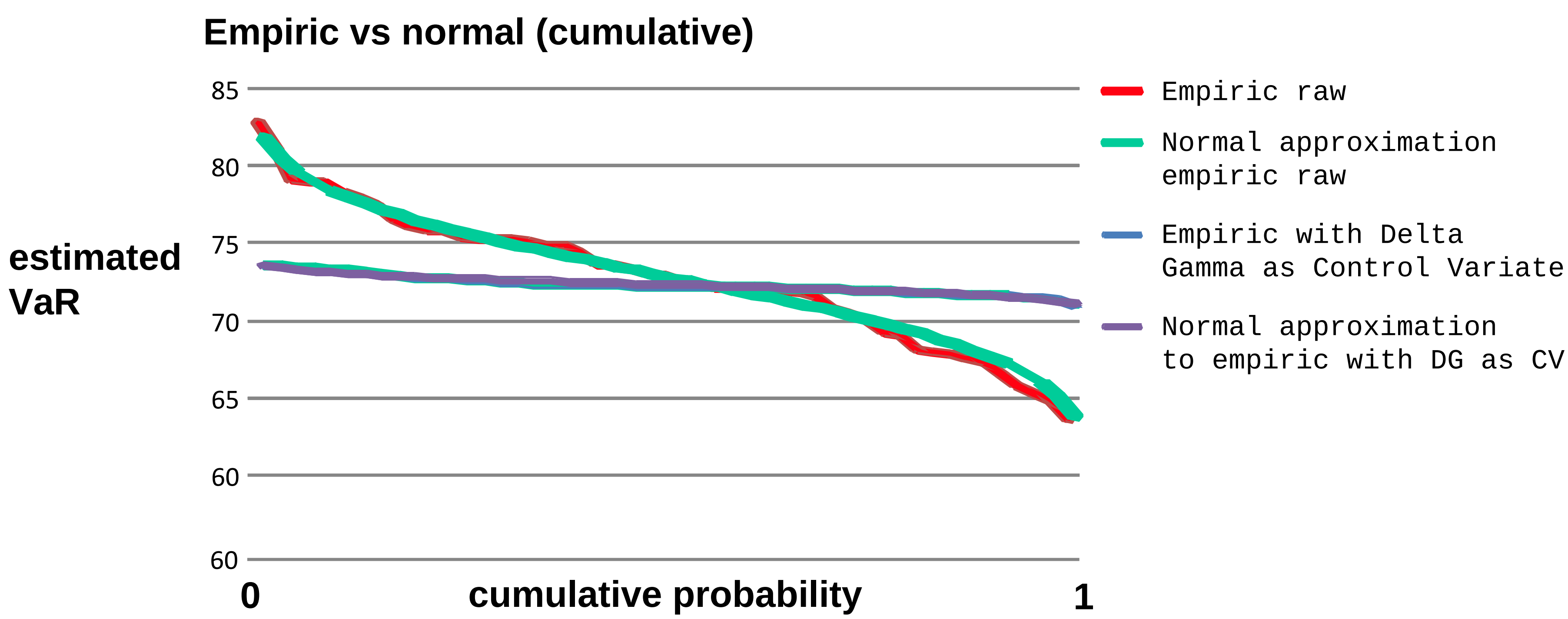
$$E(I_{L>VaR} - I_{DeltaGamma>VaR'}) \approx \frac{1}{n} \sum_{scenarios} I_{L(scenario)>VaR} - I_{DeltaGamma(scenario)>VaR'}$$

is an estimation with lower estimation error as $I_{L>VaR} - I_{DeltaGamma>VaR'}$

has lower volatility than $I_{L>VaR}$

Enhancing the full stochastic approach with the DeltaGamma-function as control-variate substantially reduces numerical error

- Example as above, Delta-Gamma-approximation as above
- Volatility is 0.85% of the VaR, i.e. 50 times less scenarios with same accuracy



Presenter’s contact details

Tigran Kalberer
Partner
Financial Risk Management

KPMG AG
Badenerstrasse 172
8026 Zürich
Tel. +41 44 249 32 14
Fax +41 44 249 33 77

tkalberer@kpmg.com