MATHEMATICAL MODELS

AND

THE CREDIT CRUNCH

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Plan

- Philosophy
- Background (brief!)
- Questions:
  - assumptions and limitations
  - model types
  - concealment of complexity and model risk
- What to do in the future

Background to this presentation

Acknowledgements to

- Numerous friends & colleagues
- Osmosis (i.e. accumulation of uncorroborated evidence over many months)
- Several speakers this week

Credit Crunch

- Turner Review (UK regulator):
  - [apparent] *misplaced reliance on sophisticated maths*
  - complexity $\Rightarrow$ *difficult for top management and boards to assess and exercise judgement over risks being taken*
  - complexity of market not matched by improvements in modelling
  - VaR partly to blame

Financial mathematicians must take some blame

Different individuals: some or all of

- Allowing models to be used inappropriately
- Not carrying out due diligence
- Not warning senior management abour risks
- Allowing bonus culture to over-rule common sense

$\longrightarrow$ operational risks

Question 1: assumptions and limitations

Did users of models understand assumptions and limitations of models?

- Hypothesis:

  nothing wrong with the underlying maths

  BUT require full specification + testing
- some models are better than others
- models must be fully scrutinised and tested
- underlying assumptions and limitations must be communicated upwards

Assumptions

- Is a specific assumption: (A) true, (B) approximately correct, (C) laughably wrong?
- What will happen if the assumption is incorrect?
- What can be done to mitigate incorrect assumptions?
- e.g. Black-Scholes model + delta hedging
  - Gamma hedging: rebalancing at discrete times, jumps in prices
  - Vega hedging: volatility changes from time to time

Limitations

- Model designed for a specific contract

  then applied to other contracts

- What about less complex contracts?

- What about more complex contracts?

- Model $\Rightarrow$ price $+$ risk management strategy

- Model might fail if market gets too big

Question 2: pricing versus risk-management models

Did users understand the difference between

- pricing models
- risk-management models
- risk-measurement models?

Pricing models

- Also known as *market models*
- e.g. Black-Scholes model
- Model a subset of all risks
- No-arbitrage assumption + dynamic hedging
- Risk-neutral pricing measure
- Simple enough to allow quick calculation of prices
- Calibration of parameters using today's market prices

Pricing models

Pros:

- Model is *consistent* with what we observe *today* in the market
- Avoids mispricing of very similar contracts

Cons:

- Model might not be consistent with historical dynamics and data
- Approach to calibration might not be consistent with model assumptions
- e.g. recalibration of σ in B-S model

Pricing models

Dangers:

- avoids mispricing of very similar contracts BUT
- extension of pricing to new, less similar contracts creates a market based on the *assumed* truth of the model
- e.g. (???) Gaussian copula model + credit market
- Reality: embryo market:

  pricing models A and B both consistent with limited data

  BUT A and B $\Rightarrow$ different prices in expanded market

Risk MANAGEMENT models

- Also known as *real-world models*
- Wider range of risks
- Calibrated to historical data
- Regular recalibration
- Rigorous statistical testing; model + parameter risk
- Economic reasonableness
- Rational economic dynamics

$\Rightarrow$ okay for risk control and optimisation

Risk management models

Pros:

- Consistent with the past
- Realistic
- Proper assessment of risk

Cons:

- Difficult to calibrate in real time
- Difficult to price derivatives
- Theoretical prices not exactly equal to market prices

Risk MEASUREMENT models

- Real-world models
- Incorporate market irrationality; inefficiency
  - information asymmetry
  - negative risk premiums
  - pro/counter cyclical dynamics
  - behavioural finance

    e.g. overconfidence; understatement of risks
- DO NOT attempt to optimise! ($\Rightarrow$ excessive leverage)
- Okay for: robustness of strategy $\Rightarrow$??? risk mitigation

Question 3: complexity and model risk

Do quants and/or traders have the incentive to

- conceal the extent of contract and model complexity from investors?
- downplay model risk?

$\longleftrightarrow$ Creating complexity in order to profit from the ignorance of others

A scenario – the illusion of understanding

- Marketing team: good idea for a new product
- Investors will only buy when
  - they think they understand the risk profile
  - they believe the product will help hedge risk or make money (alpha)
- Quants enlisted to help “educate” investors

A scenario – the illusion of understanding

- Quants enlisted to help “educate” investors
- One model, one calibration:

  Result: “enlightenment” and SALES
- Many models + parameter uncertainty

  Result: confusion and NO sales and NO bonuses

  (even when the product is good for risk reduction)

Variant – The Marketing team

- incentive to conceal complexity and model & parameter risk from senior managers and directors

Regulators: need to enforce Prudent Person Principle

- Would you sell this product to your grandmother?
- Do you understand the risks fully?
- Does your customer understand the risks fully?

What do we need to be doing in the future?

- Improved stochastic modelling
- Better availability and use of historical data
- A stronger voice for quants
- Alternatives to short-horizon quantile risk measures
- Stronger dialogue between academics, regulators + banks
- ...

Future Model Types

- Solvency II $\Rightarrow$

  Need combined Pricing $+$ risk management models

- Why?

  S-II $\Rightarrow$ need market-consistent values in 1 year

BUT: is S-II too focused on short-term balance-sheet volatility?

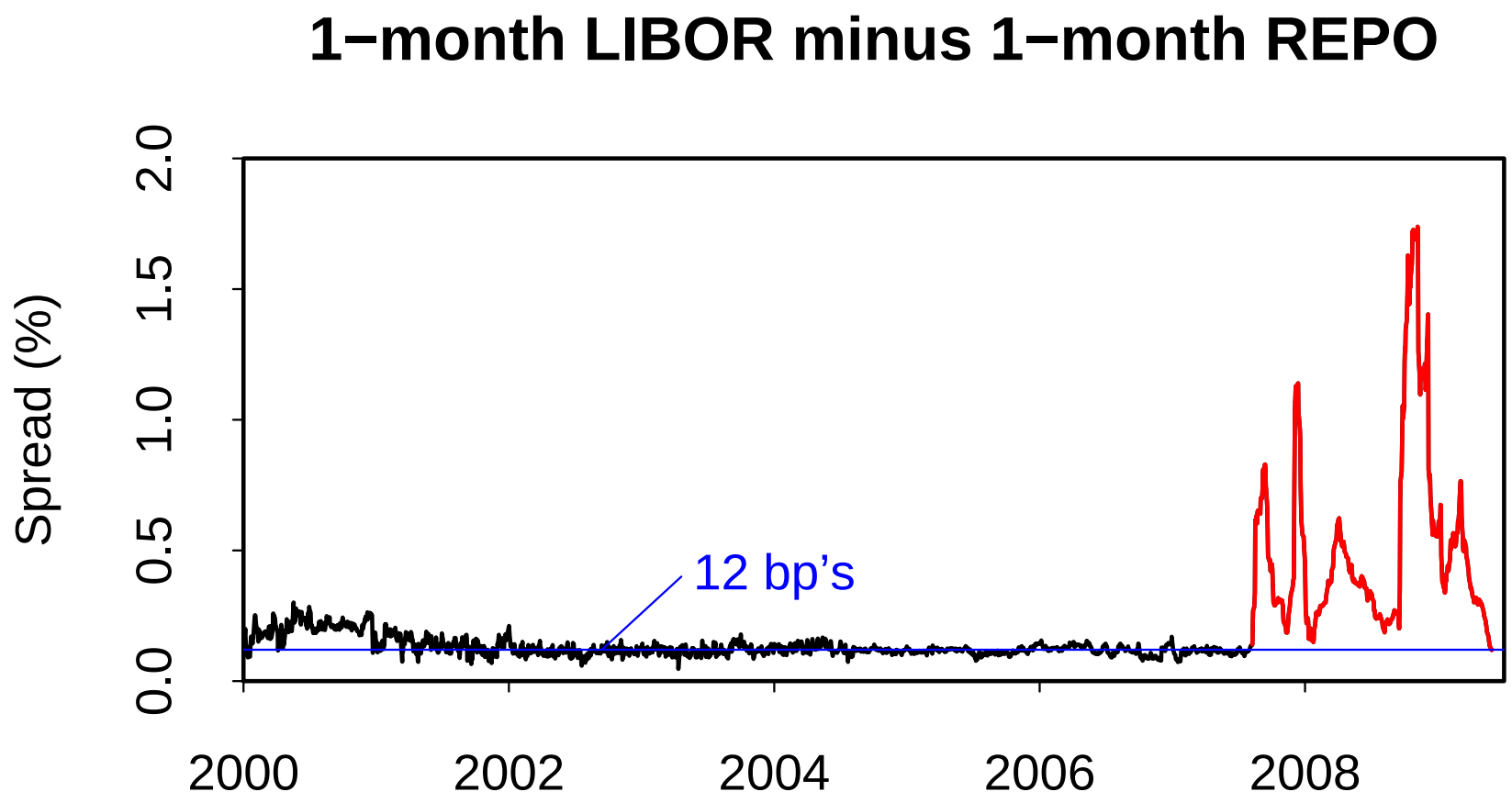
Combined pricing + RM models

Requirements:

- Realistic, multi-factor
- Process parameters ($\mu, \sigma, \rho, \ldots$) calibrated using historical data
- State variables ($S(t), r(t), \sigma(t), \ldots$) *calibrated* using market prices
- Dynamics of state variables consistent with model assumptions (c.f. pricing models)

Regime shifts

Unsecured versus collateralised short-term loans



Improved risk-management/measurement models

- Liquidity; buying/selling spreads; asymmetric info.
- Extreme regime shifts

  $\longrightarrow$ Liquidity, volatility, (perceived) information asymmetry, ...?
- Other latent variables
- Large-scale, destabilising feedback, hysteresis
- Fat tails, stochastic volatility
- Market irrationality, bahavioural finance etc.

FTSE-100, 13-17 October 2008; log returns

$$+7.94\%, \quad +3.17\%, \quad -7.43\%, \quad -5.50\%, \quad +5.09\%$$

- A: i.i.d. normal model
- B: non-central-t distribution + stochastic volatility

p-values (should be i.i.d. $\sim U[0,1]$):

Day	1	2	3	4	5
A	0.9999999999994	0.998	0.000000000001	0.0000001	0.999999
B	0.9792	0.7447	0.0610	0.1293	0.8476

Improved modelling: Augmented by

- thorough analysis of model and parameter risk

  $\Rightarrow$ discourages excessive leverage!

- scenario analysis, stress tests and black swans

A stronger voice for quants

- Quants; risk management team; external experts
- Walker Review (UK):

  Senior management and non-executive directors

  (NEDs*) $\Leftarrow$ external advice

(*) NED: *Scots slang*: Non-Educated Delinquent

A stronger voice for quants

- a greater number of NEDs should have a strong knowledge of QRM & ERM:

  enough to be able to ask the right questions

- Regulator $\Rightarrow$ "fit and proper" test
- Risk Committee NEDs should have access to middle and junior staff

The role of Value-at-Risk

- In theory: VaR $\Rightarrow$ quantile
- In practice ????

  "VaR" $\Rightarrow$ quantile $+$ i.i.d. multivariate normality
- *VaR is not a* coherent *risk measure*
  - "non-coherence" was not a cause of the crisis
  - Expected-shortfall $+$ stochastic volatility $+$ fat tails $\Rightarrow$ bigger crisis ???
  - BUT optimise VaR $\Rightarrow$ small probability, high-severity risks

Improving on traditional Value-at-Risk

- Use better models!
- How to avoid pro-cyclicality?
  - ????
  - Take the long term view

    e.g. run-off of life insurance liabilities

    ($\Rightarrow$ greater emphasis on cashflow matching)
  - Does Solvency II go far enough?

Time to stop rambling ...

Securitisation

Reasons:

- Convert illiquid into liquid assets
- Capitalise future illiquid cashflows
- Trading new risks $\Rightarrow$ $E_{\text{subj}}[\text{utility}] \nearrow$ for all

Issues:

- Complex repackaging of existing liquid traded risks $\Rightarrow$ DANGER ???
- Securitisation of OWN risks $\Rightarrow$ moral hazard
- Insurable interest
- Risk-reduction or gambling?

Better availability of data

- Longer runs of data
- Buying and selling prices (liquidity)
- Available for free (with a time lag) for non-commercial research