Sensitivity Analysis and Worst-Case Analysis – Making use of netting effects when designing insurance contracts

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IAA LIFE Colloquium 2009 in Munich, Germany

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present value of future benefits and premiums depends on ...

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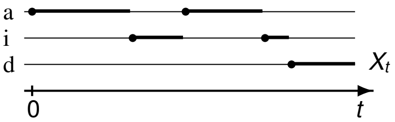

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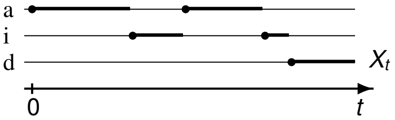

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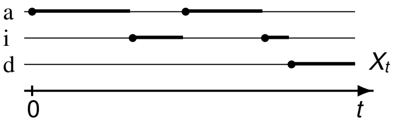

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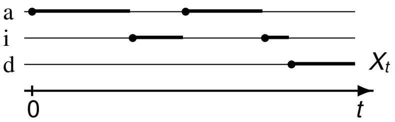

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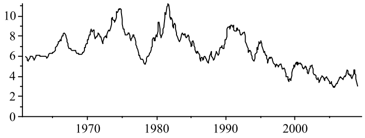
(b) investment return / interest rate $\varphi(t)$



→ financial risk (non-diversifiable)

(c) ...

Getting rid of the non-diversifiable risks?

(i) benefits guaranteed

Policyholder —————— *Insurer* —————— *Third Party*

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- enough capacity for risk transfer?

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(iv) **reality** (in Germany): compositions of (i) – (iii)

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e.g. survival benefits vs. death benefits

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Question B: which combinations give strong netting effects ?

TOOL 1:

Sensitivity Analysis

Effect of changes of the valuation basis on premiums and reserves

qualitative results:

Lidstone (1905), Norberg (1985), Hoem (1988), Ramlau-Hansen (1988), Linnemann (1993), ...

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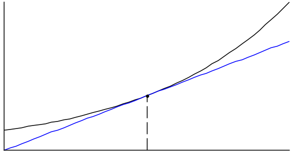

$$f(x + \Delta x) \simeq f(x) + \langle \nabla_x f, \Delta x \rangle$$

$\curvearrowright$ interpret $\nabla_x f = f'(x)$ as sensitivity of f at x

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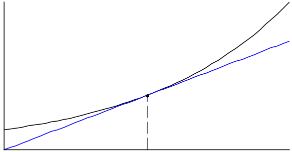

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- general approach for all valuation basis parameters and with continuous time ?

General sensitivity analysis (C., 2008)

LET

$$V_{t,a}(\varphi, \mu_{ad}, \mu_{ai}, ...) \;:=\; \text{prospective reserve at time '}t\text{' in state '}a\text{'}$$

with a valuation basis $(\varphi, \mu_{ad}, \mu_{ai}, ...)$ whose entries are either

(a) vectors of yearly interest rates, mortality probabilities, etc.

(b) or intensity functions for interest, mortality, etc.

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THEN there exists a generalized first-order Taylor expansion

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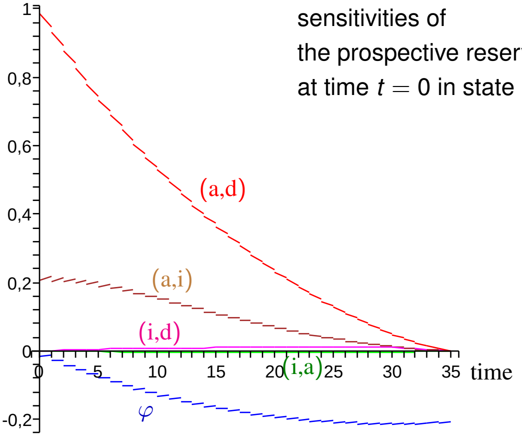
with generalized gradients / sensitivities

$$\nabla_{\varphi} V_{t,a}(s) = -\mathbf{1}_{s>t} \cdot v(t,s) \cdot \sum_k P(X_s = k | X_t = a) \cdot V_{s-,k}$$

$$\nabla_{\mu_{ad}} V_{t,a}(s) = \mathbf{1}_{s>t} \cdot v(t,s) \cdot P(X_{s-} = a | X_t = a) \cdot S@R_{ad}(s)$$

Example

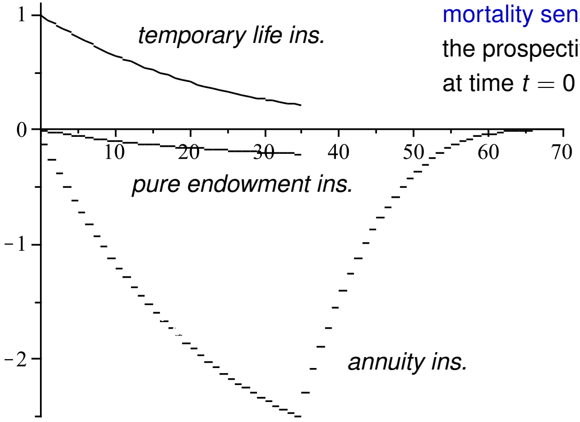
endowment insurance with disability waiver



sensitivities of
the prospective reserve
at time $t = 0$ in state $a =$'active'

Example: mortality sensitivities

policyholder at time $t = 0$ is a 30 year old male



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combinations of insurance contracts:
sensitivity of $\left(\sum_i \alpha_i \cdot \textit{policy}_i \right) = \sum_i \alpha_i \cdot \left(\text{sensitivity of } \textit{policy}_i \right)$

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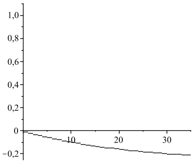

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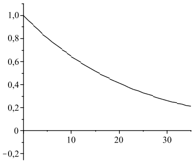

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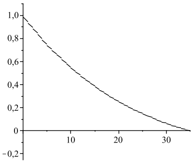

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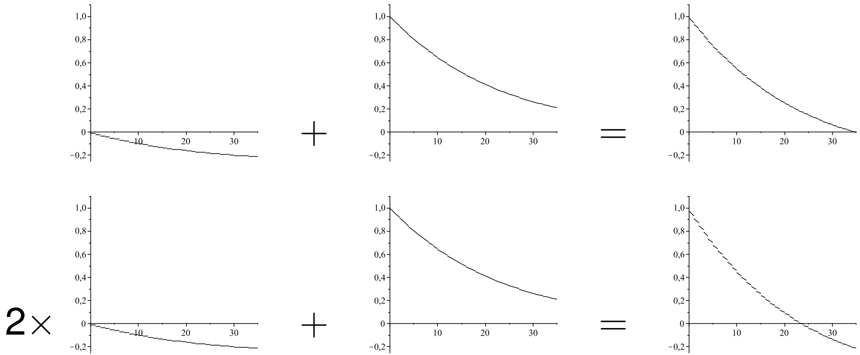

Conclusion for Tool 1: Sensitivity Analysis

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↷ how to compare policies (with respect to their systematic mortality risk) ?

Comparing policies

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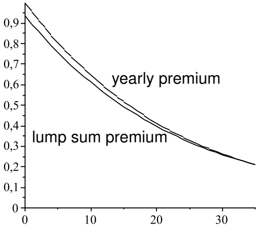

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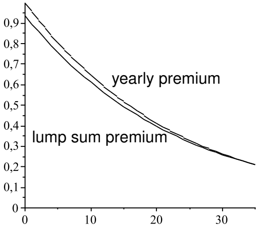

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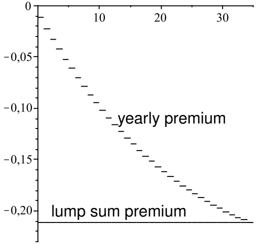

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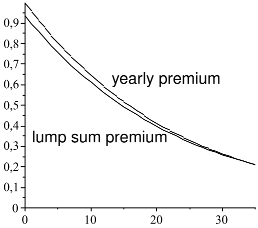

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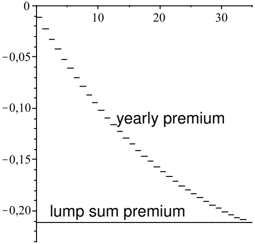

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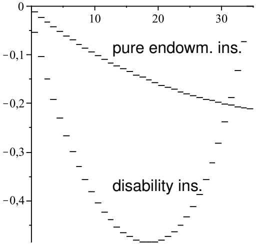

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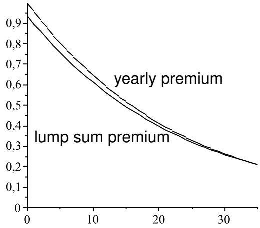

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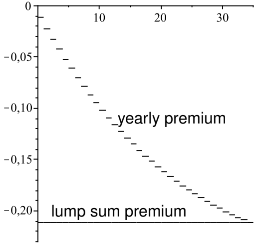

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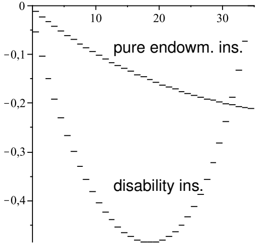

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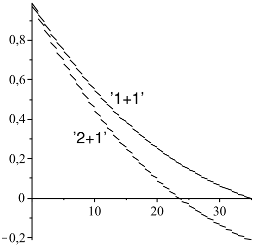

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TOOL 2:

Worst-Case Analysis

Comparing / Measuring policies

IDEA (risk measure for systematic mortality risk):
use the SCR according to the standard formula in Solvency II

$$\textit{systematic mortality risk} = \textit{Life}_{long} \;\&\; \textit{Life}_{mort}$$

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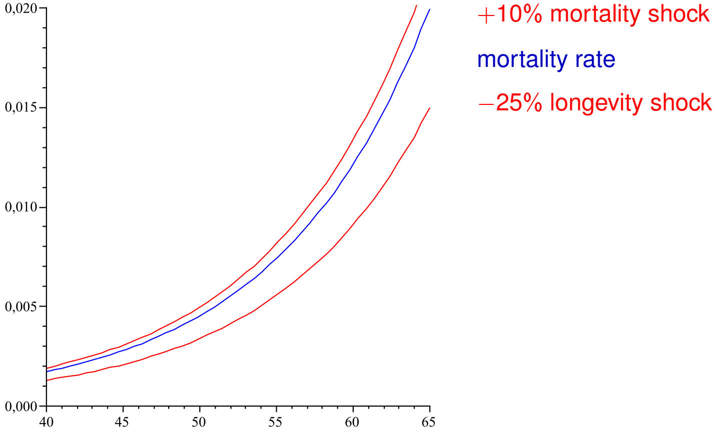
BUT risk approximation by the standard formula is too rough here !

Calculation of $Life_{mort}$ and $Life_{long}$

ΔNAV = changes in the net value of assets and liabilities due to ...



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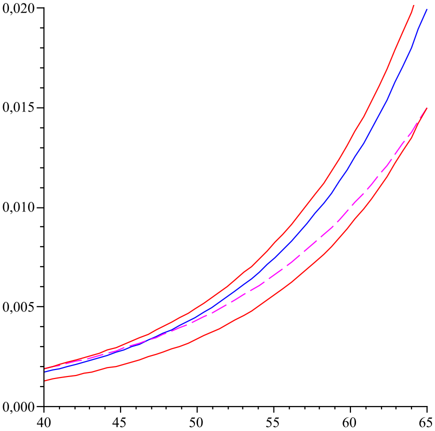

$+10\%$ mortality shock

mortality rate

-25% longevity shock

$\pm$ mixed shock

- Do we really study the crucial scenarios?

MARKT/2505/08, TS.XI.B.3 & TS.XI.C.3

For those contracts that provide benefits both in case of death and survival, one of the following two options should be chosen [...]:

1. *Contracts* [...] *should not be unbundled.* [...] *the mortality scenario should be applied fully allowing for the netting effect provided by the 'natural' hedge between the death benefits component and the survival benefits component.* [...]

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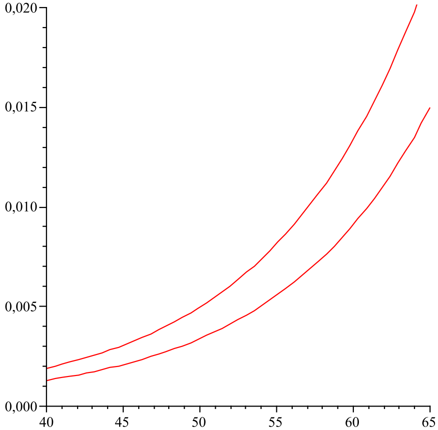
1. *Contracts* [...] *should not be unbundled.* [...] *the mortality scenario should be applied fully allowing for the netting effect provided by the 'natural' hedge between the death benefits component and the survival benefits component.* [...] → not the crucial scenarios
2. *All contracts are unbundled into 2 separate components: one contingent on the death and other contingent on the survival of the insured person(s).* [...] → no netting effect

Finding the crucial scenario(s)



+10% shock UPPER BOUND

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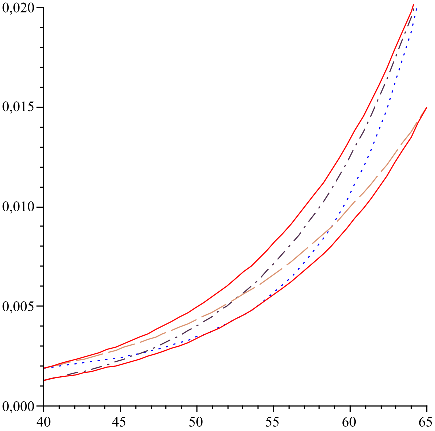

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worst/best scenario ?

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- Which scenarios lead to the highest ΔNAV ?

Optimization problem

If we focus on the liabilities only then $\Delta NAV = \Delta V_{0,a}$.

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Problem Find scenario $\overline{\mu}_{ad}$ with

$$V_{0,a}(\overline{\mu}_{ad}) = \max \left\{ V_{0,a}(\mu_{ad}) \middle| LowBound \leq \mu_{ad} \leq UppBound \right\}$$

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first-order Taylor approximation

$$V_{0,a}(\overline{\mu}_{ad} + \Delta\mu_{ad}) = V_{0,a}(\overline{\mu}_{ad}) + \langle \nabla_{\overline{\mu}_{ad}} V_{0,a} \, , \, \Delta\mu_{ad} \rangle + Remainder$$

where $\nabla_{\mu_{ad}} V_{0,a}(s) = \mathbf{1}_{s>0} \cdot v(0,s) \cdot P(X_{s-} = a \,|\, X_0 = a) \cdot S@R_{ad}(s)$

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Conclusion 1 For all s

$$\operatorname{sign}\Big(\nabla_{\overline{\mu}_{ad}} V_{0,a}(s)\Big) = \operatorname{sign}\Big(\overline{S@R}_{ad}(s)\Big)$$

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$$V_{0,a}(\overline{\mu}_{ad}) = \max\left\{ V_{0,a}(\mu_{ad}) \middle| LowBound \le \mu_{ad} \le UppBound \right\}$$

first-order Taylor approximation

$$V_{0,a}(\overline{\mu}_{ad} + \Delta\mu_{ad}) = V_{0,a}(\overline{\mu}_{ad}) + \langle \nabla_{\overline{\mu}_{ad}} V_{0,a}\,,\, \Delta\mu_{ad} \rangle + Remainder$$

where $\nabla_{\mu_{ad}} V_{0,a}(s) = \mathbf{1}_{s>0} \cdot v(0,s) \cdot P(X_{s-} = a \,|\, X_0 = a) \cdot S@R_{ad}(s)$

Conclusion 1 For all s

$$\operatorname{sign}\Big(\nabla_{\overline{\mu}_{ad}} V_{0,a}(s)\Big) = \operatorname{sign}\Big(\overline{S@R}_{ad}(s)\Big)$$

Conclusion 2 Because of the maximality property of $\overline{\mu}_{ad}$:

$\langle \nabla_{\overline{\mu}_{ad}} V_{0,a}\,,\, \Delta\mu_{ad} \rangle \le 0$ for all $LowBound \le \overline{\mu}_{ad} + \Delta\mu_{ad} \le UppBound$

Solution of the optimization problem

Hence

$$\overline{\mu}_{ad} = \begin{cases} \mathit{UppBound}_{ad} & : \ \operatorname{sign} \overline{S@R}_{ad} > 0 \\ \mathit{LowBound}_{ad} & : \ \operatorname{sign} \overline{S@R}_{ad} < 0 \end{cases}$$

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THEOREM (C., 2008)

The Worst-Case rekursion/differential/integral equation has a unique solution $\overline{V}_{t,a}$ that is maximal.

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Conclusion
solution $\overline{V}_{t,a} \quad \Longrightarrow \quad \overline{S@R}_{ad} \quad \Longrightarrow \quad \overline{\mu}_{ad}$

Example 1

Task Design a combination of

$$1\times \textit{pure endowment ins.} \;+\; \beta\times \textit{temporary life ins.}$$

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$1\times$*pure endowment ins.* $+\ \ \beta\times$ *temporary life ins.*

Question Which ratio

death benefit : survival benefit $= \beta$: 1

leads to the lowest systematic mortality risk ?

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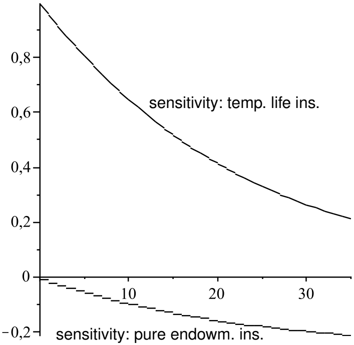

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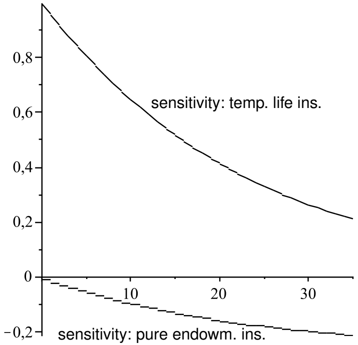

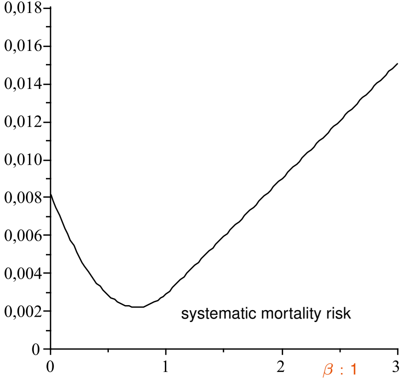

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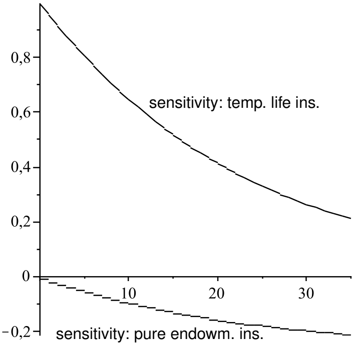

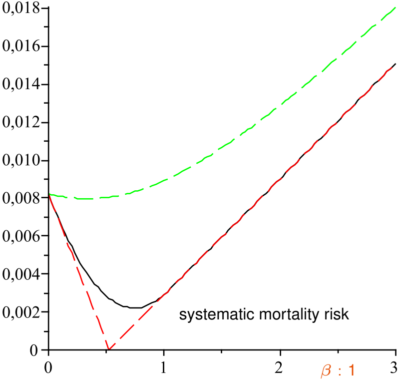

Example 2

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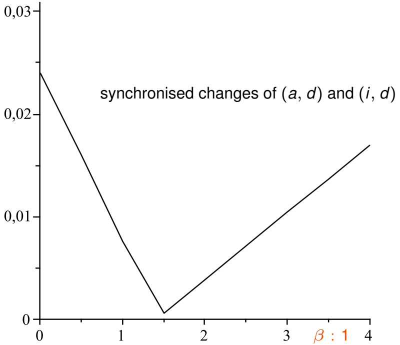

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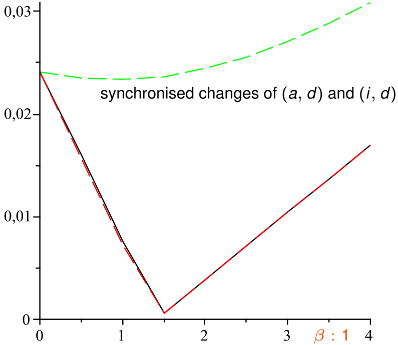

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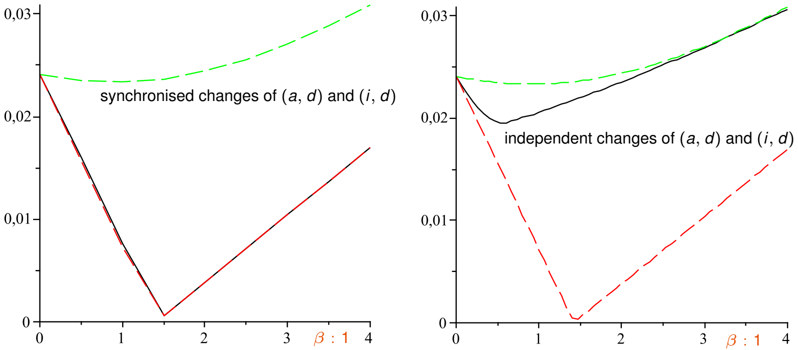

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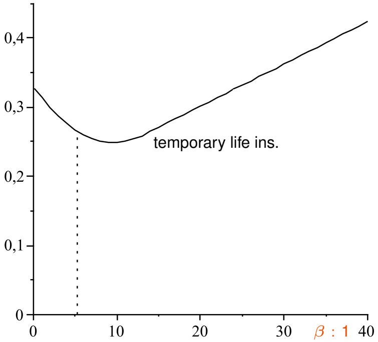

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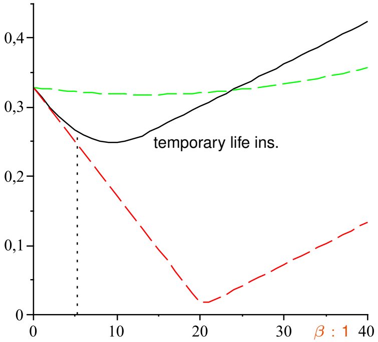

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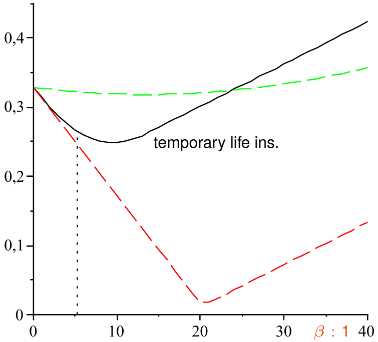

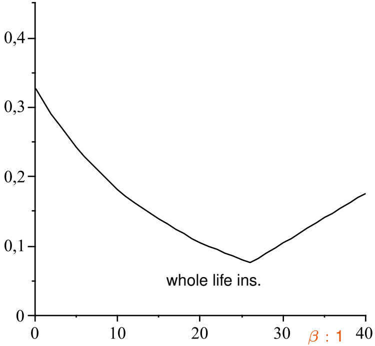

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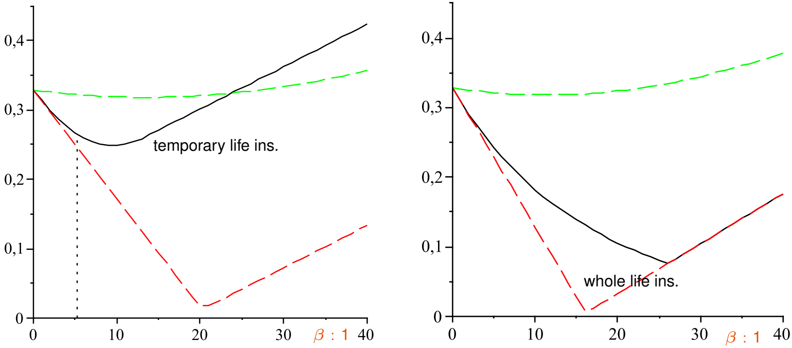

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