Coffee Break

Breakout Session Topic 9:

Asset / liability management

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AFIR LIFE MUNICH 2009

The arbitrage-free equilibrium pricing of liabilities in an incomplete market: application to a South African retirement fund

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Agenda

1. Introduction
2. Liabilities specification & model
3. Pricing method
4. Results and sensitivity
5. Conclusions

Introduction: aim

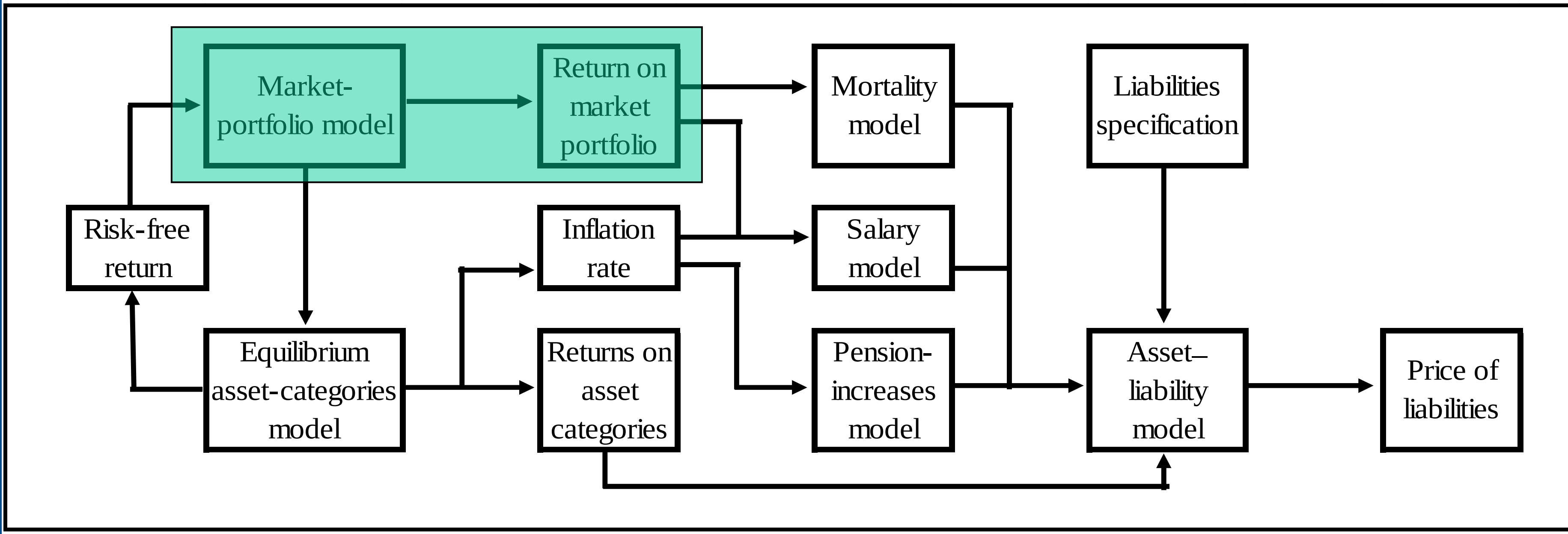
Apply the pricing method of Thomson (2005) to:

- market-portfolio model (Thomson unpublished);
- equilibrium asset-category model (Thomson & Gott, 2009);
- a DB retirement-fund model;

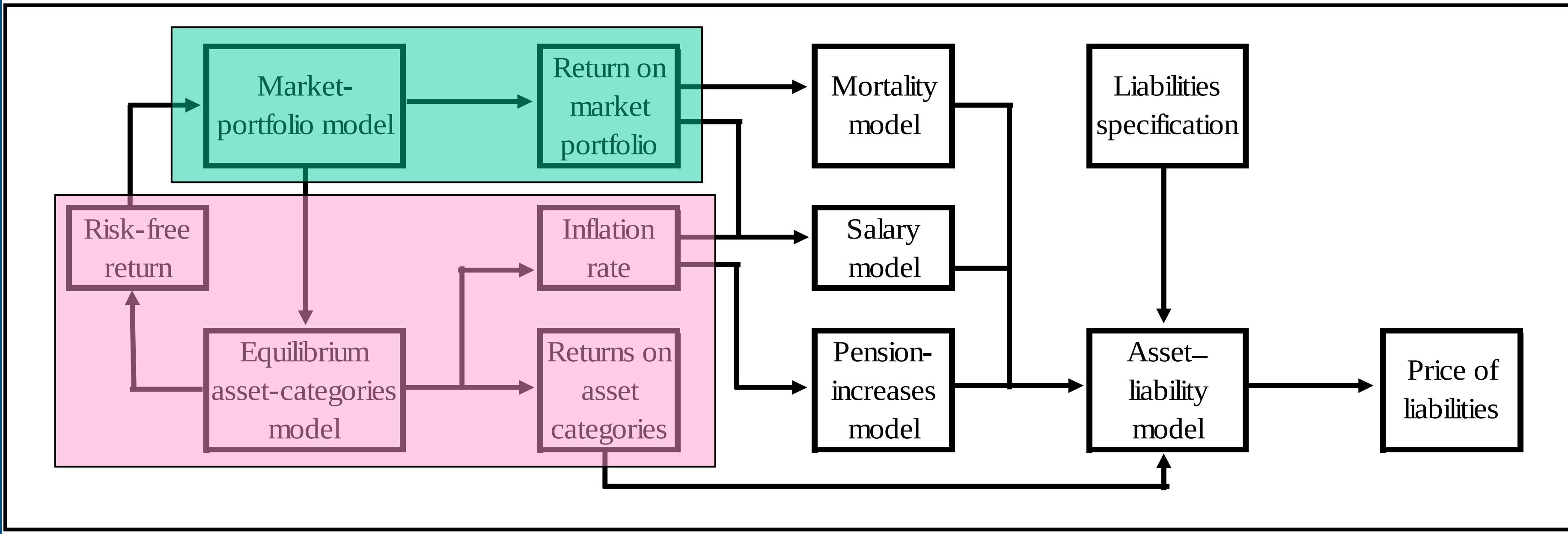
with a view to operationalising the pricing of such a fund and quantifying the effects of:

- non-additivity due to incompleteness;
- guarantees implicit in reasonable expectations of pension increases; and
- the sensitivity of the price of illustrative liabilities to sources of risk and parameters of the model.

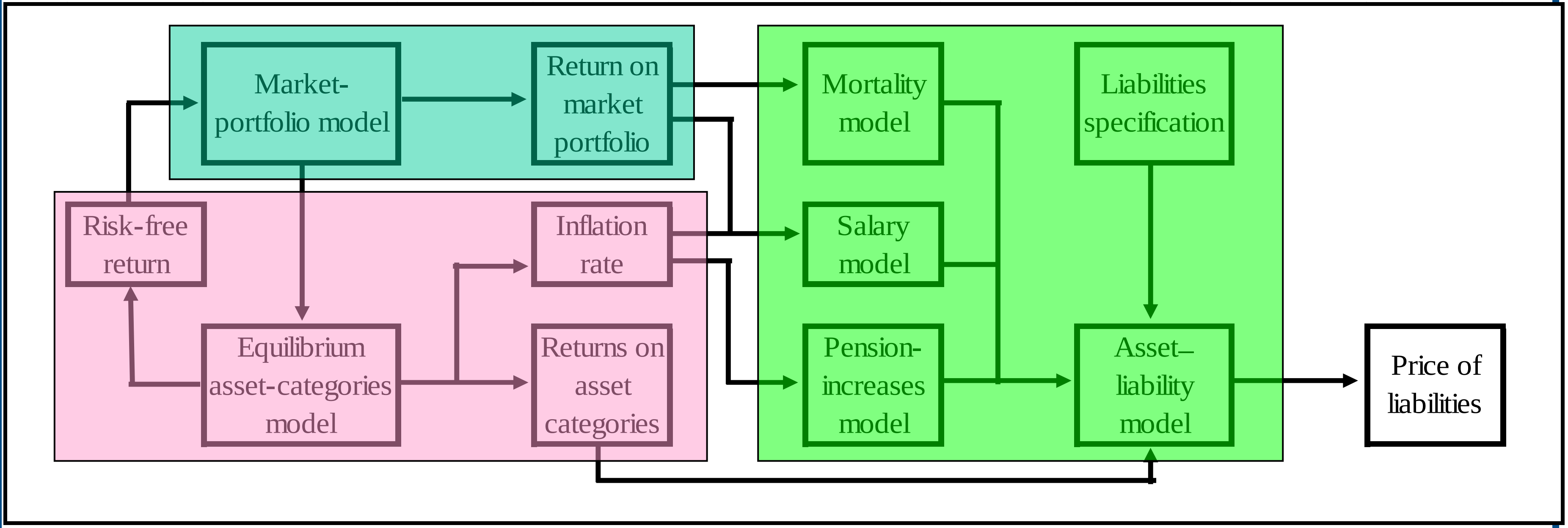
Introduction: method



Introduction: method



Introduction: method



Liabilities specification & model

- no exits before retirement
- mortality only after retirement
- projected unit method
- salaries and pensions expressed in real terms

Liabilities specification & model: salaries model

$$S_{mt} = S_{m,t-1} \exp\left(\xi_t + \bar{\zeta}_t\right)$$

$$\xi_t = \mu_\xi + b_{\xi 1}\eta_{3t} + b_{\xi 2}\eta_{7t} + \sigma_\xi \varepsilon_{\xi t}$$

$$\bar{\zeta}_t = \mu_{\varsigma x} + \sigma_{\bar{\varsigma} x}\varepsilon_{\varsigma t}$$

$$\mu_\xi = 0{,}01 \quad b_{\xi 1} = -0{,}005 \quad b_{\xi 2} = 0{,}005 \quad \sigma_\xi = 0{,}03$$

$$\mu_{\varsigma x} = \alpha_{\mu\varsigma} + \beta_{\mu\varsigma}\exp\left(-\lambda_{\mu\varsigma} x\right)$$

$$\alpha_{\mu\varsigma} = 0{,}016 \quad \beta_{\mu\varsigma} = 0{,}5 \quad \lambda_{\mu\varsigma} = 0{,}1$$

$$\sigma^2_{\bar{\zeta} x} = \frac{\sigma^2_{\zeta x}}{M_{x,t-1}}$$

$$\sigma_{\varsigma x} = \alpha_{\sigma\varsigma} + \beta_{\sigma\varsigma}\exp\left(-\lambda_{\sigma\varsigma} x\right)$$

$$\alpha_{\sigma\varsigma} = 0{,}042 \quad \beta_{\sigma\varsigma} = 0{,}5 \quad \lambda_{\sigma\varsigma} = 0{,}08$$

Liabilities specification & model: pension increases

$$P_t = P_{t-1} \exp\left\{\max\left(0, -\gamma_t\right)\right\}$$

Liabilities specification & model: pensioner mortality

$$\nu_{\{x\}}^{SAP98} = \frac{\nu_{\{x\}}^{PNL00}}{\nu_{\{x\}}^{IL00}} \nu_{\{x\}}^{SAIL98}$$

$$\nu_{\{x\}}^{SAP} = \nu_{\{x\}}^{SAP98} \exp\left(10\mu_{\nu}\right)$$

$$\nu_{\{x\}+t}^{SAP} = \nu_{\{x\}+t-1}^{SAP} \exp\left(\chi_{\nu t}\right)$$

where:

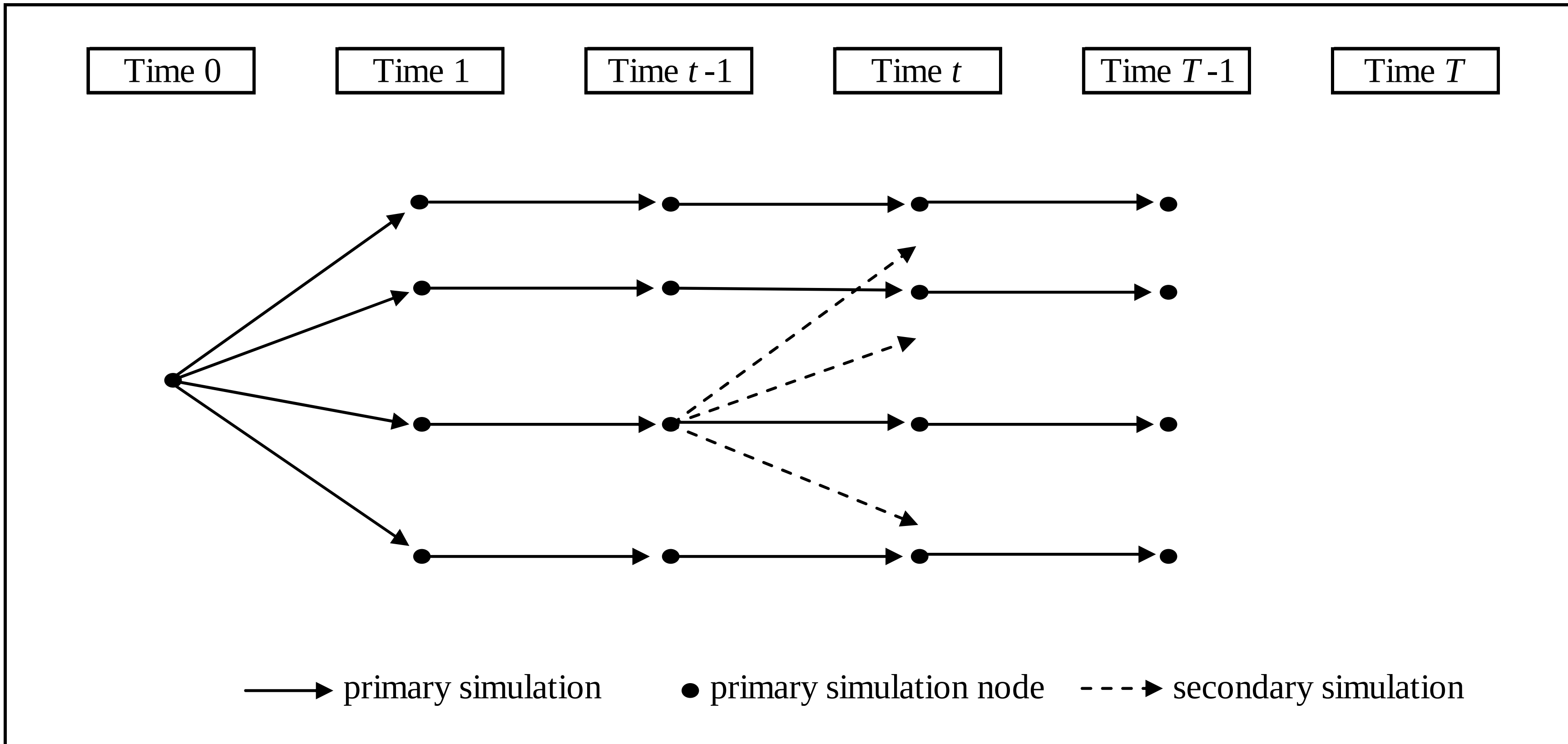
$$\mu_{\nu} = -0,004$$

$$\chi_{\nu t} = \chi_{\nu,t-1} + \mu_{\nu} + b_{\nu}\eta_{7t} + \sigma_{\nu}\varepsilon_{\nu t}$$

$$b_{\nu} = -0,001$$

$$\sigma_{\nu} = 0,005$$

Pricing method: primary & secondary simulations



Pricing method: state-space vector

$$
\boldsymbol{x}_t = \begin{pmatrix} P_{It}(s_1) \\ \vdots \\ P_{It}(s_u) \\ P_{Ct}(s_1) \\ \vdots \\ P_{Ct}(s_u) \\ \theta_t \\ P_{x_1 t} \\ \vdots \\ P_{x_N t} \end{pmatrix}
$$

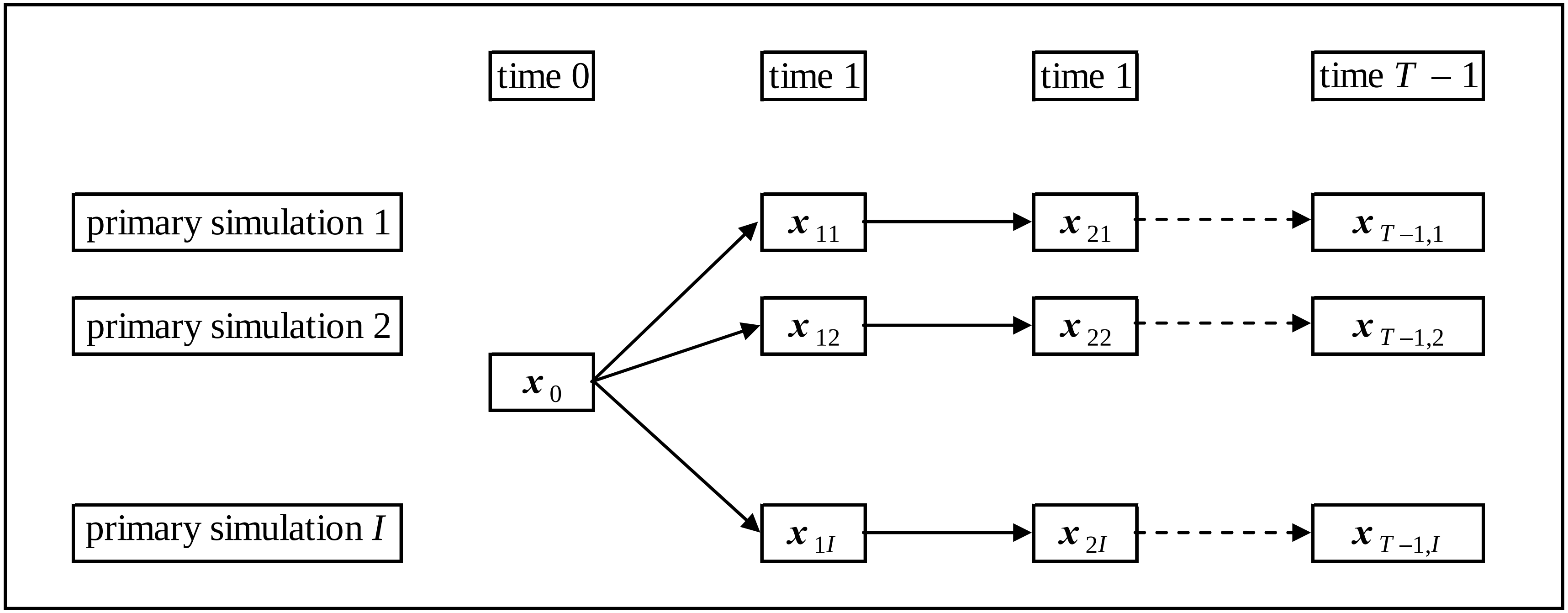
where:

$$P_{It}(s) = \exp\{-Y_{It}(s)\}$$

$$P_{Ct}(s) = \exp\{-Y_{Ct}(s)\}$$

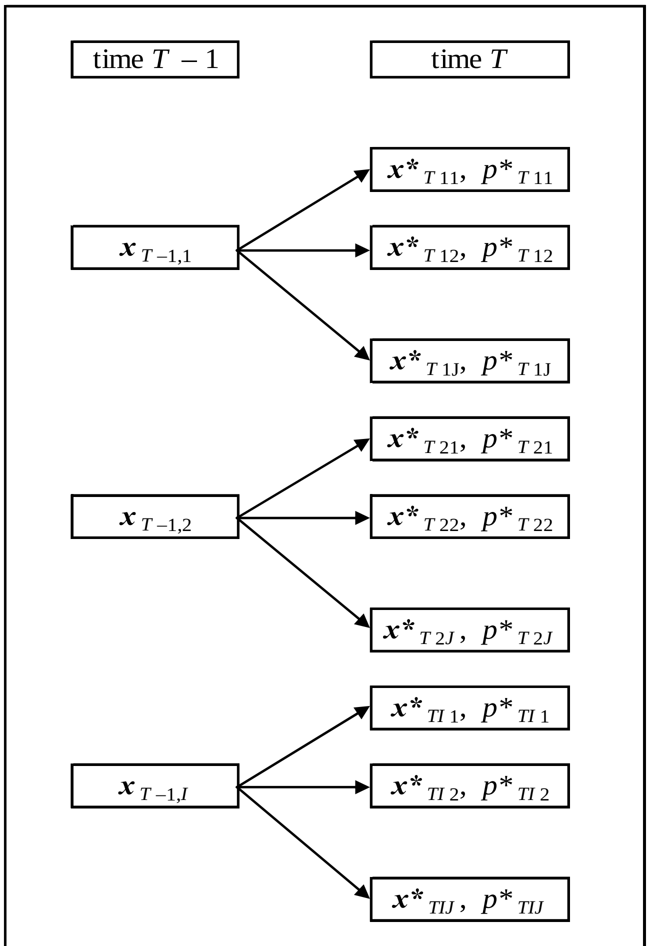
$$\theta_t = \exp(\chi_{vt})$$

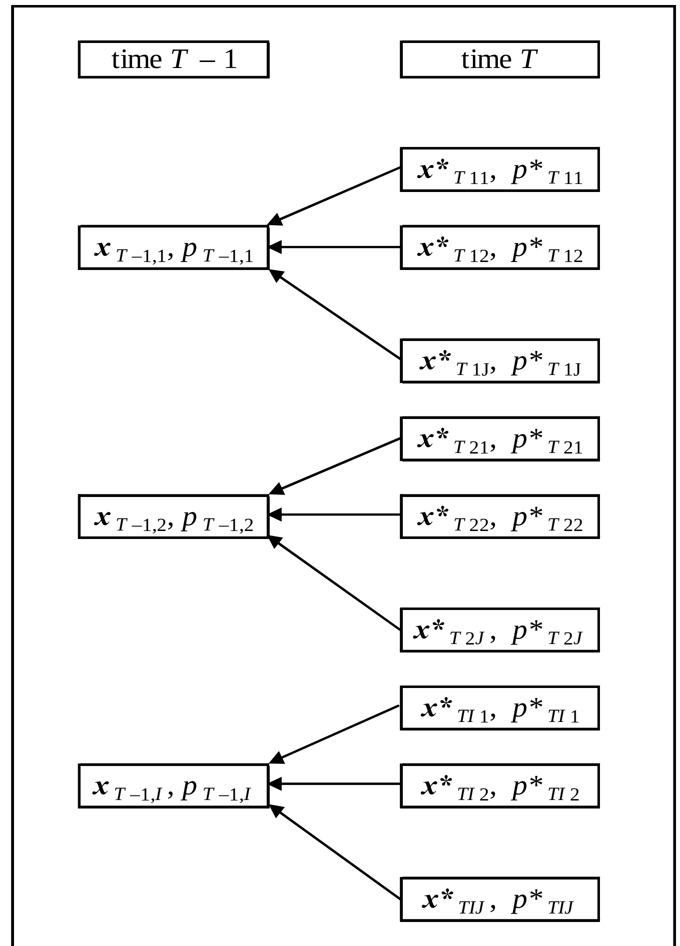
Pricing method: primary simulations of the state-space vector



Pricing method: secondary simulations

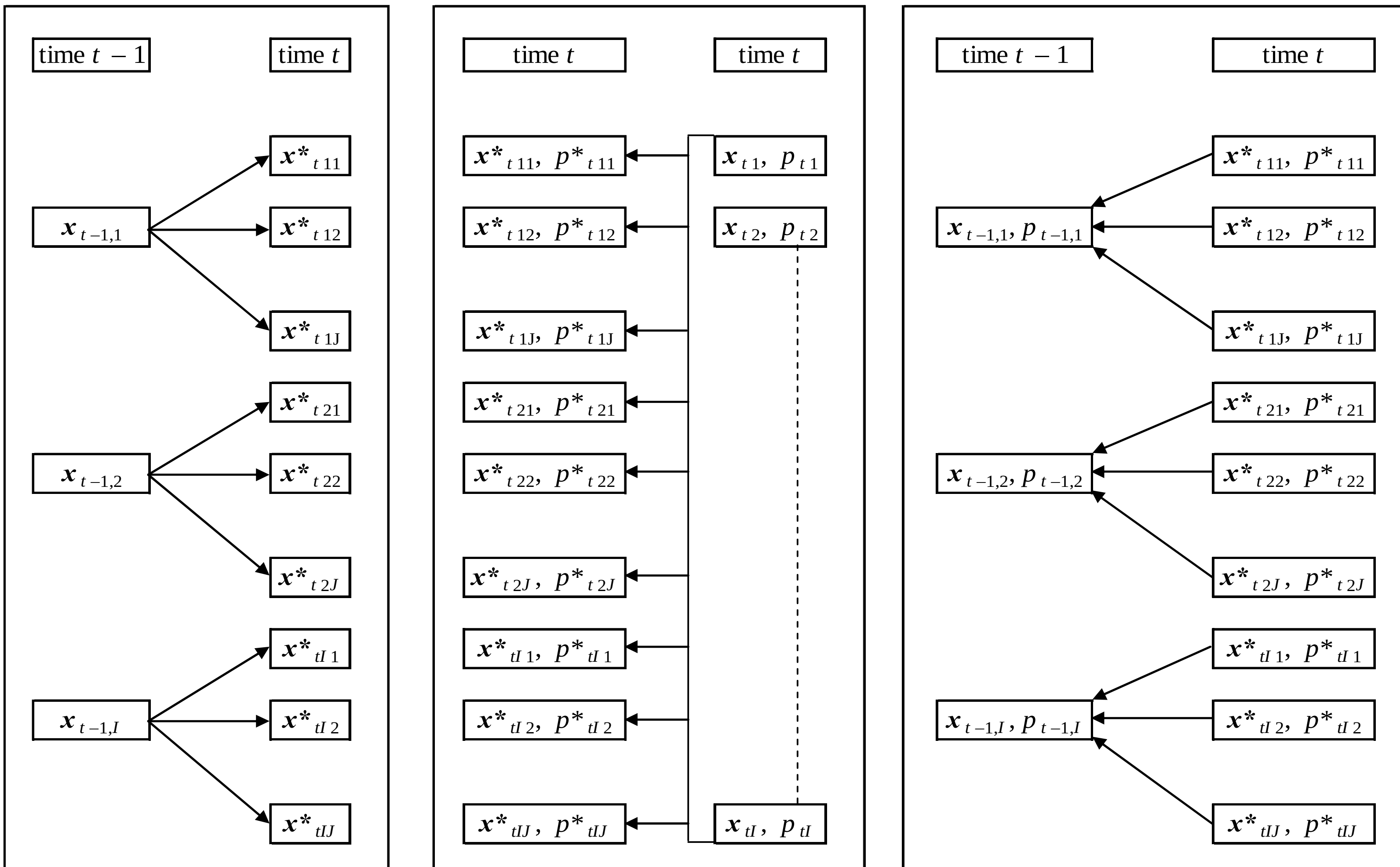
Final year





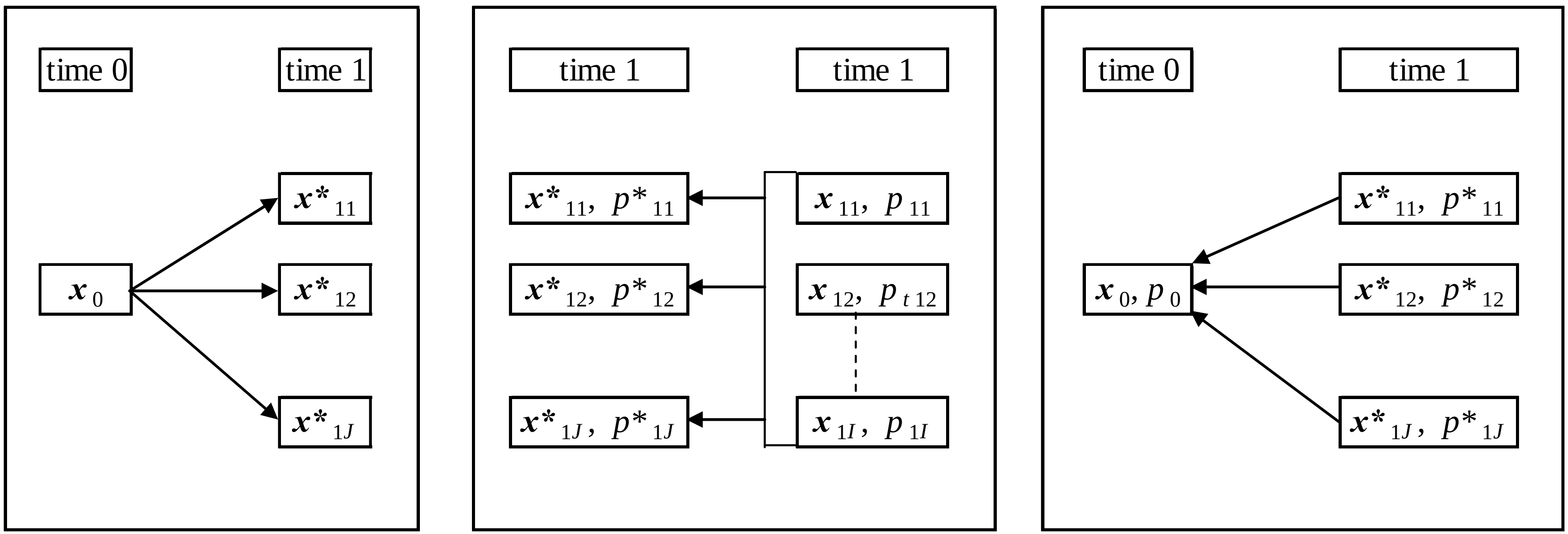
Pricing method: secondary simulations

year *t*



Pricing method: secondary simulations

year 1



Pricing method

$$\hat{\sigma}_{\varepsilon t}^{2} = \hat{\sigma}_{Ft}^{2} - \hat{\boldsymbol{\sigma}}'_{FVt}\hat{\boldsymbol{\Sigma}}_{Vt}^{-1}\hat{\boldsymbol{\sigma}}_{FVt}$$

$$\boldsymbol{z}_t = \hat{\boldsymbol{\Sigma}}_{Vt}^{-1}\left(\hat{\boldsymbol{\mu}}_{Vt} - f_t\mathbf{1}\right) \qquad \boldsymbol{m}_t = \frac{1}{\boldsymbol{z}'_t\mathbf{1}}\boldsymbol{z}_t$$

$$\hat{\mu}_{Mt} = \boldsymbol{m}'_t\hat{\boldsymbol{\mu}}_{Vt} \qquad \hat{\sigma}_{Mt}^{2} = \boldsymbol{m}'_t\hat{\boldsymbol{\Sigma}}_{Vt}\boldsymbol{m}_t \qquad \hat{\sigma}_{HMt} = \boldsymbol{m}'_t\hat{\boldsymbol{\sigma}}_{FVt}$$

$$\hat{\beta}_{Ft}^{*} = \frac{\hat{\sigma}_{HMt} + \hat{\sigma}_{\varepsilon t}\hat{\sigma}_{Mt}}{\hat{\sigma}_{Mt}^{2}}$$

$$P_{L,t-1} = \frac{1}{f_t}\left\{\hat{\mu}_{Ft} - \hat{\beta}_{Ft}^{*}\left(\hat{\mu}_{Mt} - f_t\right)\right\}$$

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Results and sensitivity

Sex	Age	Value per unit accrued pension					Aggregate value		
		deterministic valuation	stochastic price				deterministic valuation	stochastic price	
			1 member		entire cohort				
				% incr		% incr	R’million		% incr
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Female									
	25	14,11	16,00	13,4	15,84	-1,0	17	19	12,2
	35	11,94	13,46	12,8	13,36	-0,8	172	193	11,8
	45	12,00	13,49	12,4	13,39	-0,7	311	347	11,6
	55	12,67	13,70	8,1	13,62	-0,6	347	372	7,5
	65	13,36	13,78	3,1	13,78	0,0	434	447	3,1
	75	9,53	9,69	1,7	9,69	0,0	236	240	1,7
	85	5,96	6,01	0,9	6,01	0,0	62	63	0,9
	total						1 579	1 681	6,5

Results and sensitivity

	deterministic valuation	stochastic price	
	R'million		% incr
Female	1 579	1 681	6,5
Male	1 351	1 427	5,6
Total	2 930	3 109	6,1
Aggregate	2 930	3 094	5,6
Adjusted to fund data	2 912	3 074	5,6

Results and sensitivity

Deterministic valuation of model-point data	2 930
difference due to risk-free stochastic pricing	11
Risk-free stochastic price	2 941
hedge-portfolio risks	–19
Stochastic price with hedge-portfolio risks	2 922
residual risks	–20
Stochastic price: hedge-portfolio & residual risks	2 902
cost of guarantee	192
Stochastic price based on model-point data	3 094
adjustment to fund data	–20
Stochastic price based on fund data	3 074

Results and sensitivity: major effects

Parameter				Test result	
name	description	standard value	test value	price (R'million)	change in price (%)
	standard values			3 094	N/A
$b_{\xi 1}$	general salary increase: sensitivity to inflation	–0,005	0	3 098	0,14
g	return on market portfolio: sensitivity to risk-free rate	1,39	1,2	3 082	-0,37
σ_M	return on market portfolio: residual volatility	0,159	0,1	3 108	0,45
b_γ	force of inflation: residual volatility	–0,01379	0	3 061	–1,07

Conclusions

- Method computationally demanding, but not impossible: 41 hours.

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- Method computationally demanding, but not impossible.
- Convergence complicated, but Sobol numbers expedite it.
- Stochastic price 5,6% higher than deterministic.
- Without guarantee, stochastic price only 1% less than deterministic: *If the valuation of the liabilities should allow for a risk premium only to the extent that the trustees are unable to avoid risk, then the valuation basis must be much closer to a risk-free basis than that produced by the risk premiums typically used.*

Conclusions

- Method computationally demanding, but not impossible.
- Convergence complicated, but Sobol numbers expedite it.
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- Effects of non-additivity: intra-cohort 0–1% plus inter-cohort 0,5%.

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- Stochastic price 5,6% higher than deterministic: because of pension guarantee.
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- Effects of non-additivity: intra-cohort 0–1% plus inter-cohort 0,5%.
- Major sensitivities:

  volatility of force of inflation in excess of conditional ex-ante expected inflation;

  sensitivity of ex-ante expected returns on the market portfolio to positive risk-free returns;

  residual volatility of the return on the market portfolio.

Conclusions

- Method computationally demanding, but not impossible.
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- Without guarantee, stochastic price only 1% less than deterministic.
- Effects of non-additivity: intra-cohort 0–1% plus inter-cohort 0,5%.
- Major sensitivities.
- Overall effect: *Excluding uncertainties common to deterministic and stochastic valuations, an error of about 5,6% is reduced to uncertainty of about 1%.*

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