

A longitudinal study on Portfolio Optimization: Is the “Success” Time Dependent?

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Aim of the research

Compare and contrast the performance of different portfolio optimization strategies in various 5-year holding periods over a time horizon of almost three decades

Portfolio Selection Strategies

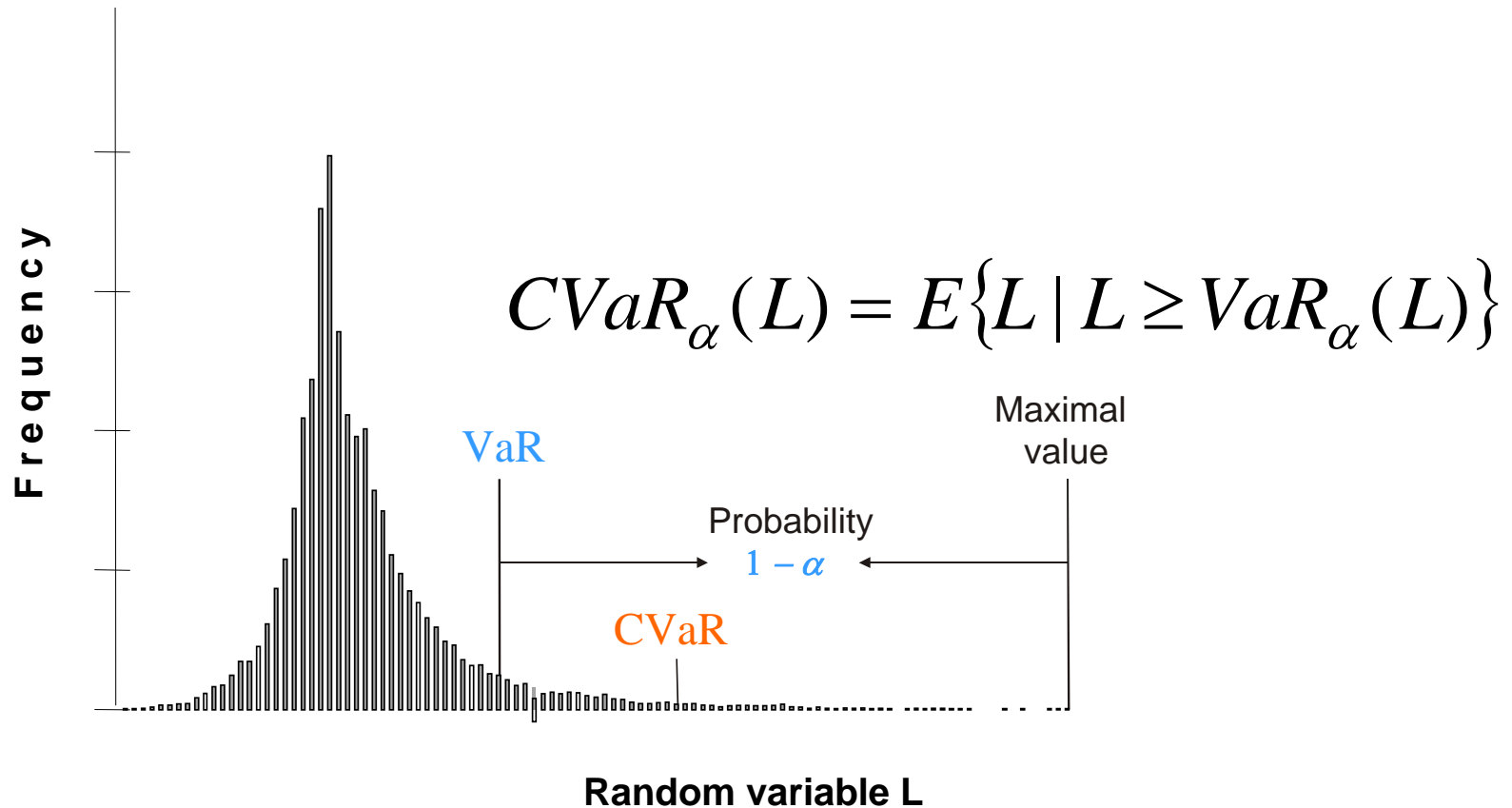
- Minimum-Variance Portfolio (MVP) / Markowitz
- Minimum-CVaR Portfolio (MCCVaR) / Rockafellar-Uryasev
- Log-Optimal Portfolio / Györfi-Ottucsák-Urbán
- Benchmarks
 - Naïve Portfolio
 - US investment
- Naïve – MVP
- Naïve - MCCVaR

Database

- MSCI equity indexes taken from Datastream
- Daily frequency
- Equity price index returns of 18 developed stock markets
 - Australia (AU), Austria (AT), Belgium (BE), Canada (CA), Denmark (DK), France (FR), Germany (DE), Hong Kong (HK), Italy (IT), Japan (JP), the Netherlands (NL), Norway (NO), Singapore (SG), Spain (ES), Sweden (SE), Switzerland (CH), the United Kingdom (GB) and the USA (US).

Value at Risk, Conditional Value at Risk

$$VaR_{\alpha} = q_{\alpha}$$



Methodology – Mean - Variance and Mean - CVaR portfolio optimization

Minimum-Variance
Portfolio (MVP)

$$\min V(\underline{x}) = \sum_{i=1}^n \sum_{k=1}^n x_i x_k \rho_{i,k} \sigma_i \sigma_k$$

**subject
to**

$$\sum_{i=1}^n x_i = 1, \quad x_i \geq 0, \quad i = 1, 2, \dots, n$$

Minimum-CVaR
Portfolio

$$\min_{\underline{x}, \zeta} CVaR(\underline{x}, \zeta) = \zeta + \frac{1}{q(1-\alpha)} \sum_{k=1}^q u_k$$

**subject
to**

$$\underline{x}^T \underline{R}_k + \zeta + u_k \geq 0,$$

$$u_k \geq 0, \quad k = 1, 2, \dots, q$$

$$\sum_{i=1}^n x_i = 1, \quad x_i \geq 0, \quad i = 1, 2, \dots, n$$

Methodology – Log-Optimal Portfolio Optimization

Log-Optimal Portfolio

$$\max E[\ln R_P(\underline{x})] = \max \frac{1}{q} \cdot \sum_{k=1}^q \ln\left(\sum_{i=1}^n x_i \cdot R_{ik}\right)$$

**subject
to**

$$\sum_{i=1}^n x_i = 1, \quad x_i \geq 0, \quad i = 1, 2, \dots, n$$

Methodology – Benchmarks, Naïve – MVP and Naïve - MCVaR

Naïve Portfolio: equally weighted portfolio
(for benchmark purposes)

US index
(for benchmark purposes)

Naïve – MVP

Extra constraint: expected return higher or equal to naïve p.

Naïve – MCVaR

Extra constraint: expected return higher or equal to naïve p.

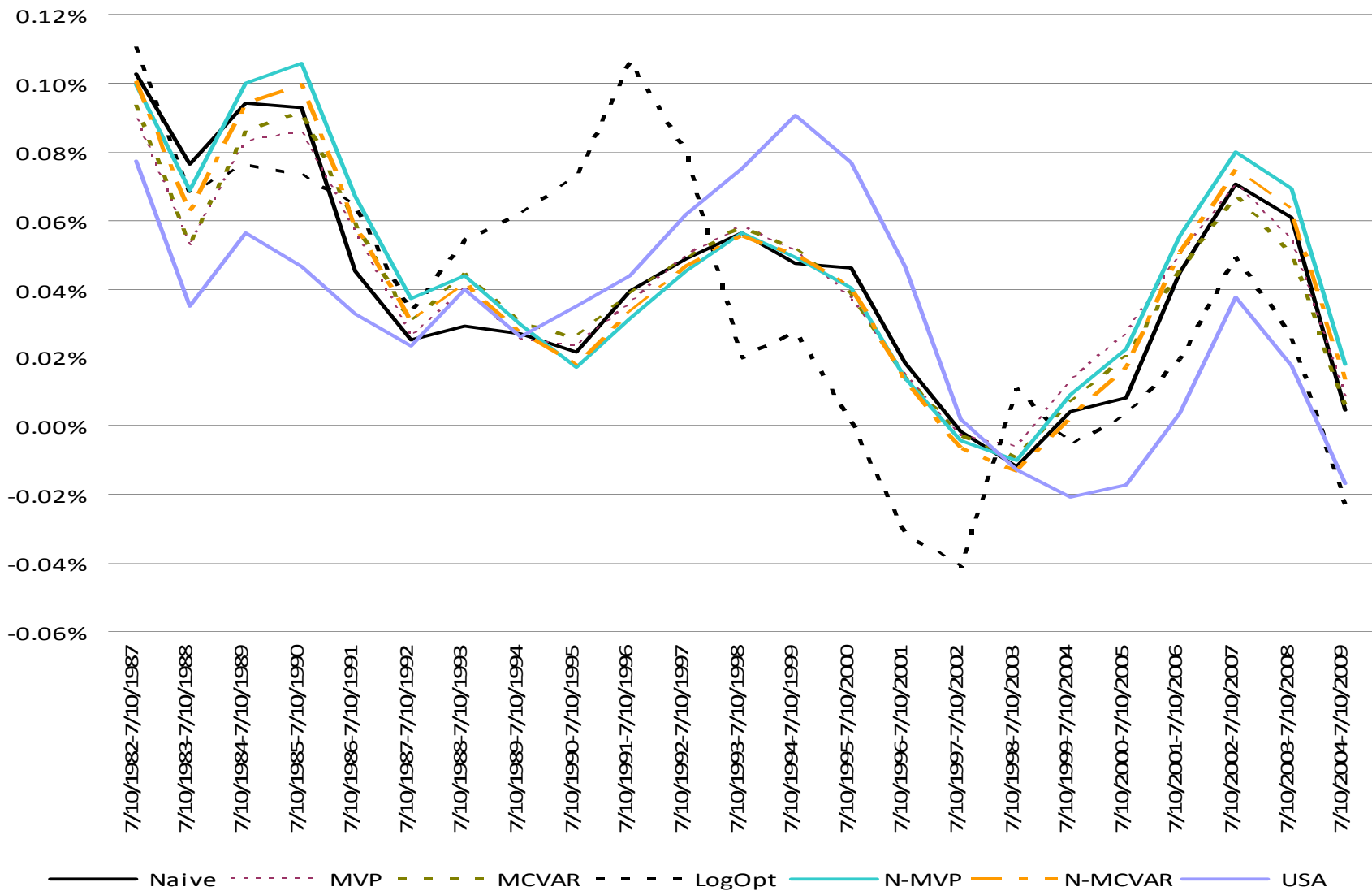
Calculations

- Ex ante analysis
 - A real life scenario is simulated
 - Fixed number of data (estimation period) is used to calculate the portfolio weights
 - The real portfolio returns are obtained for each subsequent period relying on the individual stock returns and the weights mentioned above
 - The estimation period is a „sliding window”
- Estimation periods
 - 500 days (based on daily market data)

Calculations

- Investment periods
 - 5 year long
 - Investment periods start with one year intervals
 - 23 investment periods
 - First period: 7/10/1982-7/10/1987
 - Last period: 7/10/2004-7/10/2009
 - The recent financial crisis is included

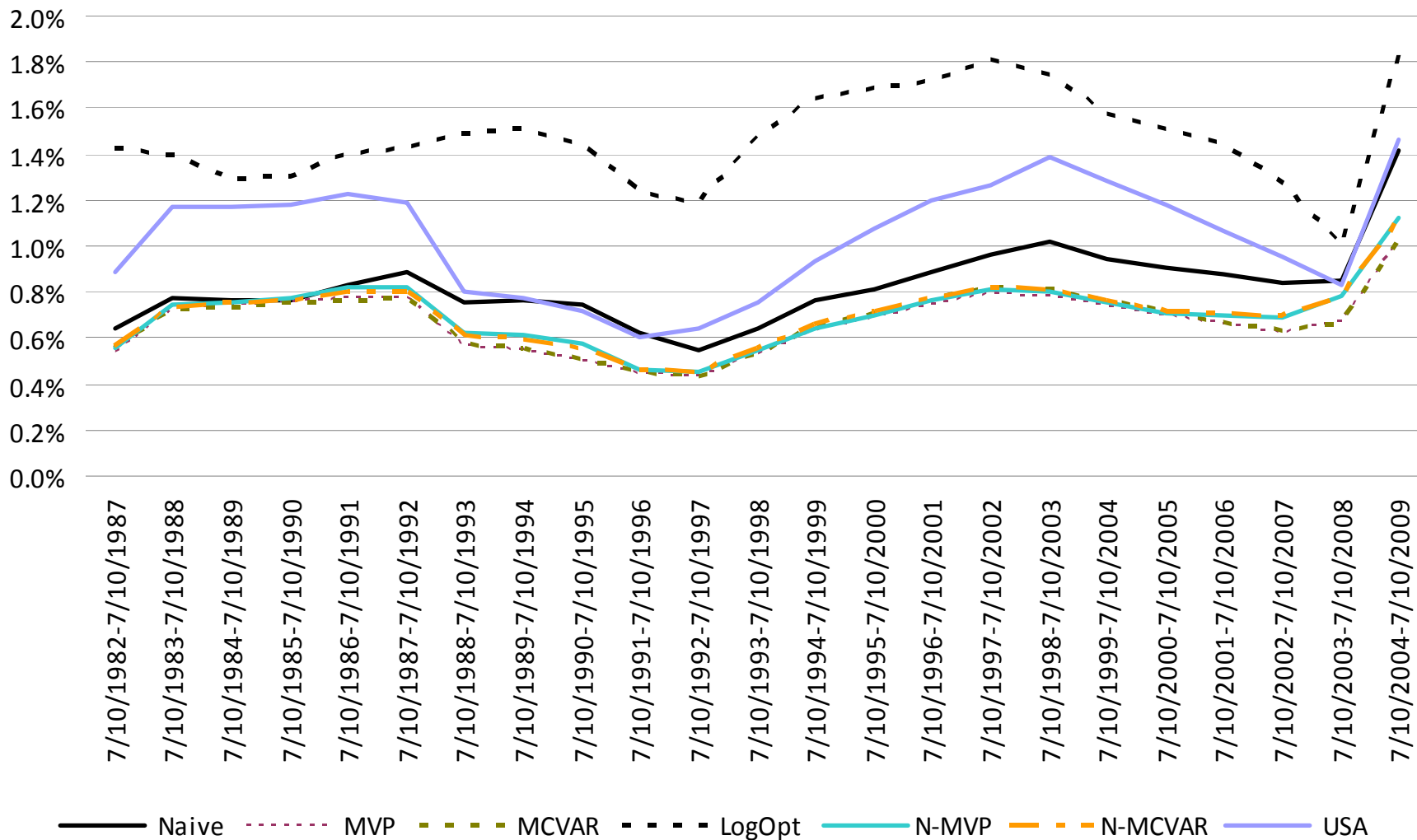
Return



Profitability

	Naïve	MVP	MCVAR	LogOpt	N-MVP	N-MCVAR	USA
Naïve	-	12	13	9	15	12	9
MVP	11	-	12	10	13	13	10
MCVAR	10	11	-	10	14	9	8
LogOpt	14	13	13	-	16	13	6
N-MVP	8	10	9	7	-	5	8
N-MCVAR	11	10	14	10	18	-	9
USA	14	13	15	17	15	14	-
Average rank	4.04	4.00	3.70	4.26	3.04	4.13	4.83

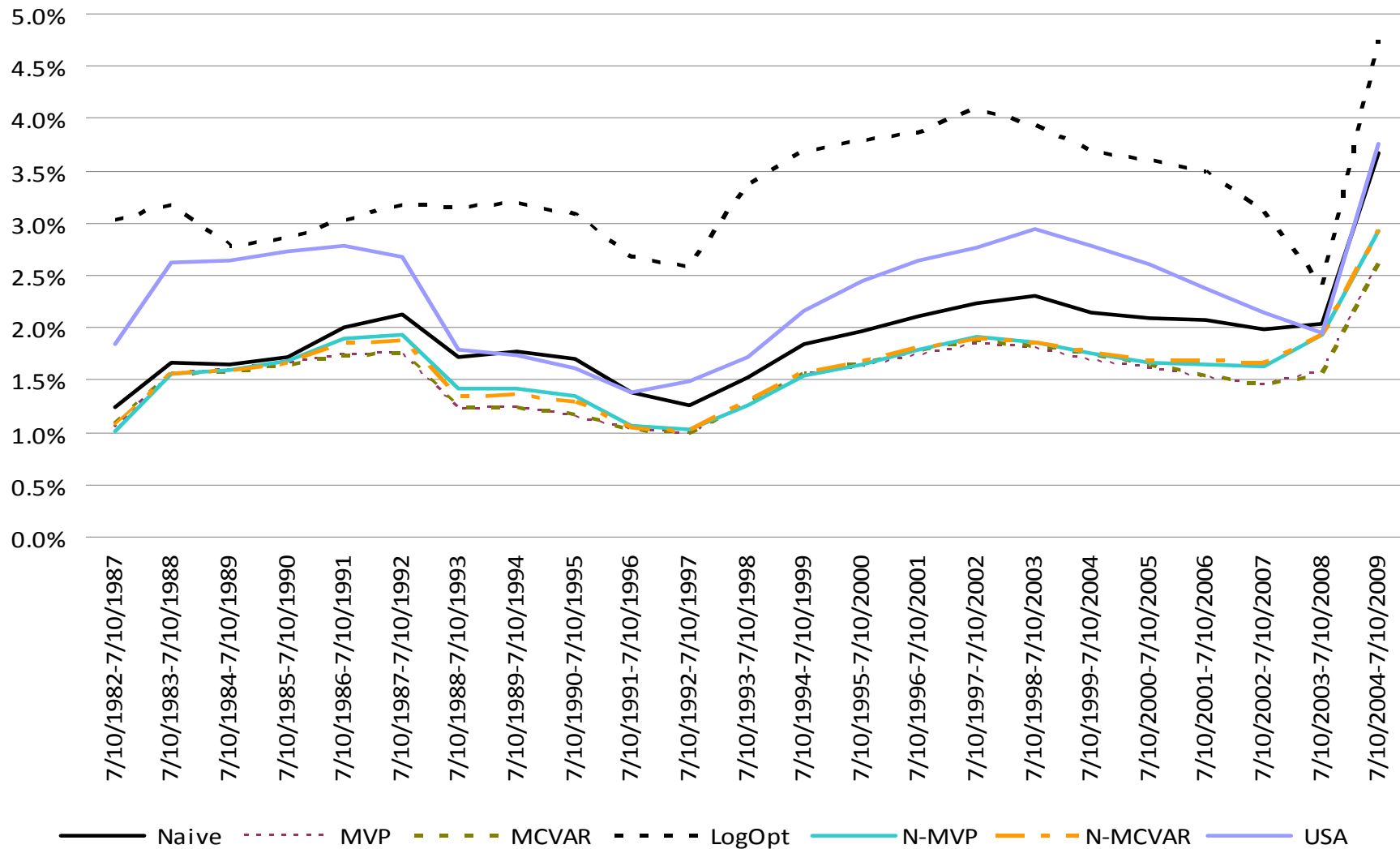
Standard Deviation



Volatility - standard deviation

	Naïve	MVP	MCVAR	LogOpt	N-MVP	N-MCVAR	USA
Naïve	-	23	23	0	22	22	3
MVP	0	-	6	0	0	0	0
MCVAR	0	17	-	0	8	0	0
LogOpt	23	23	23	-	23	23	23
N-MVP	1	23	15	0	-	10	0
N-MCVAR	1	23	23	0	13	-	0
USA	20	23	23	0	23	23	-
Average rank	5.04	1.26	2.09	7.00	3.13	3.61	5.87

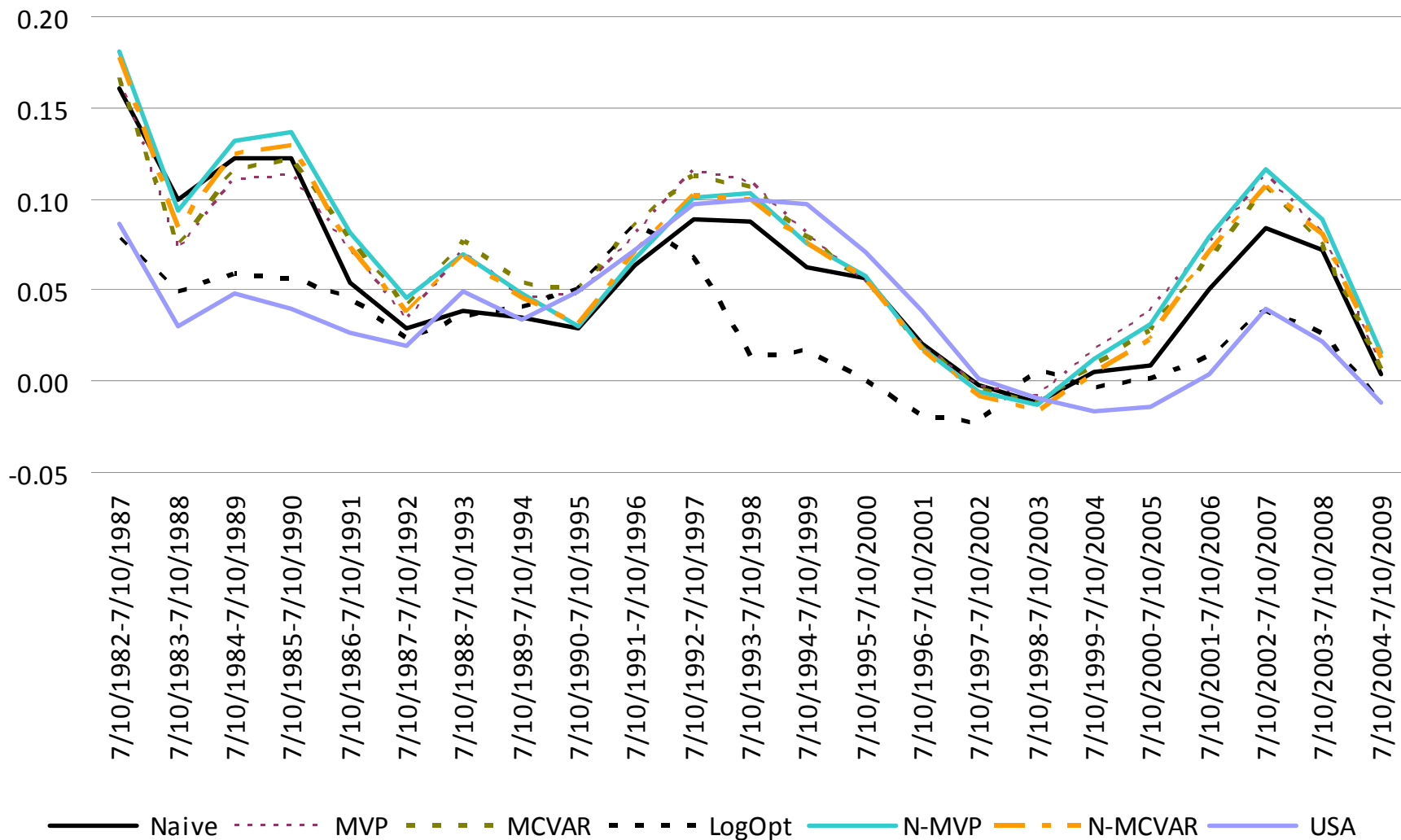
CVAR



Volatility - CVaR

	Naïve	MVP	MCVAR	LogOpt	N-MVP	N-MCVAR	USA
Naïve	-	23	23	0	23	23	3
MVP	0	-	7	0	5	2	0
MCVAR	0	16	-	0	5	1	0
LogOpt	23	23	23	-	23	23	23
N-MVP	0	18	18	0	-	11	0
N-MCVAR	0	21	22	0	12	-	0
USA	20	23	23	0	23	23	-
Average rank	5.13	1.61	1.96	7.00	3.04	3.39	5.87

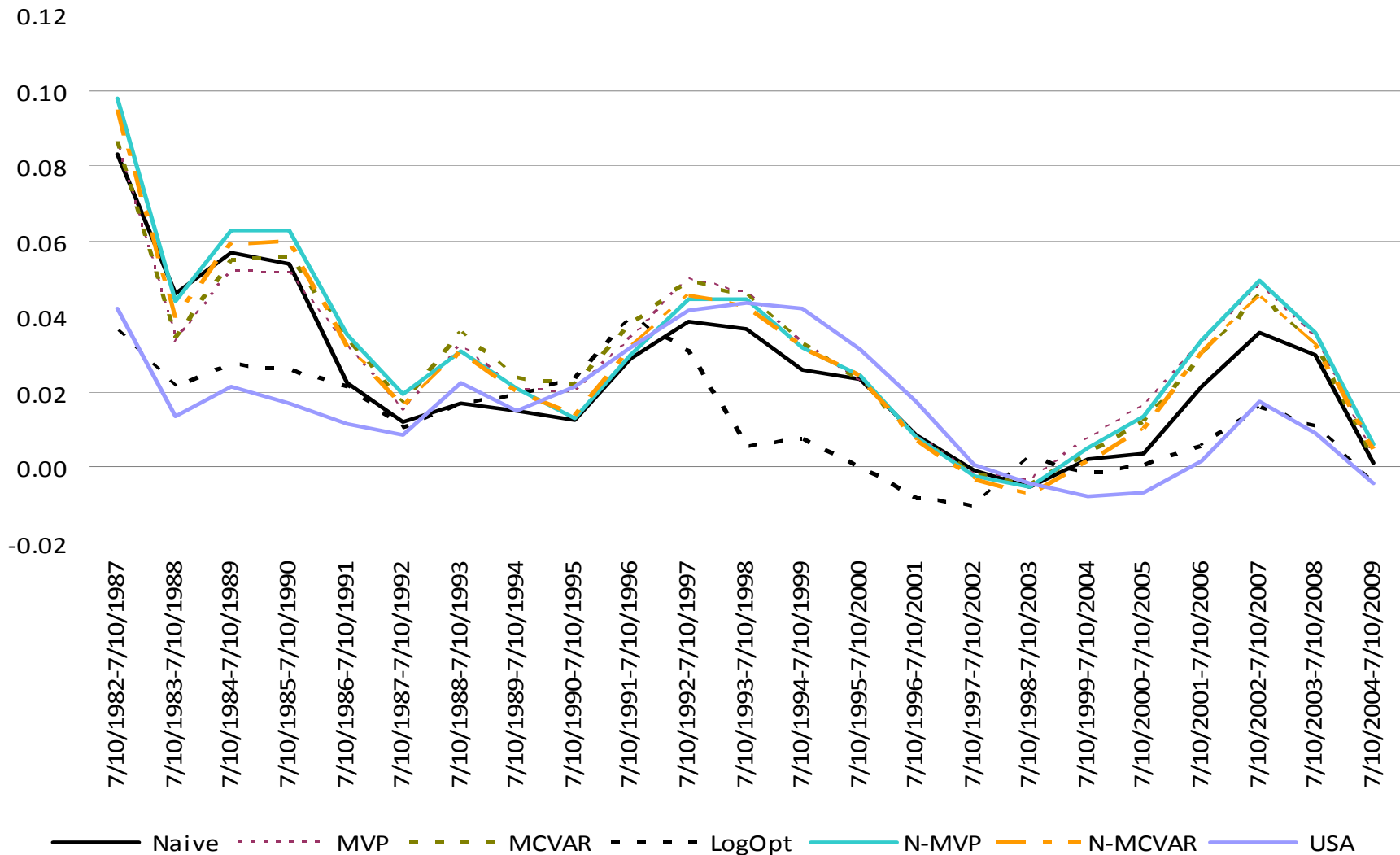
Return / Standard Deviation



Risk adjusted performance - SD

	Naïve	MVP	MCVAR	LogOpt	N-MVP	N-MCVAR	USA
Naïve	-	17	18	4	19	17	10
MVP	6	-	12	3	12	9	5
MCVAR	5	11	-	3	14	9	5
LogOpt	19	20	20	-	20	20	10
N-MVP	4	11	9	3	-	3	7
N-MCVAR	6	14	14	3	20	-	8
USA	13	18	18	13	16	15	-
Average rank	4.70	3.04	3.04	5.74	2.61	3.83	5.04

Return / CVAR



Risk adjusted performance - CVaR

	Naïve	MVP	MCVAR	LogOpt	N-MVP	N-MCVAR	USA
Naïve	-	17	18	5	19	18	10
MVP	6	-	11	3	12	7	5
MCVAR	5	12	-	3	14	8	5
LogOpt	18	20	20	-	20	20	10
N-MVP	4	11	9	3	-	3	7
N-MCVAR	5	16	15	3	20	-	8
USA	13	18	18	13	16	15	-
Average rank	4.78	2.91	3.04	5.70	2.61	3.91	5.04

Conclusions

- In terms of average realized return
 - N-MVP proved to be the best
 - US domestic portfolio was the least profitable
- Volatility
 - MVP was the best for both measures
 - The US price index and the log-optimal strategies have the highest volatility

Conclusions

- Risk adjusted performance
 - Most successful: N-MVP
 - Least successful: Log-optimal

Conclusions

- Is the „Success” Time Dependent?
 - Ranking based on return
 - Unstable – strongly time dependent
 - Ranking based on volatility
 - Stable
 - Risk adjusted performance
 - Mixed results