Market Risk Prediction under Long Memory: When VaR is Higher than Expected

Harald Kinateder
Niklas Wagner

DekaBank Chair in Finance and Financial Control
Passau University

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Motivation

Several periods of financial market stress:

- the market crash in October 1987,
- a number of accounting scandals at the beginning of the new millennium and
- the recent banking crises

have increased the regulatory and industry demand for effective (market) risk management approaches.

Despite the BIS demands no concrete method, one concept become popular: **Value-at-Risk** (VaR).

Motivation

- GARCH models generate satisfactory volatility forecasts for the **very next period**.
- **Long-term** VaR measures usually require volatility predictions for longer periods:
  - several weeks or even
  - several months.
- Despite their high practical relevance most focus has been placed on **one-day ahead** forecasts.

Motivation

Contribution of the article

- New insights into
  - risk prediction under long memory and
  - issues concerning backtesting for long-term risk measures.
- New scaling based GARCH-LM model for multi-period risk prediction.

Scaling

- In finance scaling is very important, since Basel rules of capital adequacy require banks to calculate VaR numbers for a minimum holding period of *at least* 10 days.
- Square-root-of-time rule:

$$VaR(1)\sqrt{\tau} = VaR(\tau).$$

Scaling

Premises of Square-Root-of-Time Rule

- independent and
- identically distributed (i.i.d.) returns process

Problem

Financial time series are not independent, because e.g. absolute or squared returns are highly correlated.

Consequences

- In the presence of long memory, it is not reasonable to scale by a fixed self-affinity parameter ($H = 0.5$).
- The degree of risk misspecification depends both on the risk horizon and the magnitude of long range dependence.

GARCH

GARCH-type VaR models are based on the assumption that empirical returns belong to a location-scale family of probability distributions of the form

$$R_t = \mu_t + \epsilon_t = \mu_t + Z_t \sigma_t.$$

The location μ_t and the scale σ_t are $\mathcal{F}_{t-1}$-measurable parameters and $Z_t \sim i.i.d.\ F(0,1)$.

GARCH

The **one-day ahead** GARCH VaR is obtained by

$$VaR^{\alpha}_{t,t+1} = \mu_{t+1} + \sigma_{t+1} F^{-1}_{\alpha},$$

where σ_{t+1} is the conditional standard deviation of R_t calculated by GARCH(1,1):

$$\sigma^2_{t+1} = \omega + \alpha\epsilon^2_t + \beta\sigma^2_t,$$

with $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta < 1$. When $\tau \rightarrow \infty$, the process σ^2_t is finite if and only if $\alpha + \beta < 1$, otherwise the process is non-stationary as $\sigma^2_t \rightarrow \infty$.

GARCH

The **multi-day ahead** GARCH variance prediction is obtained by

$$\sigma^2_{t+\tau} = \mathbb{E}(\epsilon^2_{t+\tau}|\mathcal{F}_t) = \sigma^2 + (\sigma^2_{t+1} - \sigma^2)(\alpha + \beta)^{\tau-1}$$

σ^2 denotes the unconditional variance of ϵ_t.

Drawbacks

- If the forecasting horizon τ rises and $\alpha + \beta < 1$ then $\sigma^2_t \to \sigma^2$.
- All relative weights on past squared returns decline at the same exponential rate $(\alpha + \beta)$.

GARCH-LM: A new Approach

The **multi-day ahead** VaR prediction in the novel setting is given by

$$VaR^{\alpha}_{t,t+\tau} = \mu_{t+\tau} + \phi(t+\tau)F^{-1}_{\alpha}.$$

In contrast to GARCH-based VaR forecasts, we substitute $\sigma_{t+\tau}$ by a scaling based variable $\phi(t+\tau)$:

$$\phi(t+\tau) = \tau^{H}\rho_{|R_t|}(\tau)^{H-\rho_{|R_t|}(\tau)}\sigma_{t+1}.$$

- H corresponds to the Hurst exponent or self-affinity parameter.
- $\rho_{|R_t|}(\tau)$ is the autocorrelation coefficient of $|R_t|$ for the time-lag τ. Assymption: $\rho_{|R_t|}(\tau) \neq 0$.

Backtesting

Special Backesting Issues

1. Which returns should be used?
   - **Overlapping returns**

     Con Autocorrelation

     Pro Backtesting criteria like Basel traffic light could be achieved easier as in case of non-overlapping returns.
   - **Non-overlapping returns**

     Pro No autocorrelation
2. **Multi-day** VaR figures exhibit an additional backtesting problem.
   - Due to higher risk horizon τ, the spread of $R_{t,t+\tau}$ increases $\Rightarrow$ the distance between $VaR_{t,t+\tau}$ and $R_{t,t+\tau}$ becomes more important in comparison to one-day ahead VaR.

Empirical Analysis

- The data contains 8,609 daily closing levels P_t from January 1, 1975 to December 31, 2007 of four international stock market indices:
  - DAX
  - Dow Jones
  - Nasdaq Composite
  - S&P 500
- We use non-overlapping continuously compounded percentage returns $R_{t,t+\tau}$ for different sampling frequencies $\tau \in \{5, 10, 20, 60\}$ days.

Long Range Dependence

In order to investigate the dependence structure of empirical returns, we calculate estimates of H for all indices, Brownian motion and test the null

- H_0: "$H = 0.5$" (**no dependence**) against
- H_1: "$H \neq 0.5$" (**dependence**).

Long Range Dependence

Index	R_t	t-value	R_t^2	t-value	$\|R_t\|$	t-value
DAX	0.520	1.91*	0.769	26.12	0.823	24.01
DOW JONES	0.476	-1.94*	0.614	8.32	0.769	19.99
NASDAQ	0.535	2.83	0.811	17.54	0.846	16.34
S&P 500	0.478	-1.95*	0.632	9.87	0.780	18.72
BM(8608)	0.490	-0.76*	0.487	-0.44*	0.495	-0.23*

Table: Empirical estimates of the Hurst exponent H for daily index data from January 1, 1975 to December 31, 2007. A theoretical estimate for simulated ordinary Brownian motion with 8,608 increments is provided for 10,000 replications. * denotes accepting the null at the 95% confidence level.

Long Range Dependence

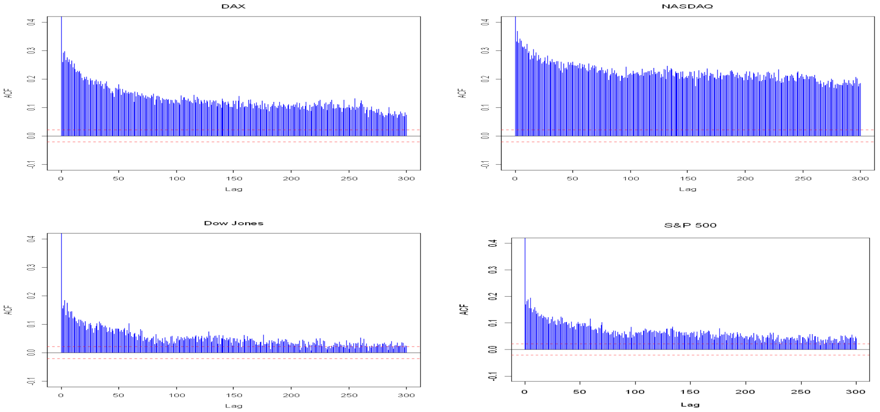


Figure: ACFs of absolute index returns. Sample period: January 1, 1975 to December 31, 2007.

VaR Forecasting Performance Results

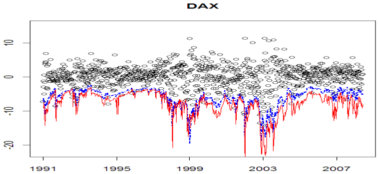
DAX

Horizon	Distribution		GARCH	LM
60 days	skewed-$t(3)$	$\overline{VaR}$	6.16	21.03
		$\%\ Viol.$	14.86	2.70
		LR_{uc}	40.37*	1.50
			[0.000]	[0.224]
		LR_{ind}	0.12	0.11
			[0.733]	[0.743]
		LR_{cc}	40.47*	1.61
			[0.000]	[0.452]
		LF	31.47	23.24

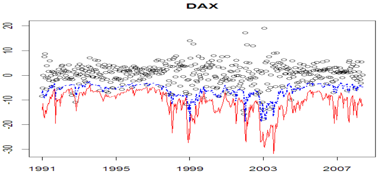
Table: 60-day ahead VaR forecasts for the GARCH and GARCH-LM model with skewed t-distribution from January 1, 1991 to December 31, 2007. * denotes rejecting the null at the 99% confidence level.

VaR Forecasting Performance Results

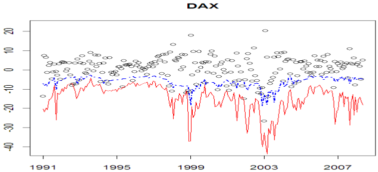
Skewed student-t distribution



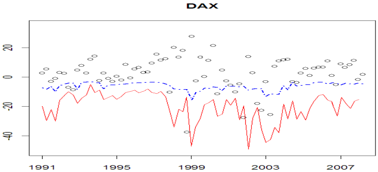
(a) 5 days



(b) 10 days



(c) 20 days



(d) 60 days