Variable Annuities - issues relating to dynamic hedging strategies

Christophe Bonnefoy[1], Alexandre Guchet[2], Lars Pralle[3]

Abstract

This article presents the concept of dynamic hedging and focuses on the risks inherent to this kind of strategies. Among these risks, the liquidity risk is treated more in detail.
We give three pragmatical methods to insert liquidity risk into internal models used for regulatory capital purpose or for price setting.

Key Words

Variable Annuities, Dynamic Hedging, Regulatory capital, Liquidity risk

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1. Mazars Actuariat, 61 rue Henri Regnault, 92075 Paris - La Défense Cédex, France, christophe.bonnefoy@mazars.fr
2. Mazars Actuariat, 61 rue Henri Regnault, 92075 Paris - La Défense Cédex, France, alexandre.guchet@mazars.fr
3. Mazars Hemmelrath, Martin-Luther-Platz 26, 40212 Düsseldorf, Germany, lars.pralle@mazars.de,
corresponding author

1. Preamble

1.1 Brief description of Variable Annuities

With Variable Annuities (VA), a product concept that combines aspects of unit-linked and traditional life insurance has found its way from the Northern American and Japanese market also to Europe. While the basic construction is unit-linked, variable annuities allow flexible product designs that move parts of the financial risk away from the insured. However, in contrast to traditional life products, the financial risk is not borne by the insurance company for free, but the customer is charged with a price for this risk transfer. Conceptually, the charges collected for this risk transfer should allow the insurance company to pass the financial risk on to the capital markets via the purchase of hedging assets.
VA products could in principle be constructed like normal unit-linked products with the flexibility inherent in the unit-linked concept, for example the possibility of regular or single premium payment, or the choice of a variety of underlying funds and the possibility of changing the fund allocation during the lifetime of the policy, either for new inflows, or even shifting accumulated investment between different funds. However, due to the guarantee component and the idea of hedging the guarantee during the policy lifetime, one has to look carefully as to how much flexibility ultimately can be allowed.
There are various types of guarantees currently offered on the market, which are characterised by the pattern of minimum benefits offered:

1.1.1 Death benefits

GMDB: guaranteed minimum death benefit. The guarantee is relevant only in the case of death and offers a defined amount to be paid out in this case even if the value of the fund would be lower.

1.1.2 Life benefits

GMAB: guaranteed minimum accumulation benefit. The guarantee offers a minimum amount at the maturity of the VA policy, even if the fund value is lower at that time.
GMIB: guaranteed minimum income benefit. The guarantee offers a minimum income stream starting at a determined time in the future.
GMWB: guaranteed minimum withdrawal benefit. This guarantee is similar to the GMIB, but does not necessarily require regular (e.g. annuitized) payments.

In respect of the amount of the guarantee, there are various proposals in the market, including

- guaranteed return of premium paid: the value of the guarantee will not drop below the sum of premiums (guaranteed minimum yield of 0)
- roll-up of premiums: additionally to the premium return, a minimum yield on the accumulated premiums is guaranteed (e. g. 3 %)
- ratchet: fund values at specific dates (e.g. policy anniversary) are locked-in as minimum value of the benefit

- combinations of lock-in and roll-up: the higher of the premium roll-up and the fund value at the lock-in dates is guaranteed

The structure and amount of the offered guarantee has a determinating impact on the construction of a hedge portfolio or strategy. However, there are certain aspects that are common to most of the hedging approaches, one of which is the requirement of a functioning, i.e. liquid and deep market for the heding assets. In the following, we will try to illustrate this further and explore ways to quantify the risk emanating from a (temporary) failure of this market.

1.2 Dynamic hedging strategies for variable annuities

1.2.1 Principle

An hedging strategy consists in building a portfolio whose fluctuations are the opposite of the liabilities of the insurer; such a portfolio is called "hedging portfolio". If the hedged portfolio suffers losses, the hedging portfolio will make profits to offset this loss; on the contrary, if the hedged portfolio makes profit, the hedging portfolio will make loss to offset this profit. A perfectly hedged portfolio is has a risk free behaviour.

There exists different strategies to build a hedging portfolio which can be split into two main categories:

- Static hedging : the portfolio is built at the beginning of the strategy and is not updated
- Dynamic hedging : the portfolio is updated at regular interval

The static hedging is an easy way to mitigate risk. An insurer selling an unit-linked product can buy the corresponding asset to have a flat net position. This kind of hedging allows to handle the major part of market risk and can be more complex for custom pay-offs.

However, the simplicity of static hedging strategies leads to drawbacks. The first one is that the market risk of complex pay-offs cannot be fully hedged with static strategy (using simple hedging instruments) and the second one is that the outstanding hedged is fixed at the initiation of the hedging strategy: any deviation from the original expectations for lapse or mortality will lead to a disparity between the liabilities and the hedging portfolio and to the market risk inherent to the net position.

The principle of a dynamic hedging strategy is to adapt the hedging portfolio to the risk profile of the liabilities over time. At regular frequency, or when triggers are activated, the portfolio manager makes the hedging portfolio fit the liability to hedge. In particular, it permits to take into account the observed lapse of the variable annuities portfolio to adapt the hedged outstanding which reduces the risk of over-hedging or under-hedging.

This kind of strategy permits very flexible ways to hedge the risk but also has drawbacks. Indeed, in many cases, the premium paid by the insured is fixed at the subscription; fluctuations of the market conditions can lead to higher hedging costs. The implementation of hedging strategy can lead to unexpected losses if such evolution of market condition have not been taken into account during the price setting process.

In addition, the implementation over time of the dynamic strategy leads to operational risk. The implementation of such strategies need a specific background. As far as the regulatory capital calculation is concerned, the dynamic hedging strategies should satisfy the constraints below:

- The dynamic hedging strategy should be documented. The documentation describes the hedge risk and the practical details of its implementation: financial instrument used, outstanding, strikes...
- An ad hoc governance which endorses the hedging strategy and monitors its implementation with a reporting at regular interval;
- The entity which implements the strategy has the technical and human resources to do it;
- The internal model used for forecasting market conditions is able to quantify the effect of the strategy (its implementation in the simulations and the valuation of the financial instruments used for hedging). In addition, all the parameters which have an influence on the prices of financial instruments should be included in the model, in particular the risk inherent to the fluctuation of the implied volatility.

The first step in the implementation of the hedging strategy is to define it. The documentation of a strategy may describe the behaviour the portfolio manager may adopt when faced to a given situation. The latitude of the trader has to follow the restrictions set in the documentation in order to have a reliable vision of the hedging portfolio in the forecast. The documentation should cover:

- the frequency ;
- the input (market factors, liabilities indicators) ;
- the kind of financial instruments to buy or sell,
- the output of the strategy: the characteristics of the financial instruments and the quantity to buy.

The next two requirements allow a secure implementation of the strategy once it is defined.
The last item required permits to quantify with accuracy the effect of the dynamic hedging strategy on the mitigation of the risk.

1.2.2 Risks linked to dynamic hedging

A recent paper (Variable Annuities, M.C. Ledlie et al, [LED08]) describes the variable annuities in the UK. In the risk management section of this article, a list of the different risks the variable annuities lead to is given. The table below presents the intensity of the relation ships between the dynamic hedging strategy and the different risks.

The table above shows that the risks inherent to dynamic hedging strategies are complex and varied. In addition to the risks listed above, one must consider liquidity risk which usually not included in the market risk. However, it is a real risk.
The academic publications describe different ways to model liquidity risk. We present three methodologies in the following section.

2. A specific risk of dynamic hedging strategies : liquidity risk; how to include it in an internal model?

The recent events highlighted risks the internal models did not consider previously for the calculation of regulatory capital, among them the liquidity risk.
This risk has three main facets depending on the point of view:

1. For a treasurer, this risk is the risk for an entity to be short of cash;
2. For a portfolio manager or a trader, the risk for a market to be inactive;
3. For a central bank, the risk of a drainage of the liquidity circulating in the economy.

As dynamic hedging strategy is concerned, the second facet is relevant: "What would happen if the market where the portfolio manager finds the hedge would disappear ?"
We present in this section three methods to quantify the liquidity risk and examples of practical implementation in internal models.

2.1 Simulation of a liquidity crisis : liquidity horizon and liquidity cost

A recent publication from the Basel committee of banking supervision [BC09] described a first methodology to include this risk in the regulatory capital. The approach proposed consists in the determination of a liquidity horizon and then the computation of a liquidity costs.

2.1.1 Liquidity Horizon

The liquidity horizon is the time needed to liquidate a position without dramatically affecting the market. Indeed, the ad-hoc liquidation of a substantial position compared with the market depth would offset market prices. Streching the liquidation of the position over a period in time, such an offset might be avoided. This constraint leading to three main recommendations for choosing an appropriate length for this period:

1. The liquidity horizon should be increasing with respect to the outstanding to liquidate or buy: the more substantial the outstanding, is longer would be the liquidity horizon; this constraint will highlight concentration issues of the liability.
2. Liquidity horizon should also consider the depth of the market;
3. The possibility to mishedge the risk with a *proxy* : if there exists a market whose behaviour is similar to the one of the real underlying, and whose depth is better that the one of the real underlying, is is possible to hedge part of the risk with this proxy. Nevertheless, it constitutes a basis risk which should also be monitored.

The Basel committee defined an arbitrary lower bound, three months, to the liquidity horizon. This threshold might be discussed between the insurance and its supervision entity.

2.1.2 Liquidity Cost

Once the liquidity horizon is determined, several methods can be used to estimate a liquidity cost. We present here two of them; the first one is the effect of the direct impact of the non implementation of the strategy and the second one is a VaR of the unhedged position.

2.1.2.1 Impact of scenario in which the hedging strategy is not implemented

The first methodology one could envisage is to forecast situations in which the hedging strategy has not be implemented for the duration of the liquidity horizon.
This methodology has the advantage of the simplicity of its implementation.
Nevertheless, in "half" of simulation, the profit and loss of the hedging strategy would have been negative and this modelisation of liquidity costs give a benefit from the non implementation of the hedging strategy which is not consistent with the market conditions of a liquidity crisis, when each entity would like to hedge their position and are confronted with inactive markets.

2.1.2.2 Quantile of the profit and loss of the unhedged position (VaR)

The use of a crash scenario is possible since it reflects a crisis. The estimation of the liquidity costs could be based on a VaR of the unhedged position.
The VaR is then defined by two main parameters :

1. the time horizon : this time horizon might be chosen equal to the liquidity horizon.
2. the probability of the crash scenario : If we consider a 95% VaR, it means that that in our simulations, there are only 5% scenarios leading to losses worse than the VaR. The probability might be a joint decision with the insurer and its supervisor.

2.2 Insertion of a risk premium : the bid ask spread

The lack of liquidity in a market can be caused by an increased risk aversion of the market participants. Investors ask for a high risk premium to carry risky positions. This premium can be compared to the bid-ask spread: a market maker who wants to avoid taking positions in the market will give high bid ask spreads so as to dissuade anybody to trade.
The insertion of this risk premium into an internal model could be done as follows. Considering the original model would forecast mid prices, then in a second step, the actual prices would be obtained inserting the bid-ask spread.
The bid-ask reveals the uncertainty of a market value, as the volatility does: the model can link the bid ask spread to the volatility of the asset. The assertions concerning the link between the volatility of the market risk factors and their bid-ask spread would have a strong impact on the tails of the distribution.

2.3 A model of the market structure

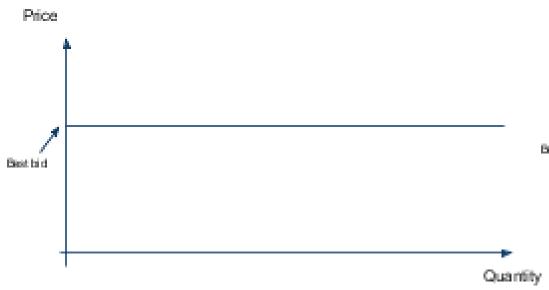
There exist different ways to valuate a portfolio. The list below gathers some methodologies:

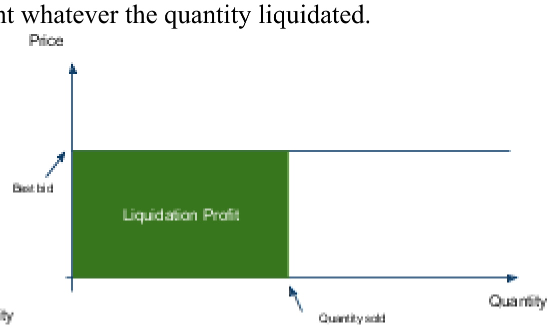
1. Use of MID prices ;
2. Use of BID price for short position and ASK price for long position ;
3. Taking into account the depth of the market.

The outcomes of these methodologies will be decreasing from mid prices to liquidation price as discussed below.

2.3.3 Definition of the liquidation cost

Usually, the value of a portfolio is given by the product of the bid price and the quantity. This calculation relies on the assumption the liquidation price is constant whatever the quantity liquidated.





The cash generated by the liquidation of the portfolio is proportional with respect to the quantity sold.
This assumption holds if the quantity sold is reasonable compared to the average volume exchanged on the market.

When the quantity to liquidate is substantial compared to the depth of the market, the assumption of constant liquidation price is not suitable. The calculation of the cash generated by the liquidation may consider the effective liquidation process.

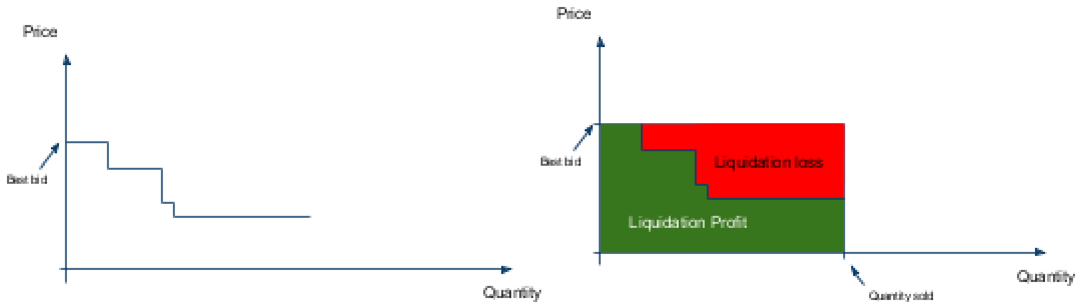
For a given instrument, the market condition is described at any time by:

- a list of decreasing prices ($p_1 > p_2 > ... > p_n$)
- a list of quantities ($q_1 > q_2 > ... > q_n$)

Each index corresponds to an offer on the market:

- Someone on the market wants to buy a quantity q_1 at price p_1;
- Someone on the market wants to buy a quantity q_2 at price p_2;
- ...

The process for a liquidation is to match the orders, in a decreasing order of price as long as the sum of the quantities sold is equal to the the quantity to liquidate.



The cash generated by the liquidation of the portfolio is the sum of $q_i p_i$ as long as the sum of the q_i is lower than the total quantity to sell: the average price of an asset is now decreasing with respect to the quantity to liquidate. This feature is consistent with the idea that if one considers the double of an illiquid portfolio, the risk more than doubles.

The model may include the different prices and the quantities to simulate a liquidity crisis of a market.

2.3.4 Model of future market condition

Whereas the internal models usually forecast the price of an asset, an expanded model could forecast the entire structure of the market, bid prices, ask prices and corresponding quantities. This granularity of the modelisation leads to huge needs of calculation resources and the model should be simplified using parametric forms.

For example, Almgren and Chriss [Alm99] introduce this feature in their model giving the asset price a behaviour with parametric form; the intensity of the decreasing trend is given by:

- The ratio of the volume sold and the market depth ;
- Parameters calibrated with an history of transactions.

With these two quantities, Almgren determines, with a closed-form formula, the liquidation cost. However, we have not seen a backtesting of this model on more recent data.

3. Conclusion

The risk management of variable annuities portfolios relies on sophisticated strategies for managing the hedging assets. When constructing or analysing these strategies, not only the specific features of the hedge instruments under normal market conditions should be taken into consideration, but attention should also be given to the limitations of the finacial markets them, in particular when dealing with large volumes. One of these limitations is the available liquidity of the market and we illustrated some approaches to the quantification of the risk coming from illiquidity or insufficient depth of the market.

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