Defined Contribution Pension Plans Management and Market Opportunities

AFIR-LIFE Colloquium 10th September 2009

Mabrouk Chetouane

University Paris Dauphine LEDa-SDFi

Outline

1. Introduction: Yet a pensions storm!
2. The literature on stock market efficiency and investment strategies
3. Empirical evidence on stock market opportunities
4. Presentation of a continuous time model
5. Two case studies
6. Conclusion and extensions

Pensions Under Pressure

- **PAYG systems are under pressure making funded schemes an interesting alternative…**

- **…but the financial downturn seriously damaged funded systems**

  - DB pension plans faced huge losses - UK DB plans recorded more than £250bn in losses, in the first quarter 2009 (source : PPF 7800 index)
  - American DC plans lost more than $1,000bn, between 2007 and 2008 (source: Federal Reserve, Flow of Funds)

- **The OECD estimates global funded pension scheme losses at $ 5,000bn**

New Challenges for Pension Investment Strategy

- **New problems and issues**
  - The crisis revealed the limitations of financial products and highlighted “overlooked” types of risk.
  - Is the sharp decline of stock market valuation an opportunity for long term investors?

- **New challenges for investment strategy design**
  - Building optimal strategies…
  - … to draw benefits from stock market properties mixing both short and long stock market dynamics

Our aim: Optimizing Equity Investment Strategies

- **Provide evidence of market opportunities**
  - Detection of stock returns momentum
  - Detection of mean reverting stock returns

- **Build a framework which takes both properties into account**
  - Inclusion of both stock return momentum and mean reversion within the stock market dynamics
  - Derivation of an optimal portfolio solution

Opportunities Are Present

- **Lo and MacKinlay (1988)**: Strongly reject the random walk assumption for stock markets returns.

- **Fama and French (1988)**: Split stock prices into a predictable permanent component and a temporary component that swings away from the fundamental value.

- **Poterba and Summers (1987)**: Stock returns exhibit positive autocorrelation in the short term and negative serial correlation in the long term.

- **Balvers, Gilliland and Wu (2000)**: OECD stock returns follow stationary processes. Chaudhuri and Wu (2004) confirm this result for emerging countries.

Structuring Solutions: Opportunities Can Be Used

- **Lo and MacKinlay (1990)** showed that benefits can be drawn from stock market overreactions, allowing thus the construction of contrarian strategies which consists in “selling winners and buying losers”.

- **For Balvers, Gilliland and Wu (2000)**, accumulated return deficits over a specific stock market can be offset within a limited period of time (due to the mean reversion properties).

- **Balvers and Wu (2005)**: Stock returns can be persistent in the short term. The combination of momentum and mean reverting properties in building financial strategies outperforms pure momentum or pure mean reverting strategies.

Detecting Market Opportunities

- **Random walk tests**
  - Lo & MacKinlay and Chow & Denning variance ratio tests
  - Runs test (non parametric test)
- **Continuation in stock return**
  - Autocorrelation function and Breush & Godfrey test
  - ARCH effects test
- **Mean reversion tests**
  - Individual unit root test (ADF – PP – KPSS)
  - Panel unit test (Levin, Lin and Shin – Maddala and Wu)

Random Walk Test

Chow and Denning Tests for the 2, 10 and 20 months holding periods

Belgium		**Hong Kong**	
Z(n)	**3.466051**	Z(n)	1.911954
Z*(n)	2.142063	Z*(n)	1.36082
Canada		**Italy**	
Z(n)	1.915782	Z(n)	**2.617628**
Z*(n)	1.27519	Z*(n)	**2.34654**
France		**Japan**	
Z(n)	1.822213	Z(n)	**3.0621**
Z*(n)	1.534671	Z*(n)	**2.8271**
Germany		**Netherlands**	
Z(n)	1.334892	Z(n)	0.900058
Z*(n)	1.013949	Z*(n)	0.6176293
Sweden		**Australia**	
Z(n)	1.8951	Z(n)	0.8658385
Z*(n)	1.5122	Z*(n)	0.7588312
Norway		**United Kingdom**	
Z(n)	**2.862553**	Z(n)	**2.3158**
Z*(n)	1.824094	Z*(n)	1.4624
Spain		**United States**	
Z(n)	**3.079421**	Z(n)	1.334922
Z*(n)	**2.716869**	Z*(n)	0.8228615

Stock return continuation test

Stock Returns* Autocorrelation function for diffrent time horizons

	Autocorrelation orders				
	1 Month	**2 Months**	**3 Months**	**4 Months**	**6 Months**
Belguim	**0,137**	0,025	0,038	**0,104**	0,031
	8,9728	9,2727	9,9528	15,191	15,714
Canada	**0,095**	-0,05	**0,084**	0,009	0,034
	4,3149	5,5121	8,8798	8,9228	10,166
France	**0,077**	-0,027	**0,118**	0,037	0,024
	2,8124	3,1604	9,794	10,463	11,202
Italy	**0,096**	-0,051	**0,116**	**0,088**	**0,114**
	4,4049	5,6486	12,111	15,816	22,528
Japan	**0,096**	-0,035	**0,093**	0,074	0,001
	4,4223	5,0119	9,1425	11,756	12,027
Norway	**0,123**	-0,001	**0,126**	-0,056	0,068
	7,2238	7,2242	14,805	16,297	18,823
Spain	**0,107**	-0,07	0,027	0,039	**0,136**
	5,4365	7,7765	8,1289	8,8431	19,045
United Kingdom	**0,12**	-0,063	0,075	0,041	-0,015
	6,8594	8,7427	11,438	12,238	17,015

* We considere arithmetics returns for the autocorelation function calculation. For each serial correlation the Ljung-Box Q statistics is provided. The Ljung-Box Q stat follows a $\chi^2(r)$ distribution.

Mean reverting stock returns test

Individual Stock Returns Unit Root test

	Augmented Dickey Fuller Test		**Philipps and Perron test**		**KPSS test based on the LM tests**
	t-statistics	***p-value***	***t-statistics***	***p-value***	***t-statistics***
Australia	-20,824	*0,00*	-20,816	*0,00*	0,0663
Belguim	-18,742	*0,00*	-18,966	*0,00*	0,1198
Canada*	-19,908	*0,00*	-19,897	*0,00*	0,0332
France	-19,935	*0,00*	-20,140	*0,00*	0,0798
Germany*	-20,821	*0,00*	-20,820	*0,00*	0,0602
Hong Kong**	-20,159	*0,00*	-20,100	*0,00*	0,2273
Italy	-19,760	*0,00*	-19,944	*0,00*	0,1140
Japan**	-19,707	*0,00*	-19,836	*0,00*	0,4066
Netherlands**	-21,134	*0,00*	-21,134	*0,00*	0,1370
Norway	-19,033	*0,00*	-19,301	*0,00*	0,0427
Spain	-19,350	*0,00*	-19,306	*0,00*	0,2473
Sweden **	-20,314	*0,00*	-20,423	*0,00*	0,0710
United Kingdom	-19,136	*0,00*	-19,116	*0,00*	0,1145
United States*	-20,675	*0,00*	-20,695	*0,00*	0,1558

* The Constant is significant at the 10% level. ** The Constant is significant at the 5% level

- A **defined contribution** pension plan investor …

… with a **positive initial wealth** (self financing framework is assumed).
There are no inflows and no outflows.

- The pension wealth can be invested in **risky and in risk-free assets**

- The pension plan member aims to benefit from market opportunities

- **Objective:** optimize financial pension wealth over a finite horizon T.

Asset Class Returns (I)

- Focusing on the stock market

$$\frac{dS_t}{S_t} = \left(\mu_t + a m_t\right)dt + \sigma_S dZ_{S,t}$$

- with

$$\begin{cases} \mu_t \equiv r_t + \mu_t - r_t \\ m_t \equiv M_t - \mu_t \end{cases}$$

- and

$$M_t = \int_0^t e^{-(t-u)} \frac{dS_u}{S_u} \Leftrightarrow dM_t = \left(1-a\right)\left(\mu_t - M_t\right)dt + \sigma_S dZ_{S,t}$$

Asset Class Returns (II)

- which leads to

$$\frac{dS_t}{S_t} = \left(r_t + \lambda_t + am_t\right)dt + \sigma_S dZ_{S,t}$$

- with λ_t the stock excess return and am_t the additional stock returns coming from the equity market's « abnormal » behaviour .

- As in Kim and Omberg (1996), the excess return is assumed to **mean revert**

$$d\lambda_t = \kappa\left(\bar{\lambda} - \lambda_t\right)dt + \sigma_\lambda dZ_{S,t}$$

- The **risk free asset dynamics** is modelled as follow

$$\frac{dB_t}{B_t} = r_t dt$$

Case 1
Solution of the Problem (I)

- **The basic case where there is a risky asset and a risk free asset**
  - The pension manager aims to optimize the pension wealth X_t
  - The problem is hence

$$J\left(X(t), M(t), \lambda(t), t, T\right) \equiv \underset{\pi_S, s\in[t,T]}{Max}\left\{E_t\left[U\left(X_T\right)\right]\right\}$$

- **Which leads to the following three separated portfolios**

$$\pi_t^* = \underbrace{-\frac{J_X}{J_{XX}X}\left[\frac{\mu_t - r_t}{\sigma_S^2} + \frac{a\left(M_t - \mu_t\right)}{\sigma_S^2}\right]}_{Part\ I} \underbrace{-\frac{J_{XM}}{XJ_{XX}}}_{Part\ II} \underbrace{-\frac{J_{X\lambda}}{J_{XX}X}\frac{\sigma_\lambda}{\sigma_S}}_{Part\ III}$$

Solution of the Problem (II)

- **Relaxing the interest rate assumption**
  - Short term interest rate dynamics are assumed to mean revert

$$dr_t = \beta\left(\bar{r} - r_t\right)dt + \sigma_r dZ_r$$

- **Let $P_K\left(t, r_t\right)$ be the zero coupon bond price. Hence $P_K\left(t, r_t\right)$ is given by**

$$\frac{dP_K\left(r_t, t\right)}{P_K\left(r_t, t\right)} = \left(r_t + a_K \sigma_r \xi_P\right)dt + a_K \sigma_r dZ_{r,t}$$

- **Thus, the pension wealth becomes**

$$dX_t = X_t\left[\pi_t\left(\left(r_t + \lambda_t + am_t\right)dt + \sigma_S dZ_{S,t}\right) + \left(1\text{-}\pi_t\right)\left(\left(r_t + a_K \sigma_r \xi_P\right)dt + a_K \sigma_r dZ_{r,t}\right)\right]$$

Solution of the Problem (III)

- The optimization problem becomes

$$J\left(X(t),M(t),\lambda(t),r(t),t,T\right)\equiv \underset{\pi_S,s\in[t,T]}{Max}\left\{E_t\left[U\left(X_T\right)\right]\right\}$$

- In this framework, the final solution is made up of 4 distinct hedging demands

$$\pi^* = \underbrace{\Omega_{Sr}^{-1}\frac{J_X}{XJ_{XX}}\left(\lambda_t+am_t-a_K\sigma_r\xi_P\right)}_{\substack{\text{Adjusted market portfolio (Merton and market timing)}\\ \text{from zero coupon bond price volatility}}} + \underbrace{\Omega_{Sr}^{-1}\frac{J_{XM}}{XJ_{XX}}\left(\sigma_S^2+a_K\sigma_{Sr}\right)}_{\substack{\text{Hedging demand against}\\ \text{unexpected changes in market}\\ \text{opportunities (momentum)}}}$$

$$+\underbrace{\Omega_{Sr}^{-1}\frac{J_{X\lambda}}{XJ_{XX}}\left(\sigma_S\sigma_\lambda+a_K\sigma_r\sigma_\lambda\rho_{Sr}\right)}_{\substack{\text{Hedging demand against unexpected}\\ \text{long term stock market changes}\\ \text{(i.e. risk premium)}}}+\underbrace{\Omega_{Sr}^{-1}\left(\sigma_{Sr}+a_K\sigma_r^2\right)\left(\frac{J_{Xr}}{XJ_{XX}}+a_K\right)}_{\substack{\text{Hedging demand against unexpected}\\ \text{interest rates changes}}}$$

Conclusion

- **Statistic tests**
  - Evidence of both momentum and mean reversion in stock returns.

- **Continuous time model for a DC investor**
  - Derive an optimal portfolio for different market configurations insisting on the role played by each source of risks within the final solutions.
  - Share of stocks held in the optimal portfolio is influenced by **momentum and mean reversion parameters**.
  - A specific fraction of the optimal portfolio consists of a **hedging demand against unexpected changes in recent stock market returns**.

The extensions

- **Moving away from the self financing framework**
  - Introducing a stochastic wage process...
  - ... and thus inflows during the accumulation phase.
  - The incomplete market framework can be considered

- **Considering jumps in stock market dynamics as another source of abnormal return**
  - Allowing the inclusion of surprises in the stock index

- **Simulating the solutions**
  - Measure the degree of efficiency: the parameter "a"
  - Evaluating the financial strategies through numerical methods

Thank you

for your attention