AFIR LIFE MUNICH 2009

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Risk–Reward Optimisation for Long-Run Investors: an Empirical Analysis

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introduction

portfolio optimisation: allocate wealth among $n_{\mathcal{A}}$ assets

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estimation problems: Jobson and Korkie (1980), Jorion (1985), Jorion (1986), Best and Grauer (1991), Chopra et al. (1993), Board and Sutcliffe (1994) and many others

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theoretical concerns: Artzner et al. (1999), Pedersen and Satchell (1998), Pedersen and Satchell (2002)

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aim of research:

- test alternative risk measures & objective functions empirically
- test alternative estimation and scenario generation methods

outline

- alternative objective functions

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- data/optimisation

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- alternative objective functions
- data/optimisation
- empirical results

alternative objective functions: building blocks

$$\underbrace{w'\mu}_{\text{reward}} - \underbrace{\frac{\gamma}{2}w'\Sigma w}_{\text{risk}}$$

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replace reward and risk by alternative functions

$$\min_{w} \frac{\text{risk}}{\text{reward}}$$

alternative objective functions: building blocks

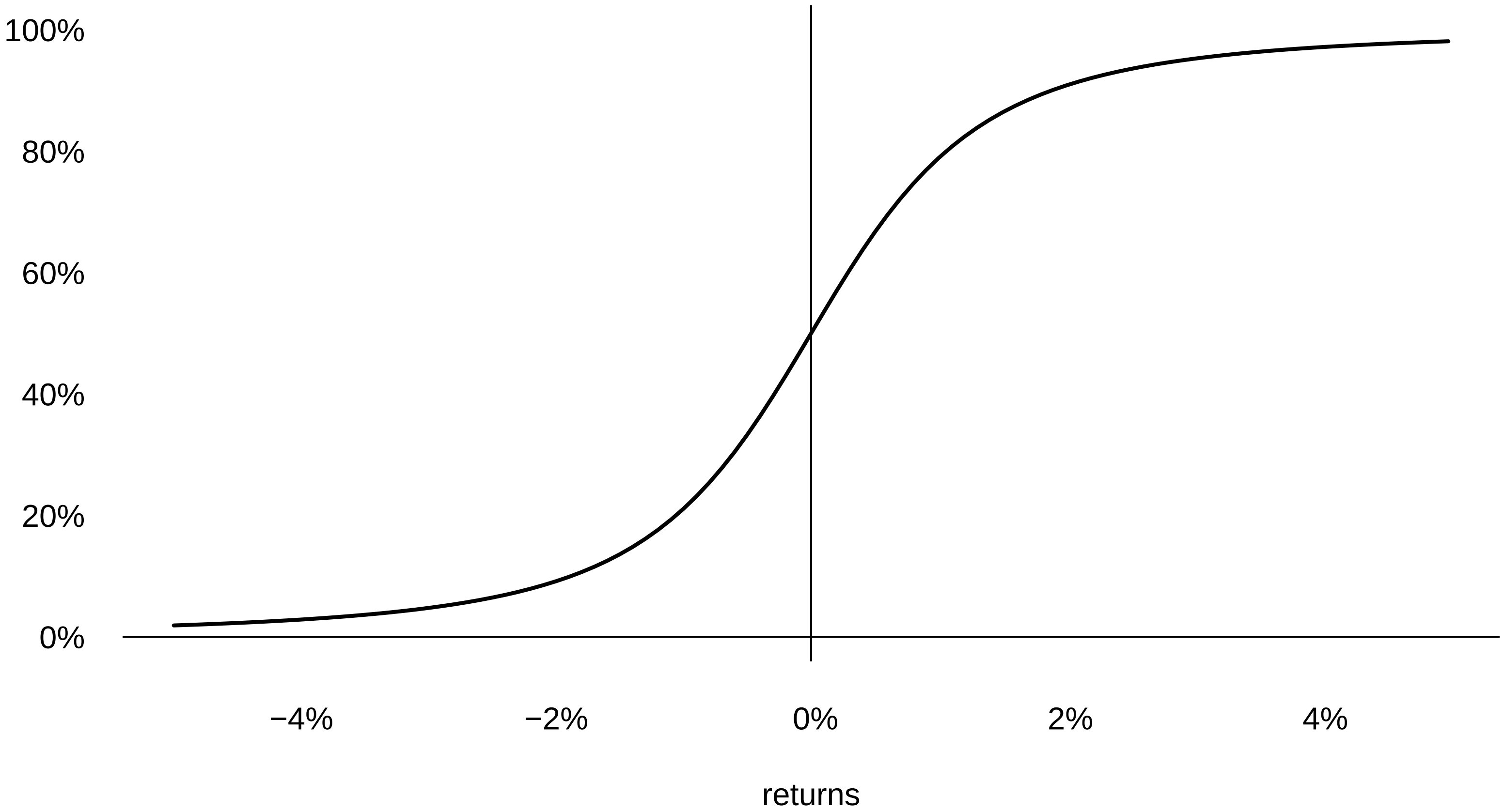
$$\underbrace{w'\mu}_{\text{reward}} - \underbrace{\frac{\gamma}{2}w'\Sigma w}_{\text{risk}}$$

replace reward and risk by alternative functions

$$\min_w \frac{\text{risk}}{\text{reward}} = \Phi$$

alternative objective functions: building blocks

based on distribution of portfolio returns



alternative objective functions: building blocks

based on distribution of portfolio returns

- moments (variance, skewness, . . .)
- conditional moments (expected shortfall, . . .), partial moments (semivariance, . . .)
- quantiles (VaR, . . .), corresponding probabilities (shortfall probability, . . .)

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based on trajectory of portfolio wealth

- drawdown ($\mathcal{D}$), time under water, . . .

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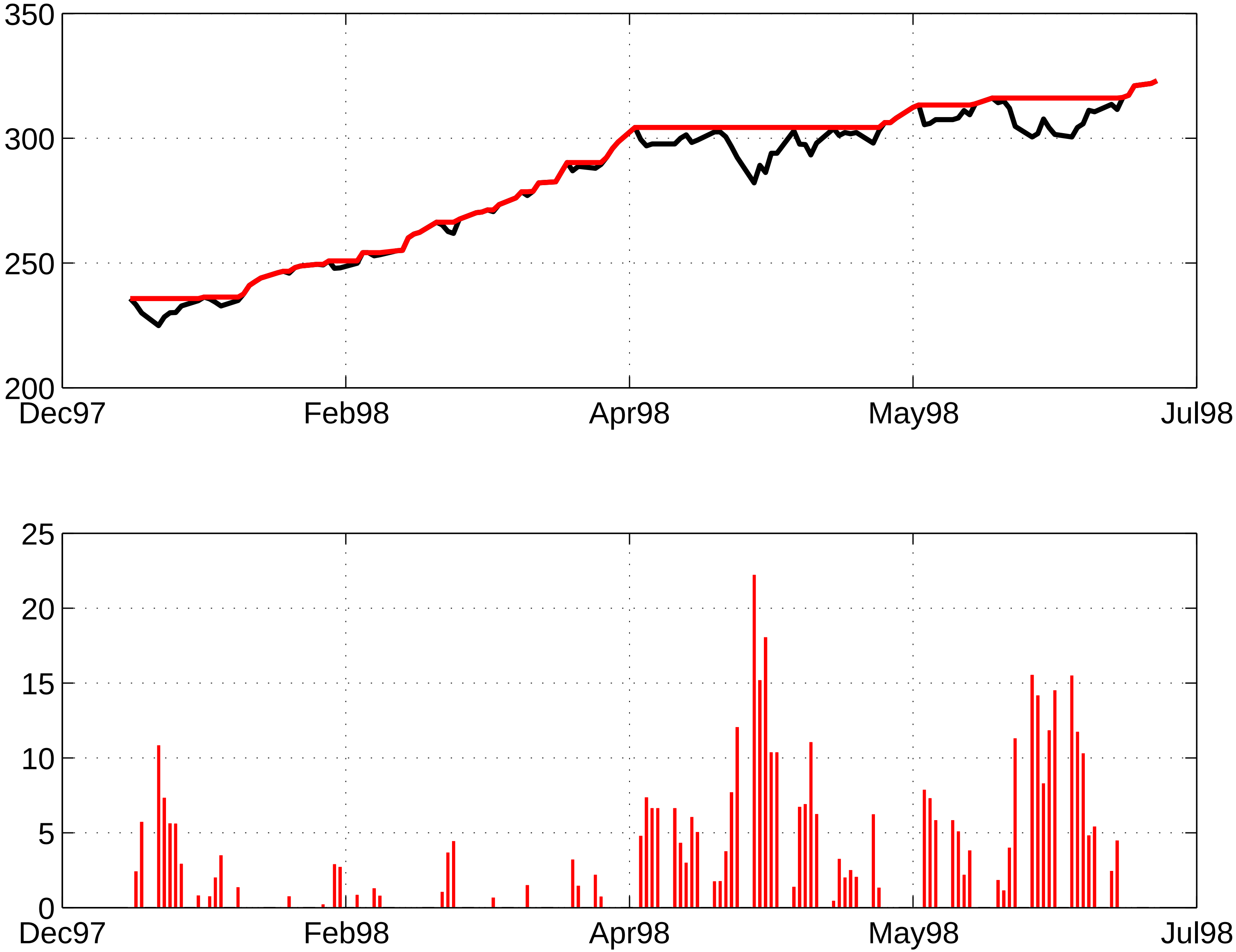

l moments

ɔrtfall probability,

alternative objective functions: building blocks

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objective function: do as you please

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based on trajectory of portfolio wealth

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objective function: do as you please

alternative objective functions: partial moments

capture non-symmetrical returns Bawa (1975); Fishburn (1977):

$$r = \underbrace{r_d}_{\text{desired return}} + \underbrace{(r - r_d)^+}_{\text{upside}} - \underbrace{(r_d - r)^+}_{\text{downside}}$$

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$$\mathcal{P}^+_\gamma(r_d) = \frac{1}{T} \sum_{r > r_d} (r - r_d)^\gamma \ ,$$

$$\mathcal{P}^-_\gamma(r_d) = \frac{1}{T} \sum_{r < r_d} (r_d - r)^\gamma \ .$$

example: semi-variance

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$$r = \underbrace{r_d}_{\text{desired return}} + \underbrace{(r - r_d)^+}_{\text{upside}} - \underbrace{(r_d - r)^+}_{\text{downside}}$$

$$\mathcal{C}_\gamma^+(r_d) = \frac{1}{\#\{r > r_d\}} \sum_{r > r_d} (r - r_d)^\gamma \ ,$$

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example: Expected Shortfall

alternative obj. functions: conditional moments

capture non-symmetrical returns:

$$r = \underbrace{r_d}_{\text{desired return}} + \underbrace{(r - r_d)^+}_{\text{upside}} - \underbrace{(r_d - r)^+}_{\text{downside}}$$

conditional vs partial moments

$$\mathcal{P}_\gamma^+(r_d) = \mathcal{C}_\gamma^+(r_d)\,\underbrace{\mathcal{P}_0^+(r_d)}_{\pi \text{ of } r>r_d}$$

$$\mathcal{P}_\gamma^-(r_d) = \mathcal{C}_\gamma^-(r_d)\,\underbrace{\mathcal{P}_0^-(r_d)}_{\pi \text{ of } r<r_d}$$

alternative objective functions: quantiles

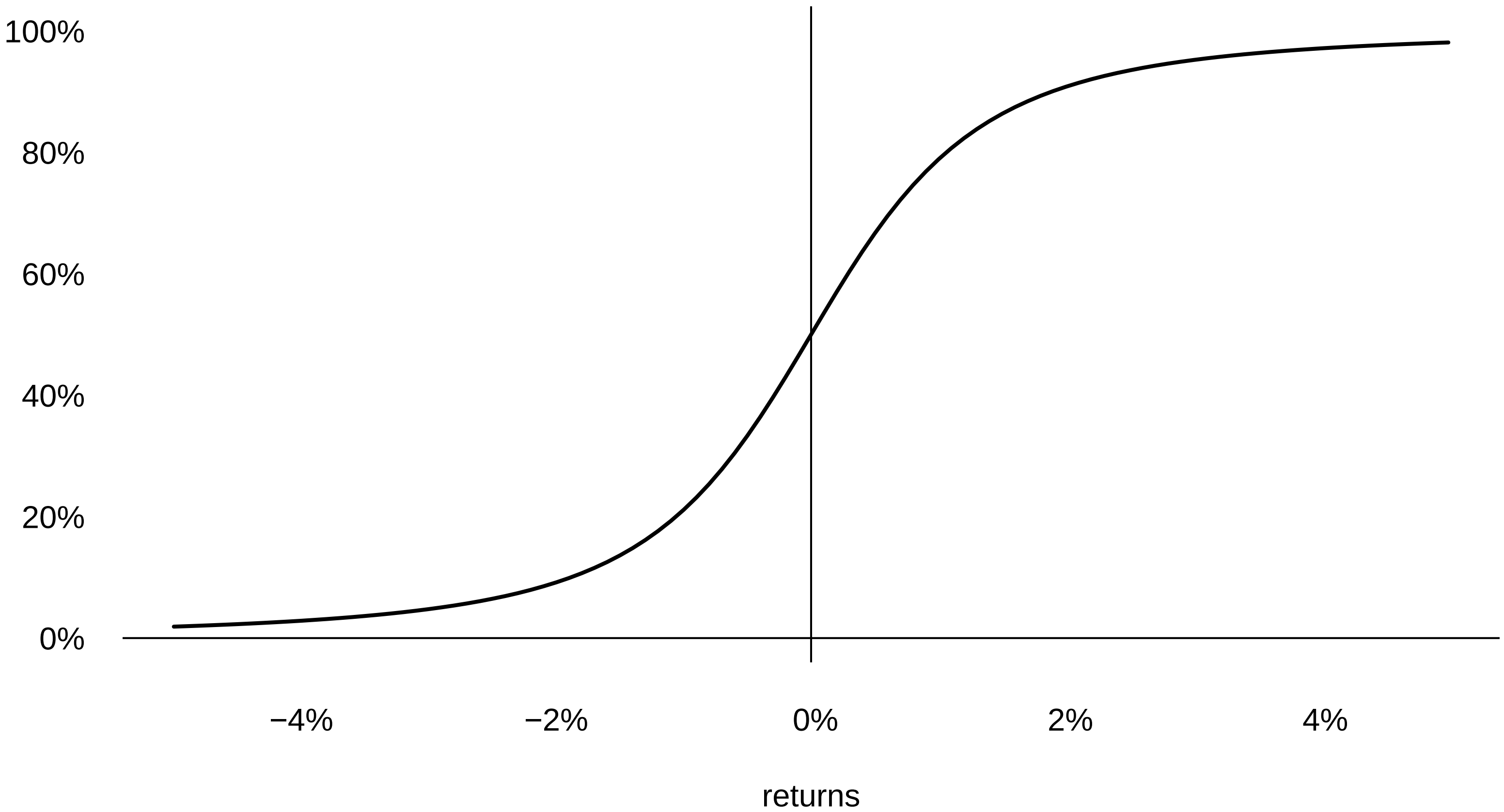
$$\mathcal{Q}_q = \mathrm{CDF}^{-1}(q) = \min\{r \mid \mathrm{CDF}(r) \geq q\}\,,$$

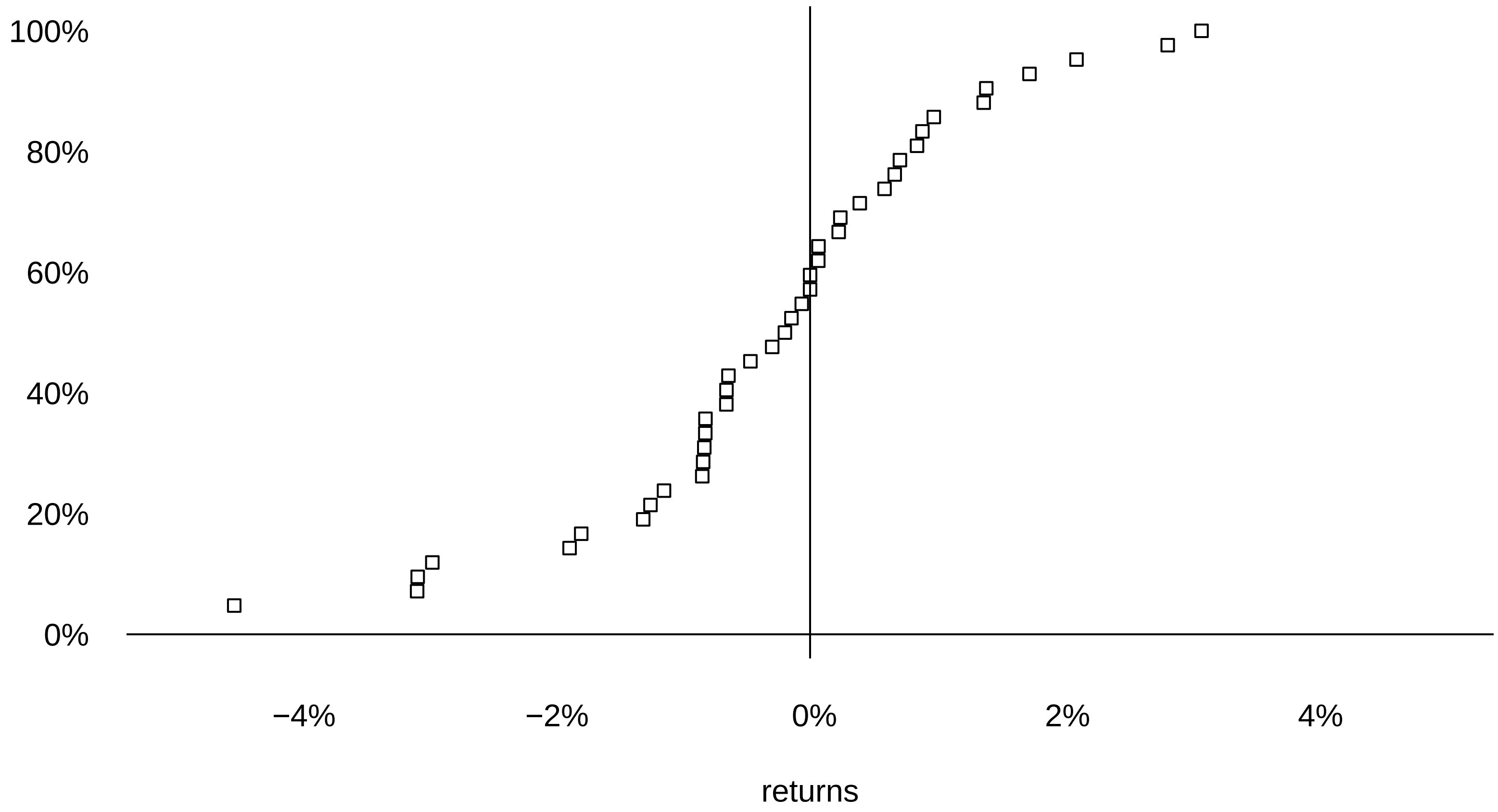
example: VaR

alternative objective functions: examples

reward	risk	
constant	$\mathcal{C}_1^-(\mathcal{Q}_q)$	minimise Expected Shortfall for qth quantile
constant	$-\mathcal{Q}_0$	minimise maximum loss
$\frac{1}{n_\mathcal{S}}\sum r$	$\sqrt{\mathcal{P}_2^-(r_d)}$	Sortino ratio
$\mathcal{P}_1^+(r_d)$	$\sqrt{\mathcal{P}_2^-(r_d)}$	Upside Potential ratio
$\mathcal{P}_1^+(r_d)$	$\mathcal{P}_1^-(r_d)$	Omega for threshold r_d
$\frac{1}{n_\mathcal{S}}\sum r$	$\mathcal{D}_{\max}$	Calmar ratio
$\mathcal{C}_\gamma^+(\mathcal{Q}_p)$	$\mathcal{C}_\delta^-(\mathcal{Q}_q)$	Rachev Generalised ratio for exponents γ and δ

estimation


100%
80%
60%
40%
20%
0%
−4%
−2%
0%
2%
4%
returns



empirical distribution of portfolio returns
(order statistics $r_{[1]} \leq r_{[2]} \leq \cdots \leq r_{[T]}$)

estimation

bootstrapping returns (r^B) from a simple regression model:

$$r_{it} = \alpha_i + \beta_i \, r_{Mt} + \cdots + \epsilon_{it} \qquad \begin{array}{l} i = 1, \ldots, n_{\mathcal{A}} \\ t = 1, \ldots, T \end{array}$$

estimation

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regressors: indices, PCA ...

optimisation

$$\min_{x} \Phi(x)$$

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$$x_j^{\text{inf}} \leq x_j \leq x_j^{\text{sup}} \qquad j \in \mathcal{J}$$

$$K_{\text{inf}} \leq \#\{\mathcal{J}\} \leq K_{\text{sup}}$$

$$\vdots$$

(x = numbers of shares, $\mathcal{A}$ = all assets, $\mathcal{J}$ = assets included in portfolio)

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Threshold Accepting: Dueck and Scheuer (1990), Winker (2001), Gilli and Schumann (in press),

Matlab code available from `http://comisef.eu`

data and methodology

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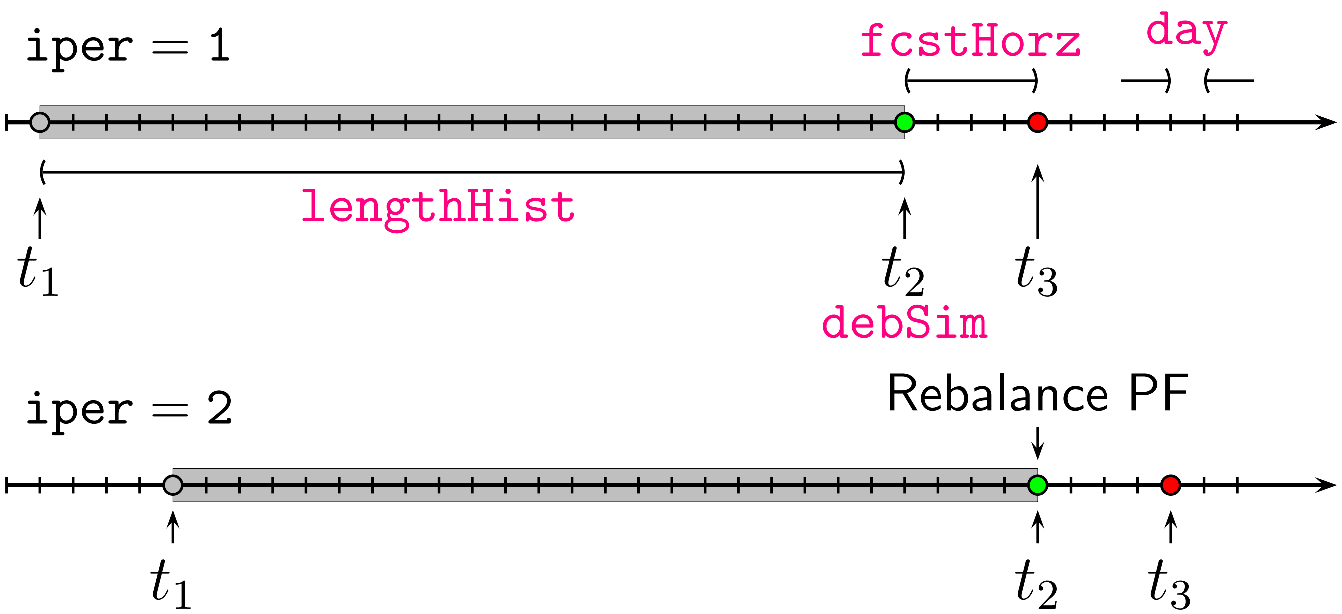

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$$0 \leq w_j \leq w_j^{\sup} \qquad j = 1, \ldots, n_{\mathcal{A}}$$

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$$
\begin{aligned}
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&\textstyle\sum_{j \in \mathcal{J}} w_j = 1 \\
&0 \leq w_j \leq w_j^{\text{sup}} \qquad j = 1, \ldots, n_{\mathcal{A}}
\end{aligned}
$$

- optimisation with maximum holding size and sector allocation constraints done with Matlab's `quadprog`.

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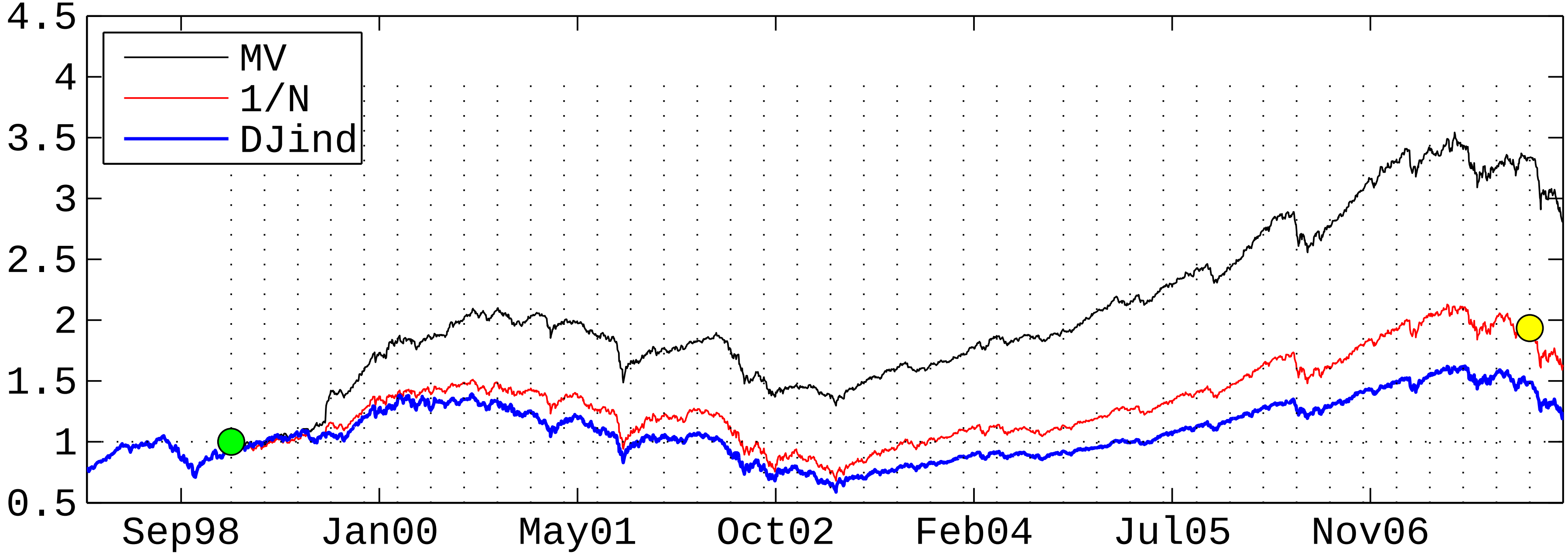

introducing uncertainty:

benchmark: minimum variance (MV)

introducing uncertainty: compute minimum-variance portfolio from jackknifed or bootstrapped time series

benchmark: minimum variance (MV)

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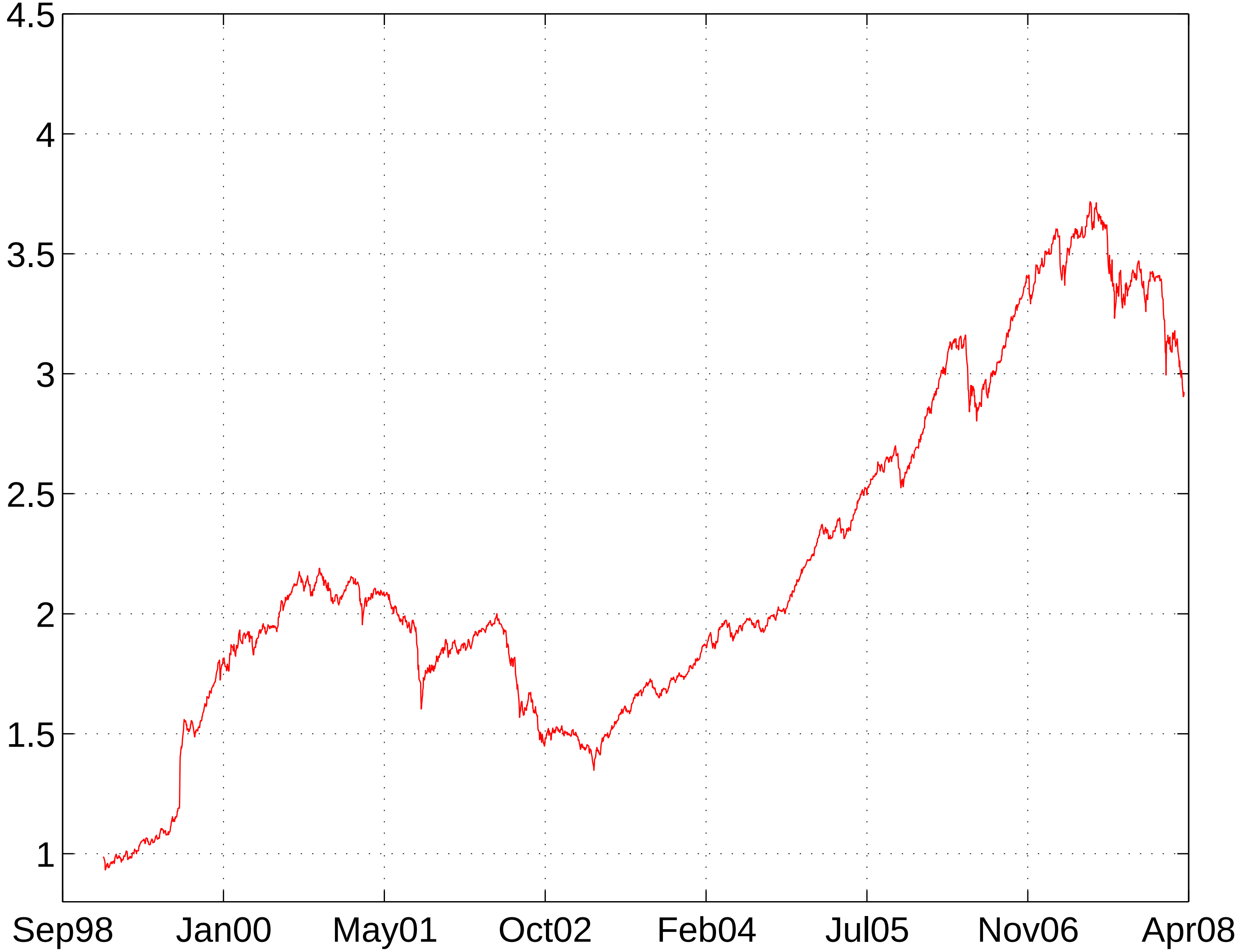

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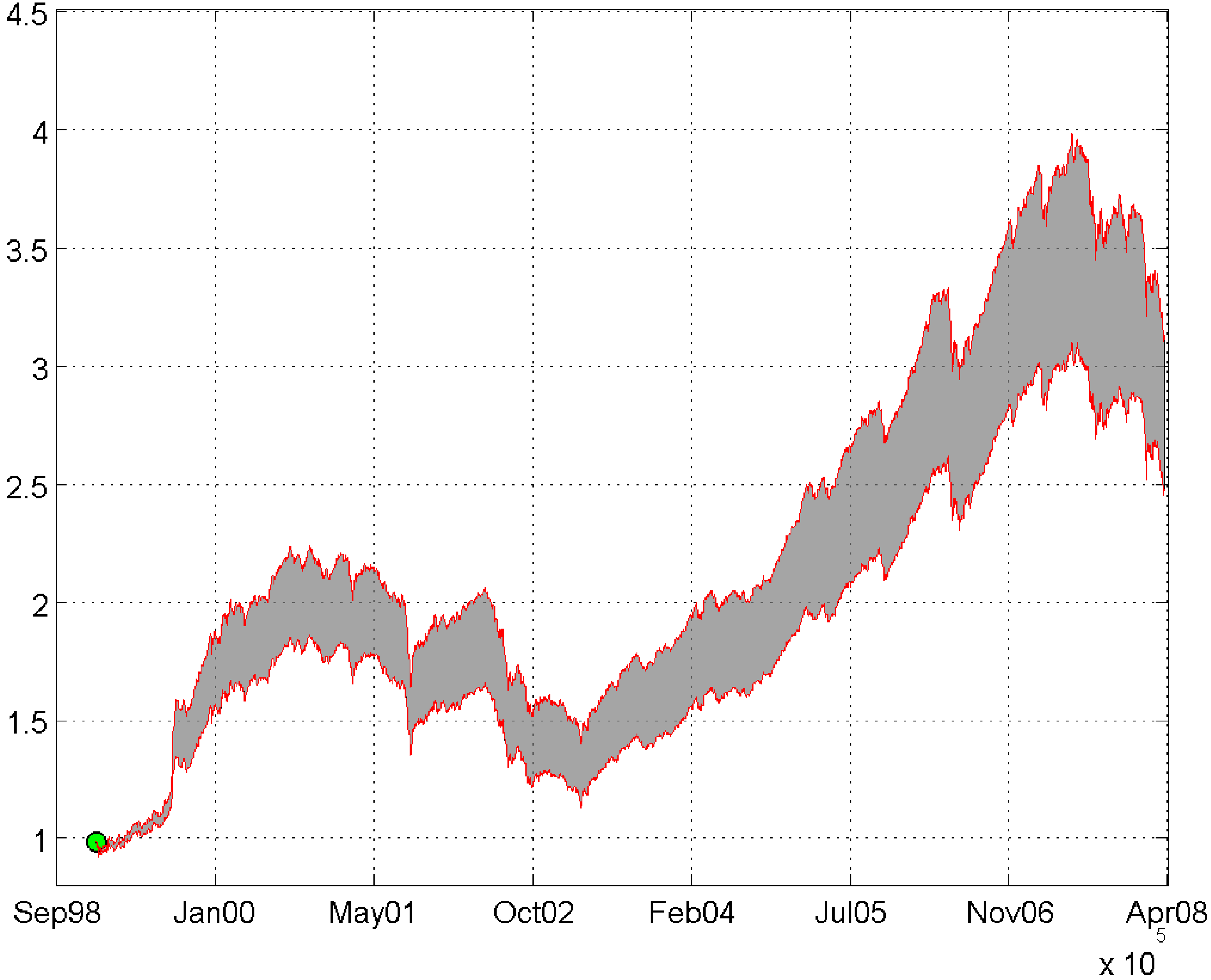

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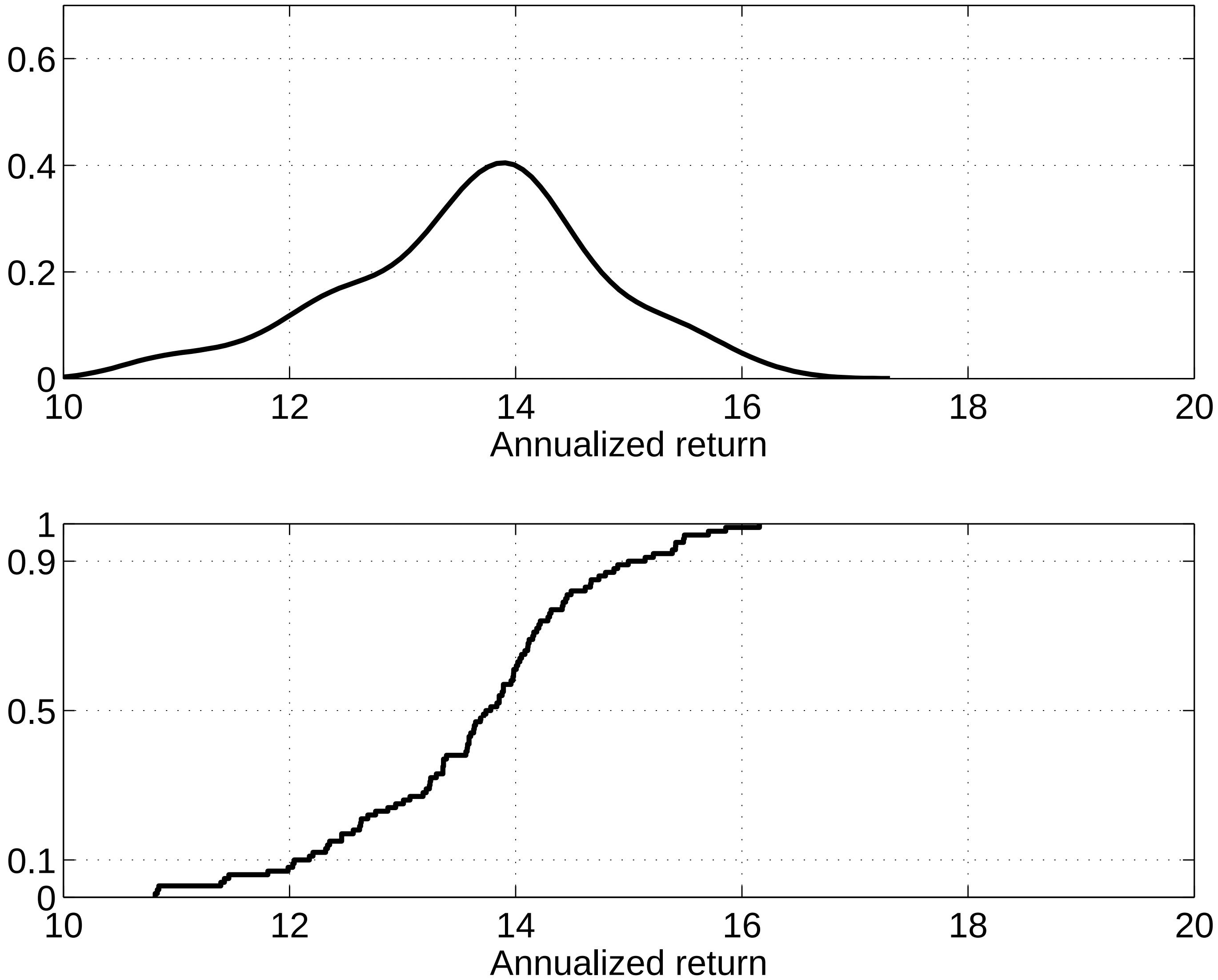

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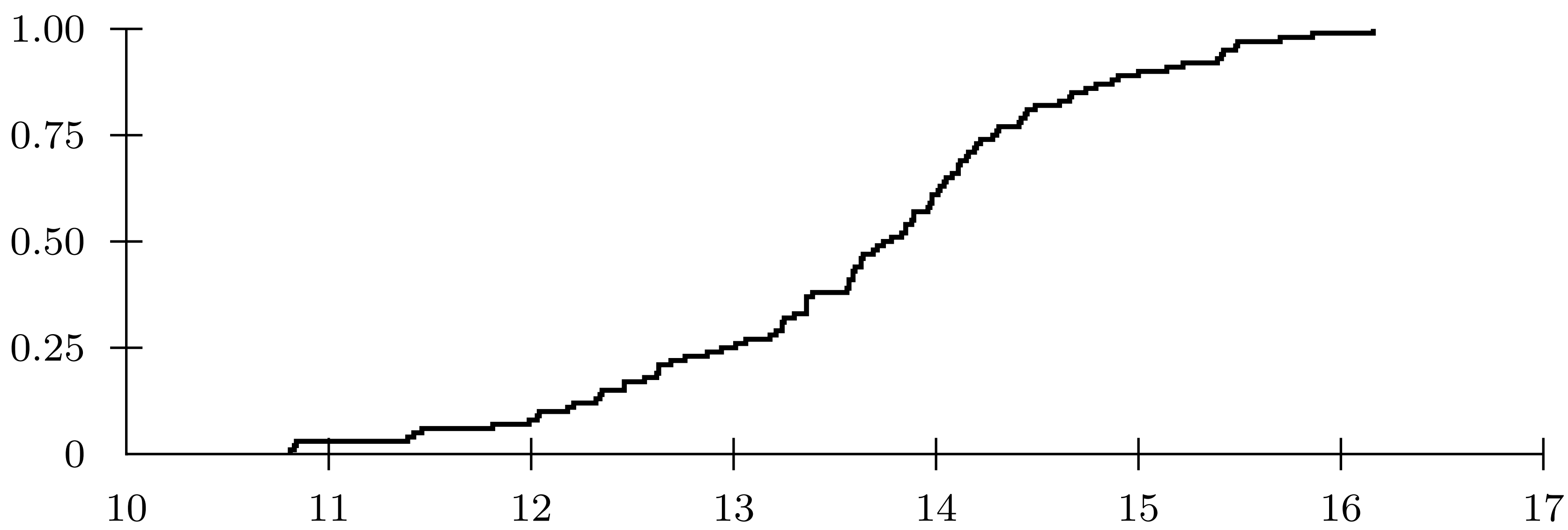

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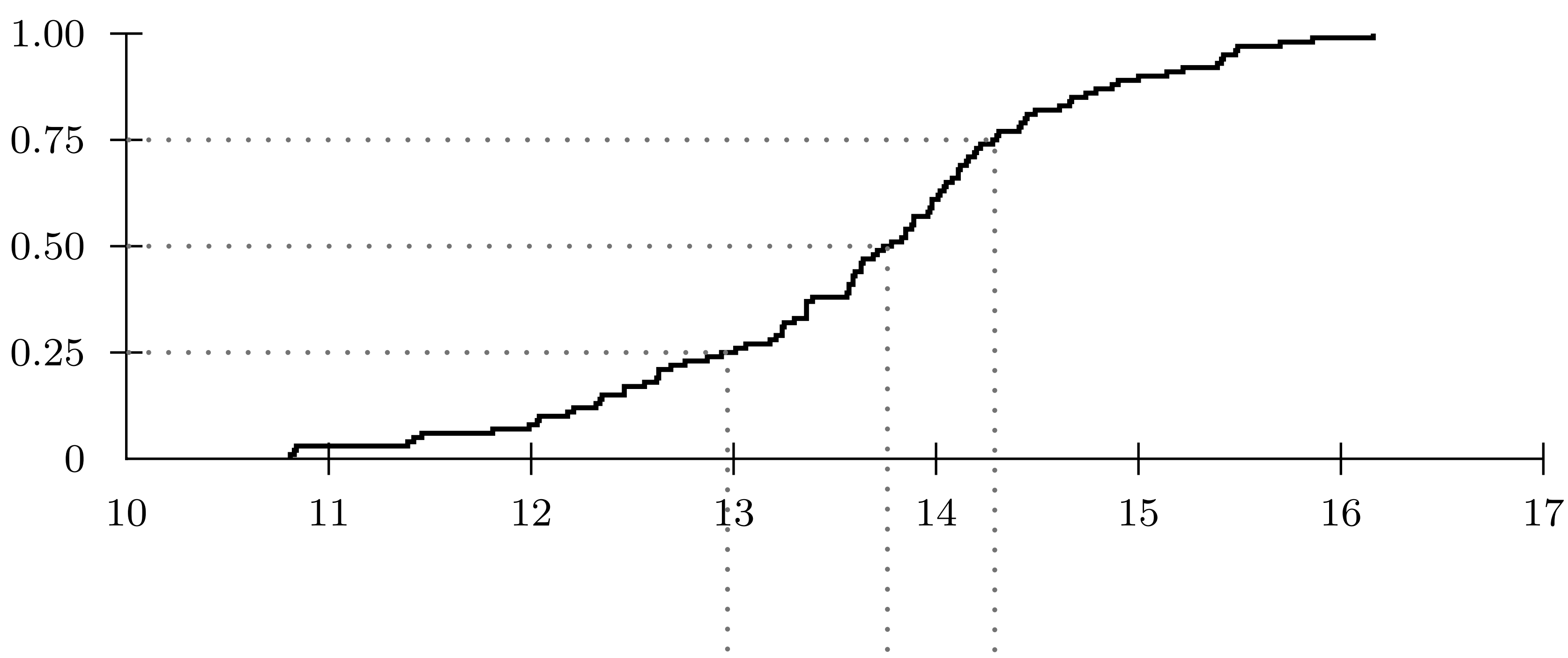

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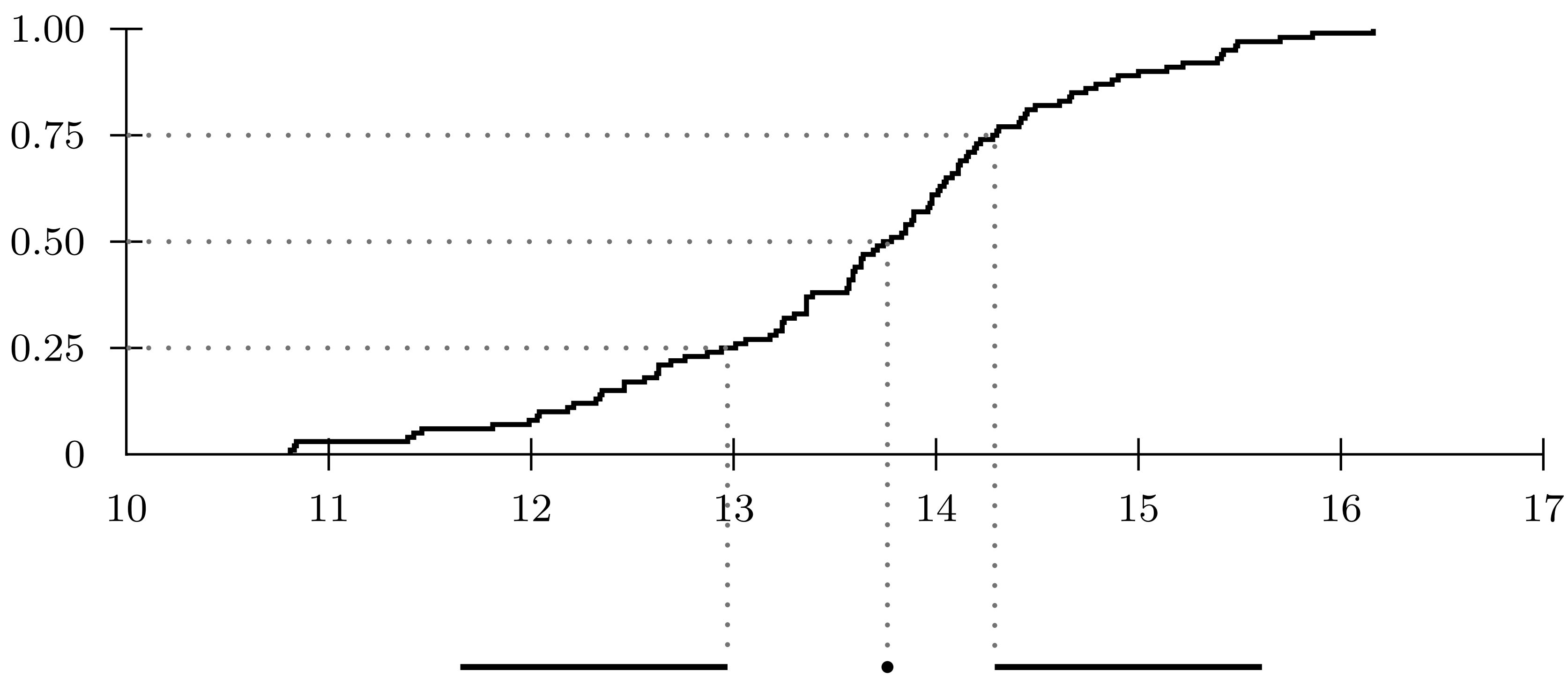

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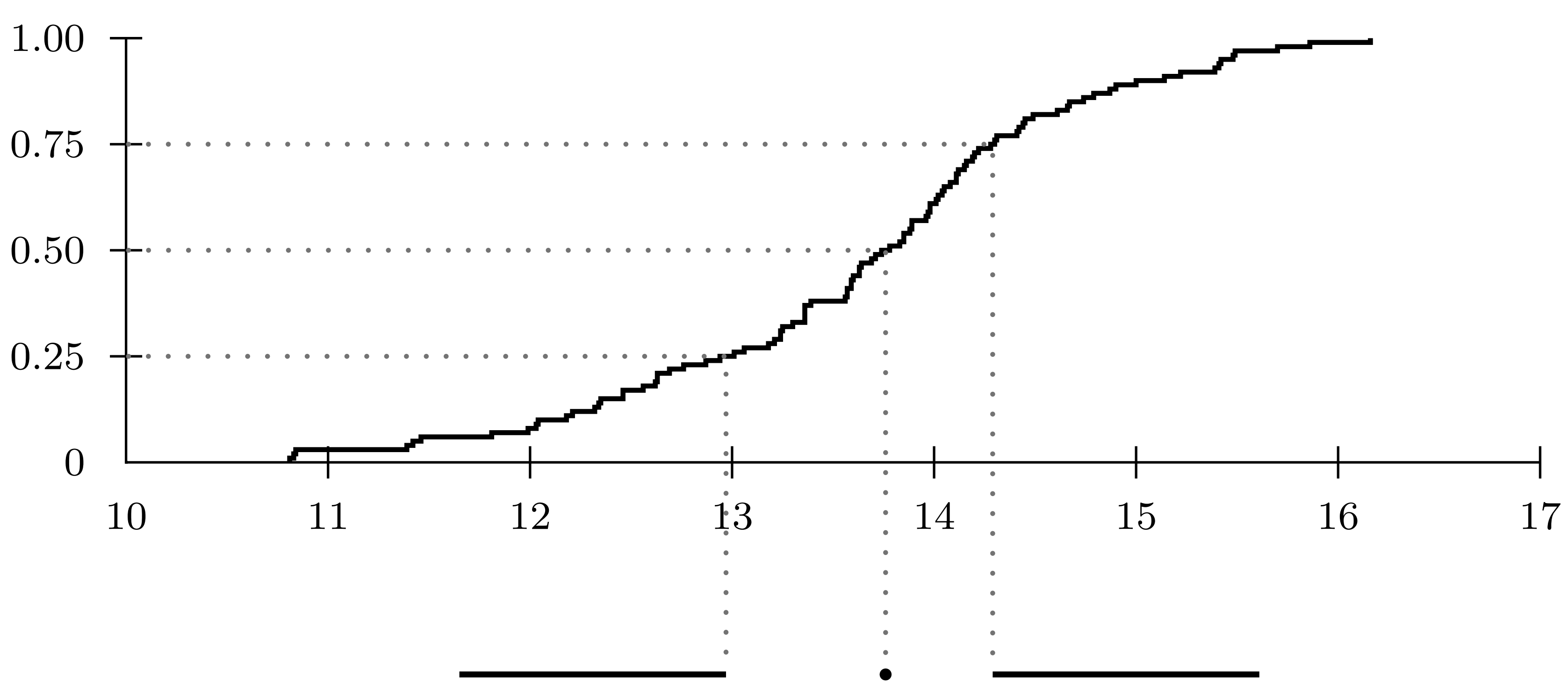
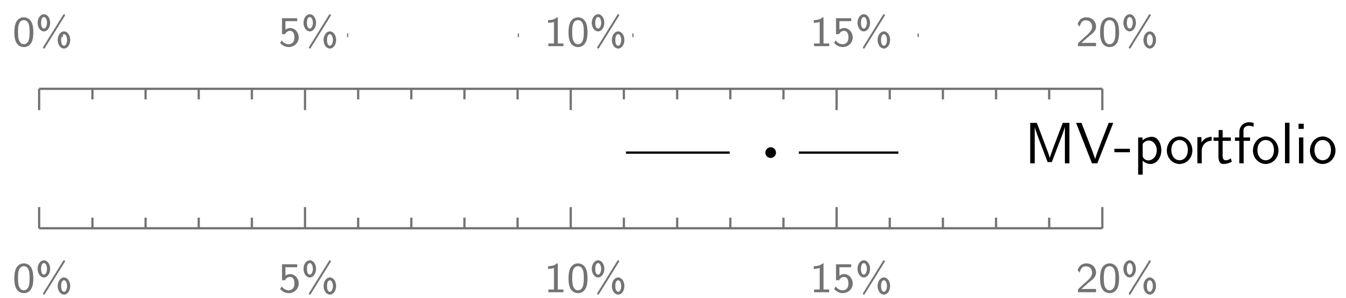


reward	risk	median
c	$\mathcal{M}_2$	13.76



results VaR, Expected Shortfall

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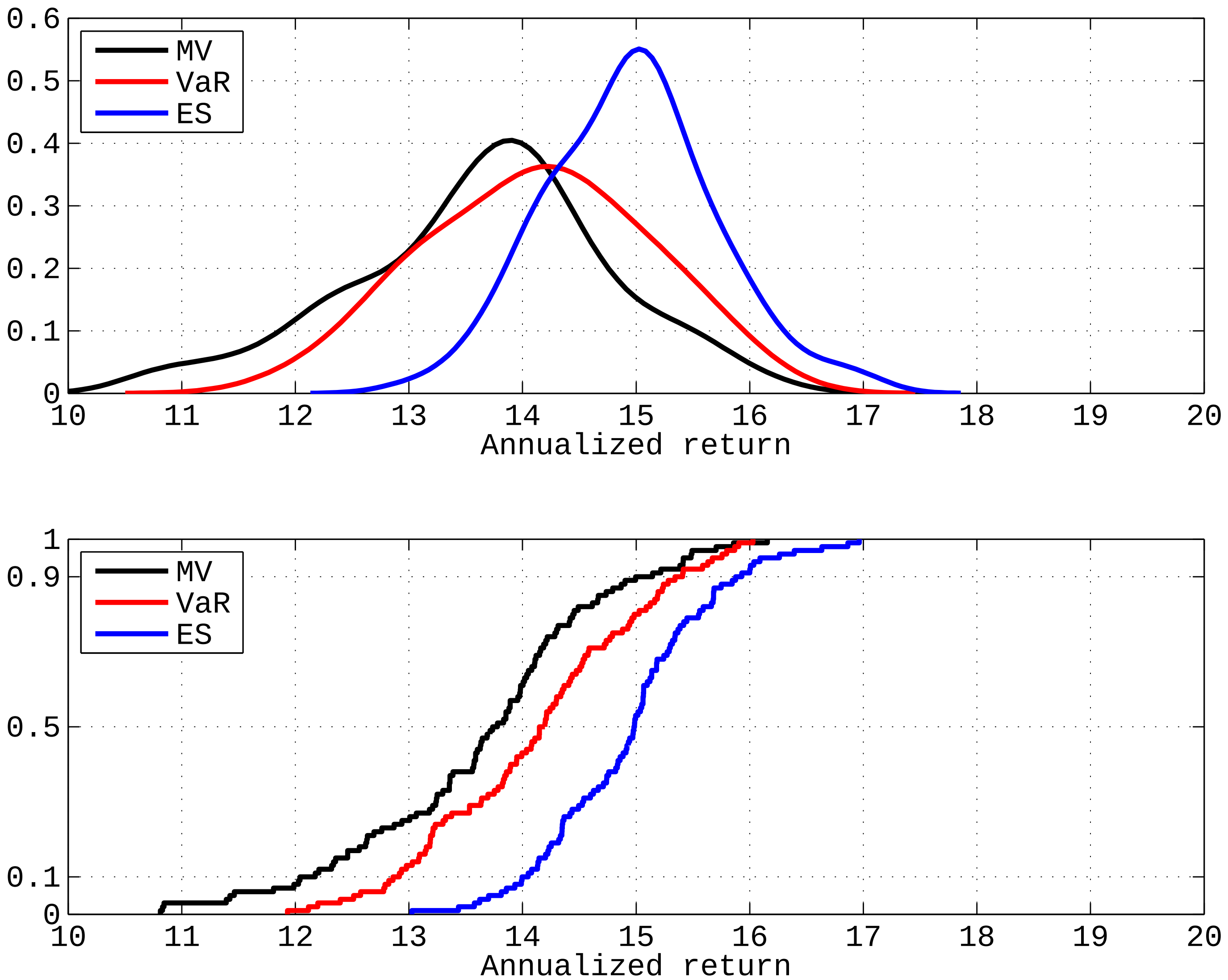

results VaR, Expected Shortfall

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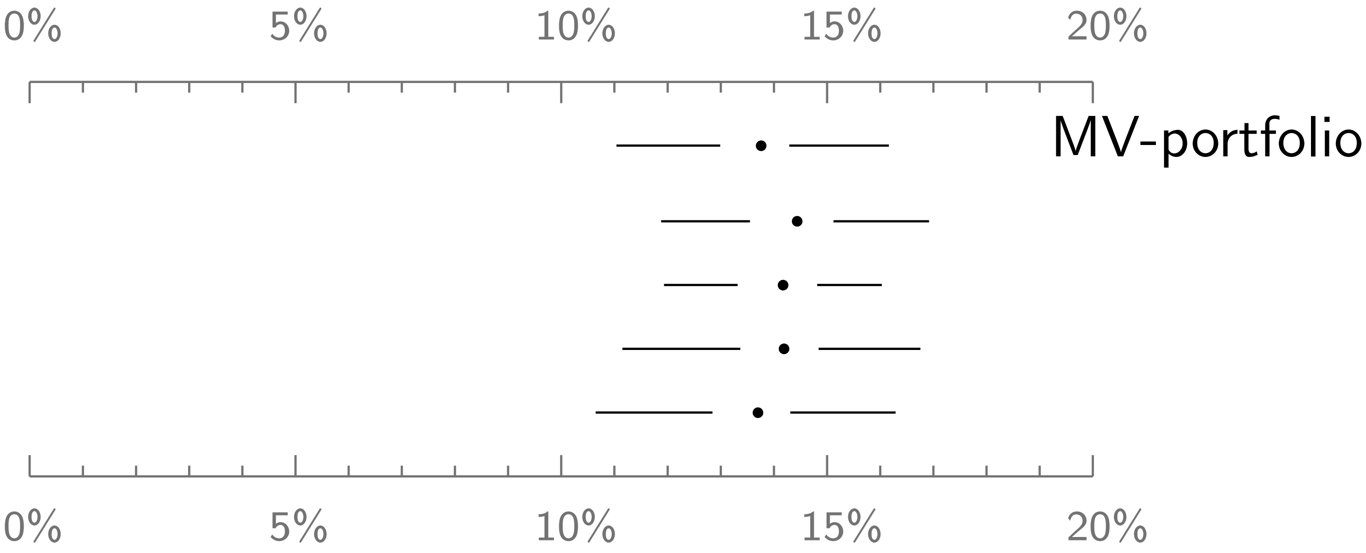
reward	risk	median
c	$\mathcal{M}_2$	13.76
c	$\mathcal{Q}_1$	14.44
c	$\mathcal{Q}_5$	14.17
c	$\mathcal{Q}_{10}$	14.19
c	$\mathcal{Q}_{20}$	13.70



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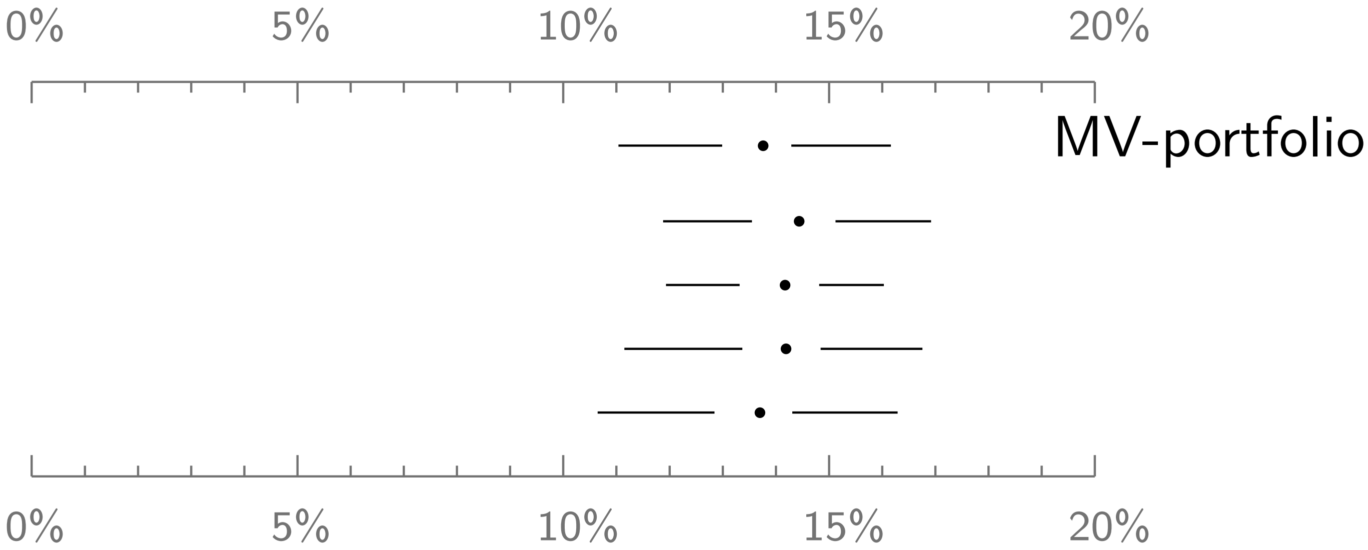
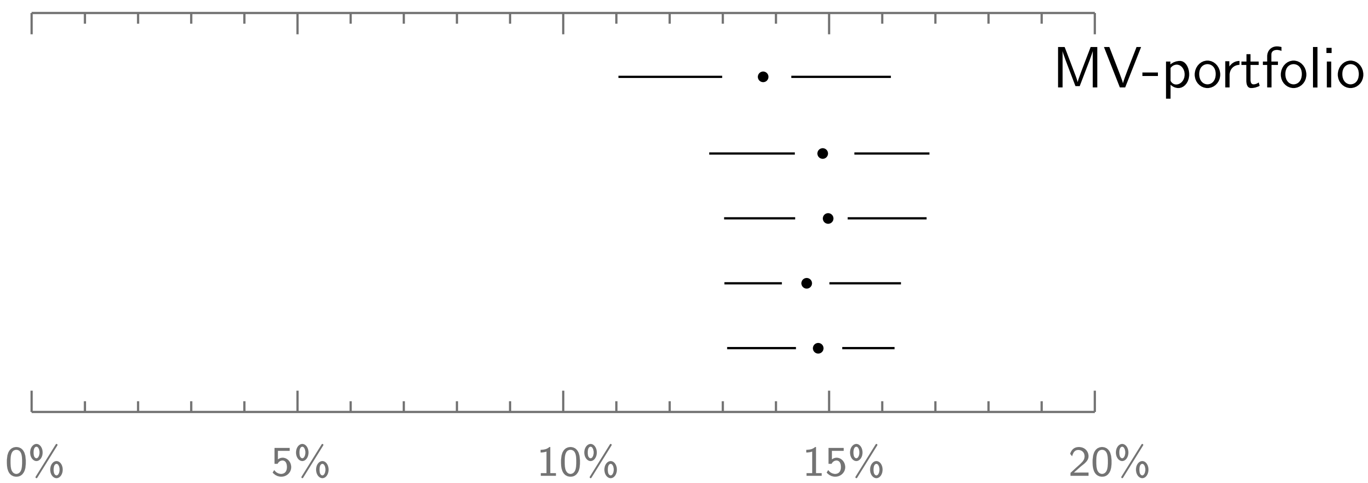


reward	risk	median
c	$\mathcal{M}_2$	13.76
c	$\mathcal{C}_1^-(\mathcal{Q}_1)$	14.88
c	$\mathcal{C}_1^-(\mathcal{Q}_5)$	14.98
c	$\mathcal{C}_1^-(\mathcal{Q}_{10})$	14.58
c	$\mathcal{C}_1^-(\mathcal{Q}_{20})$	14.80



results partial moments

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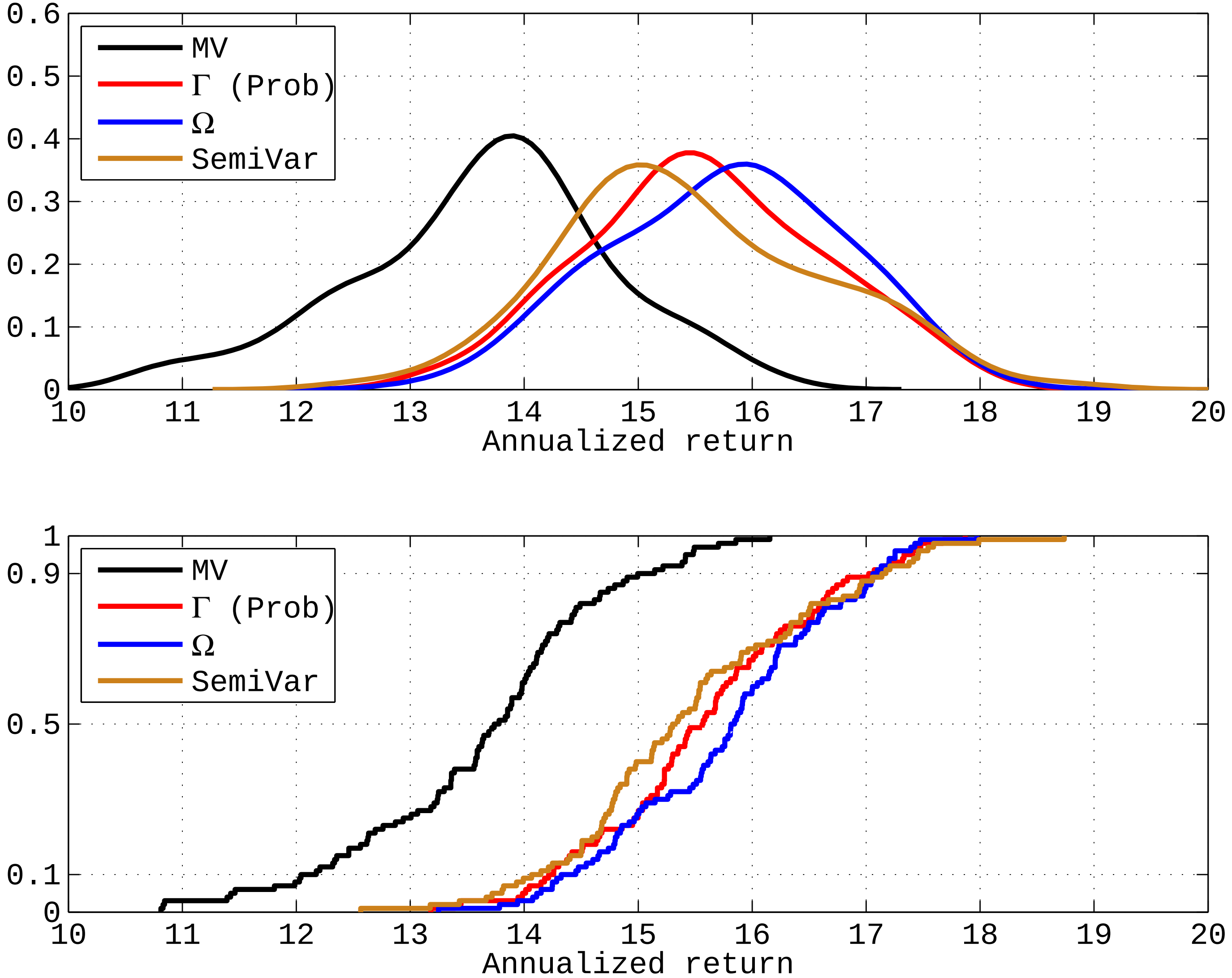

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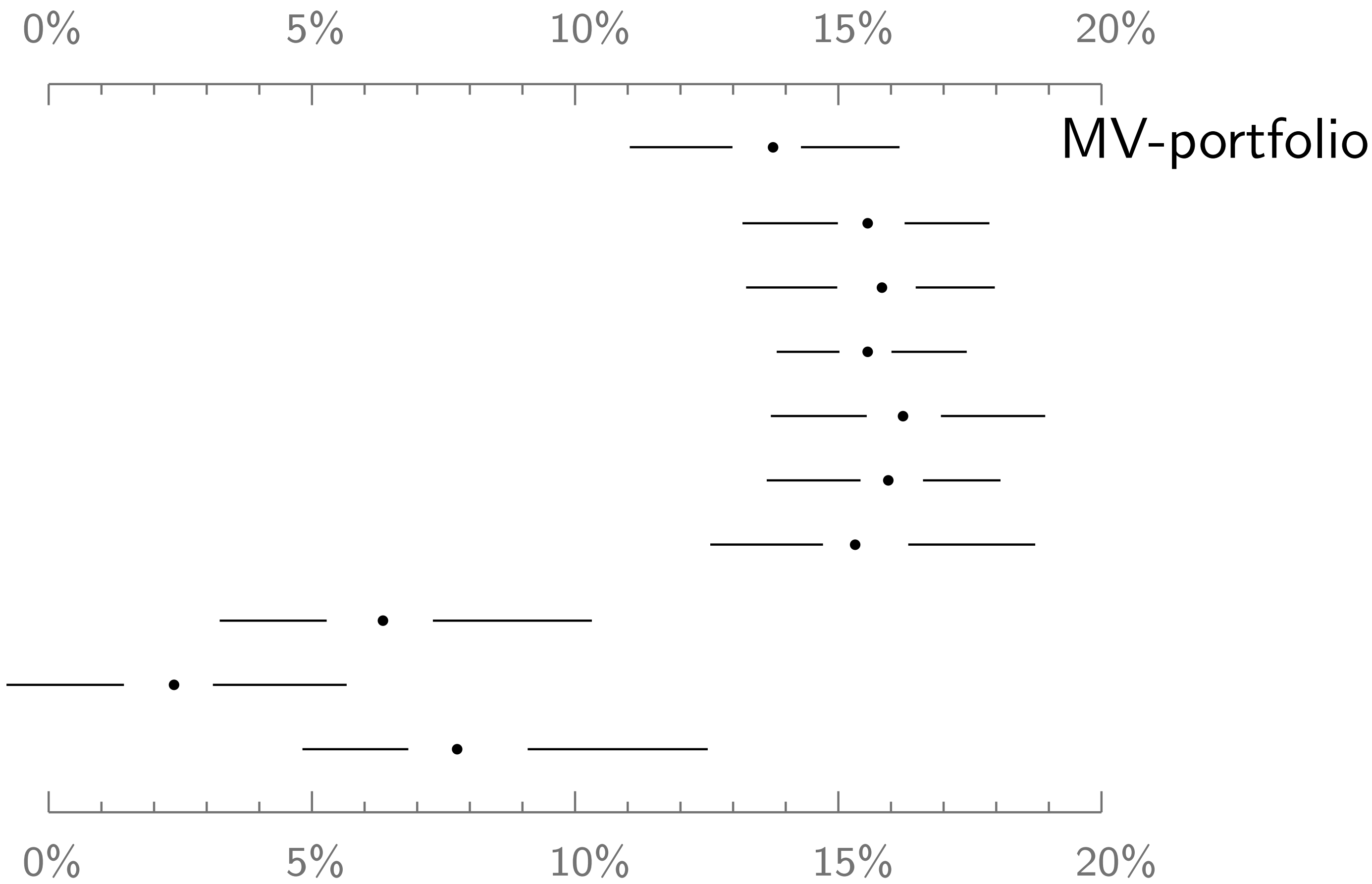
reward	risk	median
c	$\mathcal{M}_2$	13.76
$\mathcal{P}_0^+$	$\mathcal{P}_0^-$	15.56
$\mathcal{P}_1^+$	$\mathcal{P}_1^-$	15.83
$\mathcal{P}_1^+$	$\mathcal{P}_2^-$	15.56
$\mathcal{P}_{1.5}^+$	$\mathcal{P}_{1.5}^-$	16.23
$\mathcal{P}_{1.5}^+$	$\mathcal{P}_2^-$	15.95
$\mathcal{P}_2^+$	$\mathcal{P}_2^-$	15.32
$\mathcal{P}_3^+$	$\mathcal{P}_2^-$	6.35
$\mathcal{P}_4^+$	$\mathcal{P}_2^-$	2.38
$\mathcal{P}_4^+$	$\mathcal{P}_3^-$	7.76



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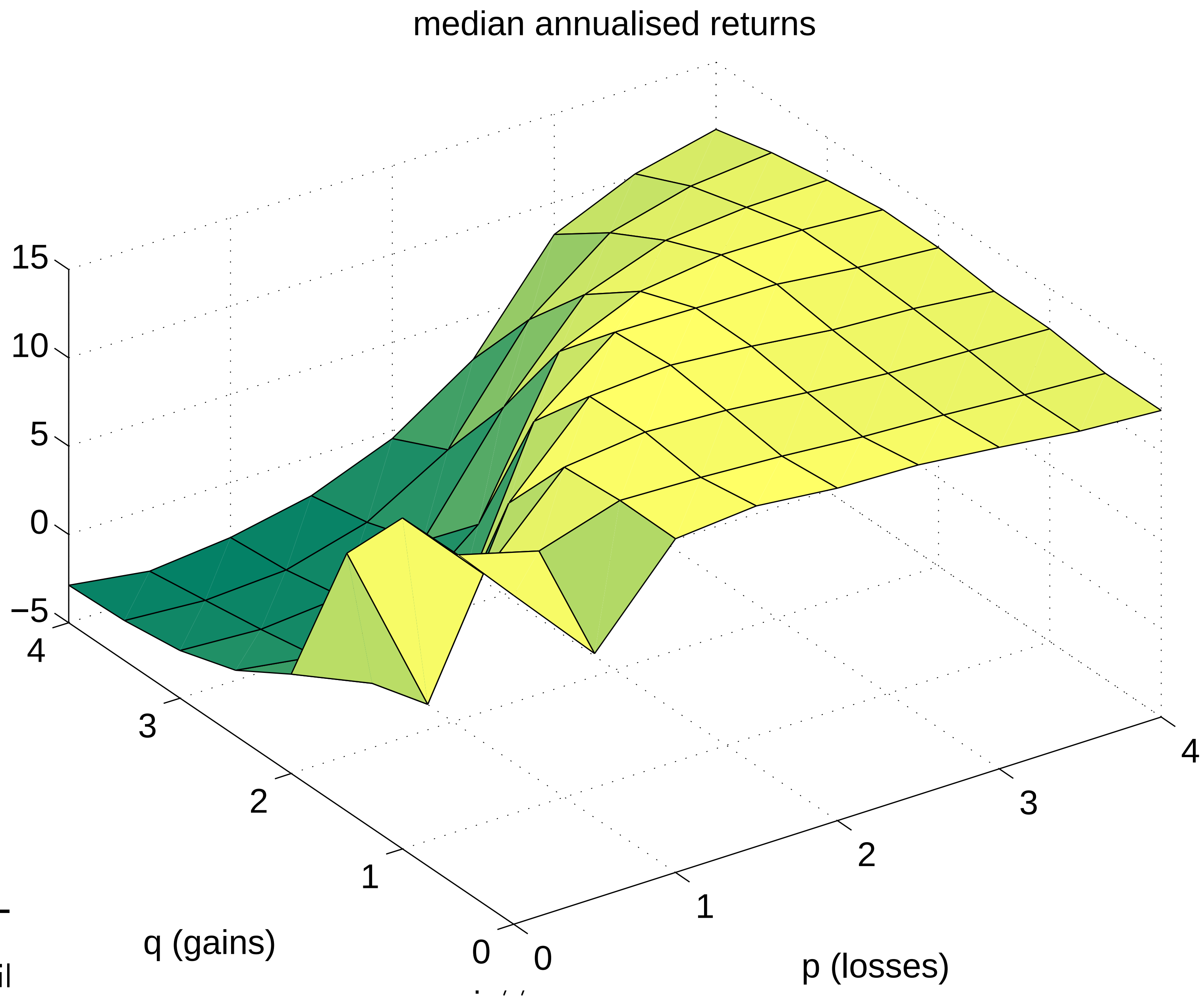

results partial moments

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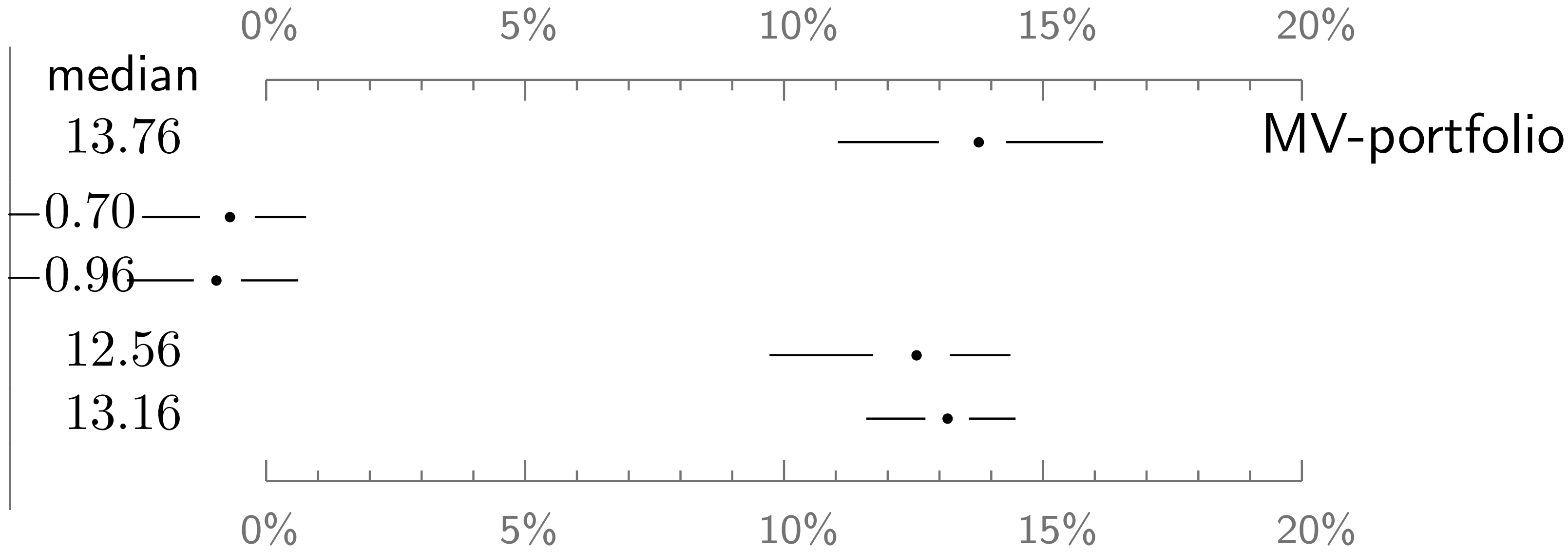
reward	risk	median
c	$\mathcal{M}_2$	13.76
$\mathcal{P}_2^+$	c	−0.70
$\mathcal{P}_3^+$	c	−0.96
c	$\mathcal{P}_2^+$	12.56
c	$\mathcal{P}_3^+$	13.16



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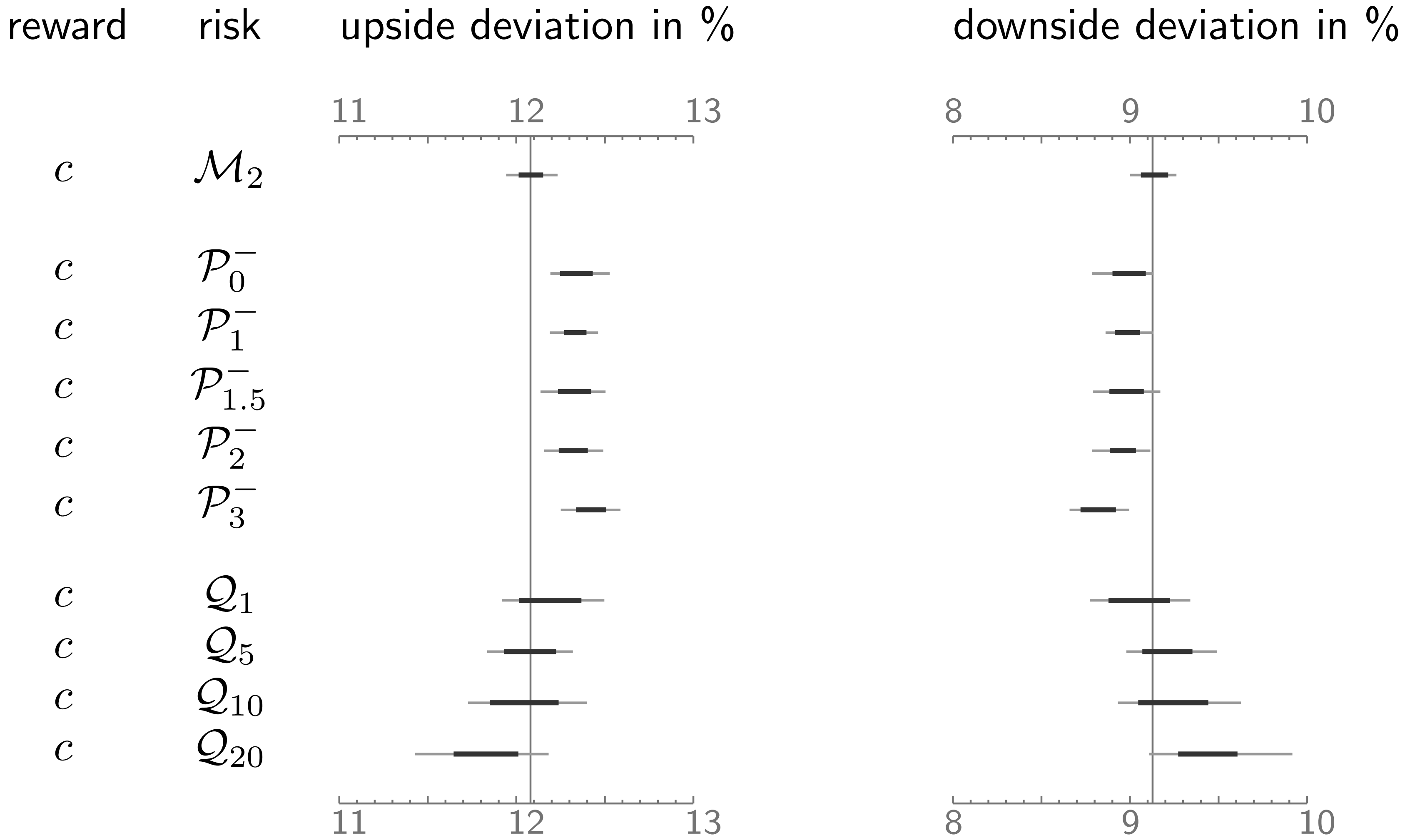

results drawdowns

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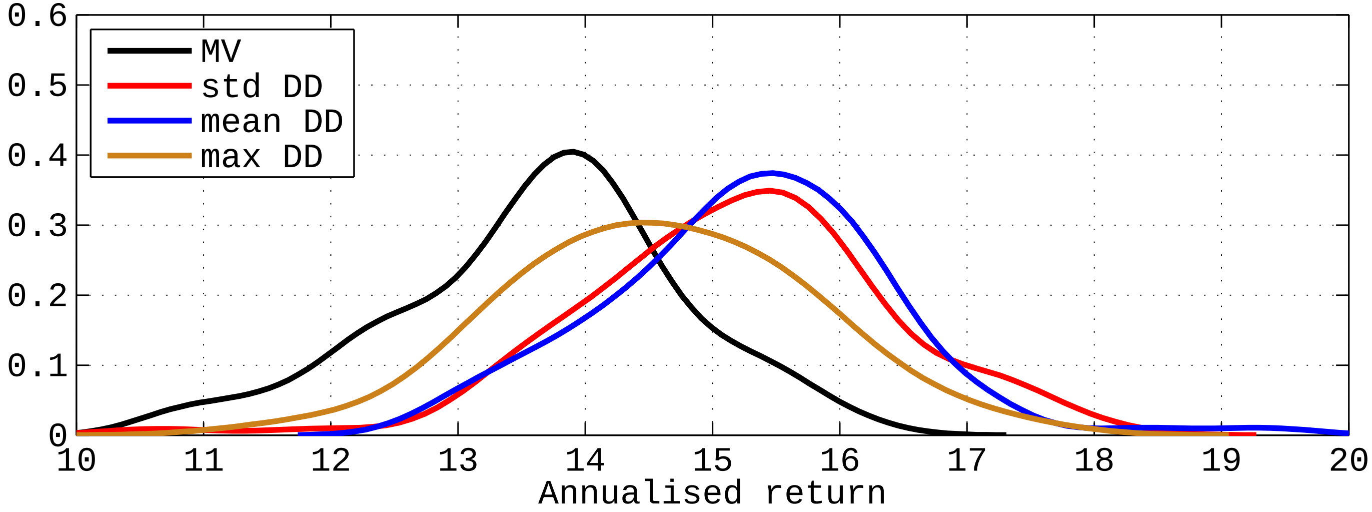

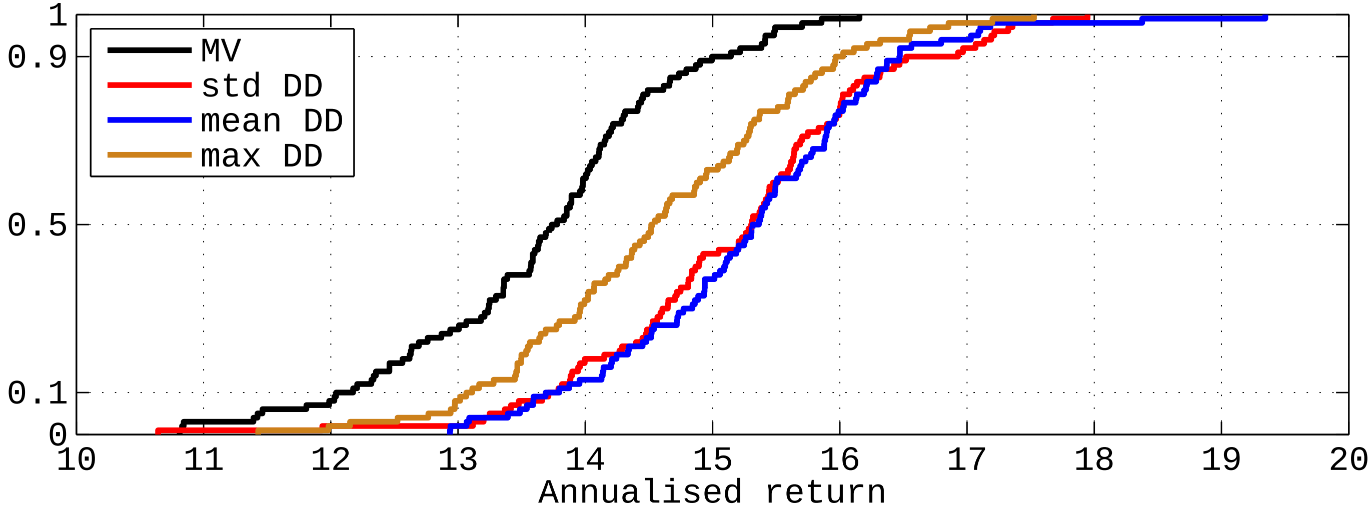

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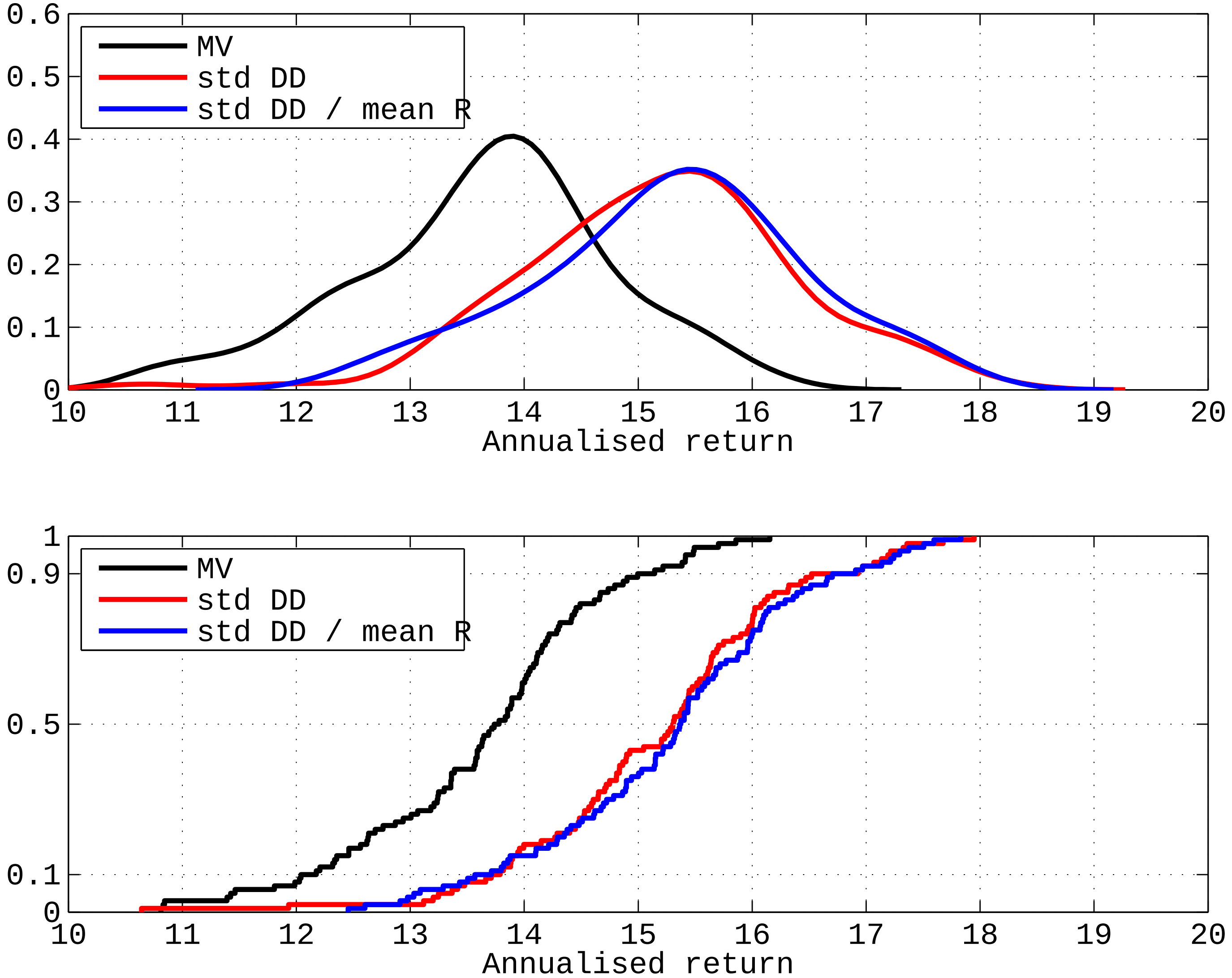

results drawdowns

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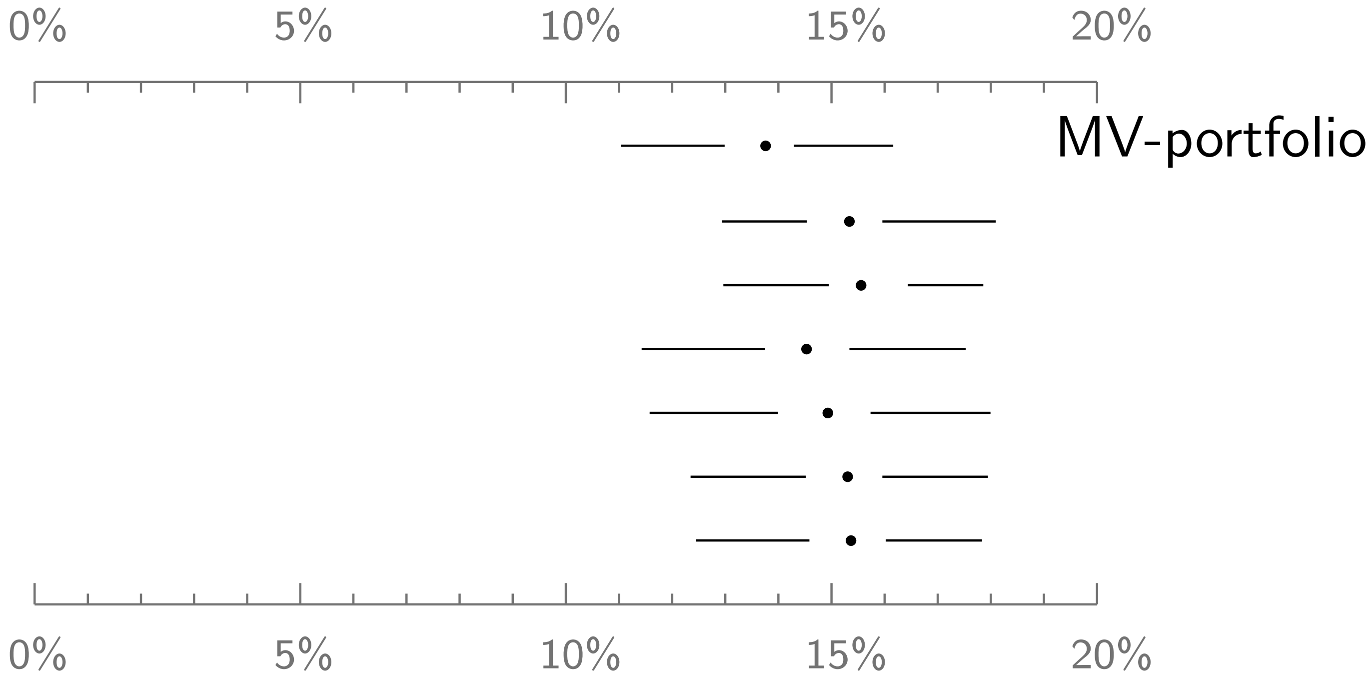
reward	risk	median
c	$\mathcal{M}_2$	13.76
c	$\mathcal{D}_{\text{mean}}$	15.34
$\mathcal{M}_1$	$\mathcal{D}_{\text{mean}}$	15.56
c	$\mathcal{D}_{\text{max}}$	14.53
$\mathcal{M}_1$	$\mathcal{D}_{\text{max}}$	14.93
c	$\mathcal{D}_{\text{std}}$	15.31
$\mathcal{M}_1$	$\mathcal{D}_{\text{std}}$	15.37



conclusions

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  - optimisation more difficult, but manageable

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  - optimisation more difficult, but manageable
  - add value over mean–variance
- problems sensitive to data changes
- minimise risk: low variability leads to well-performing portfolios
- adding reward increases return but also variability and sensitivity

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