Mortality-Indexed Annuities
Managing Longevity Risk via Product Design

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Agenda

- Introduction: Motivation and Related Research
- Mortality-Indexed Annuities as a Product Design Proposal
- Simulation Framework and Benchmark
- Selected Results
- Conclusion
Demographic Transition worldwide phenomenon (Oeppen/Vaupel, 2002)
- Decreasing birth rates (Berkel et al., 2002)
  - Reason: Changing societal and family structures
- Decreasing mortality (Willets, 1999; Kytir, 2003)
  - Reason: “better” living, working, environmental conditions; medical advances; health consciousness

Consequences:
- Changing age structures (age pyramids) (Sinn, 2004)
  - burden for PAYGO social security systems
- Globally increasing life expectancies (Vaupel, 1986; Oeppen/Vaupel, 2002)
  - Societal achievement, also holds longevity risk
• **Individual Longevity Risk**
  - Risk of individual deviations of lifetime from average. Sufficient financial means during retirement or post-working ages? *(MacMinn et al., 2006)*
  - Social security tends to provide lower benefits than initially expected *(Schmähl, 2001)*
  - Individuals challenged to adjust long-term saving/consumption to uncertain, longer lifetime *(Bloom et al., 2001)*; possibly by transferring longevity risk to insurer (life annuity)

• **Aggregate Longevity Risk**
  - Uncertainty regarding correct projection of future average mortality *(Blake/Burrows, 2001)*
  - Strong, worldwide correlation *(Zahn/Henninger, 1942)*; potential for accumulative losses
  - Hardly diversifiable or (re)insurable *(Riemer-Hommel/Trauth, 2005)*
• **Annuity Puzzle**: empirically low demand for life annuities despite theoretical optimality *(Yaari, 1965)*

  - Several explanations exist in literature *(Davidoff et al., 2005; Brown/Orszag, 2006; Van de Wen/Weale, 2006; Schulze/Post, 2006; Milevsky/Young, 2007)*

  - Among others: prices could be too high or perceived to be excessive *(Mitchell et al., 1999; Murthi et al., 1999; Finkelstein/Poterba, 2002)*
  – partly justified due to strong correlation
• Conservative Pricing
  - Limited by competition, regulation
  - Limited marketability of excessively priced products (e.g. Mitchell et al., 1999)
    (↔ tax advantages and other incentives designed to mitigate insufficient demand)

• Natural Hedging (Cox/Lin, 2007; Wetzel/Zwiesler, 2008)

• Securitization

• Leaving the annuity market (?)

• Modification of actuarial product design
Example: private health insurance in Germany
- Design similar to life annuities: recurring, constant premiums; lifelong coverage
- Policyholders bear systematic risk of increasing health expenditures (premium adjustments)

Also: Transfer of risk successful with respect to investment risk (e.g., unit-linked life insurance/life annuities)

Proposal: Mortality-Indexed Annuity (MIA) as modification of a constant life annuity

→ New: adjustments of annuity payments based on actual mortality experience: higher/lower portfolio mortality → higher/lower benefits

Result: limited risk for insurer; policyholders’ perspective?
• Immediate annuity sold against (actuarially fair) single premium; constitutes initial per-policy reserve

• Evolution of reserves due to inheritance effect and interest

• Annual adjustments of benefits according to equivalence principle (best estimate of mortality, based on actual portfolio experience)

• Regulatory requirements neglected (taxation, calculation requirements, model choices etc.)

• Further details
  – No period certain
  – Constant interest rate
  – Pure net perspective without costs or expenses; actuarially fair price
• **Monte-Carlo simulation** \((N=10,000 \text{ paths})\)
  - Consider a large portfolio of homogeneous risks over \(T\) periods

• **General** mortality follows **Lee-Carter model**
  \((\text{Lee/Carter, 1992; Brouhns et al., 2002})\)

• Best estimate of mortality for remaining periods based on Lee-Carter, accounting for mortality **experience**.

• Mortality data of British annuitants, Source: CMI

• Males, initial age \(x=60\), single premium \(\pi_0=100,000\), contract term \(T=41\) (last payment due on 100\(^{th}\) birthday)
• Idea: constant life annuity with guaranteed benefits serves as a benchmark – identical single premium $\pi_0$

• Starting point: Initially ($t=0$), benchmark benefits equal to those from MIA, as calculation based on identical assumptions; but: benefits reduced by safety loading (see below)

• Mortality correctly projected on average, but subject to uncertainty
  - Calculation sufficient on average, but underlies strong fluctuations
  - Insurer charges safety loading to reduce deficit risk to $\alpha$
• Single premium assumed fixed (identical price $\pi_0$), benefits reduced from $FV_0$ to $FV_0^\alpha$ to incorporate safety loading
  - Difference accumulated over contract term in order to reduce deficit risk

• Large potential for surplus reserves; increased by safety loading
  - Pro-rata surplus share for policyholders
    e.g. $X=75\%$ (in Germany since 2008)
Measure of “advantageousness”: actuarial present value of differences of benefits from both products, subject to actual mortality:

\[ ADV_{MIA}^\alpha = \sum_{k=0}^{T-1} \left\{ (FV_k - FV_0^\alpha) \cdot k\tilde{p}_x \cdot v^k \right\} \]

\[-X \cdot \max\{0; V_{T,\alpha}^{*,\text{conv}}\} \cdot T-1\tilde{p}_x \cdot v^{T-1} \]

Consider empirical distribution/coefficients of \( ADV_{MIA}^\alpha \)
Results – $i=3\%$, $l_{60}=100,000$, $\alpha=0.010$

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<tbody>
<tr>
<td>$FV_0$</td>
<td>5,631.67</td>
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<td>$FV_0^\alpha$</td>
<td>5,393.50</td>
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<tr>
<td>$E[ADV^\alpha_{MIA}]$</td>
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### Results – $i=3\%$, $l_{60}=100,000$

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<td>5,330.00</td>
<td>5,373.50</td>
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<td><strong>$E[ADV^\alpha_{MIA}]$</strong></td>
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<td><strong>Var $[ADV^\alpha_{MIA}]$</strong></td>
<td>220,915.5</td>
<td>229,915.0</td>
<td>238,233.3</td>
</tr>
<tr>
<td>**$E[ADV^\alpha_{MIA}</td>
<td>ADV^\alpha_{MIA} &lt; 0]$**</td>
<td>$-321.29$</td>
<td>$-492.31$</td>
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Results – $i=5\%$, $l_60=100,000$

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<td>$FV_0$</td>
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<td>6,975.56</td>
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<tr>
<td>$FV_0^\alpha$</td>
<td>6,679.05</td>
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<td>1,075.60</td>
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<td>842.33</td>
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<td>$Var\ [ADV^\alpha_{MIA}]$</td>
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<td>−448.61</td>
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### Results – \( i=3\% \), \( l_{60}=100,000 \) vs. 1,000

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<td>( FV_0^\alpha )</td>
<td>5,285.50</td>
<td>5,334.00</td>
<td>5,358.50</td>
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<tr>
<td>( E[ADV_\alpha^{\alpha\text{MIA}}] )</td>
<td>1,552.15</td>
<td>1,335.36</td>
<td>1,224.22</td>
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<td>( P[ADV_\alpha^{\alpha\text{MIA}} &lt; 0] )</td>
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<tr>
<td>( \text{Var} [ADV_\alpha^{\alpha\text{MIA}}] )</td>
<td>274,235.1</td>
<td>285,507.6</td>
<td>297,112.2</td>
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<td>( E[ADV_\alpha^{\alpha\text{MIA}}</td>
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\( i=3\% \), \( l_{60}=100,000 \) vs. \( l_{60}=1,000 \)
Conclusion

• Longevity risk creates highly correlated long-term contractual obligations for insurance companies.

• If longevity risk is considered a severe threat to insurability, alternative product design and risk (re)transfer to policyholders should be considered.

• MIA transfer a significant amount of risk to policyholders, but in return ensure insurability and offer substantial upside potential.
  - Mostly greater annuity payments, expected advantages strictly positive.
  - The more expensive the benchmark, the more advantageous the MIA.
  - The lower the interest rate, the stronger the MIA advantage.
  - A smaller insured portfolio increases the safety loading required by the benchmark product.
Conclusion: Further Research

- Refined actuarial modeling:
  - Stochastic investment returns from diversified portfolio
  - Model uncertainty: insurer does not know “true nature”
  - Benefit only adjustments beyond certain thresholds
  - Adjustments to mortality index (→ transparency vs. basis risk)

- Policyholders’ risk aversion: (transferred) risk vs. (higher) benefits
  → more accurate analysis of risk allocation effects

- More explicit modeling of defaults
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