



## Defined Contribution Pension Plans Management and Market Opportunities

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1. Introduction: Yet a pensions storm!
2. The literature on stock market efficiency and investment strategies
3. Empirical evidence on stock market opportunities
4. Presentation of a continuous time model
5. Two case studies
6. Conclusion and extensions

- **PAYG systems are under pressure making funded schemes an interesting alternative...**
  
- **...but the financial downturn seriously damaged funded systems**
  - DB pension plans faced huge losses - UK DB plans recorded more than £250bn in losses, in the first quarter 2009 (source : PPF 7800 index)
  - American DC plans lost more than \$1,000bn, between 2007 and 2008 (source: Federal Reserve, Flow of Funds)
  
- **The OECD estimates global funded pension scheme losses at \$ 5,000bn**

- **New problems and issues**
  - The crisis revealed the limitations of financial products and highlighted “overlooked” types of risk.
  - Is the sharp decline of stock market valuation an opportunity for long term investors?
  
- **New challenges for investment strategy design**
  - Building optimal strategies...
  - ... to draw benefits from stock market properties mixing both short and long stock market dynamics

- **Provide evidence of market opportunities**
  - Detection of stock returns momentum
  - Detection of mean reverting stock returns
  
- **Build a framework which takes both properties into account**
  - Inclusion of both stock return momentum and mean reversion within the stock market dynamics
  - Derivation of an optimal portfolio solution

- **Lo and MacKinlay (1988):** Strongly reject the random walk assumption for stock markets returns.
- **Fama and French (1988):** Split stock prices into a predictable permanent component and a temporary component that swings away from the fundamental value.
- **Poterba and Summers (1987):** Stock returns exhibit positive autocorrelation in the short term and negative serial correlation in the long term.
- **Balvers, Gilliland and Wu (2000):** OECD stock returns follow stationary processes. Chaudhuri and Wu (2004) confirm this result for emerging countries.

- **Lo and MacKinlay (1990)** showed that benefits can be drawn from stock market overreactions, allowing thus the construction of contrarian strategies which consists in “selling winners and buying losers”.
- **For Balvers, Gilliland and Wu (2000)**, accumulated return deficits over a specific stock market can be offset within a limited period of time (due to the mean reversion properties).
- **Balvers and Wu (2005)**: Stock returns can be persistent in the short term. The combination of momentum and mean reverting properties in building financial strategies outperforms pure momentum or pure mean reverting strategies.

- **Random walk tests**
  - Lo & MacKinlay and Chow & Denning variance ratio tests
  - Runs test (non parametric test)
  
- **Continuation in stock return**
  - Autocorrelation function and Breush & Godfrey test
  - ARCH effects test
  
- **Mean reversion tests**
  - Individual unit root test (ADF – PP – KPSS)
  - Panel unit test (Levin, Lin and Shin – Maddala and Wu)

**Chow and Denning Tests for the 2, 10 and 20 months holding periods**

<b>Belgium</b>		<b>Hong Kong</b>	
Z(n)	<b>3.466051</b>	Z(n)	1.911954
Z*(n)	2.142063	Z*(n)	1.36082
<b>Canada</b>		<b>Italy</b>	
Z(n)	1.915782	Z(n)	<b>2.617628</b>
Z*(n)	1.27519	Z*(n)	<b>2.34654</b>
<b>France</b>		<b>Japan</b>	
Z(n)	1.822213	Z(n)	<b>3.0621</b>
Z*(n)	1.534671	Z*(n)	<b>2.8271</b>
<b>Germany</b>		<b>Netherlands</b>	
Z(n)	1.334892	Z(n)	0.900058
Z*(n)	1.013949	Z*(n)	0.6176293
<b>Sweden</b>		<b>Australia</b>	
Z(n)	1.8951	Z(n)	0.8658385
Z*(n)	1.5122	Z*(n)	0.7588312
<b>Norway</b>		<b>United Kingdom</b>	
Z(n)	<b>2.862553</b>	Z(n)	<b>2.3158</b>
Z*(n)	1.824094	Z*(n)	1.4624
<b>Spain</b>		<b>United States</b>	
Z(n)	<b>3.079421</b>	Z(n)	1.334922
Z*(n)	<b>2.716869</b>	Z*(n)	0.8228615

Stock Returns\* Autocorrelation function for different time horizons

	Autocorrelation orders				
	1 Month	2 Months	3 Months	4 Months	6 Months
<b>Belguim</b>	<b>0,137</b>	0,025	0,038	<b>0,104</b>	0,031
	8,9728	9,2727	9,9528	15,191	15,714
<b>Canada</b>	<b>0,095</b>	-0,05	<b>0,084</b>	0,009	0,034
	4,3149	5,5121	8,8798	8,9228	10,166
<b>France</b>	<b>0,077</b>	-0,027	<b>0,118</b>	0,037	0,024
	2,8124	3,1604	9,794	10,463	11,202
<b>Italy</b>	<b>0,096</b>	-0,051	<b>0,116</b>	<b>0,088</b>	<b>0,114</b>
	4,4049	5,6486	12,111	15,816	22,528
<b>Japan</b>	<b>0,096</b>	-0,035	<b>0,093</b>	0,074	0,001
	4,4223	5,0119	9,1425	11,756	12,027
<b>Norway</b>	<b>0,123</b>	-0,001	<b>0,126</b>	-0,056	0,068
	7,2238	7,2242	14,805	16,297	18,823
<b>Spain</b>	<b>0,107</b>	-0,07	0,027	0,039	<b>0,136</b>
	5,4365	7,7765	8,1289	8,8431	19,045
<b>United Kingdom</b>	<b>0,12</b>	-0,063	0,075	0,041	-0,015
	6,8594	8,7427	11,438	12,238	17,015

\* We considere arithmetics returns for the autocorelation function calculation. For each serial correlation the Ljung-Box Q statistics is provided. The Ljung-Box Q stat follows a  $\chi^2(r)$  distribution.

**Individual Stock Returns Unit Root test**

	Augmented Dickey Fuller Test		Philipps and Perron test		KPSS test based on the LM tests
	<i>t-statistics</i>	<i>p-value</i>	<i>t-statistics</i>	<i>p-value</i>	<i>t-statistics</i>
Australia	-20,824	0,00	-20,816	0,00	0,0663
Belguim	-18,742	0,00	-18,966	0,00	0,1198
Canada*	-19,908	0,00	-19,897	0,00	0,0332
France	-19,935	0,00	-20,140	0,00	0,0798
Germany*	-20,821	0,00	-20,820	0,00	0,0602
Hong Kong**	-20,159	0,00	-20,100	0,00	0,2273
Italy	-19,760	0,00	-19,944	0,00	0,1140
Japan**	-19,707	0,00	-19,836	0,00	0,4066
Netherlands**	-21,134	0,00	-21,134	0,00	0,1370
Norway	-19,033	0,00	-19,301	0,00	0,0427
Spain	-19,350	0,00	-19,306	0,00	0,2473
Sweden **	-20,314	0,00	-20,423	0,00	0,0710
United Kingdom	-19,136	0,00	-19,116	0,00	0,1145
United States*	-20,675	0,00	-20,695	0,00	0,1558

\* The Constant is significant at the 10% level. \*\* The Constant is significant at the 5% level

- A **defined contribution** pension plan investor ...

... with a **positive initial wealth** (self financing framework is assumed).

There are no inflows and no outflows.

- The pension wealth can be invested in **risky and in risk-free assets**
- The pension plan member aims to benefit from market opportunities
- **Objective:** optimize financial pension wealth over a finite horizon  $T$ .

- Focusing on the stock market

$$\frac{dS_t}{S_t} = (\mu_t + am_t)dt + \sigma_S dZ_{S,t}$$

- with

$$\begin{cases} \mu_t \equiv r_t + \mu_t - r_t \\ m_t \equiv M_t - \mu_t \end{cases}$$

- and

$$M_t = \int_0^t e^{-\int_0^t (t-u) \frac{dS_u}{S_u}} \Leftrightarrow dM_t = (1-a)(\mu_t - M_t)dt + \sigma_S dZ_{S,t}$$

- which leads to

$$\frac{dS_t}{S_t} = (r_t + \lambda_t + am_t)dt + \sigma_S dZ_{S,t}$$

- with  $\lambda_t$  the stock excess return and  $am_t$  the additional stock returns coming from the equity market's « abnormal » behaviour .
- As in Kim and Omberg (1996), the excess return is assumed to **mean revert**

$$d\lambda_t = \kappa(\bar{\lambda} - \lambda_t)dt + \sigma_\lambda dZ_{S,t}$$

- The **risk free asset dynamics** is modelled as follow

$$\frac{dB_t}{B_t} = r_t dt$$

- The basic case where there is a risky asset and a risk free asset
  - The pension manager aims to optimize the pension wealth  $X_t$
  - The problem is hence

$$J(X(t), M(t), \lambda(t), t, T) \equiv \underset{\pi_s, s \in [t, T]}{\text{Max}} \left\{ E_t [U(X_T)] \right\}$$

- Which leads to the following three separated portfolios

$$\pi_t^* = \underbrace{-\frac{J_X}{J_{XX} X} \left[ \frac{\mu_t - r_t}{\sigma_S^2} + \frac{a(M_t - \mu_t)}{\sigma_S^2} \right]}_{\text{Part I}} \underbrace{-\frac{J_{XM}}{X J_{XX}}}_{\text{Part II}} \underbrace{-\frac{J_{X\lambda}}{J_{XX} X} \frac{\sigma_\lambda}{\sigma_S}}_{\text{Part III}}$$

- Relaxing the interest rate assumption
  - Short term interest rate dynamics are assumed to mean revert

$$dr_t = \beta(\bar{r} - r_t)dt + \sigma_r dZ_r$$

- Let  $P_K(t, r_t)$  be the zero coupon bond price. Hence  $P_K(t, r_t)$  is given by

$$\frac{dP_K(r_t, t)}{P_K(r_t, t)} = (r_t + a_K \sigma_r \xi_P)dt + a_K \sigma_r dZ_{r,t}$$

- Thus, the pension wealth becomes

$$dX_t = X_t \left[ \pi_t \left( (r_t + \lambda_t + am_t)dt + \sigma_S dZ_{S,t} \right) + (1 - \pi_t) \left( (r_t + a_K \sigma_r \xi_P)dt + a_K \sigma_r dZ_{r,t} \right) \right]$$

- The optimization problem becomes

$$J(X(t), M(t), \lambda(t), r(t), t, T) \equiv \underset{\pi_s, s \in [t, T]}{\text{Max}} \left\{ E_t [U(X_T)] \right\}$$

- In this framework, the final solution is made up of 4 distinct hedging demands

$$\begin{aligned} \pi^* = & \underbrace{\Omega_{Sr}^{-1} \frac{J_X}{XJ_{XX}} (\lambda_t + am_t - a_K \sigma_r \xi_P)}_{\text{Adjusted market portfolio (Merton and market timing) from zero coupon bond price volatility}} + \underbrace{\Omega_{Sr}^{-1} \frac{J_{XM}}{XJ_{XX}} (\sigma_S^2 + a_K \sigma_{Sr})}_{\text{Hedging demand against unexpected changes in market opportunities (momentum)}} \\ & + \underbrace{\Omega_{Sr}^{-1} \frac{J_{X\lambda}}{XJ_{XX}} (\sigma_S \sigma_\lambda + a_K \sigma_r \sigma_\lambda \rho_{Sr})}_{\text{Hedging demand against unexpected long term stock market changes (i.e. risk premium)}} + \underbrace{\Omega_{Sr}^{-1} (\sigma_{Sr} + a_K \sigma_r^2) \left( \frac{J_{Xr}}{XJ_{XX}} + a_K \right)}_{\text{Hedging demand against unexpected interest rates changes}} \end{aligned}$$

- **Statistic tests**
  - Evidence of both momentum and mean reversion in stock returns.
  
- **Continuous time model for a DC investor**
  - Derive an optimal portfolio for different market configurations insisting on the role played by each source of risks within the final solutions.
  - Share of stocks held in the optimal portfolio is influenced by **momentum and mean reversion parameters**.
  - A specific fraction of the optimal portfolio consists of a **hedging demand against unexpected changes in recent stock market returns**.

- **Moving away from the self financing framework**
  - Introducing a stochastic wage process...
  - ... and thus inflows during the accumulation phase.
  - The incomplete market framework can be considered
  
- **Considering jumps in stock market dynamics as another source of abnormal return**
  - Allowing the inclusion of surprises in the stock index
  
- **Simulating the solutions**
  - Measure the degree of efficiency: the parameter “ $a$ ”
  - Evaluating the financial strategies through numerical methods



Thank you  
for your attention