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Cautionary remarks about conclusions from the observation of record-life expectancy

IAA Life Colloquium 2009

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Cautionary remarks about conclusions from the observation of record-life expectancy

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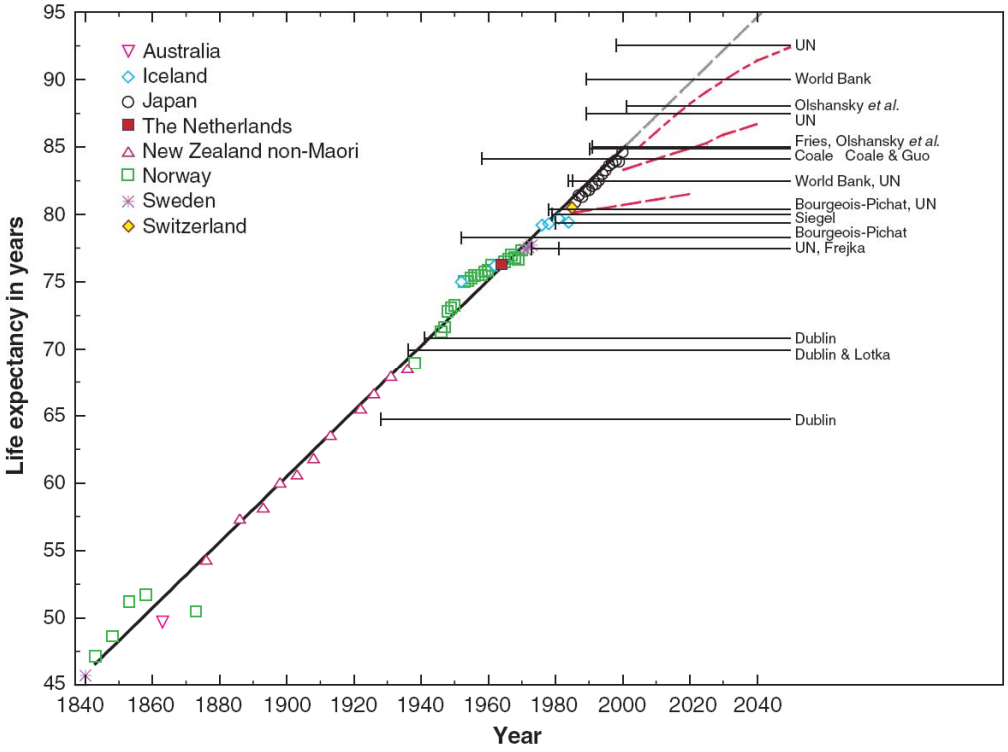
- Introduction
- Simulation results
- Record-life expectancy as forecast-tool
- Conclusions

Introduction & some definitions



Note: The LE definition of Oeppen&Vaupel is period LE at birth

Source: Oeppen&Vaupel: "Broken Limits to Life Expectancy", Science 10 May 2002:Vol. 296. no. 5570, pp. 1029 - 1031

Definitions used in the talk

- Life expectancy (**LE**) is the expected number of further years of life
- We study the time-development of **specific LEs** of nations or countries
- The **record LE** in a year is the maximal specific LE of all countries in scope
- LEs are seen as stochastic processes or **time-series**
- We study the **slopes of the regression line** of specific and record LE processes

What to expect from this presentation

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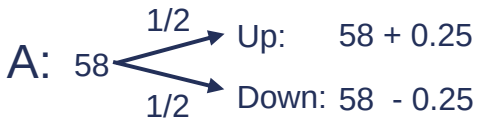
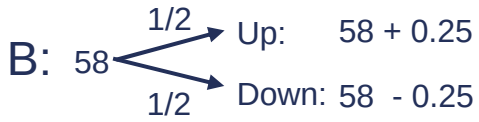
- Questions raised and (partially) answered
  - What can record LE tell us about LEs specific for a country?
  - What are drivers of record LE beyond specific LE improvements?
  - What issues need to be addressed before record LE can be used to forecast specific LEs?

- Tenor:
  - Record LE is a biased estimator of specific LEs
  - Inference on specific LEs needs to control all parameters driving the bias
  - Some of those parameters are only marginally relevant to LE development
  - To use record LE in forecasting you need a detailed model of the dependency-structure of global LE improvements

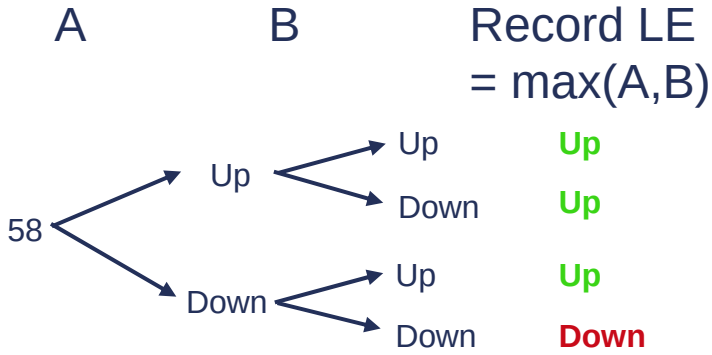
The basic issue – in a nutshell

- Country A: current LE is 58 years
  - LE has equal chance of moving up or down 3 month next year
  - Expected improvement is zero or no systematic trend
- Country B: same probabilities as A and independent





- Record LE of A and B ?
  - Probability of moving up: 3/4
  - Probability of moving down: 1/4
- **Record LE is "biased" i.e. it overstates the expected LEs of "A" and "B"**



Drivers of bias

- The simple example is already sufficient to demonstrate some key effects and drivers
- Number of countries: Increases bias
  - If three countries participate the probability of “up” is already 7/8
- Simple correlation: Decreases bias
  - Can be demonstrated in the 2-stage tree: If outcome of B does depend on outcome of A
- Initial difference of LEs: Larger initial difference lowers the bias but it is still there
  - Needs multi-step consideration but is still straightforward

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The simulation laboratory

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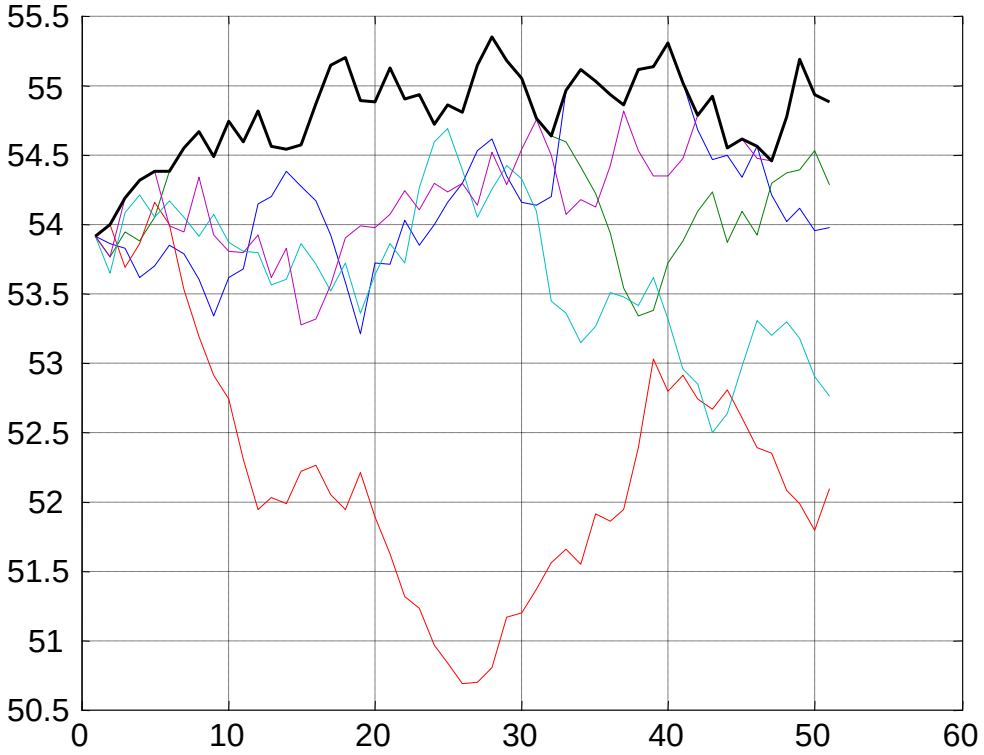
- Our goal is to demonstrate potential issues with record LE. So we strive for a simple model which is rich enough to show the effects.
- LE simulations are based on mortality rates from a Lee-Carter model[1)]
  - Force of mortality: $\mu(x,t) = \alpha_x + \beta_x \cdot \kappa_t$
  - α_x and β_x from England&Wales population data, males, 1978-2007
  - Stochastic time/series κ_t modelled as random walk with drift:

    $\kappa_t - \kappa_{t-1} = d + \varepsilon_t$ with ε_t iid, $\varepsilon_t \sim N(0,\sigma)$
- Effects can be studied by systematic variation of parameters governing the stochastic properties
  - Drift, volatility, number of time series/countries, correlation-structure
- Reasonable range of variation is fixed by comparison to historic mortality data
- For practical reasons: LEs start with age 20 and are curtailed at 90.

1) see Lee-Carter (1992) or Cairns et.al. (2007)

Simulated LEs - first example

- Generate stochastic mortality rates
- Choose a number of “countries” (i.e. Time series)
- Calculate for each time t specific LEs and their record LE

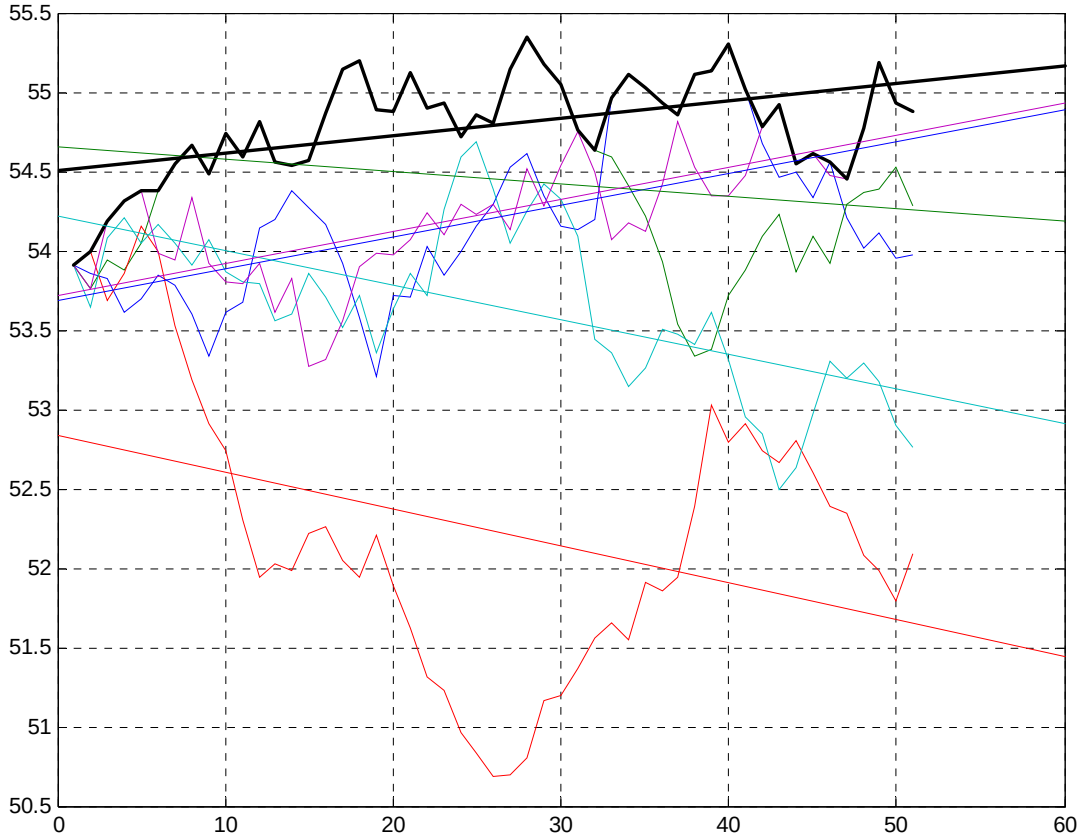


- 5 time series or “countries”
- 50 periods simulated
- Drift is zero
- Random walks are uncorrelated
- Record LE outlined in black
- Different time series make up the record LE line

nodrift_regression.m

The record LE regression line

- Regress record LE over time to find the slope of record LE



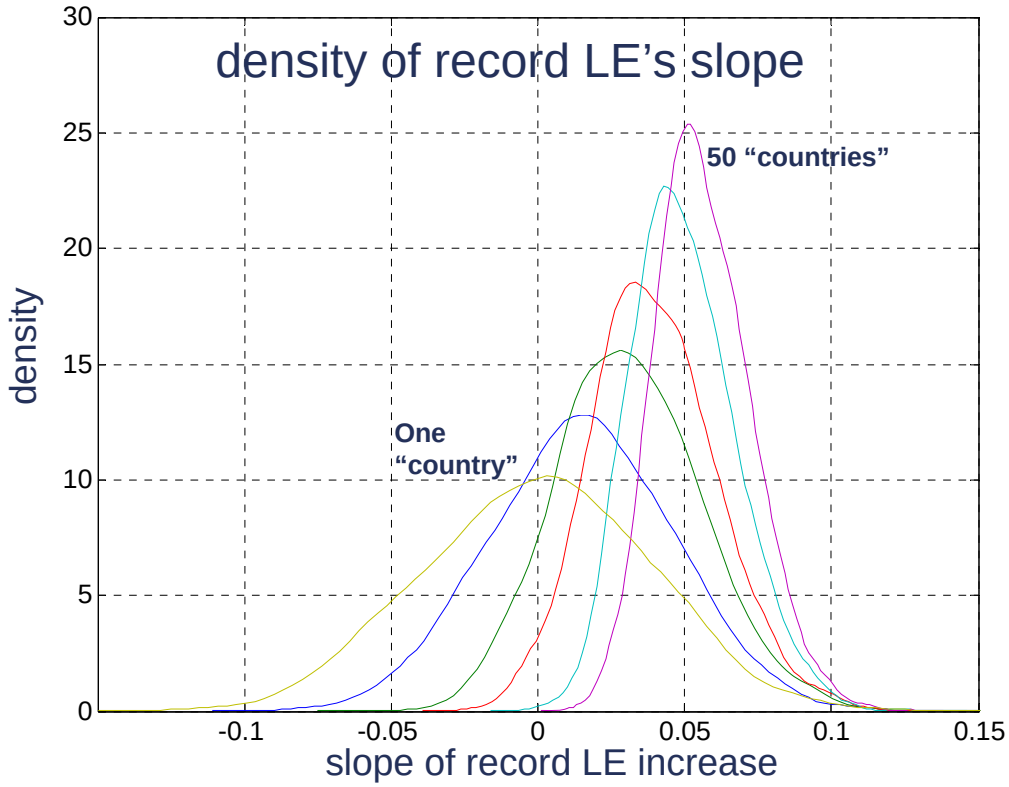
- Time series same as before
- But regression lines are included
- Sampled is one concrete outcome
- Different samples will have different regression lines

nodrift_regression.m

Improvements from nowhere

- Repeat sufficiently often to find the **distribution** of slopes of record LEs
- With sufficient countries record LE will show positive slope
- So the record LE shows improvement although all specific LEs have no drift

number of countries	Mean slope
1	-0.1%
2	1.5%
5	2.9%
10	3.8%
25	4.9%
50	5.5%



Nsim=5000
No rho, no drift, vola standard
start = +-3%
Main_sim.m

Potential amount of bias in record LE

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- More realistic scenarios varying underlying drift and volatility
- There is indeed a possibility of material bias!

kappa drift	kappa volatility (of E&W)	specific LE slope	record LE slope	over-statement due to bias
0	90%	0%	5%	NA
0	100%	0%	5%	NA
0	105%	0%	6%	NA
-0.5	90%	8%	12%	43%
-0.5	100%	8%	12%	47%
-0.5	105%	8%	12%	49%
-1	90%	15%	17%	14%
-1	100%	15%	17%	15%
-1	105%	15%	17%	16%
-1.5	90%	20%	20%	4%
-1.5	100%	20%	20%	4%
-1.5	105%	20%	20%	4%

- Comparison with data from the human mortality database[1)]
- Assumptions on drift and volatility chosen to be reasonable in comparison to 1950-2003 data
  - Historic drift between -0.5 and -1.5 (E&W: -1.1)
  - Standard deviation varies between 90% to 105% of E&W values
  - Initial dispersion of starting positions +/- 3% of average LE 2003: 57years

nsim = 5000, nperiod =50
rho = 3%, eps_init = +/-3%
bias_estimate.m

1) see www.mortality.org

How much bias in reality?

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- The analysis presented is obviously not conclusive. So we do not know the true current or historic bias in record LE
- Some obvious missing points are:
  - Young and old ages are excluded
  - Only England & Wales data was used for Lee-Carter parameters
  - Static analysis i.e. parameters are fixed in advance
  - Extremely simple dependency: Multivariate normal random walk
- What is the role of the Lee-Carter model?
  - We do not claim that Lee-Carter is a particular good model for this
  - The claim is indeed: It doesn't matter which model you use
  - Lee-Carter is just a convenient way to generate stochastic mortality rates
- Remember the nutshell example: Volatility => Bias of record LE

Further examples and conclusion

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- To decide on bias ALL parameters influencing the joint distribution need to be measured or at least their materiality estimated

- Example: Dynamic changes
  - Number of countries in scope: probably growing over time?
  - Changes in drift: catch-up to leading countries will lead to clustering, i.e. increased bias due to less differences between leaders
  - More complicated dependency, auto-regression of drift and error, changing volatility ….

- As long as there is no evidence to the contrary it is safer to assume an unknown but potentially material bias in the slope of record LE.

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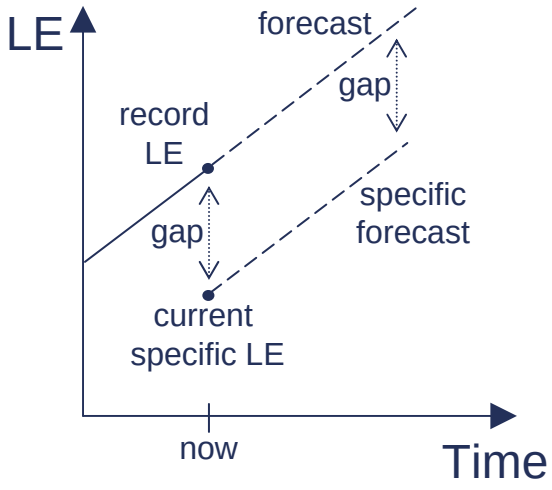
Record LE as forecasting-tool

- Record LE could be used to forecast specific LEs
- General idea
  - 1) Express any specific LE as a function of the record LE
  - 2) Forecast the record LE
  - 1) and 2) will give you immediately a forecast of your specific LE
- Example: specific LE = record LE - gap (Andreev, Vaupel 2006)



- Problem:
  - Bias in record LE is transferred to specific LE
  - Consistency is not ensured: If all countries had the same slope as record LE the record LE were different!

Are other approaches viable?

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- More complex approaches have been suggested
  - Lee (2006): Gap is not constant but decreases linearly over time
  - Torri (2008): Gap follows a stochastic process
- But any approach based on record LE is a potential victim of transfer of bias and lack of internal consistency
- Part of any proposed model should be a discussion of the consistency of assumptions between specific LE and measured/forecasted record LE
  - This most likely requires a description/analysis of the full dependency structure of all specific LEs involved
- Taking this complexity into account will limit the appeal of simplicity of the record LE approach.

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Conclusions

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- Slope of record LE is a biased estimator of slope of any specific LE.
  - This is a mathematical property of the maximum of random variables and is not particular to human longevity or its temporal development.
  - Strength of bias is influenced by a potentially wide range of parameters.
  - Some of those parameters might only be marginally related to human longevity like e.g. the number of countries in scope.

- When forecasting based on the slope of record LE,
  - Care should be taken to prevent transfer of any bias from the record LE to the specific LE to be forecasted
  - Assumptions on the relationship between the forecasted specific LEs and their ensuing record LE should be checked for consistency

- Record LE is not a simple measure and analysing/controlling all influence factors will probably reduce its appeal of simplicity

Contact details

- You are welcome to send any questions, remarks and your opinion on the matter to

guido.gruetzner@secquaero.com

- You might also want to check out my company's website (our main focus is somewhat different though)

http://www.secquaero.com

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Backup

Some sources and further reading

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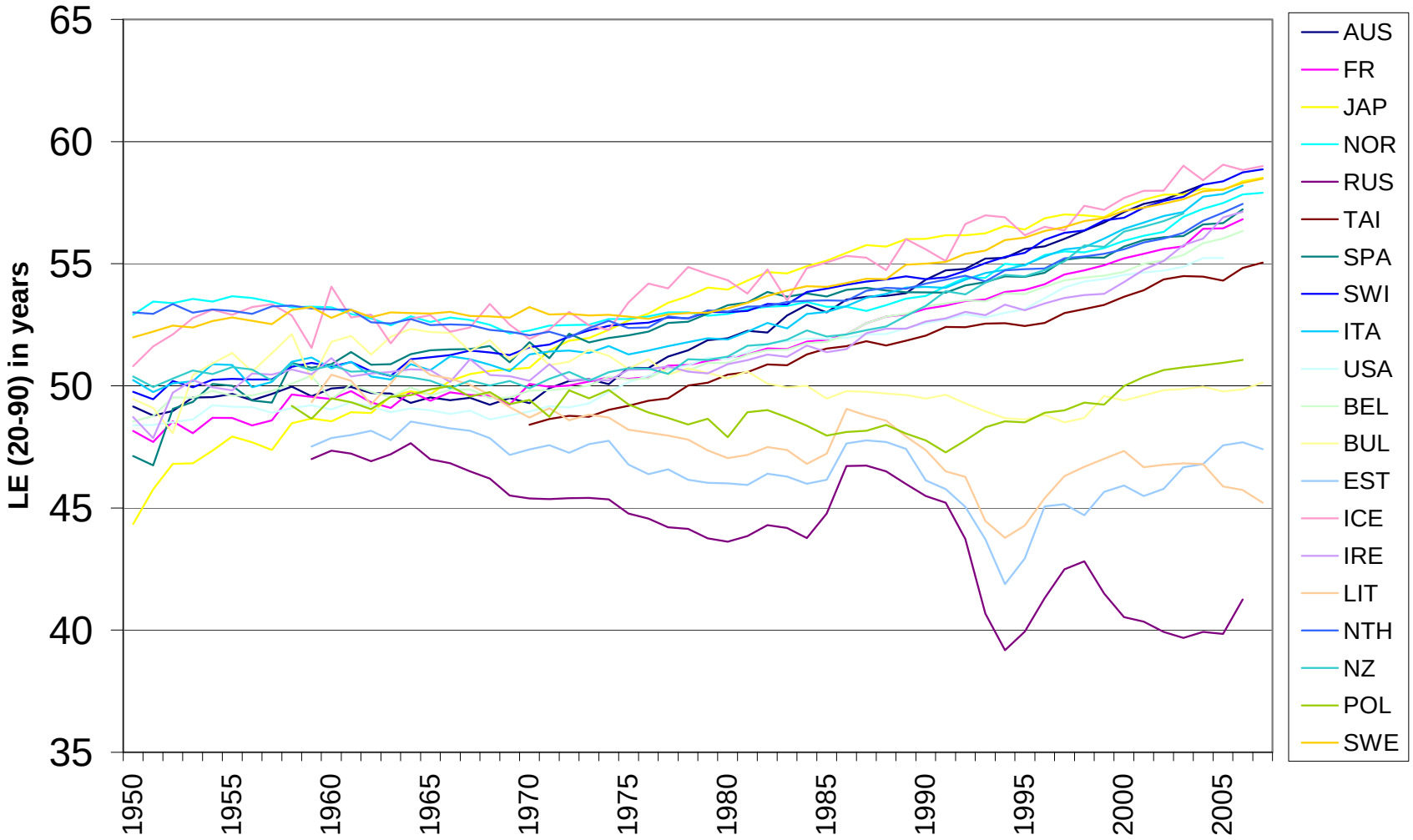
- Oeppen&Vaupel
  - “Broken Limits to Life Expectancy”, Science 10 May 2002:Vol. 296. no. 5570, pp. 1029 – 1031
- Lee, R.D., and Carter, L.R. (1992)
  - “Modeling and forecasting U.S. mortality", *Journal of the American Statistical Association,* 87: 659-675.
- Cairns, A.J.G., Blake, D., Dowd, K., Coughlan, G.D., Epstein, D., Ong, A., and Balevich, I.(2007)
  - “A quantitative comparison of stochastic mortality models using data from England & Wales and the United States", Working paper, Heriot-Watt University, and Pensions Institute Discussion Paper PI-0701.
- Andreev, K. F. and J. W. Vaupel (2006).
  - Forecasts of Cohort Mortality after Age 50. Working paper, Max Planck Institute for Demographic Research, Rostock, Germany.
- Lee, R. (2006).
  - Mortality Forecasts and Linear Life Expectancy Trends In T. Bengtsson (Ed.), *Prospectives on Mortality Forecasting. III. The Linear Rise in Life Expectancy: History and Prospects*, pp. 19-40. Stockholm:National Social Insurance Board.
- Torri, Tiziana (2008)
  - “Forecasting Life Expectancy in an International Context” Paper presented to PPA 2009 http://iussp2009.princeton.edu/download.aspx?submissionId=92339

Development of some selected LEs



Source of data: Human Mortality Database (www.mortality.org) and own calculations

Parameters

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- Parameters to vary and their implementation

Parameter	**Implementation**	
Number of countries	Number of random mortality processes/LEs simulated	
Initial dispersion of LE	Factor applied to the LE process	
Drift/Slope of LE	Parameter of κ_t distribution	
Volatility of LE	Parameter of κ_t distribution	
Correlation of LE	Parameter of κ_t distribution	

Sample data used for comparison

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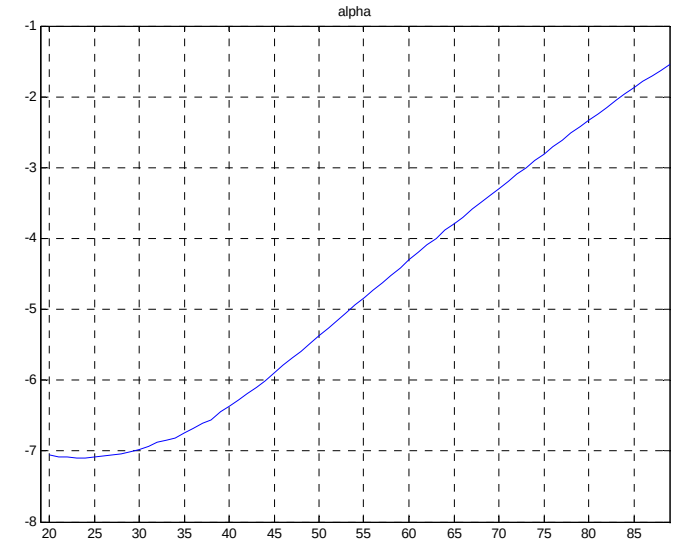
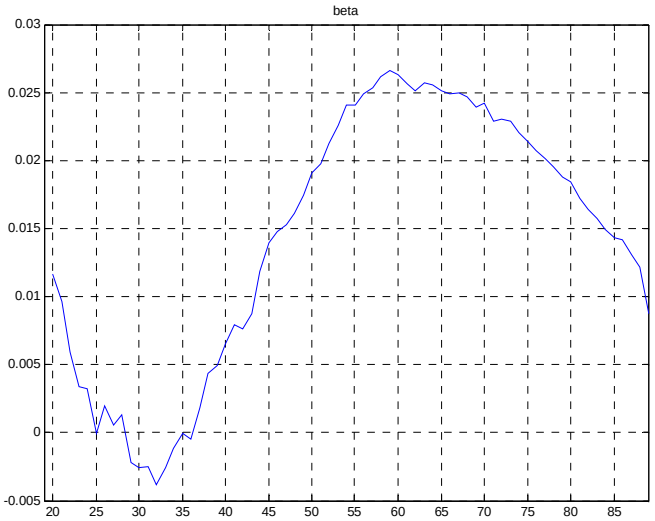
Male data	SLOPE (from 1950)	std deviation	LE 20-90 (2003)
AUS	17%	8%	57.9
FR	15%	10%	55.7
JAP	23%	10%	57.8
NOR	7%	5%	56.9
RUS	-15%	63%	39.7
TAI	17%	3%	54.5
SPA	14%	24%	56.1
SWI	15%	5%	57.7
ITA	14%	11%	57.1
USA	13%	4%	54.9
BEL	12%	7%	55.4
BUL	-4%	42%	49.9
EST	-5%	48%	46.7
ICE	12%	61%	59.0
IRE	11%	20%	55.8
LIT	-10%	42%	46.8
NTH	6%	5%	56.3
NZ	12%	9%	57.1
POL	1%	19%	50.8
SWE	10%	4%	57.6
Avg			54.2
w/o negative			56.7

Source of data:

- Mortality tables: Human Mortality Database (www.mortality.org)
- Derived values: own calculations

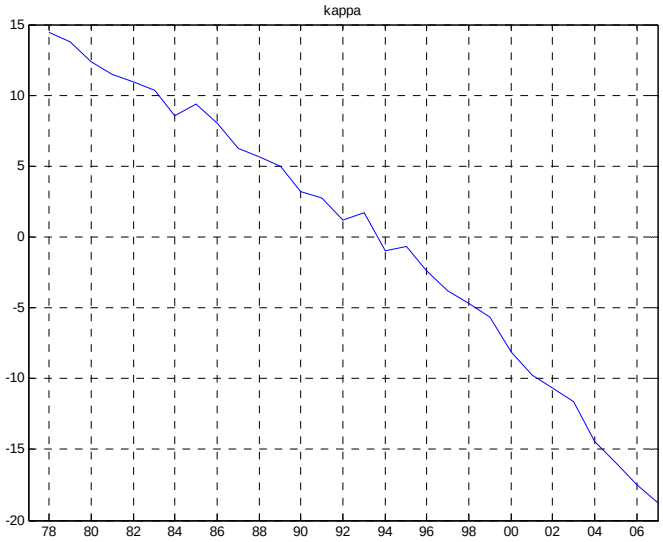
Lee Carter parameters





- Based on ONS Data
- England and Wales male population
- Calibration on years 1978 - 2007

Drift of kappa: -1.1
Standard deviation of ε: 1.4



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