A longitudinal study on Portfolio Optimization: Is the “Success” Time Dependent?

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Aim of the research

Compare and contrast the performance of different portfolio optimization strategies in various 5-year holding periods over a time horizon of almost three decades

Portfolio Selection Strategies

- Minimum-Variance Portfolio (MVP) / Markowitz
- Minimum-CVaR Portfolio (MCVaR) / Rockafellar-Uryasev
- Log-Optimal Portfolio / Györfi-Ottucsák-Urbán
- Benchmarks
  - Naïve Portfolio
  - US investment
- Naïve – MVP
- Naïve - MCVaR

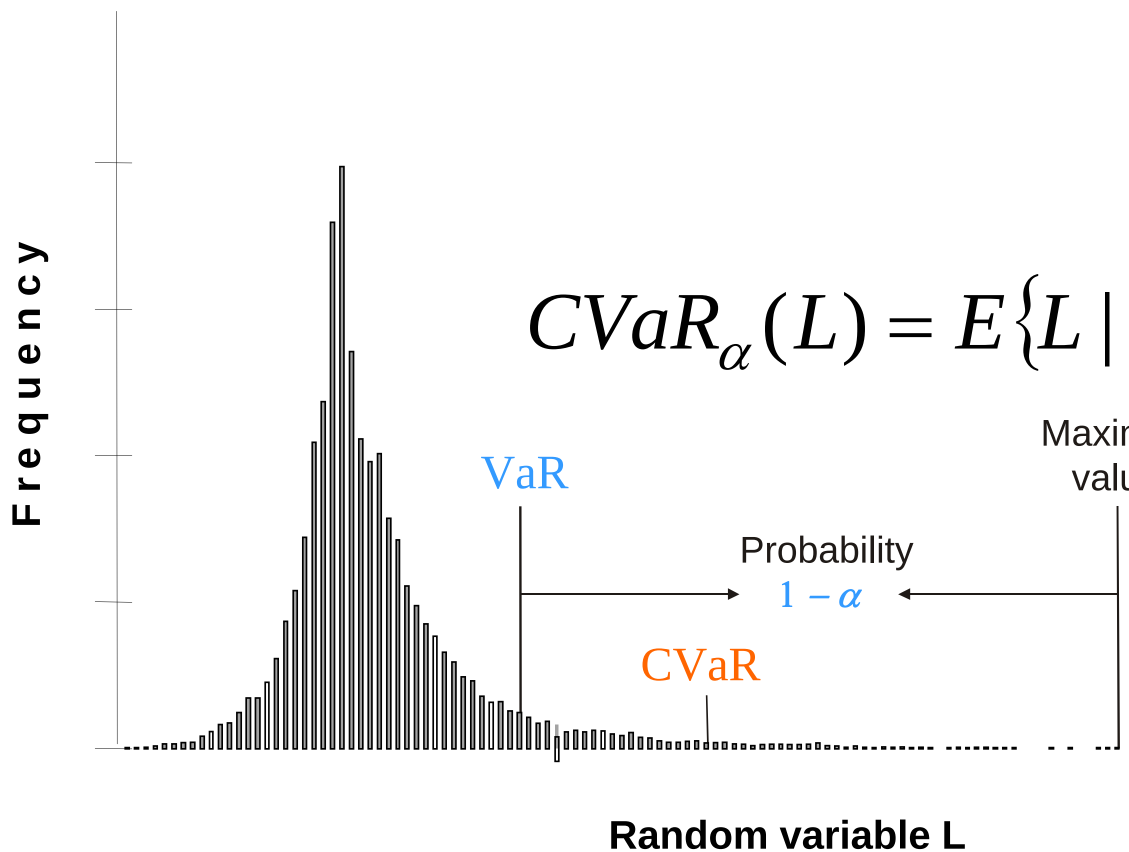
Database

- MSCI equity indexes taken from Datastream
- Daily frequency
- Equity price index returns of 18 developed stock markets
  - Australia (AU), Austria (AT), Belgium (BE), Canada (CA), Denmark (DK), France (FR), Germany (DE), Hong Kong (HK), Italy (IT), Japan (JP), the Netherlands (NL), Norway (NO), Singapore (SG), Spain (ES), Sweden (SE), Switzerland (CH), the United Kingdom (GB) and the USA (US).

Value at Risk, Conditional Value at Risk

$$VaR_{\alpha} = q\alpha$$

$$CVaR_{\alpha}(L) = E\{L \mid L \geq VaR_{\alpha}(L)\}$$



Methodology – Mean - Variance and Mean - CVaR portfolio optimization

Minimum-Variance Portfolio (MVP)

$$\min V(\underline{x}) = \sum_{i=1}^{n}\sum_{k=1}^{n} x_i x_k \rho_{i,k} \sigma_i \sigma_k$$

subject to

$$\sum_{i=1}^{n} x_i = 1, \qquad x_i \geq 0, \quad i = 1, 2, \ldots, n$$

Minimum-CVaR Portfolio

$$\min_{\underline{x},\zeta} CVaR(\underline{x},\zeta) = \zeta + \frac{1}{q(1-\alpha)} \sum_{k=1}^{q} u_k$$

subject to

$$\underline{x}^T \underline{R}_k + \zeta + u_k \geq 0,$$

$$u_k \geq 0, \quad k = 1, 2, \ldots, q$$

$$\sum_{i=1}^{n} x_i = 1, \quad x_i \geq 0, \quad i = 1, 2, \ldots, n$$

Methodology – Log-Optimal Portfolio Optimalization

Log-Optimal Portfolio

$$\max E[\ln R_P(\underline{x})] = \max \frac{1}{q} \cdot \sum_{k=1}^{q} \ln(\sum_{i=1}^{n} x_i \cdot R_{ik})$$

subject to

$$\sum_{i=1}^{n} x_i = 1, \quad x_i \geq 0, \quad i = 1, 2, \ldots, n$$

Methodology – Benchmarks, Naïve – MVP and Naïve - MCVaR

Naïve Portfolio: equally weighted portfolio

(for benchmark purposes)

US index

(for benchmark purposes)

Naïve – MVP

Extra constraint: expected return higher or equal to naïve p.

Naïve – MCVaR

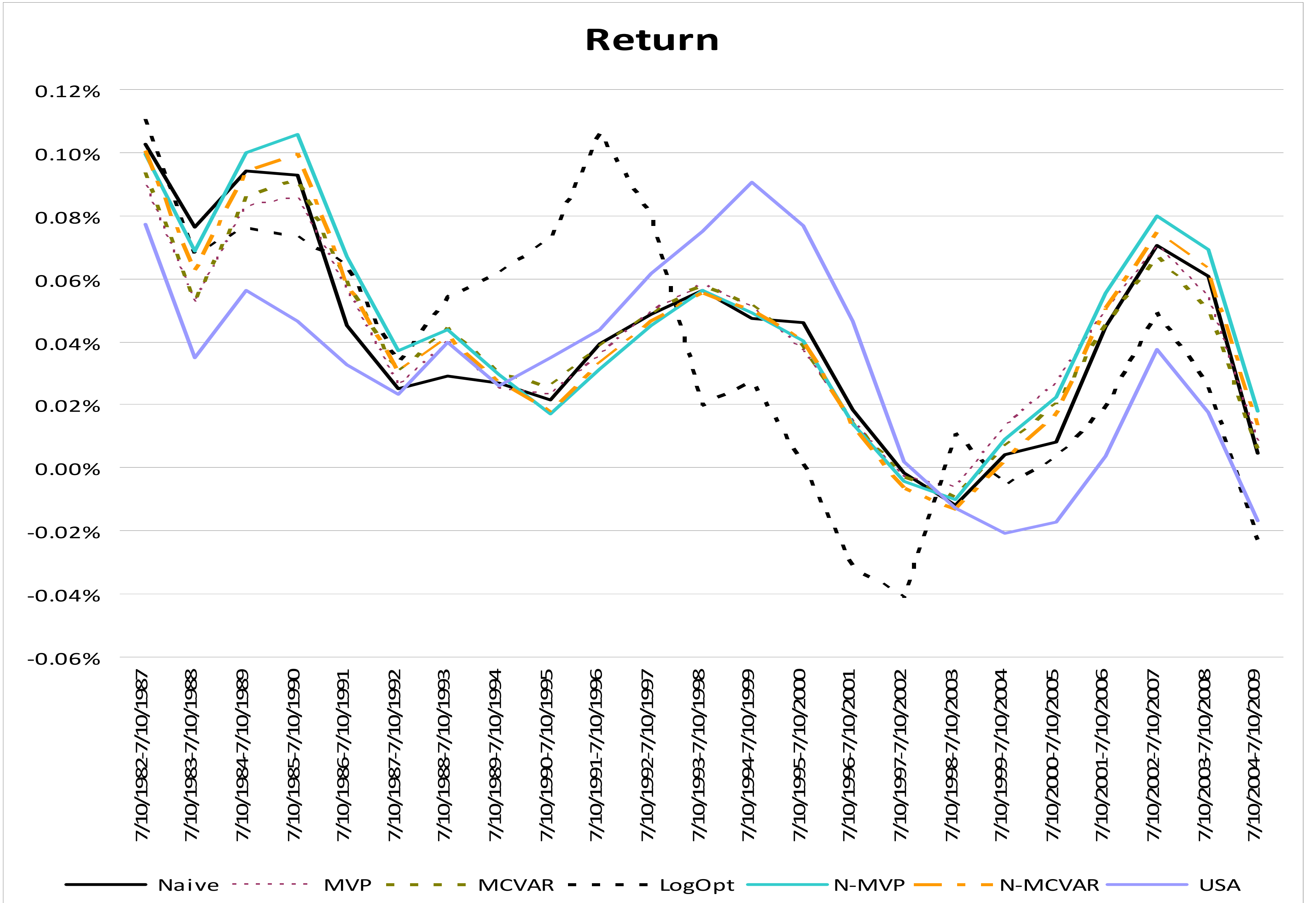
Extra constraint: expected return higher or equal to naïve p.

Calculations

- Ex ante analysis
  - A real life scenario is simulated
  - Fixed number of data (estimation period) is used to calculate the portfolio weights
  - The real portfolio returns are obtained for each subsequent period relying on the individual stock returns and the weights mentioned above
  - The estimation period is a „sliding window”
- Estimation periods
  - 500 days (based on daily market data)

Calculations

- Investment periods
  - 5 year long
  - Investment periods start with one year intervals
  - 23 investment periods
  - First period: 7/10/1982-7/10/1987
  - Last period: 7/10/2004-7/10/2009
    - The recent financial crisis is included


Return
0.12%
0.10%
0.08%
0.06%
0.04%
0.02%
0.00%
-0.02%
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7/10/1982-7/10/1987
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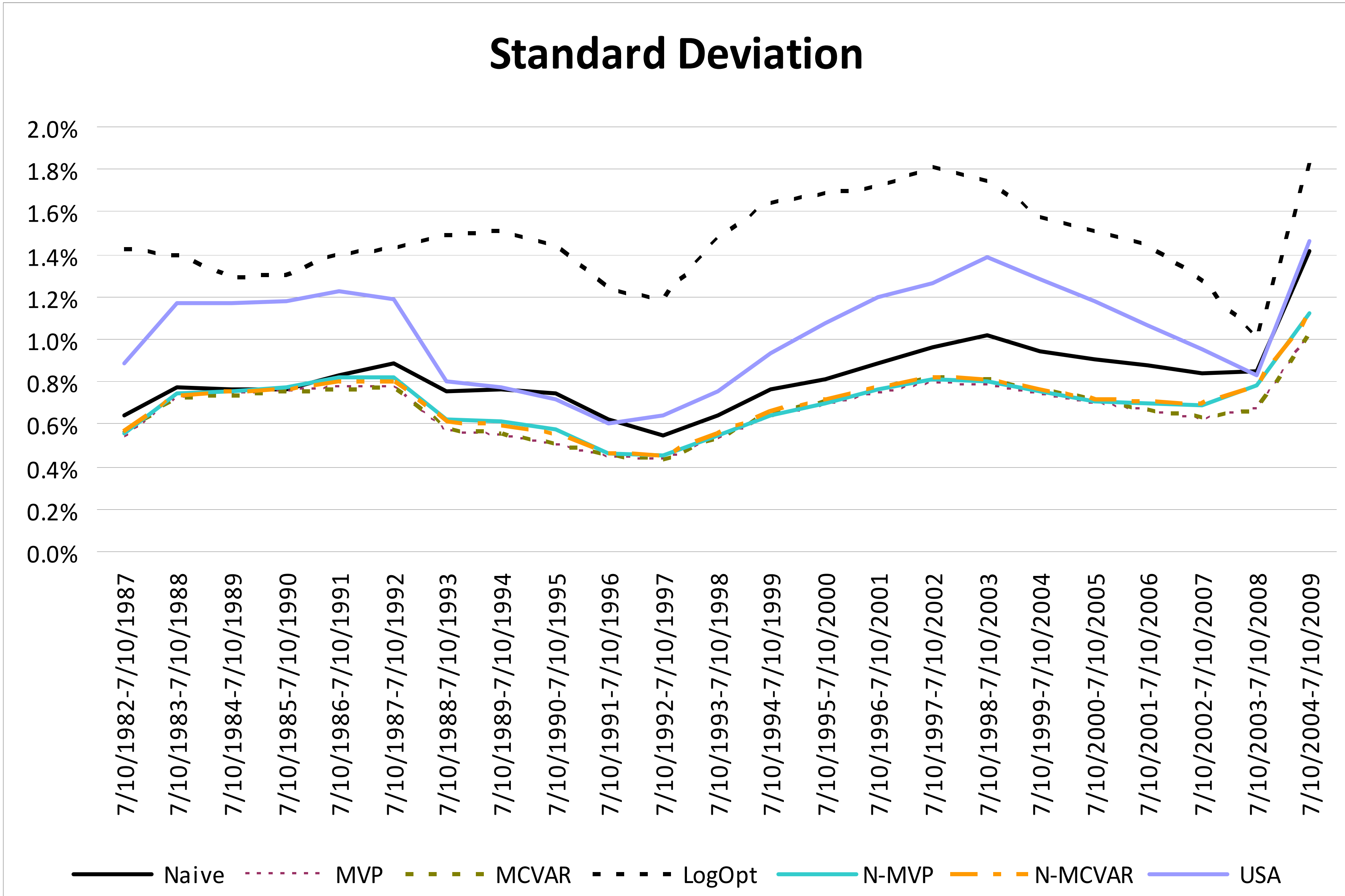
7/10/2001-7/10/2006
7/10/2002-7/10/2007
7/10/2003-7/10/2008
7/10/2004-7/10/2009
Naive
MVP
MCVAR
LogOpt
N-MVP
N-MCVAR
USA

Profitability

	Naïve	MVP	MCVAR	LogOpt	N-MVP	N-MCVAR	USA
Naïve	-	12	13	9	15	12	9
MVP	11	-	12	10	13	13	10
MCVAR	10	11	-	10	14	9	8
LogOpt	14	13	13	-	16	13	6
N-MVP	8	10	9	7	-	5	8
N-MCVAR	11	10	14	10	18	-	9
USA	14	13	15	17	15	14	-
Average rank	**4.04**	**4.00**	**3.70**	**4.26**	**3.04**	**4.13**	**4.83**


Standard Deviation
2.0%
1.8%
1.6%
1.4%
1.2%
1.0%
0.8%
0.6%
0.4%
0.2%
0.0%
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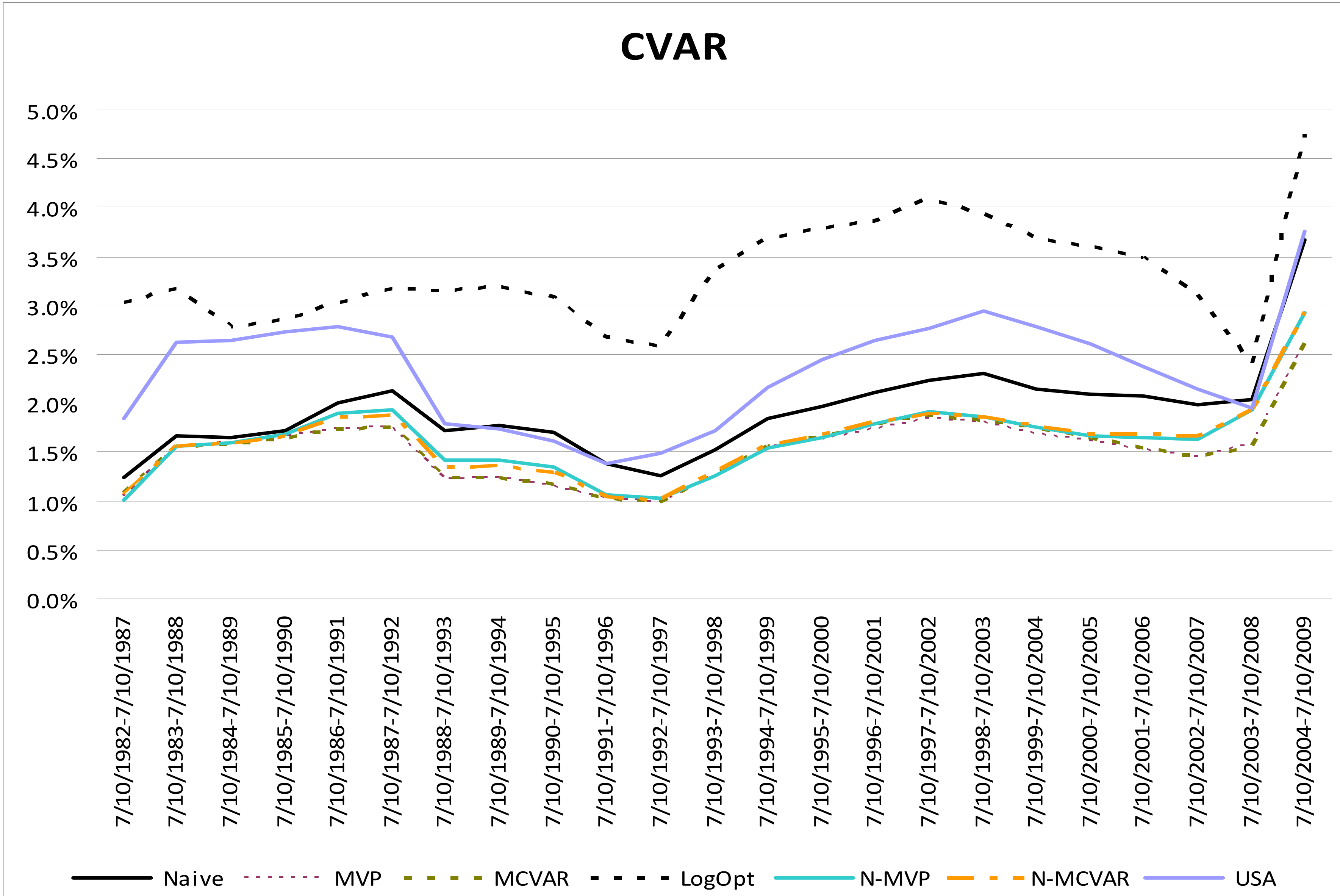
7/10/2001-7/10/2006
7/10/2002-7/10/2007
7/10/2003-7/10/2008
7/10/2004-7/10/2009
Naive
MVP
MCVAR
LogOpt
N-MVP
N-MCVAR
USA

Volatility - standard deviation

	Naïve	MVP	MCVAR	LogOpt	N-MVP	N-MCVAR	USA
Naïve	-	23	23	0	22	22	3
MVP	0	-	6	0	0	0	0
MCVAR	0	17	-	0	8	0	0
LogOpt	23	23	23	-	23	23	23
N-MVP	1	23	15	0	-	10	0
N-MCVAR	1	23	23	0	13	-	0
USA	20	23	23	0	23	23	-
Average rank	**5.04**	**1.26**	**2.09**	**7.00**	**3.13**	**3.61**	**5.87**


CVAR
5.0%
4.5%
4.0%
3.5%
3.0%
2.5%
2.0%
1.5%
1.0%
0.5%
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7/10/1982-7/10/1987
7/10/1983-7/10/1988
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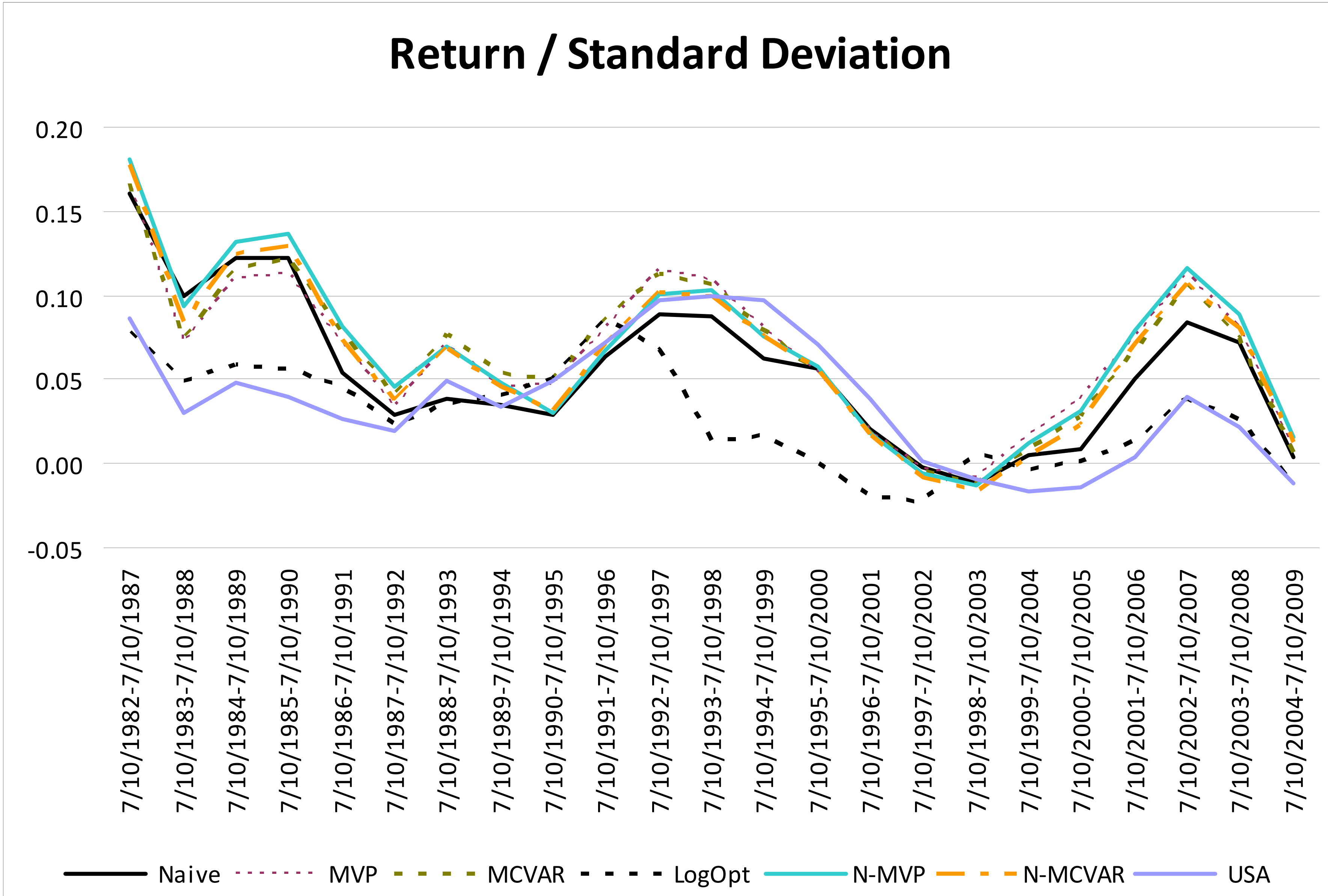
7/10/2001-7/10/2006
7/10/2002-7/10/2007
7/10/2003-7/10/2008
7/10/2004-7/10/2009
Naive
MVP
MCVAR
LogOpt
N-MVP
N-MCVAR
USA

Volatility - CVaR

	Naïve	MVP	MCVAR	LogOpt	N-MVP	N-MCVAR	USA
Naïve	-	23	23	0	23	23	3
MVP	0	-	7	0	5	2	0
MCVAR	0	16	-	0	5	1	0
LogOpt	23	23	23	-	23	23	23
N-MVP	0	18	18	0	-	11	0
N-MCVAR	0	21	22	0	12	-	0
USA	20	23	23	0	23	23	-
Average rank	**5.13**	**1.61**	**1.96**	**7.00**	**3.04**	**3.39**	**5.87**


Return / Standard Deviation
0.20
0.15
0.10
0.05
0.00
-0.05
7/10/1982-7/10/1987
7/10/1983-7/10/1988
7/10/1984-7/10/1989
7/10/1985-7/10/1990
7/10/1986-7/10/1991
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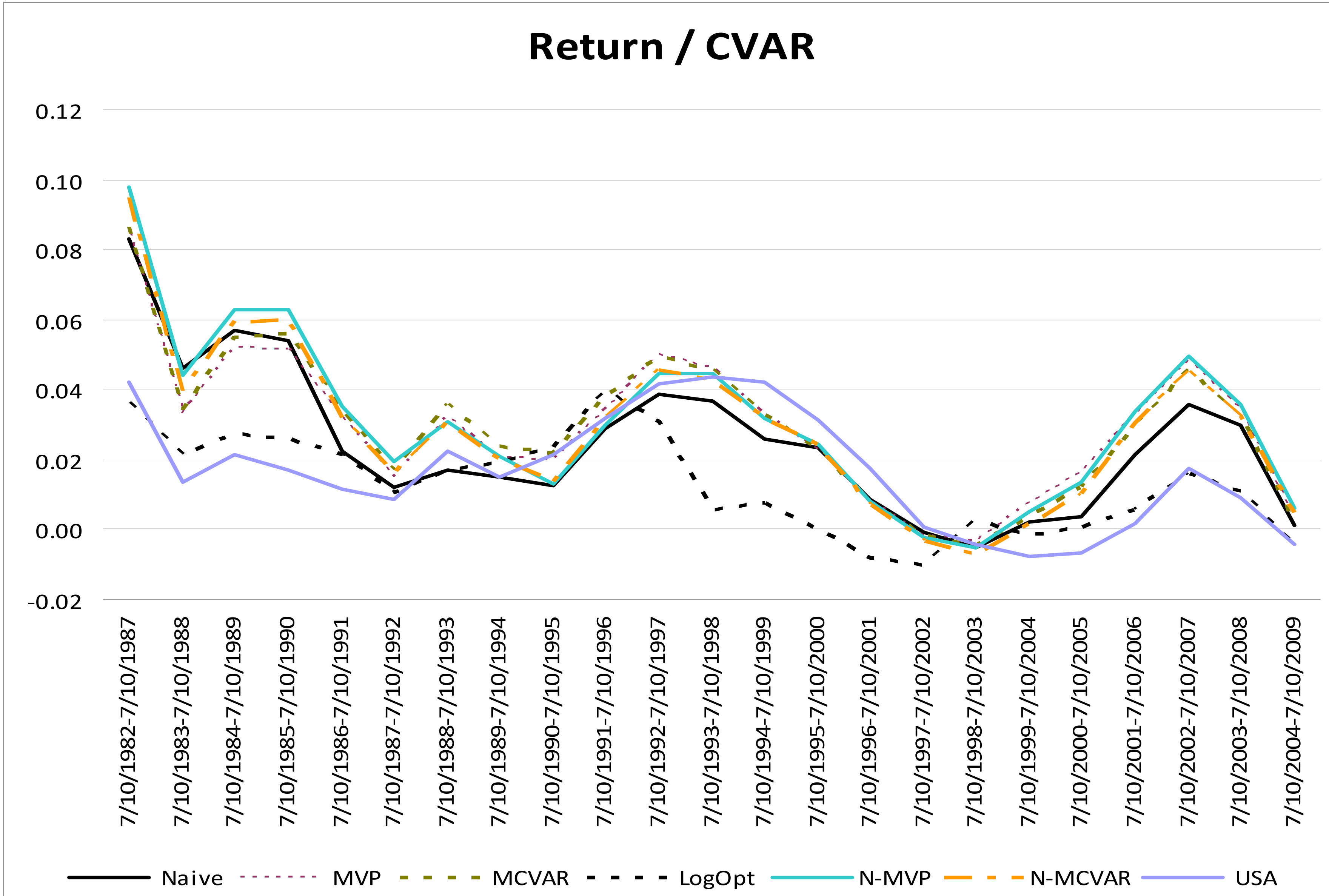
7/10/2000-7/10/2005
7/10/2001-7/10/2006
7/10/2002-7/10/2007
7/10/2003-7/10/2008
7/10/2004-7/10/2009
Naive
MVP
MCVAR
LogOpt
N-MVP
N-MCVAR
USA

Risk adjusted performance - SD

	Naïve	MVP	MCVAR	LogOpt	N-MVP	N-MCVAR	USA
Naïve	-	17	18	4	19	17	10
MVP	6	-	12	3	12	9	5
MCVAR	5	11	-	3	14	9	5
LogOpt	19	20	20	-	20	20	10
N-MVP	4	11	9	3	-	3	7
N-MCVAR	6	14	14	3	20	-	8
USA	13	18	18	13	16	15	-
Average rank	**4.70**	**3.04**	**3.04**	**5.74**	**2.61**	**3.83**	**5.04**



Return / CVAR
0.12
0.10
0.08
0.06
0.04
0.02
0.00
-0.02
7/10/1982-7/10/1987
7/10/1983-7/10/1988
7/10/1984-7/10/1989
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7/10/2004-7/10/2009
Naive
MVP
MCVAR
LogOpt
N-MVP
N-MCVAR
USA

Risk adjusted performance - CVaR

	Naïve	MVP	MCVAR	LogOpt	N-MVP	N-MCVAR	USA
Naïve	-	17	18	5	19	18	10
MVP	6	-	11	3	12	7	5
MCVAR	5	12	-	3	14	8	5
LogOpt	18	20	20	-	20	20	10
N-MVP	4	11	9	3	-	3	7
N-MCVAR	5	16	15	3	20	-	8
USA	13	18	18	13	16	15	-
Average rank	**4.78**	**2.91**	**3.04**	**5.70**	**2.61**	**3.91**	**5.04**

Concusions

- In terms of average realized return
  - N-MVP proved to be the best
  - US domestic portfolio was the least profitable
- Volatility
  - MVP was the best for both measures
  - The US price index and the log-optimal strategies have the highest volatility

Concusions

- Risk adjusted peroformance
  - Most successful: N-MVP
  - Least successful: Log-optimal

Concusions

- Is the „Success" Time Dependent?
  - Ranking based on return
    - Unstable – strongly time dependent
  - Ranking based on volatility
    - Stable
  - Risk adjusted performance
    - Mixed results