The Impact of Stochastic Volatility on Pricing, Hedging, and Hedge Efficiency of Variable Annuity Guarantees

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Research Purpose

- Variable Annuities are fund-linked annuities
  - the policyholder typically pays a single premium that is invested in one or several mutual funds
- several guarantee riders available on top of this, with the most recent one being *"Guaranteed Lifetime Withdrawal Benefits"* (GLWB)
  - the policyholder is guaranteed lifelong minimum withdrawals
    - however, the invested capital is not annuitized
      ➔ fund assets remain accessible to the policyholder
  - withdrawals are deducted from the policyholder's account value as long as it has not been depleted
    - <u>afterwards, the insurer has to compensate for the guaranteed withdrawals until the insured's death</u>
    - in return for this guarantee, the insurer receives guarantee fees deducted from the policyholder's fund assets (as long as there are any)

➔ combination of policyholder behavior, longevity and market risk that is difficult to hedge

Research Questions

– The focus of the paper lies on market risk and specifically on volatility risk in the context of GLWB options. Its key question is:

  Compared to a model that assumes deterministic equity volatility, what impact does *stochastic* equity volatility have on pricing and hedging of GLWB options?

  - How do different (dynamic) hedging strategies perform under different data-generating models?

  - How are these effects influenced by the product design of the GLWB option?

Main findings

- impact deterministic volatility ➔ stochastic volatility
  - pricing: little effect (assumed the volatility risk premium is zero)
  - efficiency of the considered hedging strategies: substantial effect
- this risk can be reduced with the additional use of standard options to hedge against volatility risk (vega hedging)
  - however, some strategies may also make things worse
- product design has a significant impact on how risky a product is to the insurer and on how well it is hedgeable

Agenda

- **Product designs**
- Market models
- Policyholder behavior
- Hedging strategies
- Hedging results

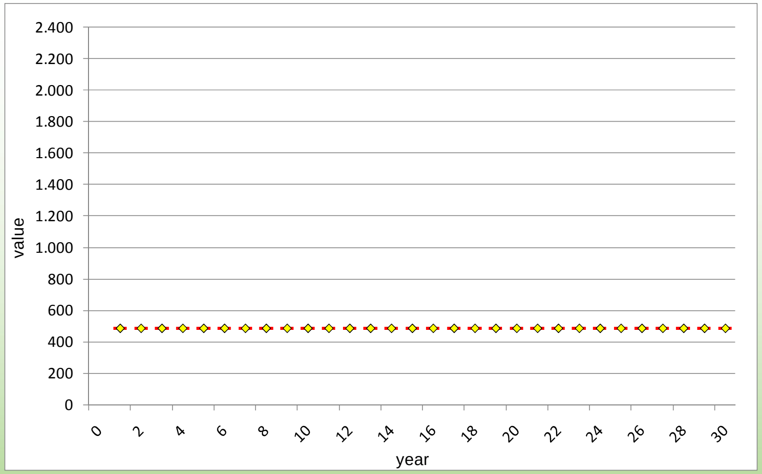
Product designs of the GLWB option

- All considered designs guarantee an annual minimum withdrawal amount for the lifetime of the insured.
  - surrender benefit = account value (contract and guarantee end)
  - death benefit = account value (contract and guarantee end)
- Depending on the product design, the guaranteed withdrawal amount can also increase if the fund performs well.

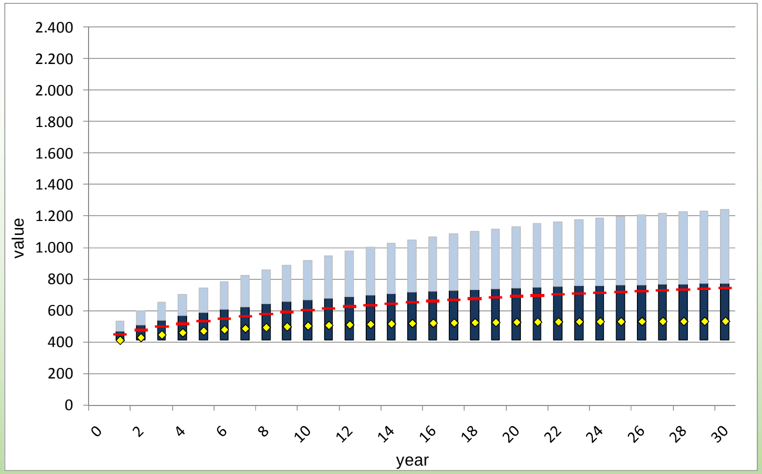
➔ Four different ratchet mechanisms considered:

1) **No Ratchet**
   - the guaranteed withdrawal amount remains constant
2) **Lookback Ratchet**
   - the guaranteed withdrawal amount is calculated as a percentage of the highest account value at all past policy anniversaries
3) **Remaining Withdrawal Benefit Base (WBB) Ratchet**
   - if the account value exceeds a certain reference value, the surplus is used to increase the guaranteed withdrawal amount for all following payments
4) **Performance Bonus**
   - if the account value exceeds a certain reference value, half of the surplus is paid out to the policyholder directly

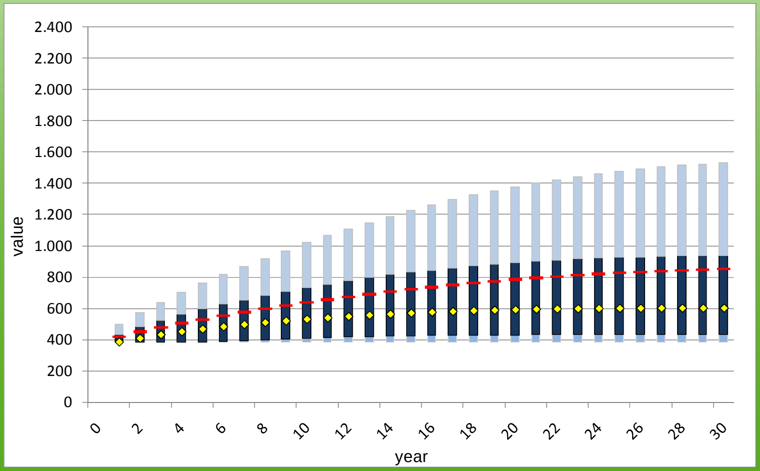
Distributions of the annual guaranteed withdrawals for all four product designs (all priced with the same guarantee fee)



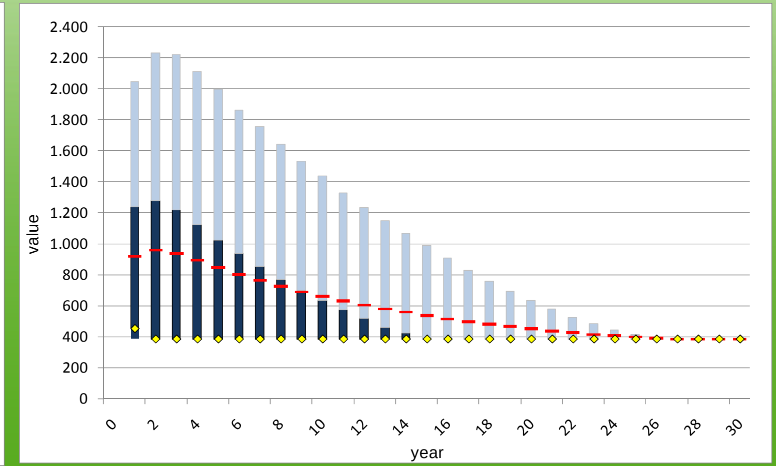
1 (No Ratchet)



2 (Lookback Ratchet)



3 (Remaining WBB Ratchet)



4 (Performance Bonus)

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Market models used for pricing, hedging and simulation

- constant interest rates
- no spreads / no transaction costs
- The dynamics of the contract's underlying fund is given by the following SDEs:
  - **Black-Scholes (1973)**

$$dS(t) = \mu S(t)dt + \sigma_{BS} S(t)dW(t), \quad S(0) \geq 0$$

  - **Heston (1993)**

$$dS(t) = \mu S(t)dt + \sqrt{V(t)}S(t)dW_1(t), \quad S(0) \geq 0$$

$$dV(t) = \kappa\big(\theta - V(t)\big)dt + \sigma_v \sqrt{V(t)}dW_2(t), \quad V(0) \geq 0$$

  - with
    - μ - drift
      σ_{BS} - Black-Scholes volatility
    - V(t) - local variance at time t
      κ - speed of mean reversion
      θ - long-term average variance
      σ_v - "vol of vol"
      $W_{1/2}$ - Wiener processes
      ρ - correlation between W_1 and W_2

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Policyholder Behavior

- only two possibilities considered:
  - policyholder withdraws the guaranteed amount
  - policyholder withdraws all of the remaining fund assets
    ➔ full surrender

- probabilistic policyholder behavior
  - policyholder performs full surrender with a certain probability in each year

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Hedging strategies

- two hedging models
  - Black-Scholes
  - Heston
- three different types of dynamic hedging strategies
  1) **no active hedging**
  2) **hedging using the underlying only**
     1) delta hedge in case of Black-Scholes
     2) local risk minimizing hedge in case of Heston
  3) **hedging using the underlying and a pre-specified (standard) option on the underlying**
     - aim: attain additional vega-neutrality of the portfolio
     - two different approaches for delta-vega hedging with Black-Scholes: modified and unmodified vega
- monthly rebalancing of the hedge portfolio

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- **Hedging results**

Hedging – Some simulation results

- values represent relative changes in risk measure (CTE90 of final P&L)
- **Black-Scholes delta hedge**
  - data-generating model: Black-Scholes ➔ Heston: +47% to +70%
- **Heston data-generating model**
  - B-S delta ➔ Heston LRM: 0% to -7%
  - B-S delta ➔ B-S delta-vega (mod. vega): -40% to -57%
  - Heston "delta" ➔ Heston "delta-vega" : -50% to -61%
  - B-S delta ➔ B-S delta-vega (unmod. vega): +166% to +282%

Thank you for your attention!

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