AFIR LIFE MUNICH 2009

Economic Values of Contribution Cashflows and Measures to Bring the EVs under Control

September 11, 2009

SHIMIZU Nobuhiro

Section Finance de l'AAI
IAA Finance Section
AFIR

Section Vie de l'AAI
IAA Life Section
LIFE

DAV
DEUTSCHE AKTUARVEREINIGUNG e.V.

Contents

1-1. Covariance pricing formula

$$q = \frac{\mathrm{E}(\mathbf{v})}{R_F} + \mathrm{cov}(\boldsymbol{\xi}, \mathbf{v})$$

- A cashflow increasing in market downturn and decreasing in market upturn correlates positively to the state price density.
- The second term thus becomes positive.

1-2. Evaluation of employer contributions

- **The EV of employer contributions would be evaluated higher than their best-estimated PV.**
- **The greater the volatility of contributions is, the higher the EV of the contributions is evaluated.**

1-3. In the case of benefit cashflows

$$q = \frac{\mathrm{E}(\mathbf{v})}{R_F + \delta}$$

- **The risk premium delta with regard to the benefit volatility originated from the beta risk of investments should be <u>positive</u>.**
- **The risk premium delta with regard to the benefit volatility originated from the *macro* longevity risk should be <u>negative</u>.**

2. A benefit design sustainable under the 'Japan Scenario'

$$\text{if } L^{(1)} \leq A \ , \ \text{then} \quad B = B^{(1)}$$

$$\text{if } A = L^{(0)} + \alpha (L^{(1)} - L^{(0)}) \quad (0 \leq \alpha < 1) ,$$

$$\text{then } B = B^{(0)} + \alpha (B^{(1)} - B^{(0)})$$

$$\text{if } A < L^{(0)} \ , \ \text{then} \quad B = B^{(0)}$$

- **B(0) and B(1) are both indexed to inflation.**
- **The portion exceeding L(0) functions as a virtual risk buffer.**

3-1. A serious deficiency in DB plans

- **No specific correspondence between individual liabilities and plan assets**
- **As a result,**
  - **the interests of beneficiaries are given the most privileged status.**
  - **no effective and timely mechanism of keeping under control the risks of active participants is incorporated.**

3-2. Framework of the payout-year-specific (PYS) funding standard

- **Divide contributions by payout year**
- **Load the decomposed contributions on the '*sequentially chained containers*'**
- **And assign each container**
  - **a payout year, and**
  - **a minimum permissible funding ratio**

3-3. Partial ring-fencing of assets

- **The assets can only be used for paying benefits in the year assigned to the container.**
- **A surplus may be used for filling up the shortfalls of other containers**
  - **a kind of *intertemporal* risk-sharing**
- **But cannot fill up a shortfall with aggravating deficiencies of other containers.**

3-4. How to decompose contributions

- **Naturally given by discounting back the accrued benefits using appropriate discount rates**
- **Then, how to determine the *appropriate* discount rates for each container?**
- **The gist of payout-year-specific (PYS) funding standard lies in this point!**

3-5. Using the expected return of the actual portfolio in not prudent

$$dA_u = r\, A_u\, du + \sigma\, A_u\, dW_u \qquad u \in [\, t\, ,\, T\,]$$

$$A_t = L_T \exp\{-r\,(T - t)\}$$

Then,

$$P(A_T > L_T) = N(-\frac{1}{2}\sigma\sqrt{T - t})$$

4-1. Main ideas of the PYS standard (1/2)

P1= the probability that the portfolio value will attain the liability value <u>at *some* time</u>

P0= probability that the portfolio value will surpass the liability value <u>at the year of maturity</u>

Then,

P1>P0

4-1. Main ideas of the PYS standard (2/2)

- Assume switching to a liability-hedging portfolio at the time of hitting the upper barrier, not waiting the year of maturity
- Determine the maximum discount rate so as to satisfy the condition:
  - P1 > p1 (given from outside)
- Need to evaluate P1

4-2. The probability of hitting the upper barrier (1/4)

$$dA_u = r\,A_u\,du + \sigma\,A_u\,dW_u \qquad u \in [t, T]$$

$$A_t = L^{(1)} \exp\{-(\mu\,\theta_t + r_F)(T-t)\}$$

$$\mu = r - r_F$$

$$dB_u = r_F\,B_u\,du \qquad u \in [t, T]$$

$$B_t = L^{(1)} \exp\{-r_F\,(T-t)\}$$

4-2. The probability of hitting the upper barrier (2/4)

$$X_u = \log \frac{A_u}{B_u}$$

Then,

$$dX_u = (\mu - \frac{1}{2}\sigma^2)\,dt + \sigma\, dW_u$$

$$X_t = \alpha_t = -\mu\,\theta_t\,(T - t) < 0$$

4-2. The probability of hitting the upper barrier (3/4)

Consider a *running maximum process*:

$$M_X(u) = \sup_{t \le s \le u} X_s$$

$F_{M(u)}(x)$: **the distribution function**

Then, probability P1 is given by:

$$P_1 = 1 - F_{M(T)}(0)$$

4-2. The probability of hitting the upper barrier (4/4)

$$F_{M(u)}(x) = N\left(\frac{(x-\alpha_t) - (\mu - \frac{1}{2}\sigma^2)(u-t)}{\sigma\sqrt{u-t}} \right)$$

$$- \exp\left\{ 2\frac{(\mu - \frac{1}{2}\sigma^2)(x-\alpha_t)}{\sigma^2} \right\} N\left(-\frac{(x-\alpha_t) + (\mu - \frac{1}{2}\sigma^2)(u-t)}{\sigma\sqrt{u-t}} \right)$$

5. Additional conditions (1/6)

- **Condition P1 > p1 only determines the maximum proportion θ_t of the excess return $\mu = r - r_F$**
  - for each combination of excess return and its volatility
- **Another condition has to be introduced, especially from the aspect of restricting the risk of underfunding**
  - to determine unique discount rates

5. Additional conditions (2/6)

(1) Restricting the severity of loss when the portfolio value could not attain the liability value *at any time*

(2) The conditional expectation of the portfolio value *at the year of maturity* is within an affordable range:

$$\frac{E[A_T \mid A_T < L^{(0)}]}{L^{(0)}} \geq q, \quad 0 < q < 1$$

5. Additional conditions (3/6)

(3) Probability P2 that the log funded ratio is absorbed into the lower barrier *at some time* is less than constant p2.

$$dC_u = r_F C_u du \qquad u \in [t, T]$$

$$C_t = L^{(0)} \exp\{-r_F (T-t)\}$$

5. Additional conditions (4/6)

$$Y_u = \log \frac{A_u}{C_u}$$

$$dY_u = (\mu - \frac{1}{2}\sigma^2)\,du + \sigma\, dW_u$$

$$Y_t = \beta_t = -\mu\,\theta_t\,(T-t) + \log(\frac{L^{(1)}}{L^{(0)}}) > 0$$

5. Additional conditions (5/6)

Consider a *running minimum process*:

$$m_Y(u) = \inf_{t \le s \le u} Y_s$$

$F_{m(u)}(y)$: **the distribution function**

Then, probability P2 is given by:

$$P_2 = F_{m(u)}(0)$$

5. Additional conditions (6/6)

$$F_{m(u)}(y) = N\left(\frac{(y-\beta_t)-(\mu-\frac{1}{2}\sigma^2)(u-t)}{\sigma\sqrt{u-t}}\right)$$

$$+\exp\left\{2\frac{(\mu-\frac{1}{2}\sigma^2)(y-\beta_t)}{\sigma^2}\right\}N\left(\frac{(y-\beta_t)+(\mu-\frac{1}{2}\sigma^2)(u-t)}{\sigma\sqrt{u-t}}\right)$$

6-1. Maximum permissible proportions of excess returns --- condition_(2)

$L^{(1)} / L^{(0)}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$T-t$	10	10	10	20	20	20	5	5	5
p_1	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
q	0.80	0.90	0.95	0.80	0.70	0.60	0.80	0.90	0.95
w_t	0.74	0.35	0.17	0.57	0.91	1.26	0.94	0.46	0.23
θ_t	0.49	0.50	0.51	0.44	0.40	0.40	0.69	0.62	0.60
r_P	7.40	6.29	5.66	6.97	7.81	8.45	7.88	6.63	5.85
$\exp\{-(r_P - r_F)(T-t)\}$	0.79	0.88	0.94	0.67	0.57	0.50	0.87	0.92	0.96
$\exp\{\alpha_t\}$	0.89	0.94	0.97	0.84	0.80	0.76	0.90	0.95	0.97

6-2. Maximum permissible proportions of excess returns --- condition_(3)

$L^{(1)}/L^{(0)}$	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50
$T-t$	10	10	10	20	20	20	5	5	5
p_1	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
p_2	0.20	0.30	0.40	0.20	0.30	0.40	0.20	0.30	0.40
w_t	0.68	0.80	0.92	0.48	0.56	0.64	0.92	1.09	1.27
θ_t	0.49	0.49	0.50	0.46	0.44	0.43	0.69	0.73	0.79
r_P	7.25	7.55	7.84	6.68	6.91	7.14	7.84	8.18	8.47
$\exp\{-(r_P-r_F)(T-t)\}$	0.80	0.77	0.75	0.71	0.68	0.65	0.87	0.85	0.84
$\exp\{\alpha_t\}$	0.90	0.88	0.87	0.86	0.84	0.83	0.91	0.89	0.87
$\exp\{\beta_t\}$	1.34	1.32	1.30	1.29	1.27	1.25	1.36	1.34	1.31

6-3. Implications for discount rates

- **The discount rates thus determined include a proportion of the expected excess return.**
- **The proportion increases progressively as the period until maturity extends.**
  - **but only if acceptable risk also increases**
- **The graph of the proportion might be hump-shaped**
  - **if there is a due limit on acceptable risks**

6-4. Implications for funding standards

- **Pension funds may take larger investment risks as investment horizon extends**
  - **but <u>only when</u> funding deficiencies can be corrected gradually spending longer periods**
- **Any funding standard should strike a right balance between**
  - **A) assuring stable employer contributions with reasonable prices, and**
  - **B) ensuring that targeted benefits are paid with reasonably high probabilities**

6-5. Implications for investment strategies

- **Any portfolio is considered as a composite of target date funds (TDFs).**
  - irrespective of the funding standards
- **Decomposition of a portfolio by payout year makes the discussion on investment horizons extremely transparent.**
- **Any portfolio should be rebalanced along with the changes in benefit cashflow estimation and the degree of risk aversion**
  - even if the prospect on the market is invariant

7. Concluding remarks

- **Any funding standard cannot continue to exist without paying proper consideration to investments and *vice versa*.**
- **The PYS funding standard is a bridge connecting the financing issues and investment issues systematically.**