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Economic Values of Contribution Cashflows and Measures to Bring the EVs under Control

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1-1. Covariance pricing formula

$$q = \frac{\mathrm{E}(\mathbf{v})}{R_F} + \mathrm{cov}(\boldsymbol{\xi}, \mathbf{v})$$

- A cashflow increasing in market downturn and decreasing in market upturn correlates positively to the state price density.
- The second term thus becomes positive.

1-2. Evaluation of employer contributions

- The EV of employer contributions would be evaluated higher than their best-estimated PV.
- The greater the volatility of contributions is, the higher the EV of the contributions is evaluated.

1-3. In the case of benefit cashflows

$$q = \frac{\mathrm{E}(\mathbf{v})}{R_F + \delta}$$

- **The risk premium delta with regard to the benefit volatility originated from the beta risk of investments should be positive.**
- **The risk premium delta with regard to the benefit volatility originated from the *macro* longevity risk should be negative.**

2. A benefit design sustainable under the 'Japan Scenario'

$$\text{if } L^{(1)} \le A \;, \quad \text{then} \quad B = B^{(1)}$$

$$\text{if } A = L^{(0)} + \alpha\,(L^{(1)} - L^{(0)}) \quad (0 \le \alpha < 1)\,,$$

$$\text{then } B = B^{(0)} + \alpha\,(B^{(1)} - B^{(0)})$$

$$\text{if } A < L^{(0)} \;, \quad \text{then} \quad B = B^{(0)}$$

- **B(0) and B(1) are both indexed to inflation.**
- **The portion exceeding L(0) functions as a virtual risk buffer.**

3-1. A serious deficiency in DB plans

- No specific correspondence between individual liabilities and plan assets
- As a result,
  - the interests of beneficiaries are given the most privileged status.
  - no effective and timely mechanism of keeping under control the risks of active participants is incorporated.

3-2. Framework of the payout-year-specific (PYS) funding standard

- **Divide contributions by payout year**
- **Load the decomposed contributions on the ‘*sequentially chained containers*’**
- **And assign each container**
  - **a payout year, and**
  - **a minimum permissible funding ratio**

3-3. Partial ring-fencing of assets

- **The assets can only be used for paying benefits in the year assigned to the container.**
- **A surplus may be used for filling up the shortfalls of other containers**
  - **a kind of *intertemporal* risk-sharing**
- **But cannot fill up a shortfall with aggravating deficiencies of other containers.**

3-4. How to decompose contributions

- **Naturally given by discounting back the accrued benefits using appropriate discount rates**
- **Then, how to determine the *appropriate* discount rates for each container?**
- **The gist of payout-year-specific (PYS) funding standard lies in this point!**

3-5. Using the expected return of the actual portfolio in not prudent

$$dA_u = r\, A_u\, du + \sigma\, A_u\, dW_u \qquad u \in [t, T]$$

$$A_t = L_T \exp\{-r\,(T-t)\}$$

Then,

$$P(A_T > L_T) = N\left(-\frac{1}{2}\sigma\sqrt{T-t}\right)$$

4-1. Main ideas of the PYS standard (1/2)

P1= the probability that the portfolio value will attain the liability value at *some* time

P0= probability that the portfolio value will surpass the liability value at the year of maturity

Then,

$$P1>P0$$

4-1. Main ideas of the PYS standard (2/2)

- Assume switching to a liability-hedging portfolio at the time of hitting the upper barrier, not waiting the year of maturity
- Determine the maximum discount rate so as to satisfy the condition:
  - P1 > p1 (given from outside)
- Need to evaluate P1

4-2. The probability of hitting the upper barrier (1/4)

$$dA_u = r\,A_u\,du + \sigma\,A_u\,dW_u \qquad u \in [t\,,T]$$

$$A_t = L^{(1)}\exp\{-(\mu\,\theta_t + r_F)(T-t)\}$$

$$\mu = r - r_F$$

$$dB_u = r_F\,B_u\,du \qquad u \in [t\,,T]$$

$$B_t = L^{(1)}\exp\{-r_F\,(T-t)\}$$

4-2. The probability of hitting the upper barrier (2/4)

$$X_u = \log \frac{A_u}{B_u}$$

Then,

$$dX_u = (\mu - \frac{1}{2}\sigma^2)\,dt + \sigma\, dW_u$$

$$X_t = \alpha_t = -\mu\,\theta_t\,(T - t) < 0$$

4-2. The probability of hitting the upper barrier (3/4)

Consider a *running maximum process*:

$$M_X(u) = \sup_{t \le s \le u} X_s$$

$F_{M(u)}(x)$: **the distribution function**

Then, probability P1 is given by:

$$P_1 = 1 - F_{M(T)}(0)$$

4-2. The probability of hitting the upper barrier (4/4)

$$F_{M(u)}(x) = N\left(\frac{(x-\alpha_t) - (\mu - \frac{1}{2}\sigma^2)(u-t)}{\sigma\sqrt{u-t}} \right)$$

$$-\exp\left\{ 2\frac{(\mu - \frac{1}{2}\sigma^2)(x-\alpha_t)}{\sigma^2} \right\} N\left(-\frac{(x-\alpha_t) + (\mu - \frac{1}{2}\sigma^2)(u-t)}{\sigma\sqrt{u-t}} \right)$$

5. Additional conditions (1/6)

- **Condition P1 > p1 only determines the maximum proportion θ_t of the excess return $\mu = r - r_F$**
  - for each combination of excess return and its volatility
- **Another condition has to be introduced, especially from the aspect of restricting the risk of underfunding**
  - to determine unique discount rates

5. Additional conditions (2/6)

(1) **Restricting the severity of loss when the portfolio value could not attain the liability value *at any time***

(2) **The conditional expectation of the portfolio value *at the year of maturity* is within an affordable range:**

$$\frac{E[A_T \mid A_T < L^{(0)}]}{L^{(0)}} \geq q, \quad 0 < q < 1$$

5. Additional conditions (3/6)

(3) Probability P2 that the log funded ratio is absorbed into the lower barrier *at some time* is less than constant p2.

$$dC_u = r_F C_u \, du \qquad u \in [t, T]$$

$$C_t = L^{(0)} \exp\{-r_F (T-t)\}$$

5. Additional conditions (4/6)

$$Y_u = \log \frac{A_u}{C_u}$$

$$dY_u = (\mu - \frac{1}{2}\sigma^2)\,du + \sigma\,dW_u$$

$$Y_t = \beta_t = -\mu\,\theta_t\,(T - t) + \log\left(\frac{L^{(1)}}{L^{(0)}}\right) > 0$$

5. Additional conditions (5/6)

Consider a *running minimum process*:

$$m_Y(u) = \inf_{t \le s \le u} Y_s$$

$F_{m(u)}(y)$: **the distribution function**

Then, probability P2 is given by:

$$P_2 = F_{m(u)}(0)$$

5. Additional conditions (6/6)

$$F_{m(u)}(y) = N\left(\frac{(y-\beta_t)-(\mu-\frac{1}{2}\sigma^2)(u-t)}{\sigma\sqrt{u-t}} \right)$$

$$+\exp\left\{ 2\frac{(\mu-\frac{1}{2}\sigma^2)(y-\beta_t)}{\sigma^2} \right\} N\left(\frac{(y-\beta_t)+(\mu-\frac{1}{2}\sigma^2)(u-t)}{\sigma\sqrt{u-t}} \right)$$

6-1. Maximum permissible proportions of excess returns --- condition_(2)

$L^{(1)}/L^{(0)}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$T-t$	10	10	10	20	20	20	5	5	5
p_1	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
q	0.80	0.90	0.95	0.80	0.70	0.60	0.80	0.90	0.95
w_t	0.74	0.35	0.17	0.57	0.91	1.26	0.94	0.46	0.23
θ_t	0.49	0.50	0.51	0.44	0.40	0.40	0.69	0.62	0.60
r_P	7.40	6.29	5.66	6.97	7.81	8.45	7.88	6.63	5.85
$\exp\{-(r_P-r_F)(T-t)\}$	0.79	0.88	0.94	0.67	0.57	0.50	0.87	0.92	0.96
$\exp\{\alpha_t\}$	0.89	0.94	0.97	0.84	0.80	0.76	0.90	0.95	0.97

6-2. Maximum permissible proportions of excess returns --- condition_(3)

$L^{(1)} / L^{(0)}$	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50
$T-t$	10	10	10	20	20	20	5	5	5
p_1	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
p_2	0.20	0.30	0.40	0.20	0.30	0.40	0.20	0.30	0.40
w_t	0.68	0.80	0.92	0.48	0.56	0.64	0.92	1.09	1.27
θ_t	0.49	0.49	0.50	0.46	0.44	0.43	0.69	0.73	0.79
r_P	7.25	7.55	7.84	6.68	6.91	7.14	7.84	8.18	8.47
$\exp\{-(r_P - r_F)(T-t)\}$	0.80	0.77	0.75	0.71	0.68	0.65	0.87	0.85	0.84
$\exp\{\alpha_t\}$	0.90	0.88	0.87	0.86	0.84	0.83	0.91	0.89	0.87
$\exp\{\beta_t\}$	1.34	1.32	1.30	1.29	1.27	1.25	1.36	1.34	1.31

6-3. Implications for discount rates

- **The discount rates thus determined include a proportion of the expected excess return.**
- **The proportion increases progressively as the period until maturity extends.**
  - **but only if acceptable risk also increases**
- **The graph of the proportion might be hump-shaped**
  - **if there is a due limit on acceptable risks**

6-4. Implications for funding standards

- **Pension funds may take larger investment risks as investment horizon extends**
  - **but only when funding deficiencies can be corrected gradually spending longer periods**
- **Any funding standard should strike a right balance between**
  - **A) assuring stable employer contributions with reasonable prices, and**
  - **B) ensuring that targeted benefits are paid with reasonably high probabilities**

6-5. Implications for investment strategies

- **Any portfolio is considered as a composite of target date funds (TDFs).**
  - **irrespective of the funding standards**
- **Decomposition of a portfolio by payout year makes the discussion on investment horizons extremely transparent.**
- **Any portfolio should be rebalanced along with the changes in benefit cashflow estimation and the degree of risk aversion**
  - **even if the prospect on the market is invariant**

7. Concluding remarks

- **Any funding standard cannot continue to exist without paying proper consideration to investments and *vice versa*.**
- **The PYS funding standard is a bridge connecting the financing issues and investment issues systematically.**