UNDERSTANDING THE DEATH BENEFIT SWITCH OPTION IN UNIVERSAL LIFE POLICIES

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ABSTRACT

Universal life policies are the most popular insurance contract design in the United States. They have either a level death benefit paying a fixed face amount, or an increasing death benefit, which additionally to a fixed benefit pays the available cash value, and both types include the option to switch from one to the other. In this paper, we are interested in the fact that—unlike a switch from level to increasing—a switch from increasing to level death benefit requires neither fees nor additional evidence of insurability. To assess the impact of the death benefit switch option, we develop a model framework of increasing universal life policies embedding the option. Consideration of heterogeneity in respect of mortality via a stochastic frailty factor allows an investigation of adverse exercise behavior. In a comprehensive simulation analysis, we quantify the net present value of the option from the insurer's perspective using risk-neutral valuation under stochastic interest rates assuming empirical exercise probabilities. Based on our results, we provide policy recommendations for life insurers.

Keywords: Increasing death benefit; Death benefit switch option; Heterogeneity in respect of mortality

1. INTRODUCTION

First introduced in 1979, universal life is now the most important individual life insurance contract type in the United States. Lifelong universal life policies offer flexibility with respect to frequency and amount of premium payments and two death benefit options to choose from (see Cherin and Hutchins (1987, p. 691) and D'Arcy and Lee (1987, p. 453)). The *level* death benefit pays a constant specified face amount; the *increasing* death benefit pays the available cash value (or policy reserve) in addition to a fixed face value. Either type of contract typically embeds the option to switch from level to increasing or vice versa (the *death benefit switch option*). A switch from level to increasing benefits requires new evidence of insurability and, possibly, an extra fee since the death benefit immediately increases by the current amount of cash value at the time the option is exercised. In contrast, when switching from increasing to level benefits, the death benefit is fixed at the current value. Thus, in the latter case, the switch does not affect the net amount at risk, i.e., the difference between death benefit and cash value, at the switch exercise time and so there are usually no special requirements or fees involved in making this type of switch (see Smith and Hayhoe (2005, p. 2); see also, e.g., www.sagicorcapitallife.com). However, development of the net amount at risk after the switch depends on premium payment behavior. Thus, there is some question as to whether insurers should be concerned about death benefit switches under otherwise unchanged actuarial assumptions.

In the present paper, we examine the death benefit switch option in a pool of increasing universal life policies with the goal of enhancing understanding of this feature. To accomplish this, we develop a model framework for increasing universal life contracts with death benefit switch option that incorporates heterogeneity in respect of mortality, switch probabilities, and stochastic interest rates. Based on this model, we evaluate the option under different premium payment assumptions after switching and for various exercise scenarios. By considering results for adverse exercise behavior depending on an insured's health status, we derive policy implications and, in particular, analyze whether a requirement of charges or evidence of insurability would be advisable.

The literature about universal life insurance is mainly concerned with the return on universal life policies (e.g., Belth, 1982; Cherin and Hutchins, 1987; Chung and Skipper, 1987; D'Arcy and Lee, 1987). Carson (1996) finds determinants for universal life cash values, and Carson and Forster (2000) examine policy yields of whole and universal life contracts. Costs of universal and term life insurance are compared in Corbett and Nelson (1992). Carson (1996), Cherin and Hutchins (1987), and Chung and Skipper (1987) empirically study the return characteristics of increasing universal life policies. However, to date there have been no attempts to develop a model of universal life contracts with increasing death benefit, much less any study of the death benefit switch option. The same is true regarding premium payment options in universal life policies. Most studies are restricted to the paid-up option (i.e., stopping premium payments) in participating life insurance contracts (Kling, Russ, and Schmeiser, 2006; Linnemann, 2003, 2004; Steffensen, 2002). In addition to the paid-up option, Gatzert and Schmeiser (2007) integrate the resumption option (i.e., resumption of premium payments after having made the contract paid-up) in their framework for participating policies.

To the best of our knowledge, increasing universal life policies and the death benefit switch option have not yet been studied. We provide an actuarial model framework of a universal life contract with increasing death benefit and incorporate the death benefit switch option. Since universal life policies are lifelong contracts that pay a death benefit, we account for mortality risk as a central risk factor. Mortality varies among insureds, and thus heterogeneity in respect to mortality is modeled by a stochastic frailty factor on a given deterministic mortality table. The concept of frailty was originally defined by Vaupel et al. (1979) in terms of the continuous force of mortality. In this paper, we use the term "frailty factor" in the discrete context in order to express the factor's stochasticity, as well as respective distributional characteristics. An examination of adverse exercise behavior with respect to an insured's health status is of importance, as exercise of the death benefit switch option does not require evidence of insurability. The new level death benefit contract is thus based on unchanged actuarial assumptions.

After switching, premium payments are adjusted since the former increasing policy premium is no longer adequate for the new level policy. Due to the full flexibility in premium payments for universal life contracts, the modification is not prescribed by the insurer; the only restriction is prevention of policy lapse.[1] An evaluation of the switch option thus necessarily involves assumptions about modified premium payment behavior after switch. It is this combination of options—the death benefit switch option and premium payment options—that can have substantial negative effects for the insurer. We consider two viable premium payment scenarios, one with constant premiums and one with flexible payments.

To gain detailed insight into the death benefit switch option of increasing universal life policies, we conduct a comprehensive investigation for different switch probabilities. In a simulation analysis, we quantify the net present value of the option using risk-neutral valuation under stochastic interest rates based on the Vasicek model. We then study the effect of adverse exercise behavior by assuming different switch probabilities depending on an insured's health status and on the time since policy inception (and thus on the amount of policy cash value). This procedure allows an investigation of the necessity of requiring evidence of insurability. Finally, we conduct a sensitivity analysis with respect to the frailty factor distribution.

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[1] A universal life policy lapses if the cash value is insufficient to pay policy costs (see Carson, 1996, p. 675). In this case, the contract is terminated without payout to the policyholder. During a one-month grace period, catch-up premium payments can be made to avoid policy lapse. After that period, reinstatement of the policy requires new evidence of insurability as well as payment of all outstanding premiums (see Trieschmann, Hoyt, and Sommer, 2005, p. 341). This understanding of policy lapse is in contrast to exercise of the surrender option, when the cash surrender value of the policy is paid out.

Results show that the value of the death benefit switch option is strongly dependent on premium payment behavior after exercise and on the health status of an exercising insured. From our findings, we derive policy implications and provide recommendations for insurers, which can be applied depending on specific—mortality and behavioral—experience in an insurance portfolio.

The remainder of the paper is structured as follows. Section 2 presents the model framework, including the model of a universal life policy with increasing death benefit and the model of the death benefit switch option. In Section 3, the valuation approach is presented and Section 4 contains numerical results. Policy implications for insurers are discussed in Section 5; a summary is found in Section 6.

2. THE MODEL FRAMEWORK

The universal life contract with increasing death benefit

We consider a lifelong universal life insurance contract with increasing death benefit. The policy is issued at time $t=0$ for an insured of age $x \in \{x_{\min}, \ldots, \omega\}$ at inception, where $x_{\min}$ is the minimum entry age admitted. The contract matures at time $T = \omega - x + 1$, where ω is the limiting age of a mortality table, i.e., the one-year probability of dying at age ω, q'_{ω}, is equal to 1. In what follows, death or survival probabilities based on the mortality table will be denoted with a prime (`) mark. The one-year table probability of death at age $x+t$ is thus given by q'_{x+t}, $t = 0, \ldots, T-1$.

In case of death during policy year t (between time t – 1 and t), the death benefit is paid in arrears at the end of the year, i.e., at time $t \in \{1, \ldots, T\}$. The increasing death benefit consists of the sum of a fixed face value Y and the cash value V_t at time t:

$$Y_t = Y + V_t, \quad t = 1, \ldots, T .$$

To focus on the pure effect of the death benefit switch option in increasing universal life policies, our model framework does not account for charges or surrenders. According to a standard actuarial valuation (see, e.g., Bowers et al. (1997) and Linnemann (2004)), for annual premium payments B_t, $t = 0,\ldots,T-1$ paid at the beginning of each year *t* in which the insured is alive, the cash value is given by the following recursive formula:

$$\left(1-q'_{x+t-1}\right)V_t = \left(V_{t-1}+B_{t-1}\right)\cdot\left(1+i\right)-q'_{x+t-1}Y_t,\ \ t=1,\ldots,T\ , \tag{1}$$

where $V_0 = 0$. We assume that a constant annual interest rate i is credited to cash value and premium. Each policy year, this amount is reduced by the cost of insurance, i.e., the product of death benefit and table probability of death. Calculations are hence based on the actuarial assumptions of a constant annual interest rate i and probabilities of death according to the mortality table. With $Y_t = Y + V_t$, the recursion formula for policy reserves in Equation (1) reduces to

$$V_t = \left(V_{t-1}+B_{t-1}\right)\cdot\left(1+i\right)-q'_{x+t-1}Y,\ \ t=1,\ldots,T. \tag{2}$$

Defining the savings premium at time $t-1$ as $B_{t-1}^{(S)} = V_t\left(1+i\right)^{-1}-V_{t-1}$ and the cost of insurance (risk premium) at the same time as $B_{t-1}^{(R)} = q'_{x+t-1}Y\left(1+i\right)^{-1}$, it turns out from Equation (2) that $B_{t-1} = B_{t-1}^{(S)} + B_{t-1}^{(R)}$. From the definition of the savings premium, we obtain the following expression for the cash value:

$$V_t = \sum_{h=0}^{t-1} B_h^{(S)}\left(1+i\right)^{t-h}.$$

Given that $B_{t-1}^{(S)} = B_{t-1} - B_{t-1}^{(R)}$, we can also write

$$\begin{aligned} V_t &= \sum_{h=0}^{t-1}\left(B_h - B_h^{(R)}\right)\left(1+i\right)^{t-h} = \sum_{h=0}^{t-1}\left(B_h - q'_{x+h}Y\left(1+i\right)^{-1}\right)\left(1+i\right)^{t-h} \\ &= \sum_{h=0}^{t-1} B_h\left(1+i\right)^{t-h} - Y\sum_{h=0}^{t-1} q'_{x+h}\left(1+i\right)^{t-h-1}. \end{aligned} \tag{3}$$

Since universal life contracts allow for flexible premium payments, we need to make certain assumptions in this regard. Universal life policies are usually paid by means of constant periodic premiums. These constant payments aim to reflect the general savings pattern of life insurance policies, where savings are accumulated during the earlier years of the contract term when the costs of insurance are low in order to finance the higher costs of insurance later in life. We, therefore, base our analysis on constant annual premium payments $B_t = B,\ \ t = 0,\ldots,T-1$. Given the premium *B*, the cash value should be positive until maturity to avoid policy lapse (see Carson (1996, p. 675)). The *minimum* (*constant* annual) premium to fulfill this condition is the amount for which the cash value at maturity equals 0. Thus, we solve $V_T = 0$ for *B* (see Equation (3)), which is equal to solving the equivalence principle, and obtain

$$B = Y \cdot \frac{\sum_{h=0}^{T-1} q'_{x+h} \left(1+i\right)^{t-h-1}}{\sum_{h=0}^{T-1} \left(1+i\right)^{t-h}}. \tag{4}$$

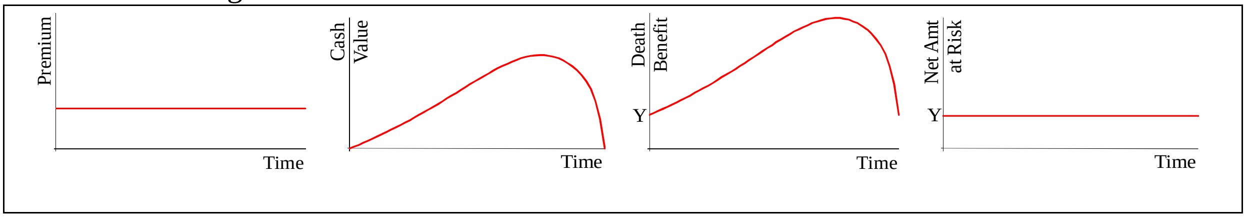
In general, the net amount at risk R_t for a universal life policy at time $t \in \{1,\ldots,T\}$ is given as the difference between the death benefit Y_t and the cash value V_t:

$$R_t = Y_t - V_t, \qquad t = 1,\ldots,T. \tag{5}$$

In the case of an increasing death benefit, the death benefit at time t is the sum of the fixed face value and the current cash value, and, thus, the net amount at risk for an increasing policy is constant and equals the face amount Y throughout the contract term.

Based on the above assumptions, Figure 1 illustrates the premiums, cash value, death benefit, and net amount at risk of a universal life policy with increasing death benefit from inception to maturity.

Figure 1: Premiums, cash value, death benefit, and net amount at risk of universal life policy with increasing death benefit



For constant annual premium payments throughout the policy term—calculated according to Equation (4)—the cash value first increases and then decreases over time until it becomes zero at maturity. The decrease in cash value near maturity is due to high costs of insurance at higher ages that exceed the interest earnings of the cash value and premiums (see Equation (2)). The death benefit is given by the sum of the fixed face value *Y* and the cash value and thus develops analogously to the latter. Hence, the term ”increasing death benefit” is employed irrespective of the fact that the death benefit may also decrease if the cash value does. Chung and Skipper (1987) account for this point and use the more precise term “non-level death benefit.” In insurance practice, however, the term “increasing” is common. It suggests that the policy—in contrast to a policy with a level death benefit—includes a dynamic component that increases death benefit coverage in the course of accumulating cash value. Since the cash value must be positive to keep the policy in force, the increasing death benefit is always at least as high as a constant level death benefit for the same face amount. The net amount at risk is equal to the fixed face value Y from policy inception to maturity.

The death benefit switch option

Increasing universal life policies typically give the policyholder the right to switch the death benefit from increasing to level without charges or additional evidence of insurability. When exercising the death benefit switch option at time $\tau \in \{1,\ldots,T-1\}$, the death benefit is switched to level and fixed at the current value $Y_\tau = Y + V_\tau$. In our model, the option may be exercised only once and at discrete exercise times, namely, at the beginning of each policy year. Exercise of the option at time τ can also be interpreted as terminating the increasing death benefit contract and, based on otherwise unchanged actuarial assumptions, purchasing a new contract with level death benefit Y_τ. The death benefit at time t given exercise at time τ is denoted by

$$Y_t^{(\tau)} = \begin{cases} Y_t, & t = 1, \ldots, \tau \\ Y_\tau, & t = \tau + 1, \ldots, T \end{cases}. \tag{6}$$

When the death benefit switch option is exercised in the accumulation phase of the cash value, the switch halts further increase of the death benefit by fixing it at the attained level Y_τ. Compared to the case without switch, future death benefit amounts are thus lower until the increasing death benefit falls below the fixed level again. A switch at or after the peak of the death benefit curve implies a higher level death benefit until maturity than under increasing policy conditions. However, in both cases, at the point in time when the switch option is exercised (and only at this point), the net amount at risk remains unchanged. This is in contrast to a switch from level to increasing, which immediately increases the death benefit, and thus the net amount at risk, by the current amount of cash value. Therefore, a switch from increasing to level does not require any charges or evidence of insurability.

However, future development of net amount at risk depends on future premiums. Hence, when evaluating the death benefit switch option, it is crucial to take into account possible changes in premium payment behavior after exercise of the option. When switching before the peak of the cash value curve, previously calculated premiums for the increasing death benefit contract (see Equation (4)) are too high for the new level policy. A switch near (some policy years before), at, or after the peak results in higher premiums due to fixing a higher death benefit than in the "increasing" case. Thus, it is not possible to simply analyze the death benefit switch option alone: we need to make assumptions about the premium payment behavior after switch, which leads to a combined examination of the death benefit switch option and premium payment options. Again, with universal life policies, policyholders are free to choose the frequency and amount of premium payments as long as the cash value stays positive.

In the following, we restrict our analysis to two premium payment scenarios that can be regarded as general cases from the insurer's perspective as they constitute minimum premium payment schedules where premiums are just high enough to avoid policy lapse. In particular, they represent the *minimum constant* annual premium payments and *minimum flexible* annual premium payments that will ensure a positive cash value throughout the contract term. Any other constant annual or flexible payments keeping the contract in force until maturity need to exceed these premium amounts.

In the first, "level premium," scenario, constant annual level premiums $B^{(\tau)}$ paid after the switch are calculated based on the equivalence principle, taking into account the present cash value V_τ at the exercise date as an additional single payment. This can be interpreted as terminating the former increasing death benefit contract and starting a new level death benefit contract with an initial premium payment in the amount of the current cash value. For universal life policies, insurers chiefly use constant annual "level" premiums to project policy values (cash value, cash surrender value, death benefit) that imply a zero cash value at maturity (so-called policy illustrations). After option exercise, updated policy illustrations are usually provided. Annual premium notices are often based on the premium values contained in these projections. Although holders of universal life policies are not forced to pay the stated premium amount, they likely do so, unless a certain event makes them depart from the prescribed premium schedule. Since a switch from an increasing to a level death benefit does not require additional evidence of insurability, mortality and interest rate assumptions remain the same. The equivalence principle requires the present value of future premium payments to equal the present value of future benefits (see, e.g., Bowers et al. (1997) and Linnemann (2004)) i.e.,

$$B^{(\tau)}\sum_{t=0}^{T-\tau-1} {}_t p'_{x+\tau}\left(1+i\right)^{-t} + V_\tau = Y_\tau \sum_{t=0}^{T-\tau-1} {}_t p'_{x+\tau} q'_{x+\tau+t}\left(1+i\right)^{-(t+1)}.$$

If the initial single premium V_τ exceeds the present value of future benefits of the new level policy, the annual premium is set to zero. Solving for $B^{(\tau)}$ thus yields

$$B^{(\tau)} = \max\left\{\frac{Y_\tau \sum_{t=0}^{T-\tau-1} {}_t p'_{x+\tau} q'_{x+\tau+t}\left(1+i\right)^{-(t+1)} - V_\tau}{\sum_{t=0}^{T-\tau-1} {}_t p'_{x+\tau}\left(1+i\right)^{-t}}, 0\right\}.$$

For simplification purposes, we do not include the scenario where, if the available cash value exceeds the present value of future benefits, the death benefit amount of a universal life policy might as well be increased in order to maintain a fair contract according to the employed technical basis. However, this assumption would be favorable from the policyholder's perspective and would increase negative effects of the switch option value for the insurer, thus implying that the obtained switch option value in the present analysis represents a lower bound to the 'actual' option value (which can already be substantial). As regards the policyholder perspective, we assume that the decision to switch may sometimes be made for other than financially rational reasons, and that, despite disadvantages in the premium amount, doing so can still be beneficial for the policyholder, despite the fixed death benefit. Premium payments at time t for a policy switched at time τ are denoted by

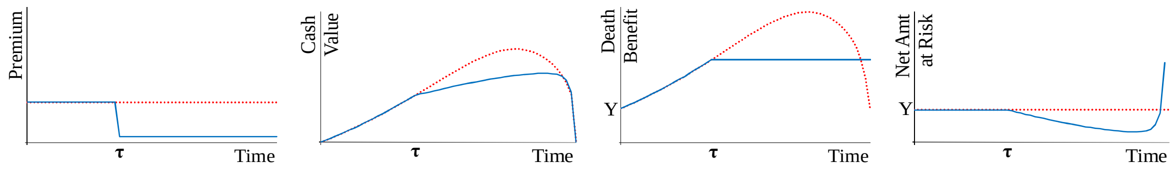
$$B_t^{(\tau)} = \begin{cases} B, & t = 0,\ldots,\tau-1 \\ B^{(\tau)}, & t = \tau,\ldots,T-1 \end{cases}. \quad (7)$$

The new death benefit $Y_t^{(\tau)}$ and new premium payments $B_t^{(\tau)}$ must be taken into account when calculating cash value $V_t^{(\tau)}$ and net amount at risk $R_t^{(\tau)}$ after exercise of the switch option, analogously to Equation (1) and Equation (5), respectively.

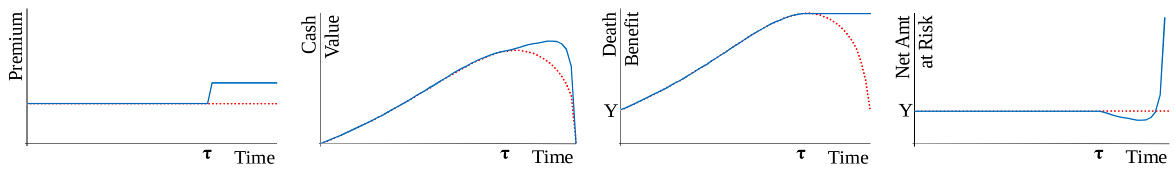
Figure 2 shows premium payments, cash value, death benefit, and net amount at risk based on the "level premium" scenario. In Part a), the switch occurs before the peak of the cash value curve; in Part b), the switch occurs at this peak.

Figure 2: "Level premium" scenario—premiums, cash value, death benefit, and net amount at risk of universal life policy with increasing death benefit switched to level at time τ

a) Switch before peak of cash value curve



b) Switch at peak of cash value curve



After the switch, the contract, in principle, works like a traditional whole life insurance contract with constant premiums. If the switch occurs at time τ before the cash value curve peaks (see Figure 2, Part a)), premiums drop to constant annual level premiums. These reduced payments result in slower growth of the cash value. A switch at the peak of the cash value curve (see Figure 2, Part b)) implies that higher level premiums are necessary, with a consequent increase of the cash value. The increasing death benefit is fixed at the switch exercise time. As the cash value increases after switch, the net amount at risk lies below the constant amount Y in the nonswitch case.

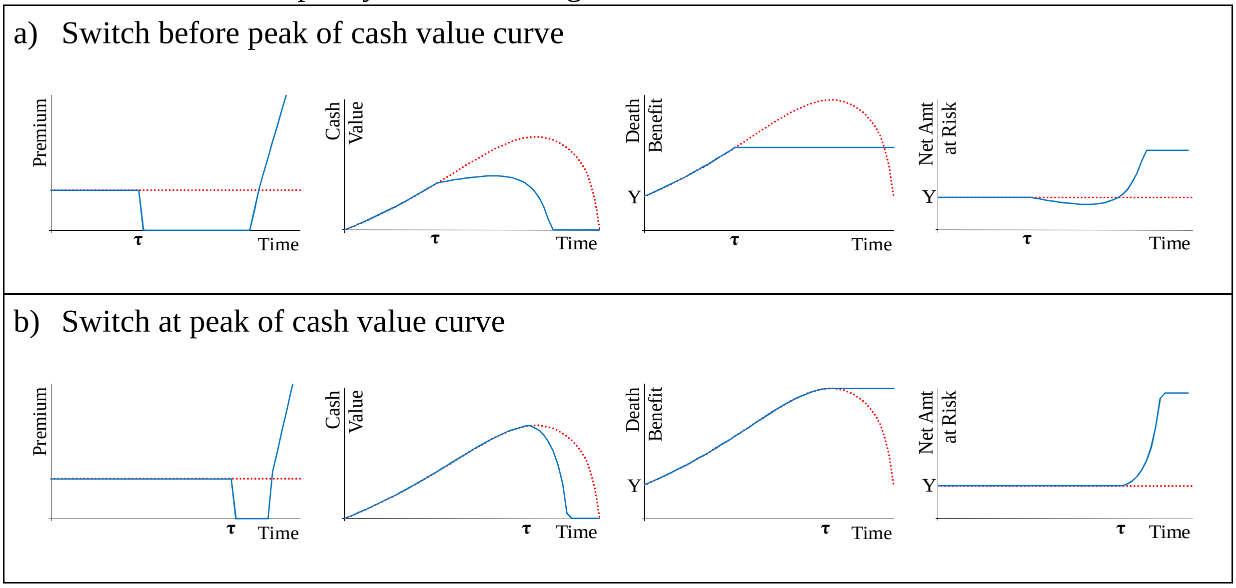
In the second premium payment scenario ("risk premium"), premium payments are stopped immediately after switch at time τ and not resumed until the cash value is exhausted. When switching from an increasing to a level death benefit, the death benefit amount is frozen at the switch exercise time, offering the policyholder the opportunity to maintain the attained death benefit level by deferring premium payments until depletion of the cash value. From then on, the risk premium is paid in only such an amount that the cash value remains zero until maturity. The premium arrangement is thus based on natural premiums, as they are related to the amount of benefit. We refer to this setting as a “risk premium scenario” to emphasize that once the cash value is exhausted, the policyholder must pay the full risk premium to keep the contract in force. This scenario is typically exercised in the secondary market for life insurance, where the policies of insureds with reduced life expectancy are traded. Life settlement companies aim to “optimize” premium payments in the sense of the risk premium scenario by paying only the minimum premium necessary to keep a policy in force, speculating on early deaths of the insureds in their portfolio (see, e.g., www.settlementwatch.com; www.lifesettlementguide.org; www.idealsettlements.eu; www.lifesettlementgrp.com).

The above assumptions imply the following formula for premium payments, which is derived from the recursive development of the cash value in Equation (1), where $V_{t+1}^{(\tau)}$ is set to zero:

$$B_t^{(\tau)} = \begin{cases} B, & t = 0,\ldots,\tau-1 \\ \max\left\{0, q'_{x+t} Y_{t+1}^{(\tau)} (1+i)^{-1} - V_t^{(\tau)}\right\}, & t = \tau,\ldots,T-1 \end{cases}. \tag{8}$$

If the cash value $V_t^{(\tau)}$ at time t exceeds the discounted risk premium for year t (i.e., $q'_{x+t} Y_{t+1}^{(\tau)} (1+i)^{-1}$), no premium payment is necessary. Once $V_t^{(\tau)}$ is less than the required risk premium for the first time, the remainder of $V_t^{(\tau)}$ is exhausted and the outstanding difference is covered by the premium payment. After the zero level of $V_t^{(\tau)}$ has been reached, it is sustained by premiums equaling exactly the amount of the discounted annual cost of insurance $q'_{x+t} Y_{t+1}^{(\tau)} (1+i)^{-1}$. Again, we illustrate the course of premiums, cash value, death benefit, and net amount at risk in Figure 3.

Figure 3: "Risk premium" scenario—premiums, cash value, death benefit, and net amount at risk of universal life policy with increasing death benefit switched to level at time τ



3. Contract Valuation

The previous section makes apparent that the effects of the death benefit switch option depend on switch exercise time and premium payment behavior after switch. When evaluating the option, however, mortality as a third (random) component also needs to be considered. Since the option value is determined by the combination of premium payment method, switch exercise time, and time of death, we account for adverse option exercise behavior. That is, we consider mortality heterogeneous insureds whose exercise behavior depends on their health status or mortality expectation. This enables a comprehensive examination of the option and an investigation of whether fees or evidence of insurability are recommended.

Option valuation can be conducted in different ways depending on policyholder exercise behavior. Generally, two approaches can be distinguished. First, under financially rational exercise, policyholders attempt to identify an optimal exercise strategy that maximizes the option value. This is implemented by solving an optimal stopping problem. In our setting, determining an optimal exercise strategy is highly ambitious because of the complex interaction between mortality and financial factors as well as further options embedded in a universal life contract. In particular, the switch exercise decision, inter alia, depends on decisions regarding frequency and amount of premium payments before and after switch, lapse option exercise, the insured's health status, and the interest rate. Therefore, tackling this problem is an extensive undertaking and requires assumptions regarding many decision variables.

In addition, even though an ever greater number of policyholders may be taking advantage of increased transparency in the insurance market and are thus making more rational exercise decisions, empirically observed exercise behavior can still vary from this assumption. Hence, the option value under rational exercise is likely to overestimate the value actually generated in insurance portfolios. From the option value based on an optimal exercise strategy, it is therefore difficult to derive policy recommendations for insurers.

For these reasons, we focus on the second valuation approach and integrate exercise probabilities into our model. The investigation is conducted from an insurer's perspective for a pool of policyholders who do not necessarily exercise their options in a rational way. Instead, exercise decisions are exogenously made for financial or other, possibly personal, reasons. In the current market situation, our model allows an assessment of the risk associated with the switch option in a portfolio of insureds as well as the derivation of policy implications, as is done in Section 4 and 5, respectively. Thus, in the following, option *value* or *net present value* of the option refers to the value of the death benefit switch option calculated using this approach.

As data regarding empirical switch exercise behavior are not available, we conduct our analysis by studying comprehensive exercise scenarios. An insurer can employ the model using its own switch exercise experience to determine the impact of the switch option in a portfolio. However, caution is needed when implementing this approach as using exercise probability estimates from historical data is not entirely without problems because deviations between actual and estimated probabilities can represent a risk for the insurer.

Heterogeneity in respect of mortality

To examine adverse option exercise behavior and thus the impact of an insured's health status, we evaluate the death benefit switch option by taking into account heterogeneity in respect of mortality. As in Hoermann and Russ (2008), we integrate heterogeneity in respect of mortality by use of a frailty model (see, e.g., Jones (1998, pp. 80–83), Pitacco (2004, p. 14), and Vaupel, Manton, and Stallard (1979, p. 440)). Since the date of the benefit payment and thus also the amount of premiums paid into the contract depend on insureds' mortality, the value of the option to switch from one death benefit scheme to another will also so depend. The introduction of a frailty factor and, in particular, its stochasticity will allow a more detailed analysis of the death benefit switch option with respect to the policyholder's individual mortality level and exercise behavior.

The one-year individual probability of death for a person age x is obtained by multiplying an individual frailty factor d with the probabilities of death q'_x of a deterministic mortality table:

$$q_x = \begin{cases} d \cdot q'_x, & d \cdot q'_x < 1 \\ 1, & x = \min\left[\tilde{x} \in \{0,\ldots,\omega\} : d \cdot q'_{\tilde{x}} \geq 1\right] \\ 0, & \text{otherwise} \end{cases} \qquad \text{for } x \in \{0,\ldots,\omega\} \text{ and } q_\omega := 1 \text{ for } d < 1.$$

If the resulting product is greater than or equal to 1 for any ages $\tilde{x}$, the individual probability of death is set equal to 1 for the youngest of those ages; for all other ages $\tilde{x}$, it is set to 0. The random variable $K(x)$ describes the remaining curtate lifetime of an individual age *x*. Its distribution function ${}_k q_x$ at a point $k \in \mathbb{N}_0$ is given by

$$F_{K(x)}(k) = \mathbb{P}\left(K(x) \leq k\right) = {}_k q_x = 1 - {}_k p_x = 1 - \prod_{h=0}^{k-1}\left(1 - q_{x+h}\right),$$

where ${}_k p_x$ is the individual k-year survival probability of an *x*-year-old and $\mathbb{P}$ denotes the objective (real-world) probability measure. The distribution of the remaining curtate lifetime thus depends on the individual frailty factor. A person with a frailty factor d less than 1 indicates that the insured is impaired with a reduced expected remaining lifetime.

The parameter d can be interpreted as a realization of a stochastic frailty factor D. The distribution F_D of D specifies the portion of individuals whose mortality is lower or higher than a certain percentage of table mortality. It is characterized as follows (see, e.g., Ainslie (2000, p. 44), Butt and Haberman (2002, p. 5), Hougaard (1984, pp. 75, 79), and Pitacco (2004, p. 15)). We assume a continuous frailty distribution such that it can represent fine differences between remaining life expectancies. It is only defined for positive values of d and for $d = 0$, it equals zero. The distribution is right-skewed, i.e., high values of d—corresponding to high mortalities—can occur. Its expected value is equal to 1, such that the deterministic mortality table describes an individual with average life expectancy.

For our analyses, we use a distribution that employs as a suitable choice of parameters for the characteristics listed above, and that is a common choice for frailty models: a gamma distribution (see, e.g., Butt and Haberman (2002, pp. 8–9), Hougaard (1984, p. 76), Jones (1998, p. 82), Olivieri (2006, pp. 29–30), and Pitacco (2004, p. 17), all of which refer to Vaupel et al. (1979, pp. 441–442)). Vaupel et al. (1979) initially chose the gamma distribution as it is one of the best-known nonnegative distributions, is convenient to work with, and is very flexible. Although some advantageous properties of the gamma frailty distribution are lost when applied to a deterministic mortality table instead of a continuous mortality law, it remains a reasonable assumption. Since mortality probabilities near zero are unrealistic, the distribution is shifted by a positive value of γ, resulting in a generalized gamma distribution, $\Gamma(\alpha,\beta,\gamma)$. For its probability density function, we employ the following formula:

$$f^{\Gamma}_{(\alpha,\beta,\gamma)}(d) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}}(d-\gamma)^{\alpha-1} e^{-\frac{d-\gamma}{\beta}}, \text{ for } d \geq \gamma, \gamma \in \mathbb{R}, \alpha, \beta > 0.$$

Switch probabilities

Let τ denote the time of switch and let $s(t)$ be the switch probability that depends on the time t since policy inception. As there is a prescribed development of the cash value in our setting, dependence on time t can be interpreted as the switch probability depending on the amount of cash value in the policy. The distribution of τ at a point in time $k \in \mathbb{N}_0$ is given by

$$F_\tau(k) = \mathbb{P}(\tau \le k) = \sum_{h=1}^{k} s(h) \prod_{\nu=1}^{h-1} (1 - s(\nu)).$$

Moreover, the switch probability can take different values depending on an insured's health status measured by the frailty factor *d*—a realization of $D \sim F_D$, i.e., $s(t,d)$. However, in the following we omit the index *d* to simplify the notation.

Short-rate process

For the short-rate process, we follow Briys and de Varenne (1994), Hansen and Miltersen (2002), and Jørgensen (2006) and use the Vasicek model (Vasicek, 1977), which is a Gaussian Ornstein-Uhlenbeck process. Under the risk-neutral measure $\mathbb{Q}$, the short-rate process $r(t)$ evolves as

$$dr(t) = \kappa(\theta - r(t))dt + \sigma dW^{\mathbb{Q}}(t),$$

where $(W^{\mathbb{Q}}(t))$, $0 \le t \le T$ is a standard Brownian motion on a probability space $(\Omega, F, \mathbb{Q})$, and $(\mathcal{F}_t)$, $0 \le t \le T$ is the filtration generated by the Brownian motion. The interest rate volatility σ is deterministic, the mean reversion level is denoted by θ, and the parameter κ determines the speed of mean reversion.

$P(0,t)$ denotes the price of a zero-coupon bond at time 0 paying \$1 at maturity *t*, where $r(0) = r$. Since the zero-coupon bond price in the Vasicek model has an affine term structure, the expectation can be represented by

$$P(0,t) = \mathrm{E}^{\mathbb{Q}}\left(e^{-\int_0^t r(u)du}\right) = \exp\left\{\left(\frac{1-e^{-\kappa t}}{\kappa}\right)\left(\theta - \frac{\sigma^2}{2\kappa^2} - r\right) - t\left(\theta - \frac{\sigma^2}{2\kappa^2}\right) - \frac{\sigma^2}{4\kappa^3}\left(1-e^{-\kappa t}\right)^2\right\}. \quad (9)$$

Hence, once all input parameters have been defined, the entire term structure can be determined as a function of the current short rate *r*.

Net present value of the death benefit switch option

Based on the above mortality assumption and the short-rate process, the net present value (*NPV*) of an increasing death benefit policy can be calculated as the expected discounted premium payments less the expected discounted death benefit, using risk-neutral valuation given a complete, perfect, and frictionless financial market (see, e.g., Björk (2004)). In the analysis, we assume independence between short-rate and mortality dynamics. Furthermore, the market is assumed to be risk-neutral with respect to mortality risk, such that the objective (real-world) probability measure $\mathbb{P}$ coincides with the risk-neutral probability measure $\mathbb{Q}$ (see, e.g., Bacinello (2003, p. 468) and Dahl (2004, p. 124)). From the insurer's perspective, pooling effects are achieved for a sufficient number of policyholders since, at the portfolio level, only expected values and thus the mortality distribution in the pool are of relevance in evaluating the contract. According to our assumptions on the frailty distribution, the expected value of the frailty factor is equal to 1, implying that, on average, mortality in the pool is described by the deterministic mortality table. For a policy with increasing death benefit throughout its term, the net present value conditional on $D = d$ under the risk-neutral measure $\mathbb{Q}$ thus results in

$$\begin{aligned} NPV(d) &= \mathrm{E}^{\mathbb{Q}}\left(\sum_{t=0}^{K(x)} B \cdot e^{-\int_0^t r(u)du}\right) - \mathrm{E}^{\mathbb{Q}}\left(Y_{K(x)+1} \cdot e^{-\int_0^{K(x)+1} r(u)du}\right) \\ &= \mathrm{E}^{\mathbb{Q}}\left(\sum_{t=0}^{T-1} B \cdot 1_{\{K(x)\geq t\}} \cdot e^{-\int_0^t r(u)du}\right) - \mathrm{E}^{\mathbb{Q}}\left(\sum_{t=0}^{T-1} Y_{t+1} \cdot 1_{\{K(x)=t\}} \cdot e^{-\int_0^{t+1} r(u)du}\right) \qquad (10) \\ &= \sum_{t=0}^{T-1} B_t \, {}_t p_x P(0,t) - \sum_{t=0}^{T-1} Y_{t+1} \, {}_t p_x q_{x+t} P(0,t+1). \end{aligned}$$

When policyholders have the option to switch the death benefit scheme, the stochastic switch exercise date τ is included in the net present value calculation. For the net present value of the policy with switch option for $D = d$, we hence obtain

$$NPV^{(\tau)}(d) = \mathrm{E}^{\mathbb{Q}}\left(\sum_{t=0}^{T-1} B_t^{(\tau)} 1_{\{K(x)\geq t\}} e^{-\int_0^t r(u)du}\right) - \mathrm{E}^{\mathbb{Q}}\left(\sum_{t=0}^{T-1} Y_{t+1}^{(\tau)} 1_{\{K(x)=t\}} e^{-\int_0^{t+1} r(u)du}\right). \quad (11)$$

The death benefit $Y_{t+1}^{(\tau)}$ is given by Equation (6) and premiums $B_t^{(\tau)}$ are given by Equations (7) and (8) for the “level premium” and “risk premium” scenario, respectively.

Equation (11) contains three sources of randomness, namely, the remaining lifetime $K(x)$, the time of switch τ, and stochastic interest rates. The equation further illustrates that $1 \leq \tau \leq K(x)$, i.e., the option can be exercised only as long as the insured is alive. Since we let the switch rate depend on an insured`s health status and thus on the frailty factor d, switch probabilities and probabilities of death are dependent. Again, assuming independence between the stochastic frailty factor and interest rates, Equation (11) can be rewritten as

$$\begin{aligned}
&NPV^{\tau}(d) \\
&= \sum_{t=0}^{T-1} \mathrm{E}^{\mathbb{Q}}\left(B_t^{(\tau)} 1_{\{K(x)\geq t\}}\right) P(0,t) - \sum_{t=0}^{T-1} \mathrm{E}^{\mathbb{Q}}\left(Y_{t+1}^{(\tau)} 1_{\{K(x)=t\}}\right) P(0,t+1) \\
&= \sum_{t=0}^{T-1}\left(\sum_{k=1}^{T} \mathrm{E}^{\mathbb{Q}}\left(B_t^{(\tau)} 1_{\{K(x)\geq t\}} \middle| \tau = k\right) \mathbb{P}(\tau = k)\right) P(0,t) \\
&- \sum_{t=0}^{T-1}\left(\sum_{k=1}^{T} \mathrm{E}^{\mathbb{Q}}\left(Y_{t+1}^{(\tau)} 1_{\{K(x)=t\}} \middle| \tau = k\right) \mathbb{P}(\tau = k)\right) P(0,t+1) \\
&= \sum_{t=0}^{T-1}\left(\sum_{k=1}^{T} B_t^{(k)} \mathbb{P}\left(K(x) \geq t \middle| \tau = k\right) \mathbb{P}(\tau = k)\right) P(0,t) \\
&- \sum_{t=0}^{T-1}\left(\sum_{k=1}^{T} Y_{t+1}^{(k)} \mathbb{P}\left(K(x) = t \middle| \tau = k\right) \mathbb{P}(\tau = k)\right) P(0,t+1) \\
&= \sum_{t=0}^{T-1}\left(\sum_{k=1}^{T} B_t^{(k)} \mathbb{P}(\tau = k)\right) \mathbb{P}\left(K(x) \geq t\right) P(0,t) - \sum_{t=0}^{T-1}\left(\sum_{k=1}^{T} Y_{t+1}^{(k)} \mathbb{P}(\tau = k)\right) \mathbb{P}\left(K(x) = t\right) P(0,t+1) \\
&= \sum_{t=0}^{T-1}\left(\sum_{k=1}^{T} B_t^{(k)} s(k) \prod_{h=1}^{k-1}\left(1 - s(h)\right)\right) {}_t p_x P(0,t) - \sum_{t=0}^{T-1}\left(\sum_{k=1}^{T} Y_{t+1}^{(k)} s(k) \prod_{h=1}^{k-1}\left(1 - s(h)\right)\right) {}_t p_x q_{x+t} P(0,t+1).
\end{aligned}$$

Thus, the expected value of Equations (10) and (11) is obtained by

$$NPV = \mathrm{E}^{\mathbb{Q}}\left(NPV\left(D\right)\right) \quad (12)$$

and

$$NPV^{(\tau)} = \mathrm{E}^{\mathbb{Q}}\left(NPV^{(\tau)}\left(D\right)\right), \quad (13)$$

respectively. In the context of heterogeneity in respect of mortality implied by a gamma distributed frailty factor, closed-form solutions are generally not feasible for the above net present values.

To assess the value of the death benefit switch option, we subtract the net present value of the increasing policy without switch in Equation (12) from Equation (13) and denote the value by NPV^{Opt}. Hence,

$$NPV^{Opt} = NPV^{(\tau)} - NPV\ . \quad (14)$$

4. NUMERICAL ANALYSIS

This section presents results from a simulation study so as to quantify the impact of the death benefit switch option. First, we consider the increasing universal life contract. Next, we integrate the switch option and illustrate effects for deterministic switch exercise times and times of death. We then derive net present values of the option from the insurer's perspective for different switch probabilities depending on the health status of insureds and for some specific exercise scenarios. In addition, a sensitivity analysis with respect to the parameterization of the frailty distribution is provided.

Input parameters

We examine a universal life insurance contract with increasing death benefit with a policy face value of Y = \$100,000 for a male insured aged x = 45 years at inception. The actuarial minimum interest rate is set at i = 3.5%. The minimum guaranteed interest rate for universal life products is usually around 4%. For newer products, it is often 3% (see, e.g., www.aegon.com). To be conservative, numerical analyses are based on the U.S. 1980 Commissioners Standard Ordinary (CSO) male ultimate composite mortality table with a limiting age $\omega = 99$. Composite means that smokers and nonsmokers are not distinguished. An older mortality table with low limiting age—like the 1980 CSO table—is conservative regarding death risk in the sense that it tends to overstate probabilities of death. In contrast, modern life tables account for mortality improvement and usually have a limiting age of 120. For the generalized gamma distribution of the frailty factor, we employ the parameterization used in Hoermann and Russ (2008), given by $D \sim \Gamma(2.0; 0.25; 0.5)$. The parameter values lead to a frailty distribution that fulfils the requirements laid out in Section 3. A shift by $\gamma = 0.5$ means that individual probabilities of death can be at most half the size of the mortality table probabilities but not less than that. We later vary distributional assumptions to examine the sensitivity of switch option values to parameterization changes.

For the stochastic interest rate, we use the input parameters given in Hansen and Miltersen (2002) with speed of mean reversion $\kappa = 0.30723$, mean reversion level $\theta = 3.7\%$, interest rate volatility $\sigma = 0.02258$, and $r(0) = 3.7\%$. As is common in the life insurance business, the interest rate credited to the account value (here, 3.5%) is slightly below the interest earned by the insurance company in the long term (this difference is larger in European countries, e.g., in Germany the minimum guaranteed interest rate is currently 2.25%). Numerical results are derived using Monte Carlo simulation with 50,000 sample paths (see Glasserman, 2004). In all simulation runs, we use the same set of random numbers to ensure comparability of results.

Value of the universal life contract with increasing death benefit

The constant annual premium for the increasing policy calculated according to Equation (4) is given by B = \$5,937. The risk-neutral net present value from the insurer's perspective results in $NPV = \$2{,}866$, as determined by Equation (12) under consideration of stochastic interest rates and the stochastic frailty factor. More precisely, in a Monte Carlo simulation, 50,000 frailty factors are generated that imply 50,000 individual mortality distributions. Based on these probabilities of death, the *NPV* can be determined. It would be zero when using $D \equiv 1$, i.e., solely the mortality table, as well as the calculation interest rate *i* instead of stochastic interest rates.

Value of the death benefit switch option by switch exercise time and time of death

The option to switch from an increasing to a level death benefit can be exercised only once during the policy term and if done, must be done at the beginning of a year until the year of death. After switch, premiums are adjusted. In the following, we evaluate the option and compare results for the two previously described premium scenarios to identify the effect of future premium payments on the switch option value. In the "level premium" case, constant annual premiums are paid after switch, which are calculated based on the equivalence principle, taking the current cash value at the time of switch as a single premium. In the "risk premium" case, premium payments are stopped at the exercise date and not resumed until the cash value is exhausted. From then on, the minimum risk premium is paid that will keep the cash value at zero and thus avoid policy lapse. The NPV^{Opt} of the death benefit switch option is given by the difference between the $NPV^{(\tau)}$ of the increasing policy with switch option and the *NPV* of the contract without switch (see Equation (14)).

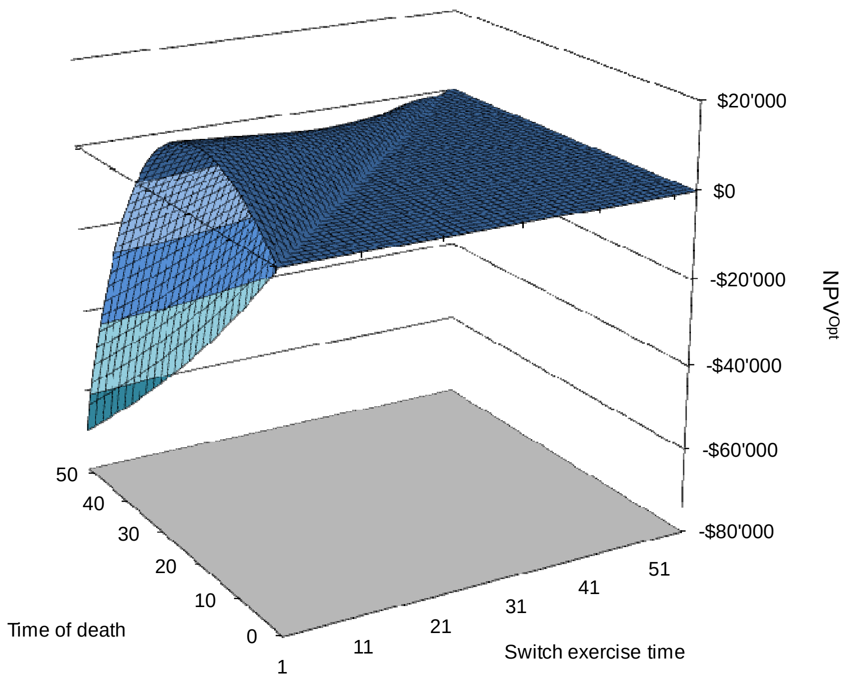
To provide a first impression of the impact of the death benefit switch option, we calculate risk-neutral values for different deterministic times of switch exercise and times of death. For deterministic switch date τ and date of death *K(x)*, Equation (11) simplifies to

$$NPV^{(\tau)} = \sum_{t=0}^{K(x)} B_t^{(\tau)} \cdot P(0,t) - Y_{K(x)+1}^{(\tau)} \cdot P(0, K(x)+1) .$$

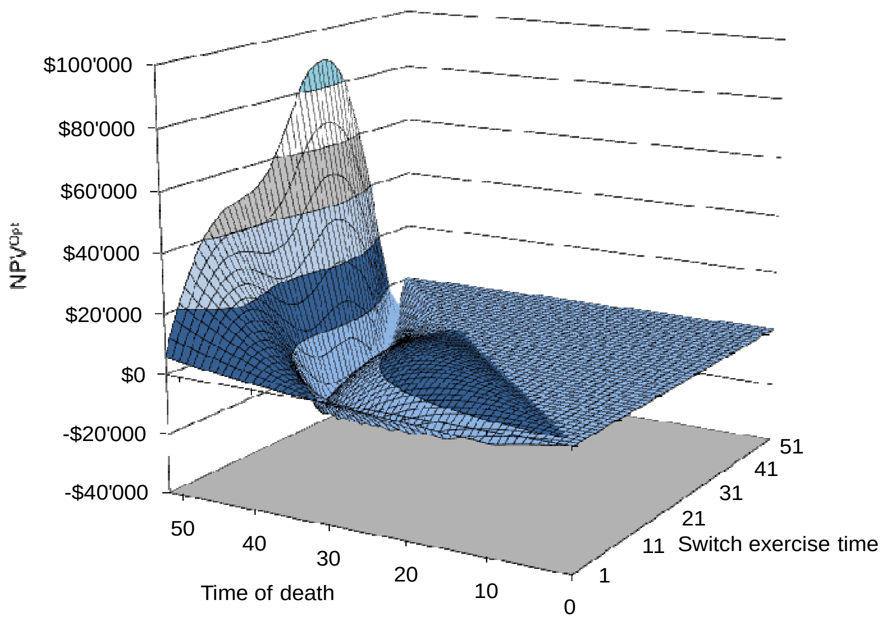
Note that in order to examine the effect of the switch option in a portfolio, these deterministic values have to be weighted with respective probabilities of switch and survival. Results are displayed in Figure 4 for "level premiums" (Part a)) and "risk premiums" (Part b)).

Figure 4: Net present value (NPV^{Opt}) of switch option by time of switch exercise and time of death for 45-year-old insureds

a) "Level premium"



b) "Risk premium"



If policyholders pay level premiums after switch as shown in Part a), the switch option value falls below zero for insureds, deceasing after about 40 policy years (age 85). This is because insureds with high life expectancy “save”—so to speak—on premiums over time when choosing to exercise the switch option in combination with level premium payments. Without switch, in contrast, they are likely to survive until the time when the death benefit decreases again toward maturity (approaching *Y*). Hence, they do not benefit from the “increasing” death benefit feature anyway. Option values are lower the earlier the year of switch and the later death occurs. In the “level premium” case, negative values are thus generated by insureds with high life expectancy, especially when exercising the switch option in early policy years.

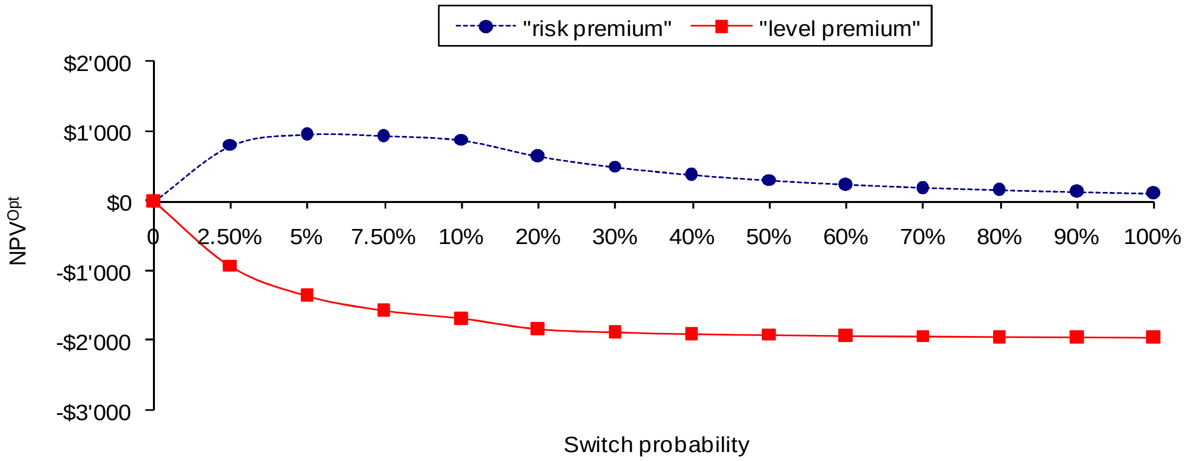
In the “risk premium” scenario, option values can also become negative from the insurer’s perspective. This is the case if death occurs early after switch, such that premiums for the remaining lifetime are covered by the available cash value and no high risk premiums become due. In contrast, option values are extremely high if death occurs late and risk premiums are paid after the cash value is exhausted.

Value of the death benefit switch option by switch probability

To obtain the NPV^{Opt} of the death benefit switch option, individual switch probabilities, as well as individual probabilities of death, need to be taken into account. Results for different constant switch probabilities between s = 0% and s = 100% are displayed in Figure 5 for risk and level premium payments.

Figure 5 shows that the two premium payment scenarios have very different outcomes. In the “level premium” case, the net present value from the insurer’s perspective is negative for all switch probabilities; however, it remains positive in the “risk premium” scenario. In the latter case, the NPV^{Opt} at most reduces to $115 as the switch probability approaches 100%, implying early switch. Hence, for risk premiums, high net present values (see Figure 4 Part b)) cause the NPV^{Opt} of the switch option to always remain positive in the example even though the probability of occurrence of such extreme events (e.g., survival until $t = 45$, i.e., age 90) is very low. This implies switch profits for the insurer if switch probabilities are constant in the portfolio of insureds. However, if insureds terminated contracts (policy lapse) instead of paying high risk premiums after depletion of the cash value, as discussed in Section 2, the NPV^{Opt} turns negative, looks similar to the “level premium” curve.

Figure 5: Net present value (NPV^{Opt}) of switch option for different switch probabilities for 45-year-old insureds



For switch probabilities higher than or equal to 20%, the “level premium” case leads to negative net present values to about $-2,000 in the calibration employed. This is due to considerably negative values for early exercise times, which are weighted more heavily for high switch probabilities (see Figure 4, Part a)). Thus, depending on the premium payment method, the switch option can have negative effects on an insurer’s portfolio even if switch probabilities are assumed to be constant over time, an assumption that we will relax in the following analysis.

Value of the death benefit switch option by switch probability and health status

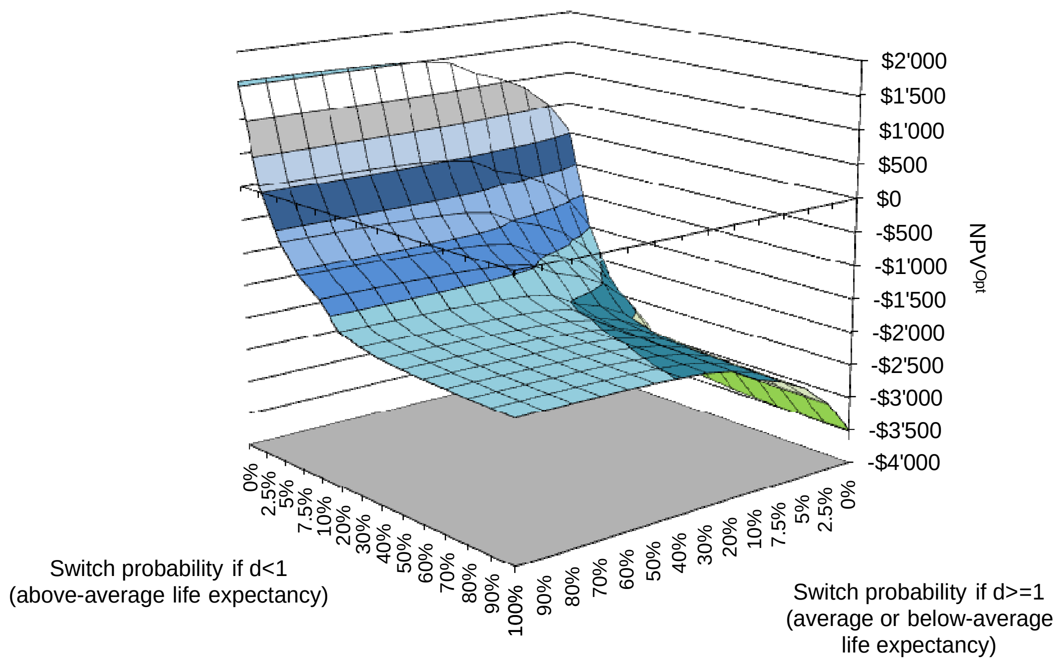
Since the value of the death benefit switch option is strongly dependent on an insured’s life expectancy, as demonstrated in Figure 4, we next examine the effect of adverse exercise behavior with respect to health status on the option's risk-neutral value NPV^{Opt}. This is done by calculating the option value for switch probabilities that vary depending on an insured's individual mortality. We distinguish persons with a frailty factor greater than or equal to one ($d \geq 1$, average or below-average life expectancy) and persons with a frailty factor $d < 1$ (above-average life expectancy). Results are displayed in Figure 6 for "level premium" (Part a)) and "risk premium" (Part b)).

The "level premium" graph in Part a) of Figure 6 reveals strong discrepancies in the NPV^{Opt} if the option exercise behavior depends on an insured's health status. In this case, from the insurer's perspective, risk-neutral values remain positive only if persons with above-average life expectancy have very low switch probabilities and thus tend to switch—if at all—late in the contract term. The value of the death benefit switch option becomes negative if they exercise the option with higher probability. This effect is more pronounced the lower the switch probabilities are for insureds with below-average life expectancy, with the NPV^{Opt} reaching negative values up to about $-3,500. This is in line with results in Figure 4 Part a), where negative values are generated for early switch times and late times of death.

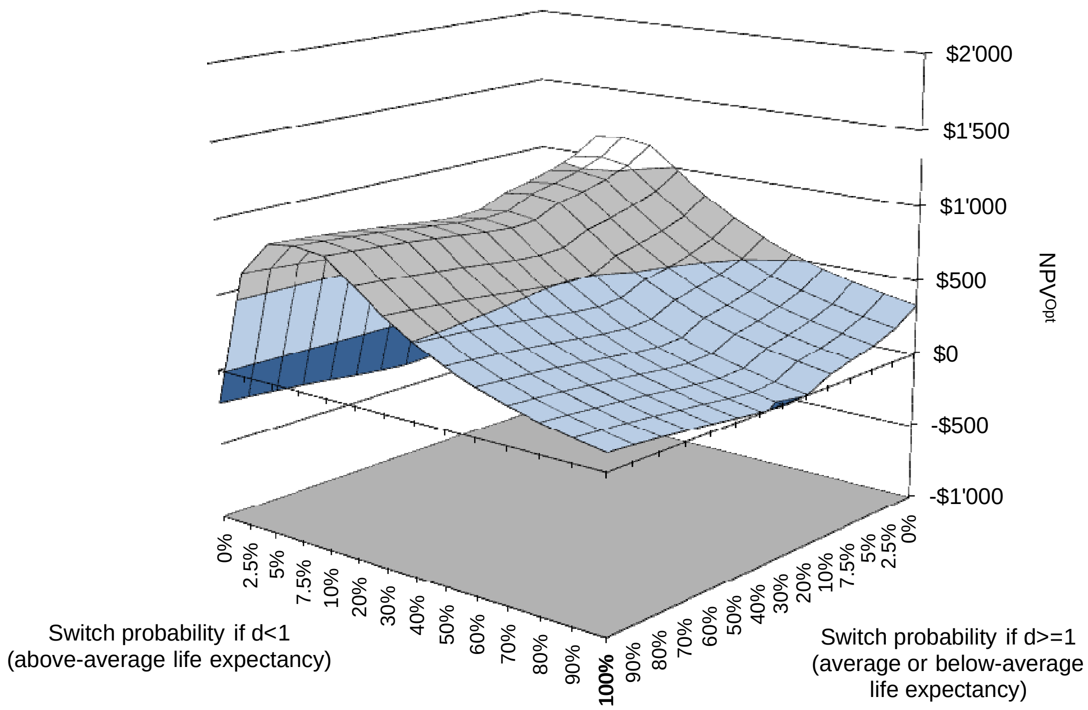
In the "risk premium" scenario, shown in Part b) of Figure 6, differences depending on the health status are less distinct, but still visible. In particular, the "risk premium" scenario generates negative values for the insurer only if persons with below-average life expectancy exercise the option and switch probabilities are zero for insureds with above-average life expectancy. This observation is in line with the reasoning that individuals with impaired health are likely not to pay high risk premiums after depletion of the cash value due to expectations of early death. If insureds survive until cash value exhaustion, increasing death probabilities imply high risk premiums and thus lead to positive net present values from the insurer's perspective. Altogether, strong adverse effects can be observed.

Figure 6: Net present value (NPV^{Opt}) of switch option for different switch probabilities depending on health status for 45-year-old insureds

a) "Level premium"



b) "Risk premium"



Additional exercise scenarios

Based on the previous analyses, certain substantial adverse effects can be seen for specific exercise scenarios that depend on health status and switch option exercise time. If the switch option is exercised around the time the cash value reaches its peak, for instance, the level death benefit for the remaining contract term is higher than in the case of the original increasing death benefit as set out in Section 2. This is because—without switch—the "increasing" death benefit would actually *decrease* in line with the cash value, down to the fixed level *Y* at maturity (see Figure 1). Hence, when switching, the new level death benefit is in fact higher than the original death benefit of the increasing contract at certain times during the contract term. Such comparably higher death benefit amounts can be obtained without having to pay additional fees or providing new evidence of insurability. Thus, depending on the insured's health status, particular exercise behavior can have a considerable influence on contract value, which may have serious consequences when considering a pool of insureds. To further emphasize the potential risk of adverse effects regarding the death benefit switch option, we study several alternative exercise scenarios (see Table 1).

Table 1: Net present values (NPV^{Opt}) of switch option for specific exercise scenarios depending on the health status for 45-year-old insureds

	s=100% at *t*=41 (peak)			*s*=10% *t*=25 to *t*=41			*s*=10% to *s*=100%* *t*=25 to *t*=41			*s*=10% *t*=5 to *t*=15		
	All	$d \geq 1$	$d < 1$	All	$d \geq 1$	$d < 1$	All	$d \geq 1$	$d < 1$	All	$d \geq 1$	$d < 1$
Level premium	-365	12	-377	-750	194	-945	-1'023	293	-1'315	-1'189	856	-2'045
Risk premium	1'324	-89	1'413	1'332	-108	1'440	1'576	-123	1'700	758	-204	962
Risk premium (lapse)	808	10	799	288	-37	325	229	-42	272	-740	128	-867

Notes: $d \geq 1$: *insureds with average or below-average life expectancy,* $d < 1$: *insureds with above-average life expectancy, *: linear increase.*

First, for insureds with above-average life expectancy, the switch option is valuable in combination with the "level premium" scenario. If exercised around the peak of the cash value curve, a high death benefit is maintained compared to the decreasing benefit that occurs without switch. Even though new level premiums are higher than the original premiums in this case, option exercise may still give rise to negative values for the insurer, which can be observed in Table 1 (first column, "Level premium"). The scenario "*s*=100% at *t*=41 (peak)" compares results when either all insureds, only insureds with average or below-average $(d \geq 1)$, or only insureds with above-average life expectancy $(d < 1)$ exercise the switch option with probability 1 at the peak of the cash value curve (i.e., at age 86).

Second, one would suspect that the switch option is especially valuable for insureds with below-average life expectancy in combination with the "risk premium" scenario. When exercised around the peak of the cash value curve at *t*=41 or age 86, persons with reduced life expectancy preserve a high death benefit without having the underlying mortality table adjusted. Furthermore, future premiums can mostly be financed from the available cash value. For insureds with higher-than-average life expectancy, on the other hand, this exercise pattern would imply high risk premium payments as the policy approaches maturity and thus switch profits for the insurer. These expectations are confirmed by the numerical results in Table 1 ("*s*=100% at *t*=41 (peak)", "Risk premium"). However, risk-neutral values are much less negative in this case than they are for adverse exercise by healthy insureds in the level premium case.

For risk premium payments, we additionally consider a scenario in which policyholders let the policy lapse, e.g., due to financial distress, as soon as risk premium amounts exceed 10% of the new level death benefit (first column, "Risk premium (lapse)" in Table 1). The value of 10% was chosen by intuition; however, further tests revealed that results remain robust with respect to changes in the percentage parameter. Compared to the risk premium scenario without lapse (second row of Table 1), such behavior has a considerable negative impact on the net present value from the insurer's perspective if only insureds with above-average life expectancy are concerned. In particular, the NPV^{Opt} is almost cut in half due to the lack of high risk premium payments. In contrast, if impaired insureds let the policy lapse when high risk premiums become due, negative effects are alleviated and the net present value is increased because soon expected death benefit payments do not have to be made.

We further extend the analysis and consider option exercise prior to cash value peak given a constant switch probability of 10% from age 70 (t=25) to age 86 (t=41) (second column in Table 1). Results show that option values are substantially affected. In particular, they are more negative from the insurer's perspective in the "level premium" case for insureds with above-average life expectancy. The latter net present value decreases even more if the switch probability is linearly raised from s = 10% at age 70 to s = 100% at age 86 (third column in Table 1). There are two effects responsible for these aggravated results. First, if the time of cash value peak is not the only possible time to switch (but instead ranges from between age 70 and age 86), option exercise, on average, occurs earlier. From Figure 4 Part a) we know that the earlier the option is exercised by insureds with long remaining lifetime, the lower are the option values. And second, observed effects are stronger due to the larger number of insureds still alive at age 70, compared to at age 86, and thus able to exercise the option.

We now turn to the case where the switch option is exercised after five to fifteen policy years, i.e., between ages 50 and 60 (fourth column in Table 1), given a constant switch probability of 10%. A reason for switching early during the term of the policy could, e.g., be the wish to reduce premium payments. In this scenario, results are even more pronounced than in the case of exercising around the cash value's peak. As discussed previously, it is particularly in the "level premium" scenario that adverse exercise behavior by insureds with high life expectancy generates negative net present values in an insurance portfolio.

These adverse exercise scenarios assume that insureds are well informed about their individual mortality, i.e., whether they have an above- or below-average life expectancy. The examples indicate that it is those exercise scenarios that are intuitively rational that pose the greatest threat to insurers: namely, if insureds with above-average life expectancy switch early and thus "save" risk premiums by making level payments, and if impaired insureds set out premium payments after switch, being aware that they will possibly not survive until high risk premiums have to be paid. In fact, the switch option will be even more valuable if insureds follow optimal exercise strategies to maximize the option value, a topic that is, however, beyond the scope of this paper.

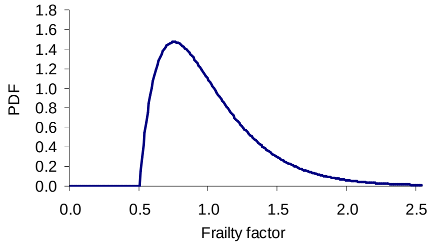
Sensitivity analysis with respect to the frailty distribution

The variance of life expectancies in a portfolio of insureds can be an important risk driver when considering policies with death benefit payments. To assess the impact of the frailty factor distribution, we compare switch profits for different parameterizations of F_D and different switch probabilities, leaving all other parameters unchanged. Part a) of Figure 7 displays the basic scenario with the gamma frailty distribution $D \sim \Gamma(2;0.25;0.5)$ with variance $Var(D)$ = 0.125 (left hand side in Figure 7) and the respective net present values NPV^{Opt} (right hand side in Figure 7) as shown in Figure 5. The net present value for an increasing policy is given by NPV = $2,866. We find that varying the frailty distribution has very little effect on switch profits in the "risk premium" case. For "level premiums," however, effects are much more dramatic.

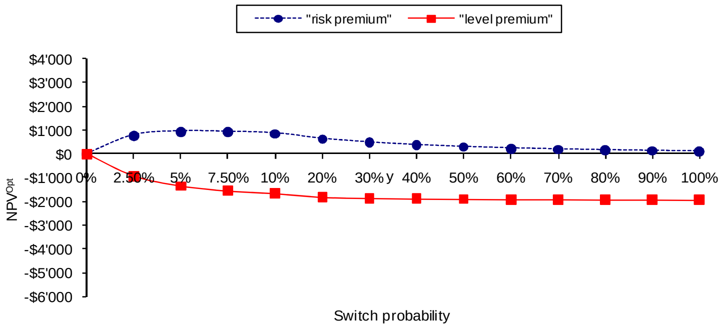
Figure 7: Net present value (NPV^{Opt}) of switch option for different parameterizations of the frailty distribution for 45-year-old insureds

a) Basic scenario

$D \sim \Gamma(2; 0.25; 0.5)$

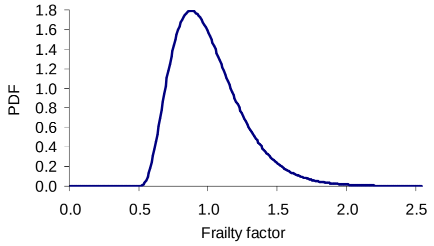


NPV^{Opt} (NPV = \$2,866)

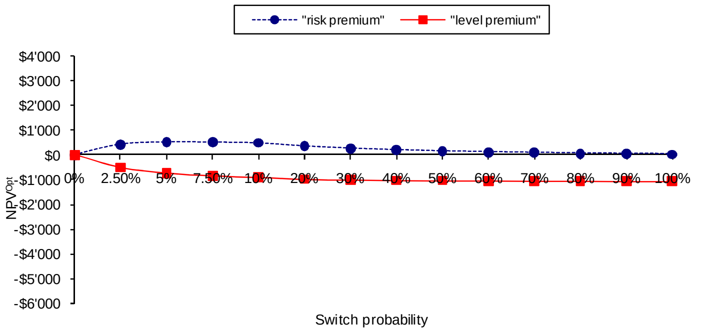


b) "Thinner"

$D \sim \Gamma(4; 0.125; 0.5)$

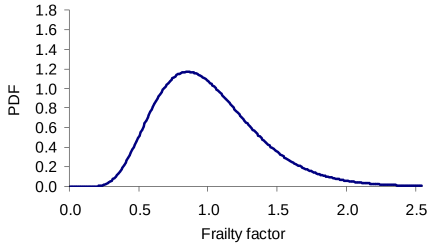


NPV^{Opt} (NPV = \$1,581)

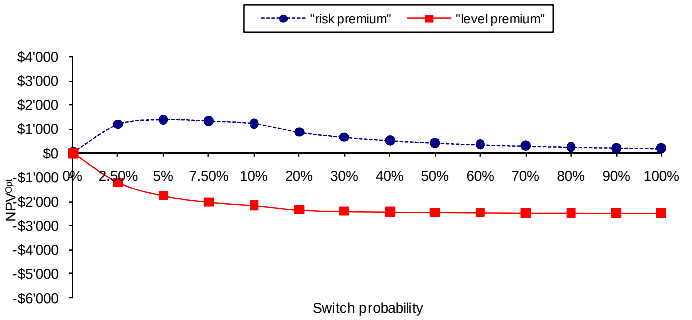


c) "Wider"

$D \sim \Gamma(6; 0.15; 0.1)$

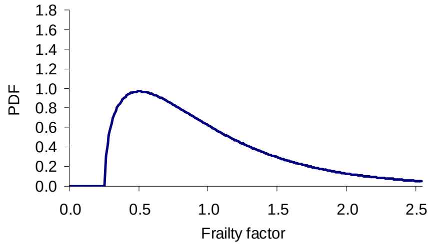


NPV^{Opt} (NPV = \$3,584)

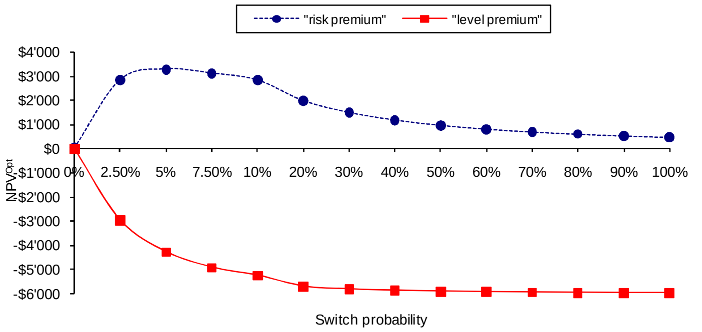


d) "Heavy tailed"

$D \sim \Gamma(1.5; 0.5; 0.25)$



NPV^{Opt} (NPV = \$8,507)



Notes: PDF = probability density function of frailty factor distribution.

Part b) of Figure 7 illustrates that a change to a "thinner" frailty factor distribution results in a much lower net present value of *NPV* = \$1,581 (compared to \$2,866). This is because individuals' probabilities of death disperse less from average death probabilities according to the mortality table ($Var(D) = 0.0625$), i.e., the effect of heterogeneity in respect of mortality is reduced. Hence, the NPV^{Opt} for high switch probabilities in the "level premium" case is not as negative as in the basic scenario and values in the "risk premium" case are closer to zero. Altogether, we find that the difference between the two premium payment scenarios is less distinct with the thinner frailty distribution.

For the "wider" gamma distribution shown in Part c) of Figure 7 where $Var(D) = 0.135$, switch values for "level premiums" decrease compared to the base case, which is particularly important for negative results at high switch rates.

Tremendous differences can be observed for the comparatively "heavy tailed" distribution in Part d) of Figure 7. This assumption implies a greater variance of life expectancies in the portfolio ($Var(D) = 0.375$). Changes can also be observed for the NPV^{Opt} in the "risk premium" case. Net present values are much higher, and a peak around a switch probability of 5% is more pronounced. The "level premium" curve decreases substantially over all switch probabilities. The *NPV* of the policy without switch option nearly triples to *NPV* = \$8,507.

Thus, even though the main results are essentially robust, this sensitivity analysis demonstrates the importance and the impact of heterogeneity in respect of mortality in a portfolio, as well as the relation between premium payment method and mortality distribution.

5. POLICY IMPLICATIONS FOR AN INSURER

Our results do not have straightforward implications for insurance companies. In particular, it turns out not to be sufficient to simply require evidence of insurability or impose additional fees in order to reduce the risk inherent in the death benefit switch option. Instead, we identified four key factors that are of relevance for the option value and that must be considered simultaneously when taking action: insureds' life expectancies, the chosen premium payment method after switch, switch probabilities (and thus the time of switch), and lapsation. It is the combination of these factors that can make the switch option either valuable or risky for an insurer.

The first question for an insurer is whether to even offer the switch option. As the demand for insurance protection can decrease or increase over time, policyholders might choose to surrender if switching is not included in the contract. Signing a new contract, however, has several disadvantages: evidence of insurability is required, updated actuarial pricing assumptions may be applied, and charges have to be paid to initiate the contract. Hence, a switch may be more attractive than surrendering the policy. From the insurer's perspective, offering the option to switch from increasing to level has the advantage of keeping those contracts in its book of business and of reducing surrender rates. In this case, careful monitoring of the four factors listed above—including empirical switch probabilities in the pool of policyholders, possible adverse exercise scenarios, and the mortality distribution in the portfolio of insureds—is vital to avoid risks in the portfolio that originate from switch option exercise.

Overall, there are several reasons why the switch option is of practical interest to insurers. First, the option can become valuable when exercised early as well as late during the contract term, depending on the respective premium payment scenarios. The latter might even become more important in the future given demographic development and longevity risk, i.e., if insureds have longer life expectancies. Second, the option is also relevant in that the opportunity to switch might prevent some policyholders from surrendering the contract. Third, our analysis of the *NPV* of the switch option shows that in a pool of insureds for given switch probabilities, the switch option can have a substantial value, even though many insureds in the pool may not survive to higher ages when the value of the option is most intuitive.

Given empirical exercise probabilities and the corresponding premium payment behavior, our model allows insurers to check whether their portfolios might be negatively affected by the switch option. For instance, if an insurer observes that, typically, constant level premiums are paid after switch with an annual switch probability of about 5%, caution is advised as negative values can result from the insurer's perspective, given the contract calibration in our examples (see Figure 5). If policyholders tend to stop premium payments after switch, implications are not as obvious and must be analyzed in more detail. In particular, adverse exercise experience may pose a risk for insurers if it is mostly the impaired individuals who exercise this way.

If monitoring reveals possible negative net present values for an insurer, action should be taken to reduce the risk by considering the four key factors. First, requiring new evidence of insurability before allowing policyholders to switch from increasing to level death benefits could help identify an insured's health status. This would, in principal, allow the adjustment of actuarial pricing assumptions and, in particular, the mortality table in the case of impaired individuals. However, since the requirement of providing evidence of insurability, and its costs, would apply to all insureds and thus penalize healthy insureds, such a requirement could have the effect of intensifying adverse effects.

To reduce negative effects originating from adverse exercise behavior of healthy insureds who pay level premiums after switch, adequate charges for the death benefit switch option could be imposed. In general, fees should be borne by the group of insureds causing the undesirable adverse effect. However, as the switch option value is strongly linked to the premium payment method after switch and to the time of switch, charges can hardly be calculated independent of these factors. A solution would be the prescription of premium payments after switch, combined with charges to avoid adverse effects. In our examples, requiring level premium payments after switching means that healthy insureds are charged higher premiums than impaired individuals. Yet, this approach would also imply a change from universal life to whole life contracts and thus a loss in flexibility for policyholders.

Furthermore, due to the dependence of the switch option value on the time the option is exercised (in our examples, negative values were predominantly generated for early exercise times), insurers could restrict switch exercises to predefined time ranges to control for adverse effects. Finally, if a shift toward rational exercise behavior is noticed, premium pricing needs to be adjusted based on the maximum option value determined as the solution of an optimal stopping problem.

6. SUMMARY

Universal life policies with increasing death benefit as well as the death benefit switch option have not been investigated in the literature to date. In this paper, we develop an actuarial model framework and conduct a detailed examination of this option. The model includes heterogeneity in respect to mortality using a frailty model and switch probabilities. We point out situations where the death benefit switch option can have considerably negative effects on an insurer and we provide policy implications to reduce the existing risk potential.

One main finding is that the value of the death benefit switch option is strongly dependent on premium payment behavior after exercise and on the health status of the exercising insured. A switch in the “risk premium” scenario has predominantly positive effects in the examples considered, but the option can actually generate severe negative net present values from the insurer’s perspective in the “level premium” case. Both scenarios share the result that option values decrease with increasing switch probability, i.e., the greater the number of insureds who switch early in the contract term, the more the option values decrease. However, the extent varies when exercise probabilities differ depending on insureds’ life expectancies. In the case of risk premium payments, negative values occur if it is only impaired persons who switch early in the contract term, while in the level premium scenario, it is insureds with good health status who generate highly negative values. Similar results are obtained if policies are switched at or near the peak of the cash value curve, logging in highest possible death benefit values. Altogether, we find that combined exercise of the switch option and premium payment options can generate substantial negative net present values from the insurer's perspective due to adverse effects regarding insureds' health status.

Results are stable with respect to parameterization of the frailty distribution. However, the spread between positive results in the risk premium scenario and negative results in the level premium scenario is enhanced with greater variance of life expectancies, i.e., heterogeneity of insureds' mortality. Hence, careful consideration and estimation of the mortality distribution in an insurance portfolio is crucial.

In summary, our findings indicate that the death benefit switch option can pose a threat to insurers in case of adverse exercise behavior with respect to insureds' health status. This result depends on the premium payment method after switching and is even intensified when additionally considering the amount of cash value as a trigger for option exercise. Overall, insurers should be aware of the potential impact the death benefit switch option can have and should consider implementing risk reduction measures. Our policy implications are based on a broad analysis from an insurer's perspective for a pool of insureds covering a wide range of possible exercise scenarios. Depending on the observed exercise behavior in an insurance portfolio, insurers could require evidence of insurability or charge fees in case of option exercise, prescribe the premium payment method after exercise, or restrict possible option exercise times. If insureds followed an optimal exercise strategy, resulting switch option values could in fact be much higher. Determination of the latter would be an interesting subject of further research, but also a very challenging one due to complex interactions between frequency and amount of premium payments before and after switch, lapsation, the insured's health status, and interest rates.

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