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Longevity: A “simple” stochastic modelling of mortality

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Agenda

Context

The model

- Stochastic process
- Parameter estimation
- Validation
- Determination of the volatility

Application

- Sample paths
- Bias on cash flows

Selected results

Conclusion

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Context

Determining Risk Based Capital

Pricing reinsurance solutions with non-linear structure

- Proportional with limits, Stop Loss

→ Require stochastic modelling due to non-deterministic nature

Most stochastic models impose an underlying mortality structure, e.g. Lee-Carter or the Cairns, Blake and Dowd family.

→ Need for a stochastic model being:

- Applicable to **any** specific future mortality assumption
- As **simple** as possible to understand and implement

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The Model

Describes how mortality rates behave around any projected expected values

Focus on the systemic risk

→ Multiplicative model

$$\hat{q}_{x,t} = q_{x,t} \times C_t$$

Where for age *x* at time *t:*

- $q_{x,t}$ expected mortality
- $\hat{q}_{x,t}$ observed mortality
- C_t stochastic process

The Model

$$\hat{q}_{x,t} = q_{x,t} \times C_t + \varepsilon_{x,t}$$

Restricted to [0,1]

Stochastic process:

$$\begin{cases} C_t = \exp(X_t) \times C_{t-1} \quad \forall t > 0 \\ C_0 = 1 \\ (X_i)_{i \in N} \sim \mathrm{N}(\mu, \sigma) \ \text{ iid} \\ \mathrm{E}[\exp(X_t)] = 1 \\ (\varepsilon_{x,i})_{i \in N} \ \text{ iid}, \quad \mathrm{E}[\varepsilon_{x,i}] = 0 \ \forall i \\ (C, \varepsilon) \quad \text{independent} \end{cases}$$

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The Model

"E[exp (X_t)] = 1"

$$\mathrm{E}\left[\exp\left(X_t\right)\right]=1 \quad \Rightarrow \quad \mathrm{E}\left[C_t\right]=1 \quad \Rightarrow \quad \mathrm{E}\left[\hat{q}_{x,t}\right]=q_{x,t} \quad \text{as} \quad \mathrm{E}[\varepsilon_{x,i}]=0$$

The results of the C_t process are centred around the projected expected values.

"$\varepsilon_{x,i}$"

- "Noise" component
- Corresponds to the diversifiable risk

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The Model

"E[exp (X_t)] = 1"

As X_t follows a Normal distribution by assumption, this is equivalent to:

$$\mu + \frac{\sigma^2}{2} = 0$$

Indeed, $$\mathrm{E}[\exp(X_t)] = \exp\left(\mu + \frac{\sigma^2}{2}\right) = 1$$

→ The model calibration depends only on one parameter, i.e. the volatility σ.

Parameter estimation

The model can be calibrated using:

$$X_t = \ln\left(\frac{C_t}{C_{t-1}}\right) \qquad \textit{where} \quad C_t = \frac{\hat{q}_{x,t}}{q_{x,t}}$$

Where $\hat{q}_{x,t}$ “**real**” observed mortality

$q_{x,t}$ “**expected**” mortality

→ Expected mortality could be **derived** from observed mortality.

Parameter estimation

From a methodological point of view we should

- Split the observation period into 2 equal intervals
- Use the first interval to forecast the expected mortality
- Determine the volatility during the second interval based on observed and expected mortality
- Use more years per interval than you aim to project

→ Impossible in practice…

But "smoothing" is possible

Parameter estimation

"Smoothing" methods

- M1: Lee-Carter
- M3: Currie "Age-Period-Cohort"
- M4: PB-Splines
- M7: Cairns-Blake-Dowd "Quadratic"

Example: England and Wales population

- Source: Human Mortality Database
- Age range 60-89, calendar years 1955-2005

→ **M1 "LC"** does not capture the „cohort effect" observed in the data

→ **M4 "PB-Splines"** results depend heavily on chosen parameters

→ **M3 "Currie APC"** and **M7 "CBD/Q"** have shown reasonably robust back-testing results

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Source: A quantitive comparison of stochastic mortality models using data from England & Wales and the United States, Pensions Institute DP PI-0701

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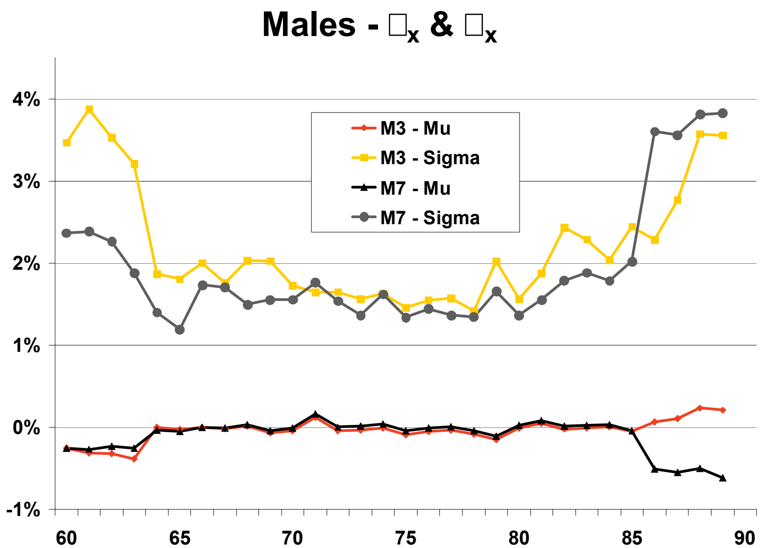
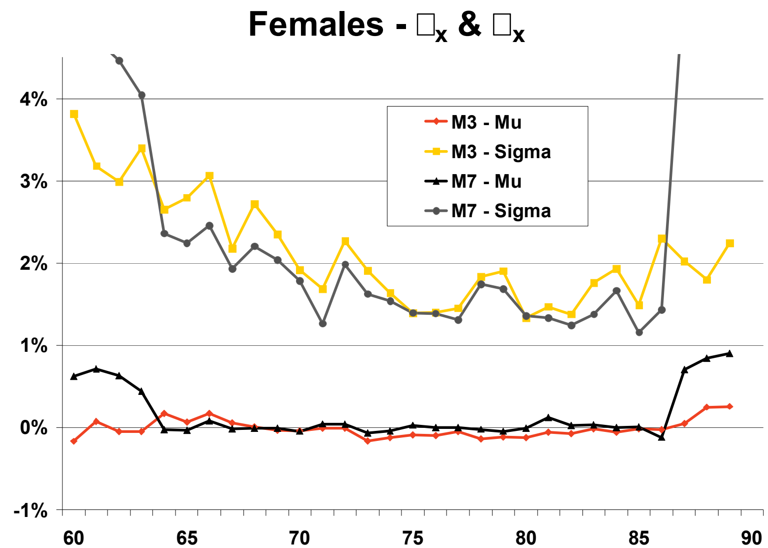
Validation

"Parameters μ and σ are independent of age"





- Reasonable for most ages
- Some effects at the extremes, partially due to smoothing

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Validation

"The X_t follow a Normal distribution"

We have used the Jarque-Bera test:

$$JB = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4}\right)$$

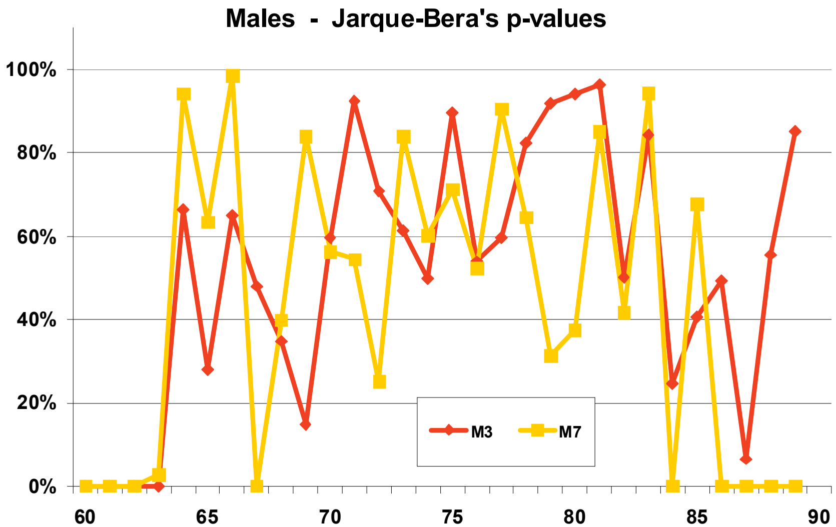
Where ***S*** is the skewness, ***K*** the kurtosis of the tested distribution and ***n*** the number of observations

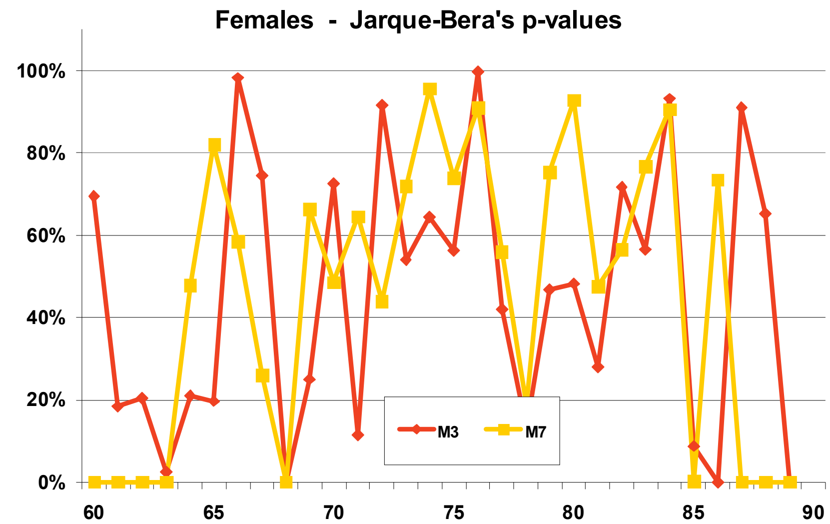
→ If the tested distribution is Normal, then

$$JB \rightarrow \chi^2(2)$$

Validation

“The X_t follow a Normal distribution”





Apart from the effects at the extremes, the p-values are ≥ 5% almost everywhere
→ We can accept the “Normal” hypothesis

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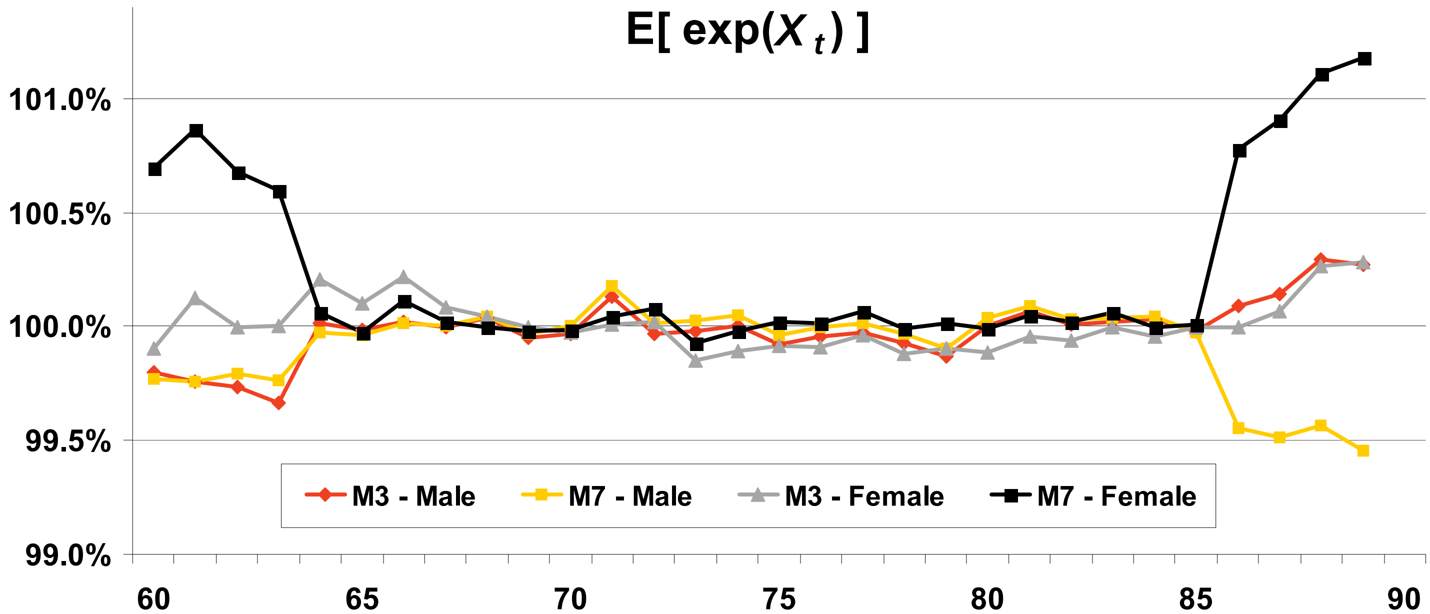
Validation

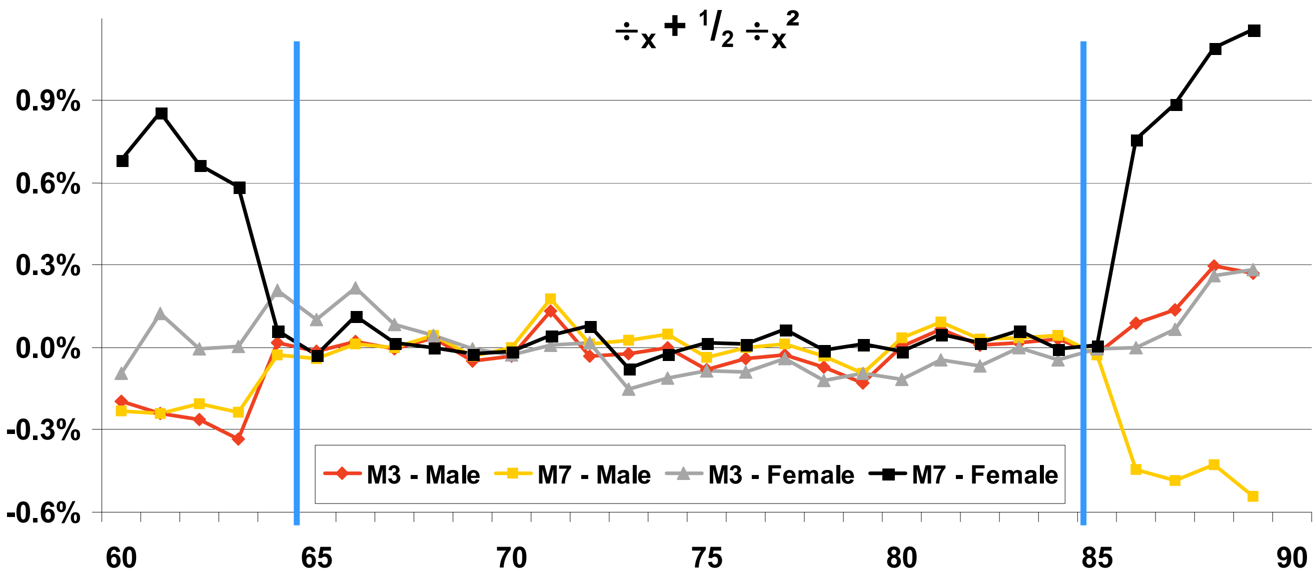
"E[exp (X_t)] = 1, i.e. around the expected mortality"



To avoid effects at the extremes, we consider only the age range 64 to 84

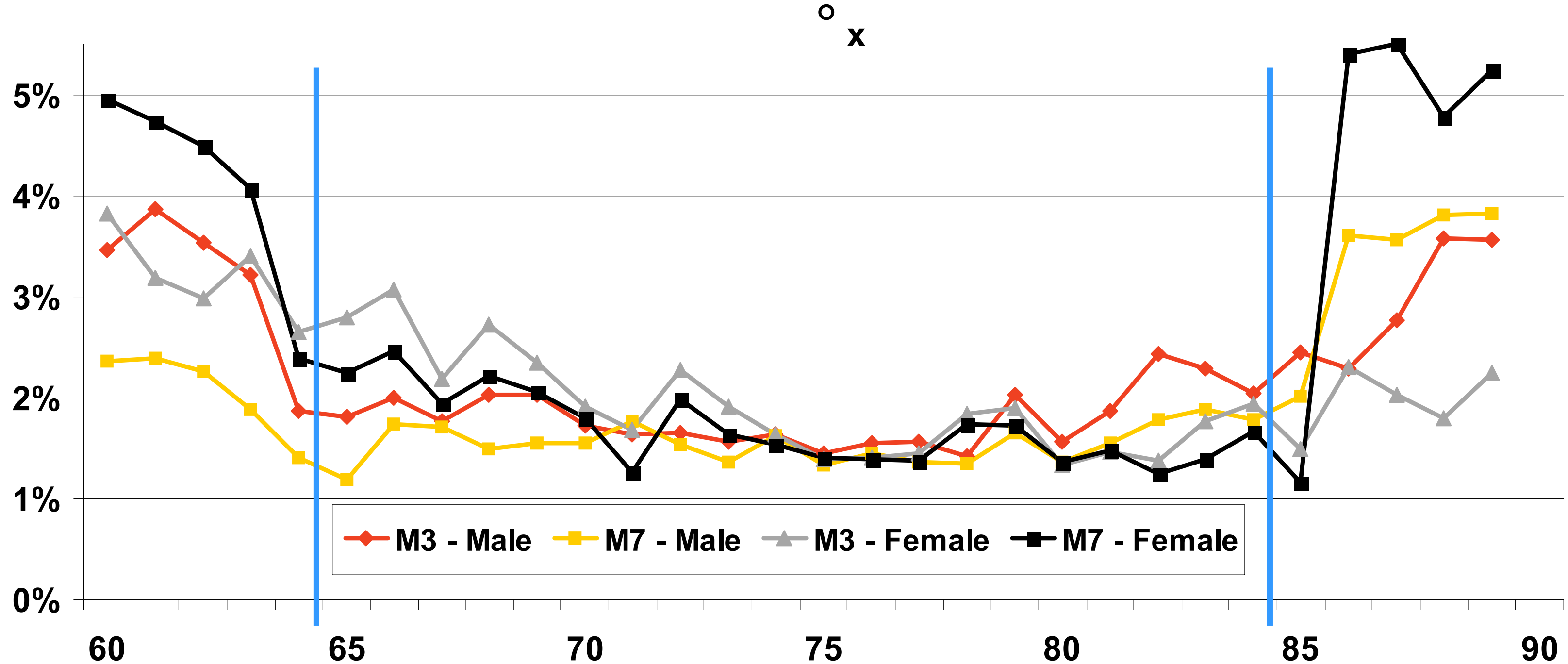
Validation

"E[exp (X_t)] = 1, i.e. around the expected mortality"



Almost 0 within the considered range, exceptions due to the effects at the extremes

Determination of the Volatility



- Volatility based on 1955-2005 data
- What do we use for the future?

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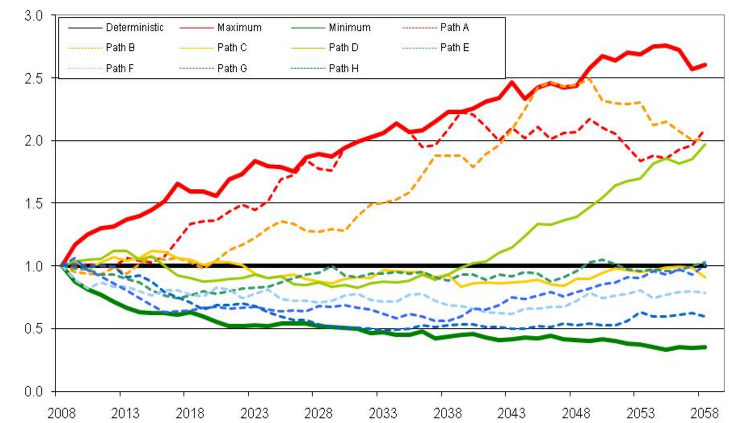
Sample paths

5'000 simulations with σ = 4%

- Some C_t paths
  - Log-normally distributed by year
- The upper and lower limits

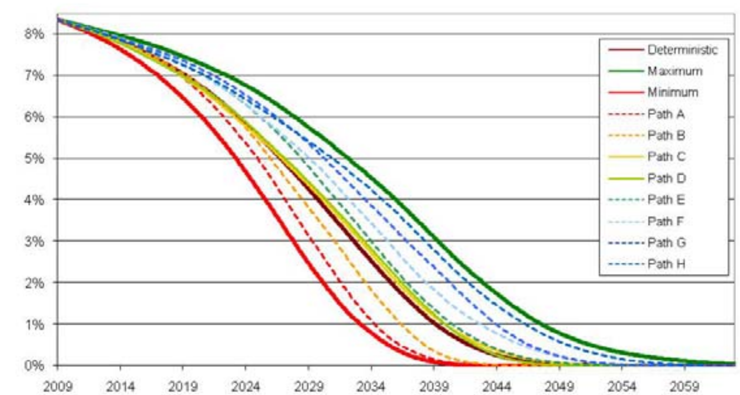


- The corresponding cash flows

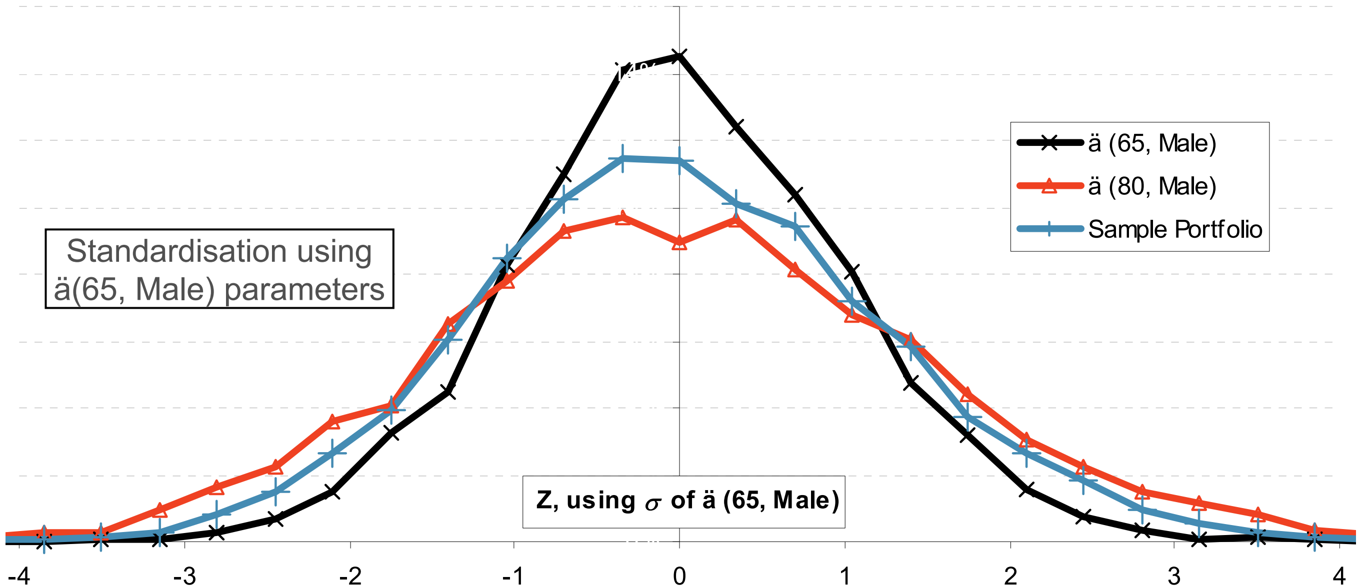
Annuity of 1 for a male aged 65, paid monthly from 2009 onwards based on:

- 100% PCMA00 in 2000
- 100% Medium Cohort from 2001 onwards

Source: CMI R-23 & WP-37



"Standardised" distributions of Present Values



- Same C_t paths and discount rate of 5%
- Depends on the portfolio composition and characteristics

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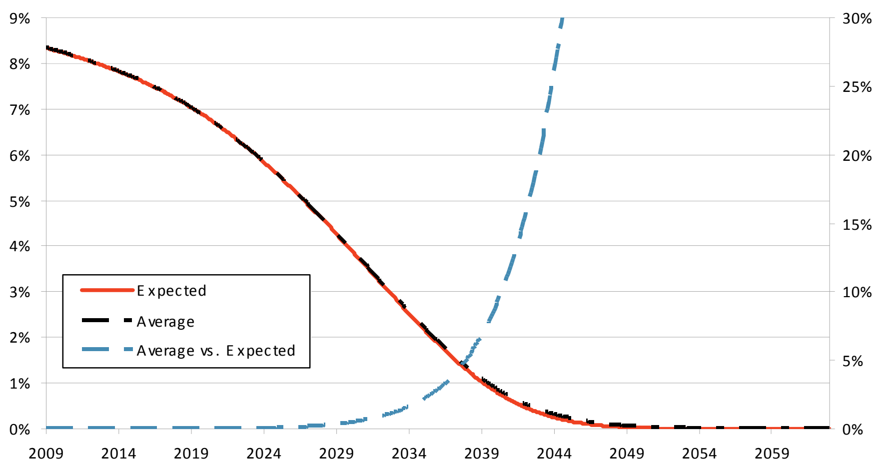
Bias on cash flows

$$E\left[\ddot{a}_{x,t}^{Stochastic}\right] \neq \ddot{a}_{x,t}^{Expected} \quad \text{as } \ddot{a}_{x,t} = \sum_{k\geq 0} (1+i)^{-k} \cdot \Pi_{0\leq l\leq k-1}\left(1-q_{x+l,t+l}\right)$$

E.g. deviation of NPVs is 0.3% at outset in the case of ä(65, Male)



Solution

The bias should be removed/ added when comparing deterministic scenarios to the stochastic distribution.

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Capital requirements

The UK regulator requests an Individual Capital Assessment (ICA) assessing capital requirements at a 99.5% confidence level for all types of risks over 1 year timeframe.

This is consistent with Solvency II QIS4's requirements.

Examples for ä(65, Male)

- Capital ~ VaR_α $Z_{99.5\%}$ = 2.70 (+8.4%) $Z_{97.5\%}$ = 1.99 (+6.2%)
- Capital ~ $TVaR_\alpha$ $Z_{99.5\%}$ = 3.10 (+9.6%) $Z_{97.5\%}$ = 2.42 (+7.5%)

Sources: MAP/GN46 V2.0 Amendment 1, BAS Recommended Practice
CEIOPS-CP-49/09

Probability of a specific scenario

A stochastic model allows you to estimate the probability of occurrence of any deterministic scenario.

Examples for ä(65, Male)

- Solvency II QIS 4 extreme scenario for longevity risk is a permanent drop of -25% on the base mortality level

  Z = 2.35 (+7.3%), i.e. the **98.9%** percentile

- Combination of a drop of -15% with additional improvements of "+1%"

  Z = 2.68 (+8.3%), i.e. the **99.5%** percentile

Source: CEIOPS-CP-49/09

Application

Pricing of reinsurance solutions for longevity

- Swap of annuity payments with limits
- Capital allocated

Enable the inclusion of shock scenarios

- Pandemics

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Advantages

- Apply to any future mortality basis
- Consistent by market or even by product
- Link with extreme scenarios
- Can include shock
- Quick implementation

Disadvantages

- Calibration
- Bias needs to be compensated for
- Computing time, as with any stochastic model

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