# MATHEMATICAL MODELS

# AND

# THE CREDIT CRUNCH

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# Plan

- Philosophy

- Background (brief!)

- Questions:

  – assumptions and limitations

  – model types

  – concealment of complexity and model risk

- What to do in the future

# Background to this presentation

Acknowledgements to

- Numerous friends & colleagues

- Osmosis (i.e. accumulation of uncorroborated evidence over many months)

- Several speakers this week

# Credit Crunch

- Turner Review (UK regulator):

  – [apparent] *misplaced reliance on sophisticated maths*

  – complexity $\Rightarrow$

    *difficult for top management and boards to assess and exercise judgement over risks being taken*

  – complexity of market not matched by improvements in modelling

  – VaR partly to blame

# Financial mathematicians must take some blame

Different individuals: some or all of

- Allowing models to be used inappropriately

- Not carrying out due diligence

- Not warning senior management abour risks

- Allowing bonus culture to over-rule common sense

$\longrightarrow$ operational risks

# Question 1: assumptions and limitations

Did users of models understand assumptions and limitations of models?

- Hypothesis:

  *nothing wrong with the underlying maths*

  BUT require full specification $+$ testing
- some models are better than others
- models must be fully scrutinised and tested
- underlying assumptions and limitations must be communicated upwards

## Assumptions

- Is a specific assumption: (A) true, (B) approximately correct, (C) laughably wrong?

- What will happen if the assumption is incorrect?

- What can be done to mitigate incorrect assumptions?

- e.g. Black-Scholes model $+$ delta hedging

  - Gamma hedging: rebalancing at discrete times, jumps in prices

  - Vega hedging: volatility changes from time to time

# Limitations

- Model designed for a specific contract

  then applied to other contracts

- What about less complex contracts?

- What about more complex contracts?

- Model $\Rightarrow$ price $+$ risk management strategy

- Model might fail if market gets too big

# Question 2: pricing versus risk-management models

Did users understand the difference between

- pricing models

- risk-management models

- risk-measurement models?

# Pricing models

- Also known as *market models*

- e.g. Black-Scholes model

- Model a subset of all risks

- No-arbitrage assumption $+$ dynamic hedging

- Risk-neutral pricing measure

- Simple enough to allow quick calculation of prices

- Calibration of parameters using today's market prices

# Pricing models

Pros:

- Model is *consistent* with what we observe *today* in the market

- Avoids mispricing of very similar contracts

Cons:

- Model might not be consistent with historical dynamics and data

- Approach to calibration might not be consistent with model assumptions

- e.g. recalibration of $\sigma$ in B-S model

# Pricing models

Dangers:

- avoids mispricing of very similar contracts BUT

- extension of pricing to new, less similar contracts creates a market based on the *assumed* truth of the model

- e.g. (???) Gaussian copula model + credit market

- Reality: embryo market:

  pricing models A and B both consistent with limited data

  BUT A and B $\Rightarrow$ different prices in expanded market

# Risk MANAGEMENT models

- Also known as *real-world models*

- Wider range of risks

- Calibrated to historical data

- Regular recalibration

- Rigorous statistical testing; model $+$ parameter risk

- Economic reasonableness

- Rational economic dynamics

  $\Rightarrow$ okay for risk control and optimisation

# Risk management models

Pros:

- Consistent with the past

- Realistic

- Proper assessment of risk

Cons:

- Difficult to calibrate in real time

- Difficult to price derivatives

- Theoretical prices not exactly equal to market prices

# Risk MEASUREMENT models

- Real-world models

- Incorporate market irrationality; inefficiency

  – information asymmetry

  – negative risk premiums

  – pro/counter cyclical dynamics

  – behavioural finance

    e.g. overconfidence; understatement of risks

- DO NOT attempt to optimise! ($\Rightarrow$ excessive leverage)

- Okay for: robustness of strategy $\Rightarrow$ ??? risk mitigation

# Question 3: complexity and model risk

Do quants and/or traders have the incentive to

- conceal the extent of contract and model complexity from investors?

- downplay model risk?

$\longleftrightarrow$ Creating complexity in order to profit from the ignorance of others

# A scenario – the illusion of understanding

- Marketing team: good idea for a new product

- Investors will only buy when

    – they think they understand the risk profile

    – they believe the product will help hedge risk or make money (alpha)

- Quants enlisted to help "educate" investors

# A scenario – the illusion of understanding

- Quants enlisted to help "educate" investors

- One model, one calibration:

  Result: "enlightenment" and SALES

- Many models $+$ parameter uncertainty

  Result: confusion and NO sales and NO bonuses

  (even when the product is good for risk reduction)

# Variant – The Marketing team

- incentive to conceal complexity and model & parameter risk from senior managers and directors

Regulators: need to enforce Prudent Person Principle

- Would you sell this product to your grandmother?

- Do you understand the risks fully?

- Does your customer understand the risks fully?

# What do we need to be doing in the future?

- Improved stochastic modelling

- Better availability and use of historical data

- A stronger voice for quants

- Alternatives to short-horizon quantile risk measures

- Stronger dialogue between academics, regulators $+$ banks

- ...

# Future Model Types

- Solvency II $\Rightarrow$

  Need combined Pricing + risk management models

- Why?

  S-II $\Rightarrow$ need market-consistent values in 1 year

BUT: is S-II too focused on short-term balance-sheet volatility?
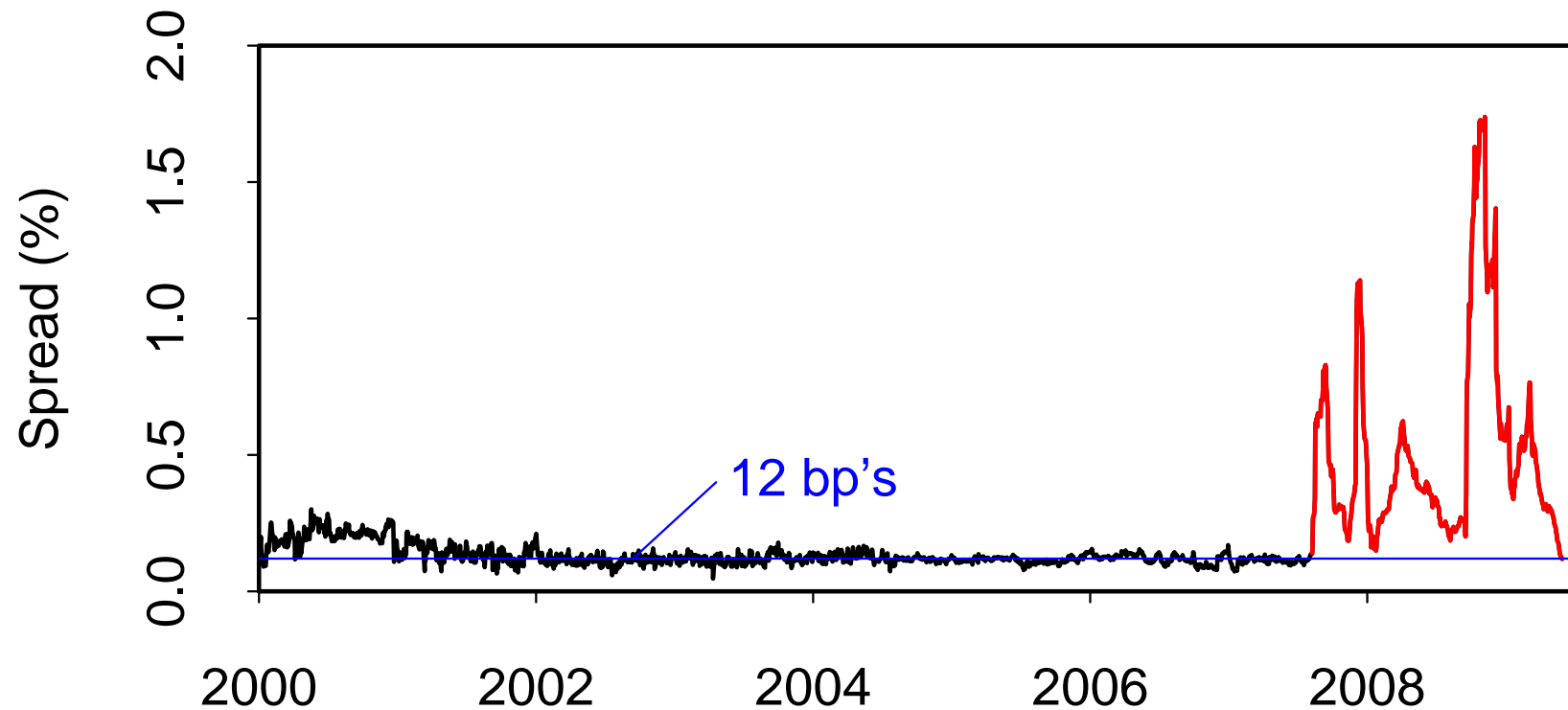
# Combined pricing $+$ RM models

Requirements:

- Realistic, multi-factor

- Process parameters $(\mu, \sigma, \rho, \ldots)$ calibrated using historical data

- State variables $(S(t), r(t), \sigma(t), \ldots)$ *calibrated* using market prices

- Dynamics of state variables consistent with model assumptions (c.f. pricing models)

# Regime shifts

Unsecured versus collateralised short-term loans



**1−month LIBOR minus 1−month REPO**

# Improved risk-management/measurement models

- Liquidity; buying/selling spreads; asymmetric info.

- Extreme regime shifts

  $\longrightarrow$ Liquidity, volatility, (perceived) information asymmetry, ...?

- Other latent variables

- Large-scale, destabilising feedback, hysteresis

- Fat tails, stochastic volatility

- Market irrationality, bahavioural finance etc.

# FTSE-100, 13-17 October 2008; log returns

$$+7.94\%, \;\; +3.17\%, \;\; -7.43\%, \;\; -5.50\%, \;\; +5.09\%$$

- A: i.i.d. normal model

- B: non-central-t distribution $+$ stochastic volatility

$p$-values (should be i.i.d. $\sim U[0,1]$):

| Day | 1 | 2 | 3 | 4 | 5 |
|-----|-----|-----|-----|-----|-----|
| A | 0.99999999999994 | 0.998 | 0.00000000001 | 0.0000001 | 0.999999 |
| B | 0.9792 | 0.7447 | 0.0610 | 0.1293 | 0.8476 |

# Improved modelling: Augmented by

- thorough analysis of model and parameter risk

  $\Rightarrow$ discourages excessive leverage!

- scenario analysis, stress tests and black swans

# A stronger voice for quants

- Quants; risk management team; external experts

- Walker Review (UK):

  Senior management and non-executive directors

  (NEDs$^{*}$) $\Leftarrow$ external advice

(*) NED: *Scots slang*: Non-Educated Delinquent

# A stronger voice for quants

- a greater number of NEDs should have a strong knowledge of QRM & ERM:

  *enough to be able to ask the right questions*

- Regulator $\Rightarrow$ "fit and proper" test

- Risk Committee NEDs should have access to middle and junior staff

# The role of Value-at-Risk

- In theory: VaR $\Rightarrow$ quantile

- In practice ????

  "VaR" $\Rightarrow$ quantile $+$ i.i.d. multivariate normality

- *VaR is not a* coherent *risk measure*

  - "non-coherence" was not a cause of the crisis

  - Expected-shortfall $+$ stochastic volatility $+$ fat tails $\Rightarrow$ bigger crisis ???

  - BUT optimise VaR $\Rightarrow$ small probability, high-severity risks

# Improving on traditional Value-at-Risk

- Use better models!

- How to avoid pro-cyclicality?

  - ????

  - Take the long term view

    e.g. run-off of life insurance liabilities

    ($\Rightarrow$ greater emphasis on cashflow matching)

  - Does Solvency II go far enough?

# Time to stop rambling ...

# Securitisation

Reasons:

- Convert illiquid into liquid assets

- Capitalise future illiquid cashflows

- Trading new risks $\Rightarrow E_{\mathsf{subj}}\big[\mathsf{utility}\big] \nearrow$ for all

Issues:

- Complex repackaging of existing liquid traded risks $\Rightarrow$ DANGER ???

- Securitisation of OWN risks $\Rightarrow$ moral hazard

- Insurable interest

- Risk-reduction or gambling?

# Better availability of data

- Longer runs of data

- Buying and selling prices (liquidity)

- Available for free (with a time lag) for non-commercial research