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# Cash-flow based valuation of pension liabilities

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joint work with  
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## Introduction

- We present a computational framework for valuing insurance liabilities in incomplete markets where the claims can not be fully hedged
- We define the value of liabilities as the minimal capital required to cover the claims at an acceptable level of risk when the capital is prudently invested in financial instruments available in the market

## Introduction

- Such a value depends essentially on the following subjective factors
  - 1) risk factors:** the description of future development of uncertain investment returns and the insurance claims
  - 2) risk preferences:** the acceptable level of risk at which the investment returns should cover the insurance claims.
  - 3) hedging strategy:** the strategy according to which the given capital is invested in financial markets.

## Valuation of liabilities in a deterministic world

- Let  $c_t$  denote the aggregate claims associated with an existing insurance portfolio during period  $[t-1, t]$
- Let  $R_t$  denote the return of the investment portfolio during period  $[t-1, t]$
- If both of these variables are assumed *deterministic*, the valuation of the insurance portfolio is straightforward

## Valuation of liabilities in a deterministic world

- The initial capital required to cover the insurance payments  $c_t$  is obtained by solving  $V_0$  from the system of equations

$$V_t = R_t V_{t-1} - c_t \quad t = 1, \dots, T,$$

$$V_T = 0.$$

- It is easily checked that the solution is given by

$$V_0 = \sum_{t=1}^T \frac{c_t}{\prod_{s=1}^t R_s}.$$

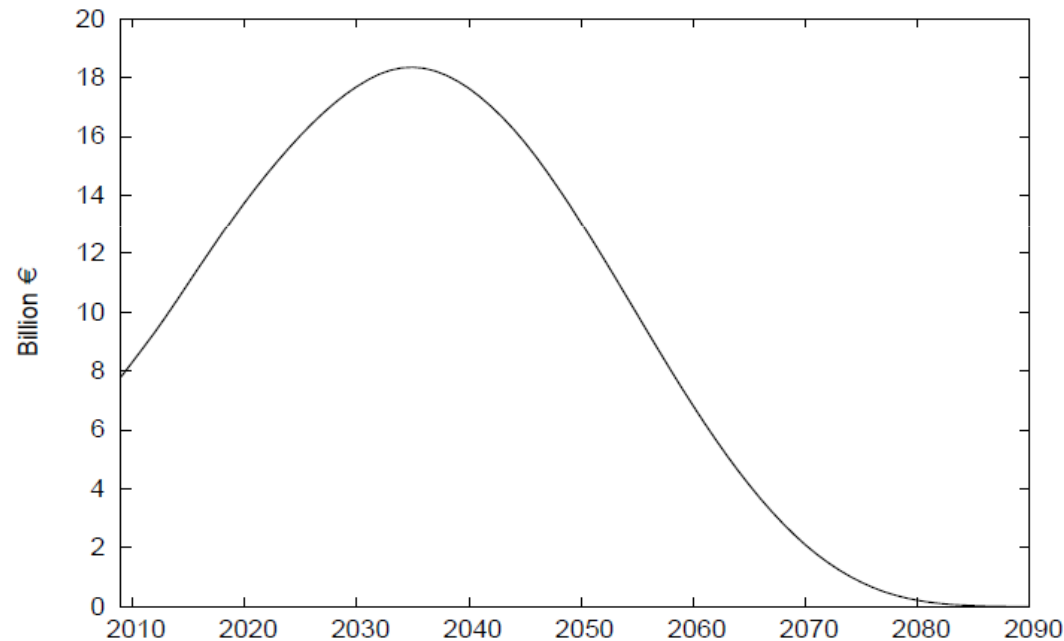
- This corresponds to the traditional actuarial present value of insurance liabilities<sup>[1]</sup>

## Case Study: Finnish private sector pension liabilities

- The liabilities consist of the current insurance portfolio of the Finnish private sector occupational pension system
- The yearly claims  $c_t$  consist of aggregate old age, disability and unemployment pension benefits that have accrued by the end of 2008 and become payable during year  $t$ .
- The liabilities are of the defined benefit type and they depend on the development of wage and consumer price indices.

## Case Study: Finnish private sector pension liabilities

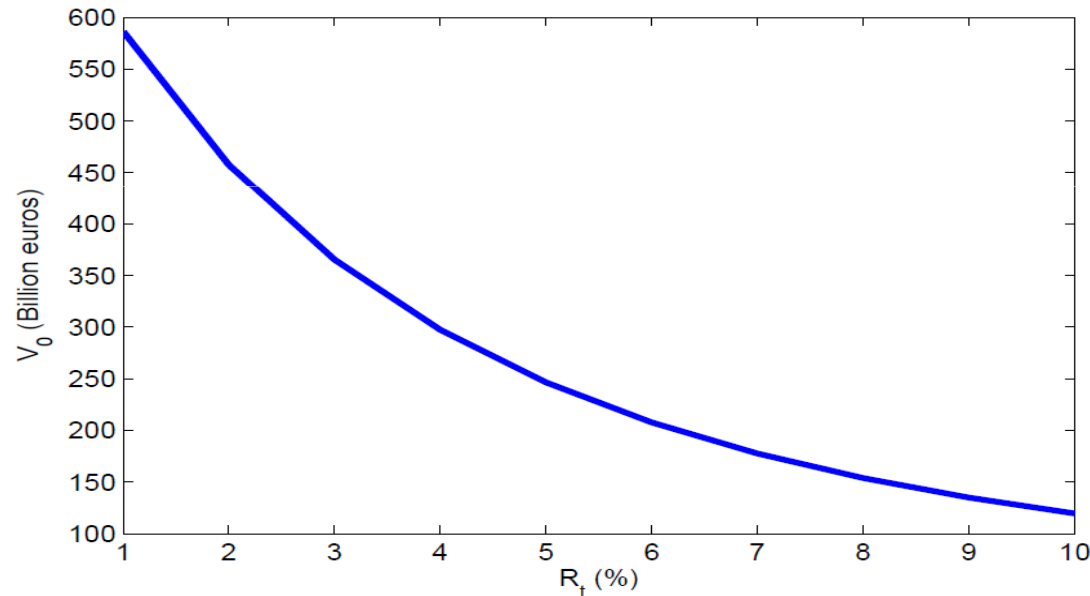
Figure 1: Evolution of the projected aggregate claim payments associated with the accrued old age, disability and unemployment pension benefits.



- The forecast is based on the assumption of constant annual wage increases of 3.8%, annual inflation of 2.0%, the current accrued pension rights and Finnish mortality tables

## Case Study: Finnish private sector pension liabilities

- Capital requirement  $V_0$  as a function of the annual investment return.



- If  $R_t = 6\%$ , the capital requirement  $V_0 = 207.7$  billion euros.
- A mere percentage point reduction in the expected annual return of 6%, increases  $V_0$  by roughly 20%, or 40 billion euros.

## Case Study: Finnish private sector pension liabilities

- The estimated capital requirement is very sensitive wrt. inflation, wage growth and especially the investment return ( $R_t$ )
- This highlights the need for a risk sensitive valuation framework that can accommodate all the relevant risk factors in the determination of adequate capital requirements.

## Valuation under uncertainty

- We extend the previous analysis to allow uncertainties in both the claims  $c_t$  and investment returns  $R_t$ .
- In an uncertain environment and in the absence of liquid markets for the insurance claims, there is always a risk that in some future scenarios the return on invested capital is insufficient to cover the liabilities.
- Thus, the valuation of insurance claims must in some way reflect the risk preferences of the insurer

## Valuation under uncertainty

- The claims and investment returns are modeled as random variables, which reflect the decision maker's views regarding the future evolution of the underlying risk factors
- The value of the insurance claims can be defined as the solution to the optimization problem

$$\begin{aligned}
 &\text{minimize} && V_0 \\
 &\text{subject to} && V_t = R_t V_{t-1} - c_t \quad t = 1, \dots, T, \quad (1) \\
 &&& \rho(V_T) \leq 0,
 \end{aligned}$$

- where  $\rho$  is a *risk measure* that quantifies the decision maker's preferences over random terminal wealth distributions

## Valuation under uncertainty

The most significant factors affecting  $V_0$  are

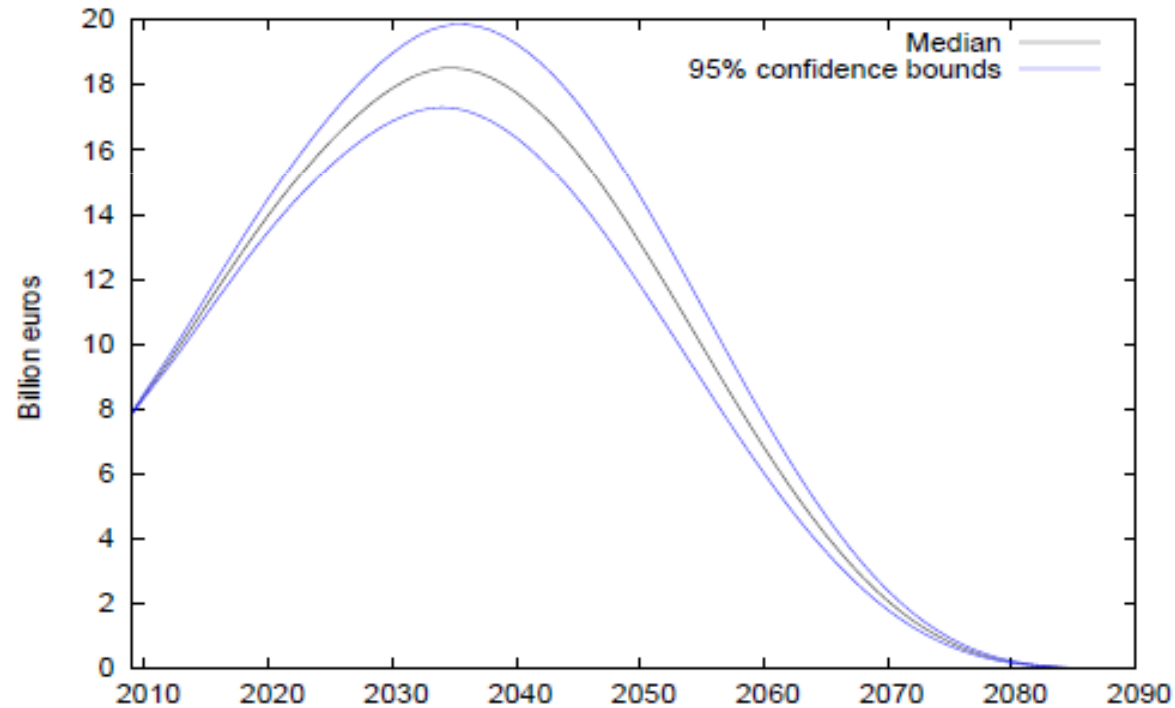
- 1) **risk factors:** Decision maker's views regarding the uncertain future development of the claims  $(c_t)_{t=0}^T$  and the investment returns  $(R_t)_{t=0}^T$
- 2) **risk preferences:** The risk measure that defines the set of acceptable distributions of the random terminal wealth.

## Case Study: Finnish private sector pension liabilities

- We modify the case study discussed earlier by allowing wages, inflation and investment return to be stochastic.
- A VEC model is used to describe the evolution of the risk factors<sup>[2]</sup>
- In the calculation of the capital requirement we will quantify the risk preferences with CV@R and V@R with varying confidence levels to illustrate the effect of risk preferences on the capital requirement

## Case Study: Finnish private sector pension liabilities

Figure 4: Evolution of the claims associated with the accrued old age, disability and unemployment pension benefits.



## Case Study: Finnish private sector pension liabilities

- After the specification of the risk measure and the probability distribution of the risk factors, the valuation of the insurance claims can be carried out numerically by
  - generating  $N$  scenarios of asset returns  $R_t$  and claims  $c_t$  over  $t = 1, \dots, T$  and
  - solving the corresponding discretized (1) by a simple line search.

## Case Study: Finnish private sector pension liabilities

		Confidence level				
		95%	90%	85%	80%	66%
Risk measure	$V@R$	289	271	259	250	232
	$CV@R$	305	288	276	268	252

Table 1: Pension liability (billion euros) with varying confidence levels.

- In the calculations  $N=200,000$
- The 66% confidence level corresponds roughly to a 99.5% annual solvency probability (required e.g. in Solvency II) until the full amortization of the pension claims in  $T = 82$  years.

## Valuation and asset management

- The construction of an appropriate hedging strategy for an insurance portfolio is one of the main tasks of an insurance company.
- In real markets where an insurer has multiple investment opportunities, the chosen investment strategy plays a significant role in the determination of the risk based capital requirement.
  - By choosing an investment strategy whose dynamic risk and return profile conforms to the cash flow profile of the liabilities it may be possible to lower the initial capital requirement.

## Valuation and asset management

- Consider an ALM problem, where the assets under management can be diversified each year among a finite set  $J$  of available asset classes.
- Denote by
  - $R_{t,j}$  the (total) return on class  $j \in J$  during period  $[t-1, t]$ .
  - $h_{t,j}$  the amount of wealth invested in class  $j \in J$  of *liquid* instruments in the beginning of year  $t$
  - $\bar{h}_j$  the amount of wealth invested in class  $j \in \bar{J}$  of *illiquid* instruments

## Valuation and asset management

- The investment strategy thus consists of a static allocation  $\bar{h}$  in the illiquid assets and a dynamic trading strategy  $h = (h)_{t=0}^T$  in the liquid assets.
- The search for the minimum capital requirement leads to an optimization problem

$$\begin{aligned}
 & \underset{h \in \mathcal{N}, \bar{h} \in \mathbb{R}^{\bar{J}}}{\text{minimize}} && \sum_{j \in J} h_{0,j} + \sum_{j \in \bar{J}} \bar{h}_j \\
 & \text{subject to} && \sum_{j \in J} h_{t,j} + c_t \leq \sum_{j \in J} R_{t,j} h_{t-1,j} + \sum_{j \in \bar{J}} \bar{R}_{t,j} \bar{h}_j \quad t = 1, \dots, T, \\
 & && h_{t,j}, \bar{h}_l \geq 0 \quad j \in J \setminus \{0\}, l \in \bar{J}, \\
 & && \rho\left(\sum_{j \in J} h_{T,j}\right) \leq 0,
 \end{aligned}$$

## Valuation and asset management

- The above problem cannot be solved analytically, except in some simple special cases.
- In practice, one has to rely on expert knowledge of the problem or numerical approximation schemes or both.
- We apply the computational procedure developed in Koivu and Pennanen [3] to the valuation problem by approximating the optimal dynamic hedging strategy by optimal diversification among a finite set of parametric trading rules

## Valuation and asset management

- The solution procedure can be summarized as follows
  - 1) generate  $N$  scenarios of asset returns  $R_t$  and claims  $c_t$  over  $t = 1, \dots, T$
  - 2) evaluate each parametric trading strategy along each of the scenarios  $i = 1, \dots, N$  and
  - 3) find the optimal combination of trading strategies and a static allocation in the illiquid assets that minimizes the initial capital requirement

## Case Study: Finnish private sector pension liabilities

- In our numerical study the liquid assets are
  - Cash
  - Bonds
  - Nordic, European, US and Asian equities and
  - Real estate
- The illiquid assets are fixed coupon bonds with a 4% annual coupon rate and maturities of 10, 20 and 30 years
- The stochastic model for the asset returns is described in [2]

## Case Study: Finnish private sector pension liabilities

- We used 529 parametric dynamic investment strategies with varying investment styles.
  - buy and hold,
  - fixed proportion and
  - constant proportion portfolio insurance.
- In the optimization we used CV@R to quantify the risk preferences of the decision maker

## Case Study: Finnish private sector pension liabilities

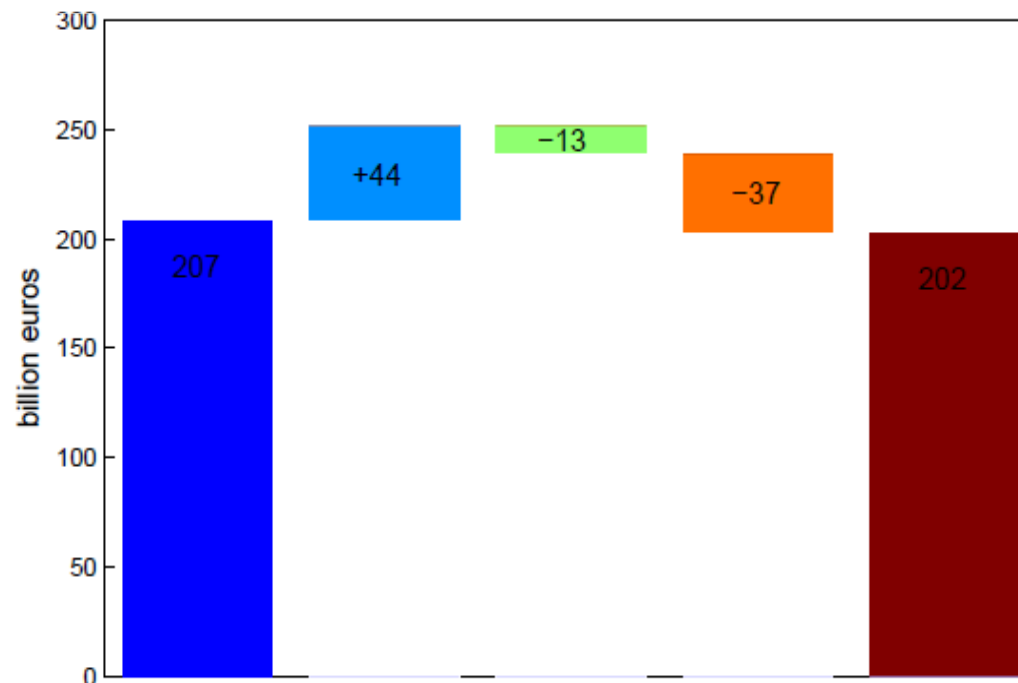
- Capital requirements with CV@R risk measure

	Confidence level				
	95%	90%	85%	80%	66%
Section 3	305	288	276	268	252
Best basis	296	284	273	261	239
Optimization	288	271	254	236	202

Table 4:  $V_0$  (billion €) with varying investment strategies and confidence levels.

## Case Study: Finnish private sector pension liabilities

Figure 6: Contributions to  $V_0$  with  $CV@R_{66\%}$  risk measure. Notes: The first bar gives the deterministic value of  $V_0$ , the second the contribution of stochastic returns and liabilities, the third reflects the diversification effects among asset classes, the fourth reflects the effects of optimal investment strategy and the last bar gives the optimized  $V_0$ .



## Conclusion

- We developed a computational framework for market consistent, cash-flow based, valuation of insurance claims in incomplete markets
- The results revealed the fundamental role of
  - uncertainty (asset returns and claims)
  - risk preferences and
  - hedging strategy
- The framework is easy to implement and adapt to varying valuation problems.

## References

- [1] N. L. Bowers, H. U. Gerber, J. C. Hickman, D. A. Jones, and C. J. Nesbitt. *Actuarial Mathematics*. Society of Actuaries, Schaumburg, IL, 2nd edition, 1997
- [2] P. Hilli, M. Koivu, and T. Pennanen. A stochastic model for assets and liabilities of a pension institution. 2nd PBSS Section Colloquium, 2007
- [3] M. Koivu and T. Pennanen. Galerkin methods in dynamic stochastic programming. Optimization, to appear.