Optimal Risk Classification and Underwriting Risk for Substandard Annuities

Nadine Gatzert, Gudrun Hoermann, Hato Schmeiser*

Abstract

Substandard annuities pay higher pensions to individuals with impaired health and thus require special underwriting of applicants. Although such risk classification can substantially increase a company's profitability, these products are uncommon except for the well established U.K. market. In this paper, we comprehensively analyze this issue and make several contributions to the literature. First, we describe enhanced, impaired life, and care annuities, and then discuss the underwriting process and underwriting risk related thereto. Second, we propose a theoretical model to determine the optimal profit-maximizing risk classification system for substandard annuities. Based on the model framework and for given price-demand dependencies, we formally show the effect of classification costs and costs of underwriting risk on profitability for insurers. Risk classes are distinguished by the average mortality of contained insureds, whereby mortality heterogeneity is included by means of a frailty model. Third, we discuss key aspects regarding a practical implementation of our model as well as possible market entry barriers for substandard annuity providers.

JEL classification: C61, G22, L11

Subject Category and Insurance Branch Category: IM12, IM22, IB13

Keywords: Risk classification, Underwriting risk, Mortality heterogeneity, Substandard annuities

1. Introduction

Substandard annuities pay higher pensions to individuals with impaired health.[1] These contracts are increasingly prominent in the U.K. insurance market where, according to Watson Wyatt (2008), more than 20% of annuities sold are based on enhanced rates. Since its development in the 1990s, the market for substandard annuities in the United Kingdom has experienced impressive growth.[2] Today, this market is well established;

* Nadine Gatzert, University of Erlangen-Nuremberg; Gudrun Hoermann, Munich; Hato Schmeiser (corresponding author), Institute of Insurance Economics, University of St. Gallen, phone +41 71 243 40 11, fax +41 71 243 40 40, email: hato.schmeiser@unisg.ch.

1 See LIMRA and Ernst & Young (2006, p. 10). The term "substandard annuity" includes enhanced, impaired life, and care annuities (for a detailed description, see Section 2). In the U.K. market, all types of substandard annuities are sometimes referred to as "enhanced annuities."

2 This may to some extent be due to mandatory partial annuitization of retirement income in the United Kingdom. See LIMRA and Ernst & Young (2006, p. 6).

there were eight providers in 2007 and at least three more entered the market in 2008. And there is still enormous growth potential with up to 40% of annuitants estimated to be eligible for increased pension payments. Outside the United Kingdom, however, substandard annuities are surprisingly rare. In the United States, for instance, only 11 providers out of 100 insurers issuing single-premium immediate annuities offer substandard annuity products, according to LIMRA and Ernst & Young (2006). Only around 4% of annuities sold in the U.S. market are based on enhanced rates.[3] It is not obvious why the substandard annuity market is so small, especially given that such a risk classification generally increases a company's profitability.[4] Furthermore, substandard annuities would make private pensions available for a broader range of the population and would thus improve retirement incomes for insureds with a reduced life expectancy. Thus, there must be important reasons behind the reluctance of many insurers to enter the substandard annuity market.

The aim of this paper is to develop a model to determine the optimal risk classification system[5] for substandard annuities that will maximize an insurance company's profits. We further include the costs of insufficient risk assessment (underwriting risk) that occurs when insureds are assigned to inappropriate risk classes. This extension is crucial, as underwriting risk is considered to be the most significant risk factor in the issuance of substandard annuities and thus should be taken into account when making informed decisions. In addition, we provide qualitative background information about underwriting and classification methods and describe underwriting risks for different types of substandard annuity products. We also discuss key aspects regarding a practical implementation of our model as well as possible market entry barriers. Since the risk classification model is formulated in a rather general way, it can as well be applied to other classification problems.

Selling substandard annuities is a challenging task and several factors in the process will influence a provider's profitability. First, a reasonable classification system must be established based on insureds' life expectancy. Second, adequate underwriting guidelines are necessary to ensure that each applicant is assigned to the proper risk class. Distinctive features of risk classes include medical conditions or lifestyle factors such as smoking, weight, geographical location, education, or occupation. Resulting classification costs need to be taken into consideration when pricing the contract. Fi-

[3] See LIMRA and Ernst & Young (2006, p. 18).

[4] See Doherty (1981).

[5] According to *Actuarial Standard of Practice No. 12*, a risk classification system is a "system used to assign risks to groups based upon the expected cost or benefit of the coverage or services provided" (Actuarial Standards Board, 2005).

nally, demand for the product is determined by the annuity amount paid to insureds in each risk class.

The literature on substandard annuities primarily deals with practical issues of substandard annuity markets. Ainslie (2000), Weinert (2006), and LIMRA and Ernst & Young (2006) provide detailed studies of substandard annuities in the United Kingdom and the United States. Information on the development, size, and potential of substandard annuity markets, different product types, underwriting methods and challenges, mortality and risk classification issues, the impact on the standard annuity market, tax considerations, distribution channels, and reinsurance can be found in Ainslie (2001), Brown and Scahill (2007), Cooperstein et al. (2004), Froehling (2007), Hamdan and Rinke (1998), Richards and Jones (2004), and Rinke (2002). Junus et al. (2004), Nicholas and Cox (2003), and Turner (2001) focus on the underwriting of substandard annuities. The impact of individual underwriting on an insurance company's profit is examined in Hoermann and Russ (2008), based on actuarial pricing. In Ranasinghe (2007), underwriting and longevity risk for impaired lives are assessed by means of a provision for adverse deviation.

Regarding risk classification within the insurance sector, Williams (1957) provides an overview of insurance rate discrimination, including its definition, various forms, economic effects, and government regulation. Doherty (1981) examines the profitability of rate classification for an innovating insurer and the associated market dynamics. This paper is an extension of previous work (Doherty, 1980), in which the author investigates rate discrimination in the fire insurance market. Christiansen (1983) draws a parallel to substandard annuities when analyzing the "fairness" of rate discrimination. Zaks et al. (2008) show the existence of an equilibrium point, when, in a portfolio consisting of several risk classes with respective price-demand functions, the premium amount and the number of policyholders in each risk class are iteratively updated. A great deal of the literature is dedicated to risk classification controversies concerning social issues. Some authors argue that competition by risk classification is inefficient, particularly if it becomes purely selective, i.e., if it makes insurance expensive or unaffordable for persons representing high risks for insurers. In contrast, others regard risk classification as essential in avoiding adverse selection. De Jong and Ferris (2006) provide the background for this discussion. Other authors addressing this topic include Abraham (1985), De Wit (1986), Feldman and Dowd (2000), Rothschild and Stiglitz (1997), Thiery and Van Schoubroeck (2006), Thomas (2007), and Van de Ven et al. (2000). In addition, De Jong and Ferris (2006) contains a demand model to investigate the effects of changes in risk classification systems. The authors determine the impact of unisex pricing in the U.K. annuity market on the expected purchase of annuity amounts depending on a person's individual mortality level, which is described by a frailty factor.

Various authors assess risk classification in insurance from a social utility point of view (see, e.g., Bond and Crocker, 1991; Crocker and Snow, 1986, 2000; Hoy, 1989, 2006; Sheshinski, 2007; Strohmenger and Wambach, 2000; Van der Noll, 2006; Villeneuve, 2000). Promislow (1987) measures the inequity that arises from considering only certain factors—and ignoring others—when setting insurance rates. There are several papers that deal with practical issues of risk classification. Kwon and Jones (2006) develop a mortality model that reflects the impact of various risk factors. Derrig and Ostazewski (1995), Horgby (1998), and Lohse (2004) focus on risk classification based on fuzzy techniques. The work of Werth (1995) provides a broad overview of preferred lives products. Leigh (1990) reviews the underwriting of life and sickness benefits, and Walters (1981) develops standards for risk classification. Today, the impact of genetics on risk classification has taken on added importance (see, e.g., Brockett et al., 1999; Brockett and Tankersleigh, 1997; Hoy and Lambert, 2000; Hoy and Ruse, 2005; Macdonald, 1997; Macdonald, 1999; O'Neill, 1997). There is also a substantial body of literature on rate classification in non-life insurance, especially in the automobile sector. For instance, Schwarze and Wein (2005) consider the third-party motor insurance industry and empirically test whether risk classification creates information rents for innovative insurers. Cummins et al. (1983, pp. 27–62) and Driver et al. (2008) focus on the economic benefits of risk classification. The authors argue that in many cases, risk classification contributes to economic efficiency, limits adverse selection, reduces moral hazard, and encourages innovation and competition within the insurance market.

In this paper, we contribute to the literature by providing a comprehensive analysis of challenges and chances for life insurers offering substandard annuity products. To this end, we combine the two strands of literature, that on substandard annuities and that on risk classification. From an insurer's viewpoint, we solve the problem of optimal risk classification for substandard annuities taking into consideration classification costs and underwriting risk, which has not been done to date. In addition, we provide a detailed discussion on the background of substandard annuities and limitations regarding risk classification within annuity products.

Section 2 provides practical background information about different types of substandard annuities, and also describes underwriting and classification issues. In Section 3, we develop a model to determine the optimal number and size of risk classes, as well as the optimal price-demand combination for each risk class that will maximize an insurer's profit. Risk classes are distinguished by the average mortality of individuals in a certain class relative to the average population mortality. We account for mortality heterogeneity and use a frailty model to derive individual probabilities of death. The profit is maximized based on given price-demand dependencies in population subgroups and classification costs. When taking into account costs of underwriting risk, a modified risk classification system might be optimal, depending on the underwriting quality. In Section 4, we provide a detailed description of market entry barriers and risks for substandard annuity providers and aspects concerning the practical application of our model. The paper concludes with a summary in Section 5.

2. Substandard Annuities and Underwriting

In general, there are three types of *substandard annuities*, as opposed to the *standard annuity*: enhanced annuities, impaired (life) annuities, and care annuities.[6] Usually, all three are immediate annuities for a single lump-sum payment, where the annual annuity amount depends on the insured's health status. In the following, we start with a definition of each annuity type and then go on to provide information about key aspects of annuity underwriting. Market size and underwriting risks are discussed in the last part of the section.

Enhanced annuities pay increased pensions to persons with a slightly reduced life expectancy.[7] Most applicants are between 60 and 70 years of age.[8] Calculation of enhanced annuities is based on environmental factors, such as postal code or geographic location, and lifestyle factors, such as smoking habits, marital status, or occupation, as well as disease factors, including diabetics, high blood pressure, high cholesterol, or being overweight.[9] When impairments are considered, this type of annuity is sometimes referred to as an impaired (life) annuity.[10] *Impaired life annuities* are typically related to health impairments such as heart attack, cancer, stroke, multiple sclerosis, lung disease, or kidney failure for annuitants in an age range of 60 to 85.[11] *Care annuities* are aimed at seriously impaired individuals between age 75 and 90 or persons who already have started to incur long-term-care costs.[12] Risk assessment for care annuities is based on geriatric symptoms such as frailty or restricted mobility, which are meas-

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6 See, e.g., Ainslie (2001, p. 16), Brown and Scahill (2007, pp. 5–6), and Cooperstein et al. (2004, pp. 14–15).
7 See Richards and Jones (2004, p. 20) and Weinert (2006, p. 6).
8 See Ainslie (2001, p. 16) and Cooperstein et al. (2004, p. 14).
9 See, e.g., Ainslie (2001, p. 17) and Brown and Scahill (2007, p. 5).
10 See Brown and Scahill (2007, pp. 5–6). Definitions of enhanced and impaired life annuities sometimes overlap. See, e.g., Nicholas and Cox (2003, p. 5).
11 See, e.g., Ainslie (2001, pp. 16–17).
12 See Ainslie (2001, pp. 15–16) and Cooperstein et al. (2004, p. 15).

ured in terms of activities of daily living (ADL) and instrumental activities of daily living (IADL); cognitive skills may also be taken into account.[13]

For standard annuities, the annuity amount is calculated based on the average mortality of one class comprising all insureds. Payments depend on an annuitant's age and gender. Based on so-called *single-class underwriting*, the insurer decides whether to accept or reject an applicant.[14] Substandard annuities require adjustment of the underlying pricing assumptions based on an individual's impairment level, necessitating the provision of medical information. It is the applicant's responsibility to provide sufficient evidence that he or she is eligible for increased annuity payments. Upon receipt of this evidence, the reduction in life expectancy is quantified by the insurer's underwriting.[15] Life expectancy can be measured either in terms of average life expectancy (ALE) or in terms of the maximum realistic life expectancy (MRLE), corresponding to the 50%- or 90%-quantile of the remaining lifetime, respectively.[16] The modified annuity amount is determined either by an age rate-up or by a rating factor.[17] The former involves an adjustment of the insured's actual age for calculation purposes. For instance, a 60-year-old impaired male may be rated to have the life expectancy of a 65-year-old and would thus receive the annuity amount based on being age 65. The rating factor is applied to the standard mortality table.[18] For example, an extra mortality of 100% would mean multiplying average mortality probabilities by a factor of "2."

Different types of underwriting techniques are employed depending on the applicant's health status and the type of annuity requested. In the underwriting process for enhanced annuities, applicants are assigned to different risk classes depending on their health status or individual mortality. This *multiclass underwriting*[19] is the most common method used in pricing substandard annuity products. Impairments or lifestyle factors are assessed by a health questionnaire.[20] Certain rules are applied (the *rules-based approach*) to determine the rating factor, the reduction in ALE, or the age rate-up.[21] This underwriting approach—in contrast to *full individual underwriting*[22]—is appropriate only for mild impairments and lifestyle factors that correspond to extra

[13] See Brown and Scahill (2007, p. 6) and Junus et al. (2004, p. 7).
[14] See Rinke (2002, p. 5).
[15] See LIMRA and Ernst & Young (2006, p. 32).
[16] See LIMRA and Ernst & Young (2006, pp. 32–33) and Nicholas and Cox (2003, pp. 5–6).
[17] See Junus et al. (2004, p. 4), LIMRA and Ernst & Young (2006, pp. 32–34).
[18] See Cooperstein et al. (2004, p. 16), LIMRA and Ernst & Young (2006, pp. 33–34), and Richards and Jones (2004, p. 22).
[19] See Rinke (2002, pp. 5–6).
[20] See Brown and Scahill (2007, p. 5).
[21] See Ainslie (2001, pp. 16–17), Cooperstein et al. (2004, p. 14), and Nicholas and Cox (2003, p. 5).
[22] See Cooperstein et al. (2004, p. 14), Richards and Jones (2004, p. 20), and Rinke (2002, p. 6).

mortalities between 25% and 50% and thus result in only slight annuity enhancements of around 10% to 15%.[23]

Impaired life annuities require a more extensive assessment of an applicant's health status due to the larger potential increase in the annuity amount, e.g., up to 50% for extra mortalities up to 150% depending on the issue age.[24] In addition to the health questionnaire, a doctor's report may be considered, implying a mixture between rules-based and individual underwriting.[25] Based on this information, the applicant is assigned a risk class (multiclass underwriting). Sometimes, full individual underwriting is required for impaired life annuities.[26] However, there is a tradeoff between the additional costs of such and the increased accuracy thus derived.[27] According to Nicholas and Cox (2003), the impaired life expectancy is measured in terms of both ALE and MRLE.[28]

Care annuities are individually underwritten based on a doctor’s report,[29] meaning that individual life expectancy is calculated for each applicant and no risk classes are established.[30] To obtain a more precise specification, the MRLE is used.[31] Extra mortalities between 250% and 300% yield annuity enhancements of up to 125%.[32]

The market for enhanced annuity products is large, whereas the market for impaired life annuities is of moderate size. Care annuities have a small niche market.[33] Some providers focus solely on restricted market segments, whereas others cover the full range of standard and substandard annuity products.[34] Sometimes, companies merely add one substandard (enhanced or impaired life) product to their standard annuity portfolio.[35]

It is often claimed that accurate underwriting is the crucial factor in offering substandard annuities. In particular, there is substantial risk that the underwriting will not cor-

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[23] See Ainslie (2001, p. 16), Cooperstein et al. (2004, p. 14), and Weinert (2006, p. 8).
[24] See Ainslie (2001, p. 16) and Weinert (2006, p. 8).
[25] See Ainslie (2001, pp. 16–17) and Brown and Scahill (2007, pp. 5–6).
[26] See Richards and Jones (2004, p. 20).
[27] See Rinke (2002, pp. 5–6).
[28] See Nicholas and Cox (2003, p. 6).
[29] See Brown and Scahill (2007, p. 6) and Richards and Jones (2004, p. 20).
[30] See Rinke (2002, pp. 5–6).
[31] See Nicholas and Cox (2003, p. 6).
[32] See Ainslie (2001, p. 16).
[33] See Ainslie (2001, p. 16) and Cooperstein et al. (2004, p. 15).
[34] See Froehling (2007, p. 5).
[35] See Weinert (2006, p. 12).

rectly assess an applicant's mortality level.[36] LIMRA and Ernst & Young (2006) list several causes of underwriting risk, such as the pressure of competition, the lack of adequate underwriting procedures and experience, and insufficient mortality data. The latter factor is also discussed in Lu et al. (2008), who emphasize the risk of making ratings based on small-sample medical studies. The lack of mortality data—especially for higher age groups—may be partly responsible for the slow development of the substandard annuity market. This problem could be solved by outsourcing underwriting to reinsurers, who have more data.[37] There is also the danger for the point of view of an annuity provider that the life expectancies of impaired persons can improve dramatically due to developments in the medical field. Therefore, it is vital that underwriters carefully monitor the mortality experience in their book of business as well as developments in medical research.[38] The former is also important with respect to adverse selection, especially when insurers offer both standard and substandard annuities.[39] Another risk factor has to do with using lifestyle characteristics as a basis for underwriting; this practice can increase the risk of adverse selection if an insured improves his or her life expectancy by changing behavior, for example, by quitting smoking or losing weight.[40]

In this paper, we focus on the large enhanced and impaired life annuity market, where insureds are categorized in risk classes with differing mortality by means of rating factors. We determine the optimal risk structure for an insurer offering substandard annuities and explicitly model and integrate costs related to underwriting risk, which is of great concern to insurers.

3. The Model Framework

a) Basic model

We consider a *general population* consisting of $N \in \mathbb{N}$ potential risks, i.e., potential policyholders of a given gender and at a specific age $x \in \{0,\ldots,\omega\}$. ω is the limiting age of a population mortality table describing the average mortality in the general population. The entry q_x' thus specifies the average one-year probability of death for a

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[36] See, e.g., LIMRA and Ernst & Young (2006, p. 31), and Richards and Jones (2004, p. 20).

[37] See Cooperstein et al. (2004, p. 13).

[38] See Cooperstein et al. (2004, p. 16), Junus et al. (2004, p. 5), Nicholas and Cox (2003, p. 8), Richards and Jones (2003, p. 20), Sittaro (2003, p. 9), and Weinert (2006, p. 17).

[39] See Cooperstein et al. (2004, p. 16).

[40] See LIMRA and Ernst & Young (2004, pp. 42–43).

person age x out of the general population, where the prime (') mark indicates population mortality.[41]

Mortality heterogeneity in the general population is considered by means of a frailty model.[42] To obtain individual probabilities of death, we apply a stochastic frailty factor to the average mortality probability. The one-year individual probability of death $q_x(d)$ for an x-year-old is thus given as the product of the individual frailty factor $d \in \mathbb{R}_0^+$ and the probability of death q'_x from the population mortality table:

$$q_x(d) = \begin{cases} d \cdot q'_x, & d \cdot q'_x < 1 \\ 1, & x = \min\left[\tilde{x} \in \{0,\ldots,\omega\} : d \cdot q'_{\tilde{x}} \geq 1\right] \\ 0, & \text{otherwise} \end{cases} \qquad \text{for } x \in \{0,\ldots,\omega\}. \tag{1}$$

If the resulting product is greater than or equal to 1 for any ages $\tilde{x}$, the individual probability of death is set equal to 1 for the youngest of those ages; for all other ages $\tilde{x}$, it is set to 0. For $d < 1$, we let $q_\omega(d) := 1$.

The frailty factor specifies an individual's state of health. A person with a frailty factor less than 1 has an above-average life expectancy, a frailty factor greater than 1 indicates that the individual is impaired with a reduced life expectancy, and a frailty factor equal to 1 means the person has average mortality.

The individual frailty factor d is a realization of a random variable D.[43] The distribution F_D of D represents the distribution of different states of health and thus of different life expectancies in the general population. For its characteristics, we follow the assumptions in Hoermann and Russ (2008): we let F_D be a continuous, right-skewed distribution on $\mathbb{R}_0^+$ with an expected value of 1, such that the mortality table describes an individual with average health. As probabilities of death approaching zero are not realistic, the probability density function f_D is flat at zero with $\mathrm{f}_D(0) = 0$.

We assume that the insurer is able to distinguish a maximum of H different *subpopulations* that aggregate to the total general population (see Figure 1). Subpopulations differ by health status of contained risks and are ordered by their mortality,

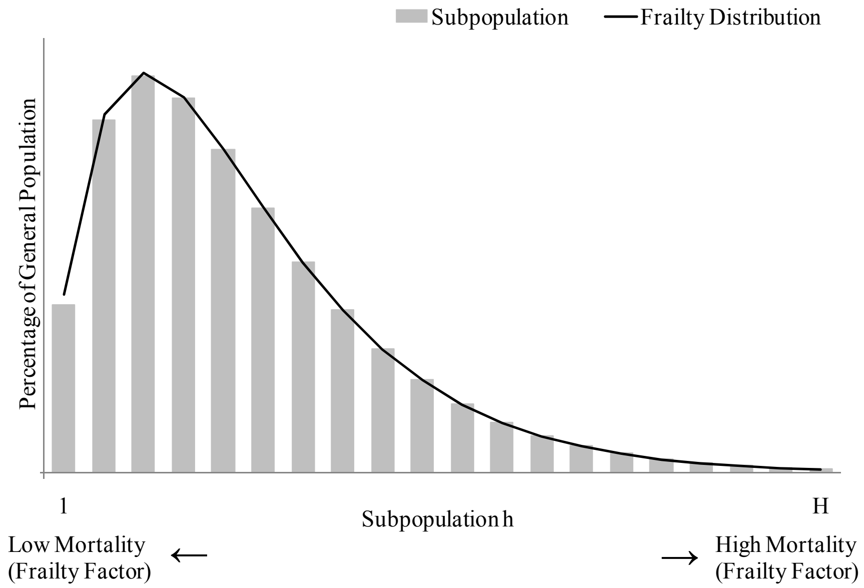
41 To be directly applicable to insurance data, we use a discrete model. By substituting annual mortality probabilities by the continuous force of mortality, a continuous model could also be employed. General results remain unchanged, however.

42 See Hoermann and Russ (2008).

43 See, e.g., Jones (1998, pp. 80–83), Pitacco (2004, p. 15), and Vaupel et al. (1979, p. 440).

where $h=1$ is the subpopulation with the lowest mortality and $h=H$ is the subpopulation with the highest mortality. Risks belong to a given subpopulation if their individual mortality lies in a corresponding frailty factor range. Subpopulation h comprises all persons with a frailty factor lying in the interval $\left[d_h^L, d_h^U\right)$, where d_h^L defines its lower and d_h^U defines its upper limit for $h=1,\ldots,H$ and $d_h^U = d_{h+1}^L, h=1,\ldots,H-1$. All intervals combined—corresponding to the H subpopulations—aggregate to the positive real axis, i.e., $\bigcup_{h=1}^{H}\left[d_h^L, d_h^U\right) = [0,\infty)$. Thus, the whole range of positive real-valued frailty factors is covered.

Figure 1: Segmentation of the general population into H subpopulations depending on mortality level (described by the frailty distribution)



The number of risks in subpopulation h is denoted by N_h. Risks in all subpopulations sum up to the total number of risks N of the general population, i.e., $\sum_{h=1}^{H} N_h = N$. N_h depends on the frailty distribution. It can be derived as the percentage of risks out of the general population with a frailty factor between d_h^L and d_h^U. Thus, it is calculated as the product of the total number of risks N and the probability of the frailty factor lying in the interval $\left[d_h^L, d_h^U\right)$. The latter can be expressed in terms of the frailty distribution F_D, leading to

$$N_h = N \cdot \mathbb{P}\left(d_h^L \le D < d_h^U\right) = N \cdot \left(\mathrm{F}_D\left(d_h^U\right) - \mathrm{F}_D\left(d_h^L\right)\right).$$

Each subpopulation h is further characterized by two functions. First, its cost function $g_h(n)$ describes costs of the insurance of survival risk for individuals in subpopulation h. Second, its price-demand function $f_h(n)$ specifies how many risks n would acquire (one unit of) annuity insurance for a given price $P_h = f_h(n)$. Both functions are defined for the number of insureds $n = 1,\dots,N_h$ in each subpopulation $h = 1,\dots,H$.

Since the insurer cannot distinguish beyond given subpopulations, they are treated as homogeneous with respect to the mortality. The cost function is independent of the number of sales, i.e., $g_h(n) = P_h^A, n = 1,\dots,N_h$, where P_h^A describes the actuarial premium for covering the cost of (one unit of) annuity insurance for the average potential insured in subpopulation h. The actuarial premium is based on the average mortality in a subpopulation, which can be derived from the frailty distribution. The average frailty factor $\bar{d}_h$ for subpopulation h is given as the truncated expected value of frailty factors in the corresponding interval $\left[d_h^L, d_h^U\right)$:

$$\bar{d}_h = \mathbb{E}\left(D \cdot 1\left\{D \in \left[d_h^L, d_h^U\right)\right\}\right) = \int_{d_h^L}^{d_h^U} z \cdot \mathrm{f}_D(z)\,dz,$$

where $1\{.\}$ represents the indicator function. Therefore, the average k-year survival probability for a person age x in subpopulation h is given by

$${}_t p_x^h = \prod_{l=0}^{t-1}\left(1 - q_{x+l}\left(\bar{d}_h\right)\right),$$

with $q_x\left(\bar{d}_h\right)$ as in Equation (1). The actuarial premium P_h^A for one unit of annuity insurance equals the present value of future annuity payments and thus results in

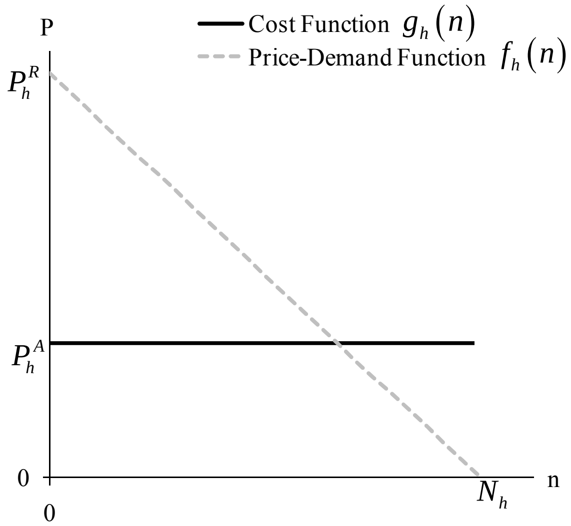
$$P_h^A = \sum_{t=0}^{\omega - x} {}_t p_x^h v^t,$$

where v denotes the discount factor. Since the average frailty factor is increasing for ascending risk classes, i.e., $\bar{d}_1 < \bar{d}_2 < \dots < \bar{d}_H$, the cost of insurance and hence the actuarial premium is decreasing, i.e., $P_1^A > P_2^A > \dots > P_H^A$.

The price-demand function $f_h(n)$ is monotonously decreasing in the number of risks n, i.e., the lower the price, the more people there are willing to buy insurance. Its first derivative $f_h'(n)$ with respect to the demand n (the price elasticity of demand) is hence negative for all n. In addition, we assume that the reservation price P_h^R, i.e., the price for which the demand is zero $(P_h^R = f_h(0))$, increases with decreasing mortality probabilities in a subpopulation. Consequently, a subpopulation with high life expectancy contains individuals, who would be expected to pay more for one unit annuity insurance compared to individuals in a subpopulation with low life expectancy. As the actuarial fair value of one unit annuity insurance is higher for healthy persons, it makes sense that their willingness to pay will also be higher. In addition, Turra and Mitchell (2004) found that annuities are less attractive to poorer risks with uncertain out-of-pocket medical expenses. In terms of reservation prices, this means that $P_1^R > P_2^R > \cdots > P_H^R$. At a price of zero, in contrast, everyone in the general population would purchase insurance, i.e., $f_h\left(N_h\right) = 0, h = 1,\ldots,H$.

An illustration of the determinants for one subpopulation h is provided in Figure 2; for illustration purposes, a linear price-demand function is displayed.[44]

Figure 2: Constant cost function g_h and linear price-demand function f_h (in terms of the price P and as a function of the demand n) in subpopulation h



Notes: P_h^R = reservation price, P_h^A = actuarial premium for (one unit of) annuity insurance, N_h = number of risks in subpopulation h.

b) Optimal risk classification

We now consider profit-maximizing insurers who intend to introduce rate-discriminating annuity products. A *risk class* is comprised of insureds subject to a specific range of risks in regard to their remaining life expectancy. Thus, annuity prices will vary by risk class. For instance, an insured with reduced life expectancy will obtain a higher annuity for a given price or, vice versa, pay a lower price for one unit of annuity insurance, as described in the previous section.

[44] See, e.g., Baumol (1977, pp. 401–402).

A combination of population subgroups to risk classes is called a *classification system*. Offering a standard annuity product corresponds to addressing the total general population. This is consistent with a classification system that aggregates all existing population subgroups to one single risk class. In our setting, subgroups are sorted by decreasing average life expectancies and, hence, only adjacent subgroups can be merged with each other into a risk class.

Let M be the set of all possible classification systems. A classification system $m \in M$ consists of I_m risk classes. However, conducting risk classification is associated with classification costs. This includes the costs for distinguishing between risk classes as described in the previous section, e.g., costs for establishing underwriting guidelines for each additional risk class (beyond the total general population). We assume these costs to be proportional to the number of distinctions and thus set them to $k\left(I_m - 1\right)$, where $k \in \mathbb{R}_0^+$.[45] When offering only one standard class, i.e., $I_m = 1$, no classification costs are incurred.

In each classification system m with I_m risk classes, a risk class $i, i = 1,\ldots,I_m$ is composed of S_i subpopulations, where $\sum_{i=1}^{I_m} S_i = H$. To simplify notation, in the following, we omit the index m when focusing on a specific risk class i (within a classification system m). Hence, all H subpopulations of the general population are assigned to I_m risk classes. Subpopulations contained in one risk class are ordered by increasing average mortality; this is indicated by the index s. The total number of individuals in risk class i is given as the sum of the number of persons N_s in each contained subpopulation s:

$$N_i = \sum_{s=1}^{S_i} N_s .$$

As for each subpopulation, a risk class i is characterized by its price-demand function $f_i\left(n\right)$ and its cost function $g_i\left(n\right)$ for $n = 1,\ldots,N_i$. If risk class i contains exactly one subpopulation h (and thus $S_i = 1$), then $f_i = f_h$ and $g_i = g_h$. Otherwise, f_i and g_i are aggregated functions of the price-demand and cost functions of the S_i subpopulations. The aggregation process is complex and must be conducted stepwise by means of inverse functions. f_h^{-1} denotes the inverse function of f_h, which is defined on the inter-

[45] For a more general specification, one might define classification costs by any monotonously increasing function in the number of risk classes I_m. Thus, disproportionately high classification costs can be represented, for instance.

val $\left[0, P_h^R\right]$. The price-demand function f_i of risk class i is aggregated based on the price-demand functions f_s $\left(s=1,\ldots,S_i\right)$ of the S_i underlying subpopulations. The aggregated function f_i will exhibit breaks when it becomes equal to the reservation price P_s^R of one of the contained subpopulations s, since each reservation price represents the point at which the next subpopulation will start buying the policy. In each risk class i, we have S_i reservation prices. Hence, there are S_i intervals on the x-axis, on each of which the aggregate price-demand function is defined differently. The intervals I_ν are given by

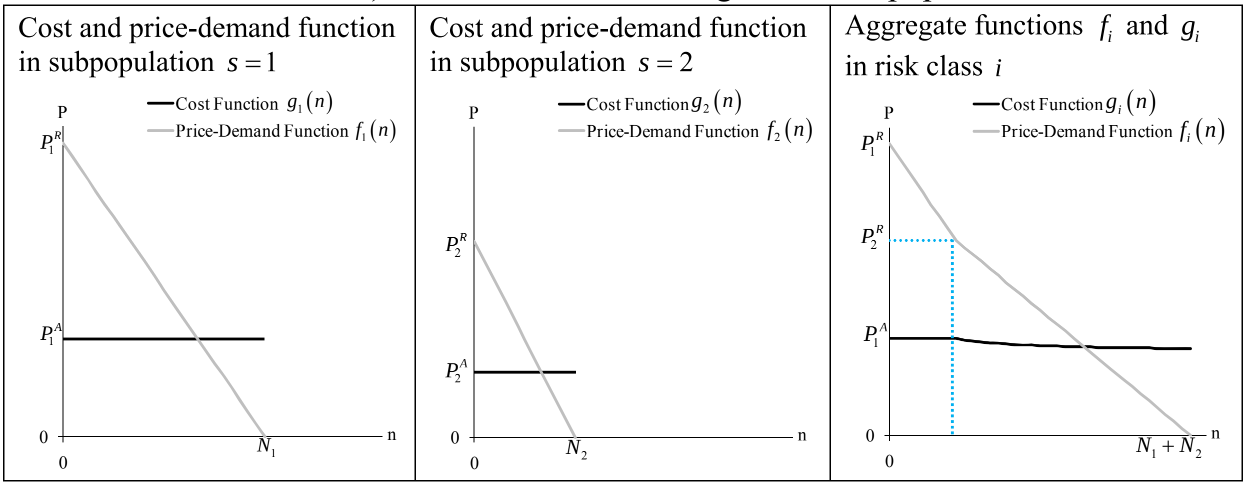
$$I_\nu = \begin{cases} \left[\sum_{s=1}^{\nu} f_s^{-1}\left(P_\nu^R\right), \sum_{s=1}^{\nu+1} f_s^{-1}\left(P_{\nu+1}^R\right)\right) & \text{for } \nu = 1,\ldots,S_i - 1 \\ \left[\sum_{s=1}^{\nu} f_s^{-1}\left(P_\nu^R\right), N_i\right] & \text{for } \nu = S_i. \end{cases} \tag{2}$$

On each interval I_ν, the aggregated price-demand function is defined by

$$f_i(n) = f_i\left(f_i^{-1}(P_i)\right) = f_i\left(\sum_{s=1}^{\nu} f_s^{-1}(P_i)\right), \text{ if } n \in I_\nu,\ \nu = 1,\ldots,S_i.$$

Thus, for any given price $P_i := f_i(n)$, the number of insured risks n in a risk class is the sum of the number of insured risks in each contained subpopulation s for which $f_s^{-1}(P_i)$ is defined. Graphically, the aggregate price-demand function of a risk class is received by horizontal addition of the price-demand functions of belonging subpopulations. It starts at the highest reservation price $P_{s=1}^R$ and whenever the function passes another reservation price, there is a bend as the demand of another subpopulation is added. Figure 3 sketches the aggregation process for risk class i consisting of two subpopulations, $S_i = 2$.

Figure 3: Aggregate cost and price-demand function (in terms of the price P and as a function of the demand n) in risk class i consisting of two subpopulations $s=1,2$



Notes: $P_s^R, s=1,2$ = reservation price, $P_s^A, s=1,2$ = actuarial premium for (one unit of) annuity insurance, $N_s, s=1,2$ = number of risks in subpopulation s.

In our setting, the cost function in a subpopulation is constant in the number of insureds. This means that no inverse function of the cost function exists. Therefore, the aggregate cost function g_i of risk class i must be derived based on the associated price-demand function f_i. It is defined piecewise in sections analogously to f_i (see Equation (2)). For a given demand n in risk class i, the price is given by $f_i(n) =: P_i$. For this price, the corresponding number of persons in each contained subpopulation s is determined by the inverse function $f_s^{-1}(f_i(n)) = f_s^{-1}(P_i) =: n_s$, if existent. The number of persons in each subpopulation is then weighted with the corresponding costs $g_s(P_i) =: P_s^A$ in that subpopulation. Finally, average costs are determined for the total of $n = \sum_{s=1}^{\nu} f_s^{-1}(f_i(n)) = \sum_{s=1}^{\nu} f_s^{-1}(P_i) = \sum_{s=1}^{\nu} n_s$ insureds. The resulting formula for the aggregate cost function on the interval I_ν is thus descriptive and given by

$$g_i(n) = \frac{1}{n}\sum_{s=1}^{\nu} f_s^{-1}(f_i(n)) \cdot g_s(f_i(n)) = \frac{1}{n}\sum_{s=1}^{\nu} f_s^{-1}(P_i) \cdot g_s(P_i) = \frac{1}{n}\sum_{s=1}^{\nu} n_s \cdot P_s^A$$

if $n \in I_\nu$, $\nu = 1,\ldots,S_i$. Since the cost function is lower for higher subpopulation indices s, i.e., $g_s(n) = P_s^A > P_{s+1}^A = g_{s+1}(n)$, and since only adjacent subpopulations may be merged into risk classes, the aggregate cost function is generally decreasing (as exemplarily illustrated in Figure 3). The profit Π_i in risk class i is calculated as the difference of earnings and costs, i.e.

$$\Pi_i(n) = E_i(n) - C_i(n) = n \cdot f_i(n) - n \cdot g_i(n), n = 1,\ldots,N_i,$$

where $E_i(n) = n \cdot f_i(n)$ describes the earnings and $C_i(n) = n \cdot g_i(n)$ describes the costs for n insured risks. Hence, a risk class is profitable as long as the market price $P_i = f_i(n)$ is higher than the actuarial premium $P_i^A = g_i(n)$, setting aside any additional (classification) costs.

The total profit from classification system m is given as the sum of the profit in each risk class $i, i = 1, \ldots, I_m$ less classification costs $k(I_m - 1)$. It is denoted by

$$\Pi\left(n_1, \ldots, n_{I_m}\right) = \sum_{i=1}^{I_m} \Pi_i(n_i) - k(I_m - 1)$$

with n_i being the number of insured risks in risk class i. The insurer aims to determine the optimal classification system and optimal price-demand combinations in the corresponding risk classes such that the profit is maximized:

$$\max_{m \in M} \max_{\left\{\left(n_1, \ldots, n_{I_m}\right)\right\}} \Pi\left(n_1, \ldots, n_{I_m}\right). \tag{3}$$

The optimal number and composition of risk classes depend on classification costs. If classification costs are zero, profit is maximized for the maximum number of distinguishable risk classes, i.e., if each risk class i corresponds to a subpopulation h for all $h = 1, \ldots, H$.[46] In the presence of classification costs, this pattern changes, depending on the costs.

The maximization process must be undertaken in recurrent steps. For each classification system $m \in M$, optimal price-demand combinations must be derived for each risk class $i = 1, \ldots, I_m$, which is done by setting the first derivative of the profit $\Pi\left(n_1, \ldots, n_{I_m}\right)$ with respect to the number of risks n_i equal to zero:[47]

$$\frac{\partial \Pi\left(n_1, \ldots, n_{I_m}\right)}{\partial n_i} = \frac{\partial \Pi_i(n_i)}{\partial n_i} = f_i(n_i) - g_i(n_i) + n_i\left(f_i'(n_i) - g_i'(n_i)\right) \overset{!}{=} 0.$$

Since f_i and g_i are defined in S_i sections, for each interval $I_\nu, \nu = 1, \ldots, S_i$ as specified in Equation (2), the number of insureds in risk class i is implicitly given by

---

[46] See, e.g., Doherty (1980, 1981).

[47] See, e.g., Baumol (1977, pp. 416–417), assuming that the functions are differentiable.

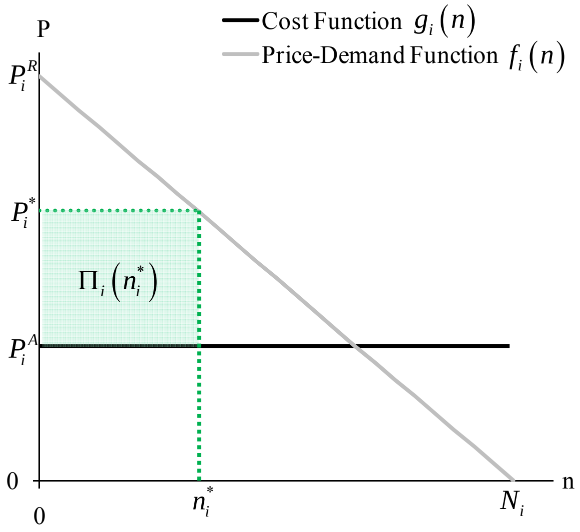
$$n_i^\nu = -\frac{f_i\left(n_i^\nu\right) - g_i\left(n_i^\nu\right)}{f_i'\left(n_i^\nu\right) - g_i'\left(n_i^\nu\right)}\text{, for } n_i^\nu \in I_\nu,\ \nu = 1,\ldots,S_i.$$ [48]

To determine the optimal price-demand combination for risk class i, we need to compare the profit based on the number of risks n_i^ν for each section $\nu = 1,\ldots,S_i$. It is maximized for[49]

$$n_i^{\nu*} = \underset{n_i^\nu \in I_\nu,\, \nu=1,\ldots,S_i}{\arg\max}\ \Pi_i\left(n_i^\nu\right).$$

Figure 4 is an illustration of the optimal price-demand combination in the case of a risk class i that equals a subpopulation h. Since the number of underlying subpopulations is $S_i = 1$ in this case, the functions exhibit no breaks, which means that no distinctions need be made between different intervals.

Figure 4: Optimal profit-maximizing price-demand combination in risk class i



Notes: P_i^R = reservation price, P_i^A = actuarial premium for one unit of annuity insurance, n_i^* = optimal number of insureds, P_i^* = optimal market price, $\Pi_i\left(n_i^*\right)$ = maximum profit, N_i = number of risks in risk class i.

48 A solution n_i^ν is valid only if it lies in the corresponding interval I_ν.

49 In this context, we assume that price discrimination is not possible within a subpopulation. Hence, for policyholders in one subpopulation—i.e., policyholders do face an identical risk situation—, the same price P_i^* will be asked for (for this point cf. the Chapter "Fairness in Risk Classification" in Cummins et al., 1983, pp. 83–92). In an extreme case, where first-degree price discrimination is possible, an insurer would like to charge a different price (customer's reservation price) to each policyholder in one subpopulation (see, e.g., Pindyck and Rubinfeld, 2008, p. 393–403 and Baumol, 1977, pp. 405–406).

In the second step, the total profit from each classification system $m \in M$ is determined by taking into account classification costs $k\left(I_m - 1\right)$. The classification system that yields the highest profits for the insurer is optimal, i.e.,

$$m^* = \underset{m \in M}{\arg\max}\, \Pi\left(n_1, ..., n_{I_m}\right),$$

implying a maximum total profit of

$$\Pi\left(n_1^{\nu *}, ..., n_{I_{m*}}^{\nu *}\right) = \sum_{i=1}^{I_{m*}} \Pi_i\left(n_i^{\nu *}\right) - k\left(I_{m*} - 1\right).$$

From this formula, in some cases, one can derive the maximum classification costs that can be incurred such that the optimal risk classification system will still be profitable.

$$\Pi\left(n_1^{\nu *}, ..., n_{I_{m*}}^{\nu *}\right) = 0 \Leftrightarrow k = \frac{\sum_{i=1}^{I_{m*}} \Pi_i\left(n_i^{\nu *}\right)}{I_{m*} - 1}.$$

However, it needs to be verified that the classification m^* is still optimal under the modified classification costs *k*, since a different classification with less or more risk classes may be optimal then. If the optimization problem yields the same result, an insurer may compare this amount with the estimated costs of increased underwriting effort.

c) Optimal risk classification and costs of underwriting risk

One of the main reasons why insurers are reluctant to engage in risk classification has to do with the costs of underwriting risk as laid out in Section 2. The effect of costs of underwriting risk in connection with risk classification on an insurer's profit situation has never been modeled. In the following, we propose a model that allows a general assessment of costs of underwriting risk.

We start from the insurer's optimal risk classification system m^* as presented in Subsection 3 b), including the optimal number of risk classes I_{m^*} as well as optimal price-demand combinations $\left(n_i^*, P_i^*\right)$ for $i = 1, \ldots, I_{m^*}$ that satisfy Equation (3). Here, and in the following, we omit the superscript ν to facilitate notation. We model underwriting risk by assuming that a policyholder who actually belongs to risk class i is wrongly assigned to a higher risk class $j > i$ with error probability $p_{ij} \geq 0$ ($\sum_{j \geq i} p_{ij} = 1$). For $j = i$, the underwriting classification is correct. We define costs of underwriting risk in terms of underwriting errors, which lead to a reduction of profit. In practice, of course, mistakes in classification can be either to the advantage or disadvantage of the insurer. Our approach can be interpreted as the excess negative effect. Furthermore, one can generally assume that the error probability decreases with increasing distance between i and j as it becomes more likely that individuals will be wrongly classified in adjacent risk classes.

Since the policyholder is actually in risk class i with the associated price-demand function $f_i(n)$, the optimal price-demand combination targeted by the insurer to maximize profit is given by n_i^* and P_i^*, representing the optimal number of policyholders n_i^* buying insurance for the price P_i^*. However, errors in underwriting imply that wrongly classified policyholders out of risk class i are charged the lower price P_j^*, which is optimal only for insureds in risk class j. The latter are representatives of a higher risk class with higher average mortality probability and thus lower costs of insurance. As a first effect, this error leads to a reduction of profits Π_i in risk class i, since for the percentage $p_{ij}, j \neq i$ of individuals out of risk class i, the lower price $P_j^* < P_i^*$ is charged:

$$\Pi_i := \Pi_i\left(n_i^*\right) = n_i^* f_i\left(n_i^*\right) - n_i^* g_i\left(n_i^*\right) = n_i^*\left(P_i^* - P_i^A\right) \geq \sum_{j \geq i} p_{ij} n_i^*\left(P_j^* - P_i^A\right).$$

As a second effect, the requested premium P_j^* creates a new demand $n_{ij} = n_i^* + \Delta n_{ij}$ with probability p_{ij}, which in turn leads to additional changes in the insurer's profit. n_{ij} denotes the number of policies sold in risk class i for the price P_j^*, which is optimal in risk class j:

$$n_{ij} = f_i^{-1}\left(P_j^*\right).$$

Δn_{ij} thus describes the difference between the actual number n_{ij} and the optimal number n_i^* of insureds in risk class i. The above notation is simplified in that it does not explicitly consider that f_i and its inverse function may be defined in sections. The profit in risk class i with insureds wrongly classified to risk class j changes to

$$\Pi_i\left(n_{ij}\right) = n_{ij} f_i\left(n_{ij}\right) - n_{ij} g_i\left(n_{ij}\right)$$

with probability p_{ij}. The expected profit in risk class i is then given as the average profit when wrongly classifying insureds to risk classes $j = i+1,...,I_m$, weighted with the respective underwriting error probabilities for each risk class:

$$\tilde{\Pi}_i = \sum_{j \geq i} p_{ij} \Pi_i \left(n_{ij} \right). \tag{4}$$

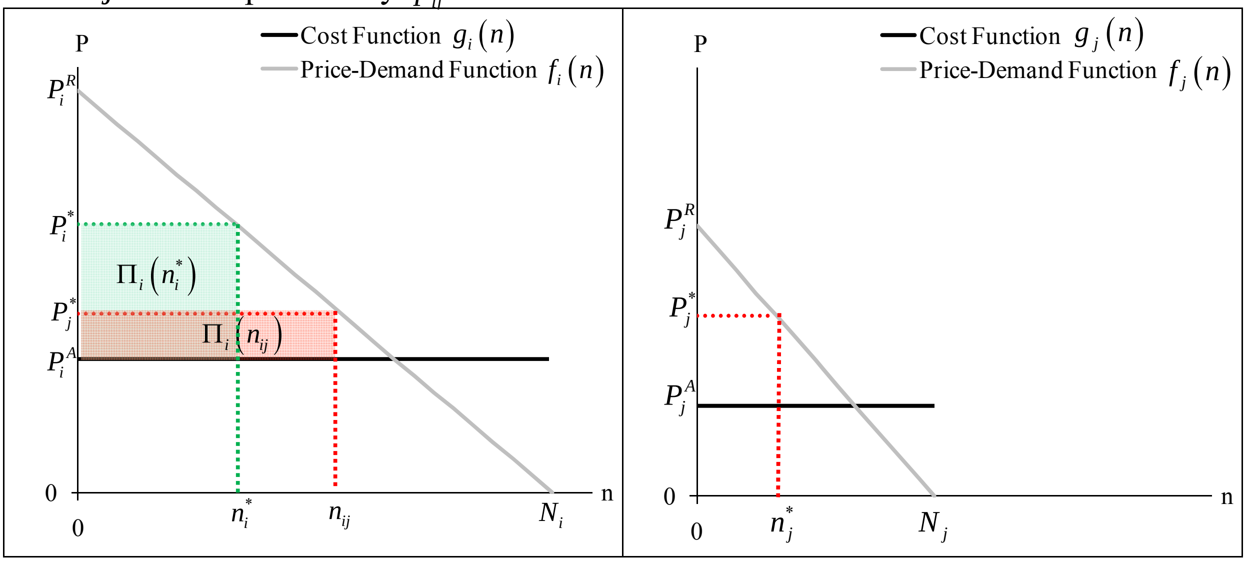
Figure 5 illustrates the effect of underwriting errors by means of two risk classes.

Since original price-demand combinations in risk class i are optimal for a profit-maximizing insurer, the modified profit $\tilde{\Pi}_i$ after accounting for underwriting errors will be lower than the original profit, i.e., $\tilde{\Pi}_i < \Pi_i$. The difference between the two amounts is the cost of underwriting risk in risk class i:

$$\varepsilon_i = \Pi_i - \tilde{\Pi}_i.$$

The cost will be influenced by various factors, such as the extent of underwriting error probabilities, the distance between two risk classes in terms of the difference between optimal risk class prices, and the cost function in each risk class.

Figure 5: Change of profit in risk class i when insureds are wrongly classified to risk class $j > i$ with probability $p_{ij} = 1$



Notes: P_i^R, P_j^R = reservation price, P_i^A, P_j^A = actuarial premium for (one unit of) annuity insurance, n_i^*, n_j^* = optimal number of insureds, P_i^*, P_j^* = optimal market price, N_i, N_j = number of risks in risk class i or j, respectively; $\Pi_i\left(n_i^*\right)$ = maximum profit in risk class i, $\Pi_i\left(n_{ij}\right)$ = profit in risk class i for demand n_{ij}.

We illustrate the effect of these different factors on the cost of underwriting risk for a case including only one erroneous classification from i to j. With $n_{ij} = n_i^* + \Delta n_{ij}$, one can reformulate the expression for costs of underwriting risk as follows:

$$
\begin{aligned}
\varepsilon_i &= \Pi_i - \tilde{\Pi}_i = \Pi_i\left(n_i^*\right) - \left(p_{ij}\Pi_i\left(n_{ij}\right) + \left(1-p_{ij}\right)\Pi_i\left(n_i^*\right)\right) = p_{ij}\left(\Pi_i\left(n_i^*\right) - \Pi_i\left(n_{ij}\right)\right) \\
&= p_{ij}\left(n_i^* f_i\left(n_i^*\right) - n_i^* g_i\left(n_i^*\right) - n_{ij} f_i\left(n_{ij}\right) + n_{ij} g_i\left(n_{ij}\right)\right) \\
&= p_{ij}\left(n_i^* f_i\left(n_i^*\right) - n_i^* g_i\left(n_i^*\right) - \left(n_i^* + \Delta n_{ij}\right) f_i\left(n_{ij}\right) + \left(n_i^* + \Delta n_{ij}\right) g_i\left(n_{ij}\right)\right) \\
&= p_{ij}\left(n_i^*\left(\left[f_i\left(n_i^*\right) - f_i\left(n_{ij}\right)\right] - \left[g_i\left(n_i^*\right) - g_i\left(n_{ij}\right)\right]\right) + \Delta n_{ij}\left[g_i\left(n_{ij}\right) - f_i\left(n_{ij}\right)\right]\right) \\
&= p_{ij}\left(n_i^*\left(\left[P_i^* - P_j^*\right] - \left[g_i\left(n_i^*\right) - g_i\left(n_{ij}\right)\right]\right) + \Delta n_{ij}\left[g_i\left(n_{ij}\right) - P_j^*\right]\right).
\end{aligned}
$$

As discussed above, the first term is positive, $\left[P_i^* - P_j^*\right] \geq 0$, since the price in a higher risk class j is lower compared to the price in risk class i. Therefore, the greater the distance between two risk classes, i.e., the bigger the difference between P_i^* and P_j^*, the higher the costs of underwriting risk. At the same time, however, the error probability p_{ij} will decrease with increasing distance between j and i, which dampens the effect of the difference between P_i^* and P_j^* on the overall cost ε_i. The second term represents the difference between the costs for the optimal number of policyholders n_i^* and the actual number n_{ij}. Since the cost function in risk class i is aggregated, it may be decreasing. In general, the cost for $n_{ij} > n_i^*$ individuals may thus be lower than the cost for n_i^* individuals, i.e., $\left[g_i\left(n_i^*\right) - g_i\left(n_{ij}\right)\right] \geq 0$. Hence, the difference will reduce costs of underwriting risk. The last term represents the difference between the actual costs for the insurer and the actual price paid, $\left[g_i\left(n_{ij}\right) - P_j^*\right]$. If the price paid does not cover actual costs in risk class i, costs of underwriting risk will be even larger. If the price paid does exceeds the costs of covering the insurer's expenses, the term will be negative, thus implying a reduction in costs of underwriting risk.

The profit $\tilde{\Pi}_i$ can even become negative if costs in risk class i are higher than the price P_j^* paid by the insured ($P_j^* < g_i\left(n_{ij}\right)$) and if, for example, the underwriter classifies insureds in risk class i to risk class j with probability $p_{ij} = 1$. In this special case, the insurer will suffer a loss from erroneous underwriting, which can be seen as follows:

$$
\tilde{\Pi}_i = \Pi_i\left(n_{ij}\right) = n_{ij}\underbrace{\left(P_j^* - g_i\left(n_{ij}\right)\right)}_{<0} < 0 .
$$

In general, costs of underwriting risk should be taken into consideration when making risk classifications. In particular, error probabilities will differ depending on the classification system. For instance, the more risk classes that are established, the smaller will be the differences between them, and the greater the probability of wrongly classifying insurance applicants. Furthermore, it can be assumed that the probability of a wrong classification diminishes with decreasing number of risk classes I_m contained in a classification system m.

In the presence of underwriting risk, the insurer again faces the problem of finding the optimal classification system. An optimal classification system m^{**} that takes underwriting risk under consideration may differ from the optimal classification system m^{*} that does not. The extent of the difference will depend on the error probability distribution and on classification costs. Calculations can be conducted based on empirically observed underwriting error probabilities or by making reasonable assumptions. The optimal classification system m^{**} solves the following equation based on the formula for modified expected profits in Equation (4):

$$m^{**} = \arg\max_{m \in M} \tilde{\Pi}\left(n_1, \ldots, n_{I_m}\right)$$

implying an optimal total profit of

$$\tilde{\Pi}\left(n_1, \ldots, n_{I_{m^{**}}}\right) = \sum_{i=1}^{I_{m^{**}}} \tilde{\Pi}_i\left(n_i\right) - k\left(I_{m^{**}} - 1\right).$$

Depending on the error probability distribution, the optimal classification system will be comprised of more or fewer risk classes, which will, of course, have an impact on classification costs. It is vital for an insurer to take all these factors into consideration so as to avoid losses from underwriting risk. By using our proposed approach and given estimation of error probabilities or sound assumptions on empirical erroneous underwriting, these risks can be quantified and appropriately taken into account.

An summary of the risk classification process along with terms and definitions introduced in this section is provided in Figure 6.

Figure 6: The process of risk classification for substandard annuities

A risk classification process for substandard annuities

Identify general population

General population consists of N potential policyholders ("risks") willing to buy annuity insurance with a given gender and at a specific age

⇩

Calibrate the frailty model

Use a frailty distribution to model mortality heterogeneity in the general population (see Figure 2). The frailty factor specifies an individual's state of health. A person with a frailty factor less than 1 has an above-average life expectancy; a frailty factor greater than 1 indicates that the individual is impaired with a reduced expected remaining lifetime; a frailty factor equal to 1 corresponds to a person with average mortality.

⇩

Divide general population into subpopulation

Subpopulations differ by health status of contained risks and are ordered by their mortality (higher subpopulations have higher average mortality). Assume that an insurer is able to distinguish only a limited number of different subpopulations (that aggregate to total general population). Risks belong to a given subpopulation if their individual mortality lies in the specified frailty factor range.

⇩

Determine price-demand function

Price-demand functions are defined for the number of insureds in each subpopulation; specify how many risks would acquire (one unit of) annuity insurance for a given price; is monotonously decreasing in the number of risks, i.e., the lower the price, the more persons are willing to buy insurance; can be estimated using, e.g., conjoint analysis.

⇩

Determine the cost function

The cost function describes the actuarial premium for covering the cost of (one unit of) annuity insurance for the average potential insured in a subpopulation; independent of the quantity of sales (insurer cannot distinguish beyond subpopulations.

⇩

Establish risk classes

A risk class comprises one or more subpopulations; addresses a specific range of risks in regard to their remaining life expectancy. If a risk class consists of more than one subpopulation, the aggregated price-demand function for the risk class has to be determined based on the price-demand functions in the underlying subpopulations. Only adjacent subpopulations can be merged to risk classes since subpopulations are specified by decreasing average life expectancies (in our setting).

⇩

Define a classification system

A combination of population subgroups to risk classes is called a classification system (a standard annuity product addresses total general population and thus corresponds to a classification system where all subpopulations are merged into one single risk class).

⇩

Calculate classification costs

A classification system consists of several risk classes. Classification costs include costs for distinguishing between risk classes (e.g., costs for establishing underwriting guidelines for each additional risk class beyond the total general population).

⇩

Account for underwriting risk costs

Model underwriting risk by assuming that a policyholder is wrongly assigned to a higher risk class with a given error probability (can be estimated or based on sound assumptions).

⇩

Solve the optimization problem

Proceed in two steps: First, find the optimal price-demand combination in each possible risk class that maximizes the profit (earnings less costs) in this class. Second, find the optimal classification system (i.e., a combination of subpopulations to risk classes) that maximizes the total profit for the insurer (total profit from a classification system is given as the sum of the profit in each risk class less classification costs). Account for underwriting risk in the process.

4. Model Application and Market Entry

This section discusses additional key issues regarding substandard annuities. For insurers trying to decide whether engaging in risk classification would be a practical and profitable pursuit, or only offering a standard tariff, our model is very useful. For practical implementation, estimating price-demand and cost functions for each risk segment is vital. Other important aspects to be considered in making such a decision include market entry barriers and the general risks and advantages of providing substandard annuities.

To ensure adequate model application and sound results, the following issues need to be addressed. First, innovating insurers need knowledge about the structure, size, and potential of their target market,[50] which includes information about the number and mortality profiles of potential annuitants. Second, estimates must be made of how product price will affect demand in each market segment. Therefore, the maximum number of risks, their reservation price and price elasticity of demand need to be derived empirically by means of, e.g., surveys leading to the respective price-demand function. Third, insurers needs to choose the (likely) most profitable market segment in which to conduct business, a choice made easier by employing our model with calibration as specified above.

[50] See Ainslie (2001, p. 17).

In addition, estimation of classification costs and an assessment of the underwriting quality is crucial, since risk classification and the associated underwriting are considered the most hazardous risk for insurers offering substandard annuity products, as discussed in Section 2. Proper underwriting demands sufficient expertise and, preferably, a sound IT-backed underwriting and classification system,[51] which, in turn, has to be accounted for in terms of classification costs. When it comes to underwriting quality, one difficulty is in estimating current mortality probabilities due to a lack of credible mortality data. In addition, the risk of future mortality improvements made possible by developments in the medical field cannot be ignored.[52] Moreover, insurers need to make sure that risk factors are not controllable by annuitants, and they must try to prevent insurance fraud, which, at least compared to life insurance, may be fairly challenging. For example, in the life insurance sector, fraud will be detected, once and for all, when the payment becomes due—either the insured is dead or not. Enhanced annuity payments, in contrast, must be made as long as the insured lives, giving much more opportunity for fraud.[53] Privacy and regulatory issues with respect to information about risk classification characteristics also need to be kept in mind.[54]

The above issues need to be taken into account by insurers considering the introduction of risk classification. However, as outlined above, calibration of the model is complex and prone to a relatively high degree of uncertainty such that implementation may present an obstacle to innovation. Moreover, there are other barriers and risks having to do with the target market and product design, as described below.

Substandard annuity markets are very competitive.[55] Applicants "shop around" for the best rates by submitting underwriting requests to several insurance companies simultaneously.[56] Insurers face a tradeoff between staying competitive and maintaining actuarially sound criteria for qualifying applicants as substandard risks. A provider's profitability can be negatively affected if the placement ratio, i.e., the ratio of sales to underwriting requests, becomes too low.[57] In addition, there is competition in the form of other financial products,[58] and not much market awareness of substandard annuities.

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[51] See Weinert (2006, p. 15).

[52] See, e.g., Cardinale et al. (2002, p. 16), Cooperstein et al. (2004, p. 13), LIMRA and Ernst & Young (2006, p. 28), and Richards and Jones (2004, p. 20).

[53] See Junus et al. (2004, p. 20).

[54] See Brockett et al. (1999, p. 11).

[55] See LIMRA and Ernst & Young (2004, p. 7).

[56] This practice is mainly observed in the United Kingdom. See, e.g., Cooperstein et al. (2004, p. 13) and Ainslie (2001, p. 19).

[57] See LIMRA and Ernst & Young (2006, p. 20).

[58] See LIMRA and Ernst & Young (2006, p. 22).

Therefore, if an insurer decides to enter the substandard annuity market, it will be in need of a distribution system strong enough to generate sufficient market awareness.[59]

Product design will need to be attractive to sales force and clients, efficient, and innovative.[60] Except for the application itself, which requires the provision of additional health information, sales processes are similar to those of standard products.[61] In line with standard annuities, a substandard annuity provider is required to maintain minimum capital requirements and account for longevity and interest rate risk.[62] In addition, the impact on existing portfolios needs to be investigated.[63]

Daunting as these barriers and risks sound, there are also substantial advantages to selling substandard annuities. For one thing, according to Cooperstein et al. (2004) and Watson Wyatt (2008), the market potential is huge. Turra and Mitchell (2004) also find support for considerable demand for these products. Thus, entering the substandard annuity market is likely to be an attractive alternative for new market players with a solid business plan, giving them the opportunity to reaching a broader population and/or meeting a niche market need.[64] For an established market player with an existing standard annuity portfolio, the situation is not as clear-cut,[65] although—except for the underwriting—offering substandard annuities may require only modest modifications of organization, product design, and distribution system.[66] Yet, there is the danger of destabilizing one's market position by becoming more competitive in the substandard market but, at the same time, less profitable in the standard annuity business, which could result in some reputational damage, too. However, if a standard insurer expects the substandard annuity market to grow, it is advisable to become active in it early on and thus avoid being forced, for defensive reasons, into quickly developing a substandard product later. Early market engagement will allow an insurer to enjoy the benefits of competitive advantage and avoid problems of adverse selection.[67]

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[59] See, e.g., LIMRA and Ernst & Young (2006, p. 22) and Weinert (2006, p. 15).
[60] See Froehling (2007, p. 5), Werth (1995, p. 6), and Weinert (2006, p. 15).
[61] See Cooperstein et al. (2004, p. 13) and LIMRA and Ernst & Young (2006, pp. 8, 37–38).
[62] See, e.g., Froehling (2007, p. 5), LIMRA and Ernst & Young (2006, p. 7), and Weinert (2006, p. 15).
[63] See Werth (1995, p. 6).
[64] See LIMRA and Ernst & Young (2006, p. 6).
[65] The following points are analogously discussed in Werth (1995) for the case of introducing preferred life products in the life insurance market.
[66] See LIMRA and Ernst & Young (2006, p. 21).
[67] See, e.g., O'Neill (1997, p. 1088) and Swiss Re (2007, p. 13).

In this context, adverse selection means that standard annuity providers will be left with a greater proportion of healthier lives in their portfolios if those with a reduced life expectancy tend to buy substandard annuities.[68] This situation leads to a reduction in profit for standard annuity portfolios, which has been quantified by Ainslie (2000) and Hoermann and Russ (2008).

5. Summary

In this paper, we comprehensively examined key aspects of substandard annuities and developed a model for an optimal risk classification system that includes consideration of underwriting risk. We began with a description of different types of substandard annuity products, their respective underwriting, potential market size, and associated underwriting risk, the latter of which is considered crucial for success in the substandard annuity sector. Supported by extant research, we focused on multiclass underwriting implemented by risk classification via rating factors. We proposed a model for a risk classification system in a mortality heterogeneous general population, which is described by a frailty distribution. The optimal number and size of risk classes as well as the profit-maximizing price-demand combination in each risk class were then derived as the solution of an optimization problem. As an extension, we solved for the optimal risk classification system when taking into account costs of underwriting risk. We modeled these costs by assuming error probabilities for wrongly classifying insureds into a higher risk class, thus underestimating the true costs of insurance. We then discussed the practical application of our model, along with market entry barriers and risks and advantages inherent in being a substandard annuity provider. Due to the generality of the model, applications to classification problems other than substandard annuities are possible as well.

In conclusion, extended risk classification in annuity markets not only increases the profitability of insurance companies, it benefits society at large as the introduction of substandard annuities makes it possible for many formerly uninsurable persons to secure for themselves a private pension.

[68] See, e.g., Watson Wyatt (2008).

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