Mortality-Indexed Annuities

Avoiding Unwanted Risk

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Abstract

Longevity risk has become a major challenge for governments, individuals, and annuity providers in most countries, and especially its aggregate form, i.e. the risk of unsystematic changes to general mortality patterns, bears a large potential for accumulative losses for insurers. As obvious risk management tools such as (re)insurance or hedging are less suited to manage an annuity provider's exposure to aggregate longevity risk, the current paper proposes a new type of life annuities where benefits are contingent on actual mortality experience, and details actuarial aspects of implementation. Similar adaptations to conventional product design exist in investment-linked annuities, and a role model for long-term contracts contingent on actual cost experience is found in German private health insurance so that the idea is not novel in general, but it is in the context of longevity risk.

By not or re-transferring the systematic longevity risk insurers may avoid accumulative losses so that the primary focus in an extensive Monte-Carlo simulation is placed on the question of whether such products could also be advantageous for policyholders and to what extent this would be the case in contrast to a comparable conventional annuity product.

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1 Motivation and Introduction

For quite a while scholars and practitioners – not only with a focus on insurance but also with backgrounds such as sociology, demography, health sciences, and other areas – have been discussing a globally observable phenomenon commonly named *demographic transition*.[1] Strong, sustaining changes of mortality, fertility, migration and other factors have tremendously affected age structures of most countries' population, and a rapid increase of average individual lifetime is a predominant consequence.[2] Inseparably linked to a reduction of mortality rates at virtually all ages, it may be considered more than welcome from each individual's point of view, and it can also be recognized as a societal achievement and major success of well-functioning public health and old-age care systems, improved working conditions, and also healthier lifestyles.

The often cited *longevity trend* can be traced back approximately 100–150 years for most Western countries. Looking at female life expectancies in several countries, Oeppen and Vaupel (2002) impressively show that each individual country has experienced a steady growth of lifetime during the past 150 years which is a strong argument supporting this notion. Even more interestingly, when taking together all countries, the existence of a *global* linear trend also seems to be supported.

The risk of outliving one's savings or other financial resources to cover expenses during retirement could also be understood as some sort of *individual* longevity risk, but the same term has been used in the literature also to refer to individual fluctuations around a general average mortality, i.e. the risk of living shorter or longer than the average member of the respective group of individuals. In fact, life and pension insurance are based on a balancing of risks across sufficiently large portfolios, and they make explicit use of the individual fluctuations around an average value without which the entire functioning of insurance would have to be put into question. While some annuity contracts terminate sooner some cease later so that the entire portfolio of contracts is eventually balanced, and with an increasing number of policies results stabilize.[3]

However, the term *aggregate* longevity risk, has been used rather to refer to the additional uncertainty about changes in the underlying patterns, and this particular risk can be considered a major concern for insurers. Because, given a sufficiently prudent selection of the insured portfolio, an insurer can well diversify the *unsystematic*, individual form of this risk and should on average not suffer losses nor generate gains from mortality differences. On the other hand the aggregate form can also be considered a *systematic* risk since it refers to unforeseeable random changes representing strong correlations between single contracts and thus potential accumulative losses.

Of course, while differences between single countries seem to exist, the aggregate form of this risk becomes even more threatening when seen from an international perspective as seems suitable for a global primary insurance group or most reinsurers.

Increasing life expectancy has not been paralleled by a similar prolongation of working life, and individuals tend to survive a substantial, increasing number of years beyond retirement age.

[1] See Montgomery (2007) for a depiction and explanation of different stages of demographic transition.
[2] For a discussion not limited to Western industrialized nations see e.g. United Nations (2005).
[3] See for instance Leinert and Wagner (2001) for an analysis of intergenerational effects in the German private annuities market caused by the longevity trend.

In fact, individual provision for an adequate and sufficient standard of living has become one of the most pressing challenges for 21st century societies. Insurance contracts as the traditionally dominating instruments for transferring financial risks offer the possibility to transfer longevity risk exposure by means of life annuity contracts.[4]

Here, the insurer can be considered to be in a situation similar to assuming the short position in a forward contract on the annuitants' survival, and as is the case with securities as underlyings, the actual remaining number of years to live is uncertain for both the annuitant and the providers. In the view of the longevity trend it thus seems to be quite risky for an insurer to promise nominally fixed annuity payments contingent on the policyholder's survival – yet this might be the most desirable contract design for the (potential) annuitant. Insurers are thus challenged to reconcile exposure to longevity risk and their customers' growing demand for adequately designed insurance products.

Curiosity has driven research for adequate mortality models for quite some time[5] and several attempts have been made, yet none of them has been universal enough to fit actually observed mortality patterns for *all* countries and *all* time periods, and it is quite unlikely that such models will ever exist. For a detailed overview and a categorization of several stochastic mortality models see e.g. Cairns et al. (2006). One of the most prominent stochastic models for mortality was proposed by Lee and Carter (1992, henceforth LC), and it has rapidly gained acceptance in academia and among practitioners, which could partly be due to its relative simplicity.[6] However, in its original form it has a number of shortcomings, such as the homoscedasticity of error terms, which Alho (2000) criticized as somewhat unrealistic, or a lack of simple implementation.

Among a large number of extensions (see, for example, Lee (2000) or Renshaw and Haberman (2003)) is an approach presented by Brouhns et al. (2002) who resort to the notion that the number of deaths can be regarded as a Poisson counting process. This implies that error terms are heteroscedastic, which seems to be more realistic, and their model has also the advantage that parameters can be estimated iteratively, thus greatly facilitating the computational implementation. This alternative approach is thus adopted here and will be denoted by PML (*Poisson Maximum Likelihood*).

It needs to be mentioned here that two major concerns persist when applying the already existing models: *model uncertainty*, i.e. the uncertainty of whether a specific model is at all suitable for describing the underlying data, and *parameter uncertainty*, i.e. the question of whether – given a specific model – the respective parameters are correctly estimated just by using observed data from the past. These potential drawbacks must be accepted as inherent to fitting models to data from the past in order to obtain projections for the future. As they cannot be completely avoided a they need to be kept in mind.

It is a hope that the accuracy and detailedness of models will be further increased, and some restrictions which have existed in the past (such as technical limitations or implementation issues) have become negligible. Ultimately, a wide range of models and sufficient knowledge

[4] An extensive investigation for how people in Germany have made provisions for their old-age income is provided e.g. by GDV (2004).

[5] Note for instance the well-known models proposed by De Moivre (1756), Gompertz (1825), or Makeham (1860).

[6] For example, the Lee-Carter model served as a starting point for mortality forecasting models used by the U.S. Social Security Administration; see for instance Lee and Tuljapurkar (1998) or Lee et al. (2003) and Edwards et al. (2003).

about when to use which of them can be expected.

Among the typical instruments implemented within a structured, iterative risk management process techniques such as hedging or risk transfer, e.g. through insurance, are widely used. However, due to the strong correlation and thus potential accumulative losses it may be questioned whether the aggregate form of longevity risk can be insured without restrictions. Even when pooled with large reinsurance companies, the systematic risk associated with the longevity trend still prevails and cannot be offset by larger portfolio sizes, i.e. the Law of Large Numbers is likely to fail and not to alleviate the threat affected insurers might be facing.

Successful hedging on the other hand requires that an insurer engages in financial transactions that effectively cancel or offset the exposure to the particular risk in question. Though often proposed, *natural hedging*, i.e. a balancing of exposures to mortality (life insurance) and longevity risk (annuities), seems at least questionable due to many potential drawbacks. For instance contract terms, relevant ages, or (anti-)selection effects do all speak rather against such an opportunity, but advances in this field could of course provide insurers with an additional tool to mitigate aggregate longevity risk.

Eventually, it may seem worthwhile to resort to another risk management tool: risk avoidance. More precisely, by avoiding to assume the aggregate or systematic portion of longevity risk in annuity contracts, an insurer may effectively reduce this particular exposure. However, since it is usually an inseparable part of the risk transferred in life annuities the strategy of avoidance effectively means that not the entire risk would be transferred or, equivalently, to some extent ex-post shifted back to the policyholders. Riemer-Hommel and Trauth (2005) claim that such adjustments would be “uncompetitive” but do not further investigate such innovations and instead concentrate of reinsurance and hedging, among others.

The present contribution is designed to address the question of how exactly providers could make adaptations to the still predominant “classic” product designs. For this purpose, an annuity type with benefits linked to actual mortality experience is proposed including an underlying actuarial model for calculation and reserving purposes. By not or re-transferring the systematic longevity risk insurers may avoid accumulative losses so that the primary focus in an extensive Monte-Carlo simulation is placed on the question of whether such products could also be advantageous for policyholders and to what extent this would be the case in contrast to a comparable conventional annuity product.

2 Risk Avoidance through Product Redesign

2.1 Introductory Remarks

As mentioned before the choices of hedging or insurance as common risk management tools, which in many other cases are straightforward, seem to be either unavailable or unsuitable for an affected insurance company to adequately manage this type of exposure. If instead annuity providers seek to reduce or diminish their exposure through avoidance of aggregate mortality risk, it was already mentioned earlier that the most drastic way would of course be to not sell life annuities and similar products at all. Consequently also all the benefits these products can provide would be forgone. Considering that this “solution” would deprive individuals or insurees of the opportunity of transferring any part of their personal longevity risk, it seems interesting from a welfare point of view whether less drastic risk management options exist that still allow for a transfer of at least the non-systematic portion of the risk. One way of tackling this could be to exclude (or reduce) the exposure to systematic longevity risk as already briefly mentioned before. Apart from the immediate fact that insurance companies disposing off this particular risk may have a preference for such a curtailed product, it is less obvious whether or not such adaptation is disadvantageous for policyholders, challenging the asserted "uncompetitiveness" as e.g. claimed by Riemer-Hommel and Trauth (2005).

A specific redesign of conventional life annuities will be proposed and further analyzed below. Note at this point that annuity markets have already seen similar (successful) adaptations to conventional product designs which will be briefly presented in Section 2.3, and this may serve as a justification for turning away from conventional, established products in the present context which aims at reducing an insurer's exposure to systematic longevity risk. Also note that potential role models for the exclusion of a specific, "systematic" part of risk otherwise transferred to insurers exist in other lines of insurance, and one such model further discussed in Section 2.3 exhibits similarities to the problem of undesirable aggregate longevity risk exposure in conventional life annuities.

2.2 Previous Modifications of Conventional Annuities

"Traditional" life annuities in Germany (and many other countries) have long included a valuable, yet conservative guarantee of a minimum interest rate earned on the premiums paid. As actual returns on the insurers' investments often exceeded these minimum yields the excess amounts were consequently largely distributed to policyholders.[7] This mechanism obviously requires little or no action by policyholders, and as long as capital market yields were substantially higher than minimum guarantees, insurers could well use the argument of (historically) quite attractive surplus participation for marketing purposes.

However, more innovative products such as *unit-linked* or *variable annuities* have been designed and successfully introduced in the insurance markets.[8] A distinct feature compared to conventional annuities is the fact that the investment risk is not born by the insurer but by the policyholder who may often choose from only a limited range of investment bundles offered by the insurer. With an increase of private persons' general interest in capital market investments and awareness of its potential for higher returns against a (higher) risk investment-linked annuities have gained a sizeable share in the pension products market.[9]

[7] German statutory regulation requires insurers to redistribute at least 90% of investment returns earned from the policyholders' reserves in excess of the minimum guarantee. Such provisions for Germany, applicable to different "generations" of life insurance and life annuities contratcs are governed by *Verordnung über die Mindestbeitragsrückerstattung in der Lebensversicherung (Mindestzuführungsverordnung – MindZV)* as of April 04, 2008 (see BGBl. I S.690), *Verordnung über die Mindestbeitragsrückerstattung in der Lebensversicherung (ZRQuotenV)* as of July 23, 1996 (see BGBl. I S.1190), and *Verordnung über die Berechnung und Höhe des Rückgewährrichtsatzes, des Normrisikoüberschusses und des Normzinsertrages in der Lebensversicherung (Rückgewährquote-Berechnungsverordnung – RQV)* as of March 28, 1984 (see BGBl. I S.496), respectively. As an industry practice, the actual fraction of redistributed execess returns had often been even higher.

[8] The term *unit-linked annuities* generally refers to deferred annuities where during the accumulation phase the premiums paid are accrued according to the performance of a certain type of investment, chosen or at least influenced by the policyholder, whereas *variable annuities* also link the actual amount of benefits during the payout phase to an investment performance indicator. In a sense, both products do not promise nominally fixed annuity benefits as would be the case with conventional annuities, but in the former case the amount of benefits can ultimately be determined at the inception date of the payout phase. For a discussion of especially the latter annuity type see e.g. Mutschler and Ruß (2001).

[9] Relevant statistical data for e.g. Germany can be found in the series "Statistical Yearbook of German Insurance" which is regularly published by the German Insurance Association (GDV), the most recent being GDV (2008).

Despite the admittedly different circumstances of risk non-transfer, just the transition from guaranteed investment yields to an insured person's option to flexibly choose investments and consequently bear the performance risk does not only give a counterexample to the notion that policyholders would in general reject any changes to traditional products but also that under certain circumstances they might even be interested in taking on a significant share of the risk. Of course, policyholders would only accept to bear this additional amount of risk if they are adequately compensated for, or rather if such an alternative features lower prices or additional benefits in comparison to the "conventional" product.[10]

2.3 An Example for Policies Shifting Insurance Risk Back to the Insured: German Private Health Insurance

The above mentioned idea of linking annuity benefits not to investment performance but to actual mortality experience – be it that of a given portfolio of insured, an entire insurance company, the insurance industry as a whole, or a specific country's population – is not a novel idea in the sense that products accounting for loss experience do exist. However, due to the extreme longterm character of life annuity contracts, which can easily exceed 40 years, it is not a trivial problem to connect the actual realizations of the insured risk to future benefits.

A comparable concept can for instance be found in insurance contracts as they are common in the German private health market.[11] With the exception of civil servants, freelancers, and higher-income earners these products are only supplemental for the vast majority of the population, as those persons are generally covered by the health branch of the German social security system. However, the basic actuarial concept of such contracts as longterm coverage is similar to life annuities, and this is a distinct feature in comparison to many countries where health contracts bear more similarities to property/casualty insurance.[12] In exchange for regular *constant* premium payments health care costs are covered for the entire lifetime of the insured person. This way, excessive costs for increasingly needed medical procedures at older ages are avoided at the cost of premiums that are higher than necessary at younger ages.

Another similarity is the fact that it would be a special challenge to adequately predict the future development of health care costs at the time of contract inception. Generally speaking, the health care cost inflation has historically been higher than e.g. the consumer price index development, and further innovations in medical procedures and techniques also lead to more expensive treatments. This special type of uncertainty can best be compared to the aggregate form of longevity risk, where a general tendency can be expected but where also the specific extent of its future development is hard to determine.

To enable insurers to still provide lifelong coverage, premiums may be increased (and also have to be lowered) in the case and to the extent that actual benefits of a portfolio of policies paid exceed (or under-run) presumed amounts, i.e. if current premiums turn out to be (more than or) insufficient. Of course, as it does not seem advisable to make such adjustments every time the actual expenses in a period even marginally differ from previous projections, legislation in Germany requires expenses to be above or below a certain threshold in order

[10] The rare case of risk lovers longing to forgo guarantees without any compensation from the annuity provider seems quite absurd here, as such individuals might be inclined to refrain from buying insurance anyway.

[11] See Brünjes (1990) for specific technical details regarding Germany and Timmer (1990) for a multi-country comparison.

[12] This is the case e.g. in the US where health insurance is mostly sold on an annual basis without premium guarantees but typically with an option for the insured to annually renew the policy. For further details see e.g. Black and Skipper (2000, p.140ff).

for premium adjustments to be permissible. Effectively, policyholders thus bear the risk of systematic health care cost increases which the insurer may completely pass on, and in this context it has also been named *premium risk*; see e.g. Kifmann (2000, 2002). It is also obvious that the extent of premium increases or decreases highly depends on whether a certain trend of health care cost development is already incorporated in the rate making process and how much it is. On the other end, the scope for such decisions is also limited by the marketability and competitiveness in the health insurance market.

Contrasting the challenge for annuity providers to manage or mitigate their exposure to the systematic portion of longevity risk this concept of shifting back the systematic risk to policyholders could also be transferred to annuity markets, and this notion will be further discussed in the following.

2.4 Implementation Issues and Appraisal

Recurring to the other major alteration of conventional annuity design, i.e. investment-linked products where part of the risk is shifted back to the policyholders, despite some similarities there are also some pronounced differences in comparison to annuities accounting for a portfolio's actual mortality experience that will be briefly pointed out.

While in the case of mortality-linked or health insurance products it is the insurance industry that has a strong incentive to dispose itself of the highly correlated risks of health cost inflation or – analogously – aggregate longevity (or brevity, i.e. shorter than expected lifetime, for that matter) to avoid potential accumulative losses, this is less likely the case with investment-linked products. Rather a general interest of policyholders in participating in higher capital market returns may be the driving force here.

However, investment performance can hardly be influenced by policyholders, especially when thinking of the general average individual and the extremely long terms which for immediate life annuities may easily exceed 30 or 40 years. Thus moral hazard does not seem to play a role in this case. On the other hand, in health insurance there is a great potential for such unobservable policyholder behavior which must be thoroughly accounted for. The observable increase of health care costs may in part be due to ex-post moral hazard arising from the fact that policyholders once in need for (covered) medical procedures consume (motivated by themselves; see Feldstein (1973, p.252)) or receive (through medical providers; see e.g. Feldstein (1970, 1971))) more or more expensive treatments than they would without insurance coverage, i.e. when paying out of their own pocket, which can be considered a quantity effect.[13] But note that in incomplete repair markets there is also a price effect due to a limited price elasticity, and this contributes additionally towards increasing health care costs.

Fortunately, these effects would not be present in the case of (indexed) life annuities. In particular, it is quite unlikely that policyholders have a possibility to intentionally expand their respective individual lifetimes once insurance coverage is provided.

However, another type of asymmetric information is equally inherent to both health insurance and life annuities: In neither case does the insurer know as much about the health status (or the "true" life expectancy) as the individual herself. In health insurance, it is thus common to use extensive questionnaires or even require medical examination reports prior to the

[13] Nell et al. (2009) investigate the effect of insurance on "repair markets" which includes the demand for "medical repairs". Since it is impossible to write complete insurance contracts often rather actual expenses are compensated for, and this has an effect on repair markets; see e.g. Feldstein (1970), Zweifel and Crivelli (1996) or Pavcnik (2002).

contract acceptance, and the intention is to overcome this asymmetry and offer accordingly discriminated premiums that account for the individual health status. In the annuities business, however, such premium discrimination is largely inexistent with few exceptions so far.[14] Instead, premiums are calculated based on almost the "worst case", i.e. assuming that insured persons experience rather long lifetimes. Presuming that individuals have a relatively precise understanding of what their individual life expectancy is (which could well be questioned) those with a poorer than average health status are likely to be driven out of these markets. Consequently, conventional products rather attract persons with higher life expectancies, and eventually the portfolio of insured is entirely composed of individual risks representing the poorest quality from the provider's perspective.[15] The extent to which such anti-selection occurs in the annuity market might also be dependent on the characteristics of the annuity contracts offered. In particular the fact whether immediate or deferred annuities at variable or fixed annuitization conditions are offered or whether they include a lump-sum payout option at some age(s) will have an influence on the degree of adverse selection.

While premium adjustments in the German private health insurance business are triggered by the experience of a suitably confined portfolio of contracts it is up to the insurer to determine this partitioning, and for outside stakeholders in general or policyholders in particular it may be difficult to retrace this decision. Furthermore, the credibility of actual cost experience from the policyholders' point of view is at least limited unless industry cost figures are drawn on. A similar consideration of course applies to the case of actual mortality experience as a basis for indexed annuities. Unlike the question of what health care costs exactly are to be considered, it seems hardly disputable how deaths are to be counted. However, the maximum possible degree of objectivity and traceability can only be reached when the general population mortality as published by a governmental statistics agency is the basis for further considerations, or optionally an official actuarial authority, such as the Government Actuary's Department (GAD) in the UK, which *inter alia* is in charge of publishing life tables. Naturally, the general population's or insureds' mortality is not necessarily identical to a specific annuity provider's experience. The opposite extreme would be to base any sort of adjustments on the experience of an insurance company or even a specific portfolio, thus completely dispelling basis risk at the cost of slightly more limited transparency.[16] No general recommendation on this particular issue can be made here but the trade-off between transparency and basis risk has to be pondered in each case.

The bottom line of the above deliberations is that neither of the two presented cases exactly represent the challenge of retransferring aggregate longevity risk in life annuities to the policyholders through redesign of conventional product features. However, both do, firstly, give a strong example of successful changes to conventional products and, secondly, may serve as a guideline for how actual losses or expenditure experience can be used for adjustments of

[14] *Impaired* or *enhanced* annuities aim at providing higher benefits for individuals with a significantly poorer health status upon sufficient proof by the policyholder. Schumacher (2008) further analyzes these products, which seem to be established only in the UK annuity market but are not yet common elsewhere.

[15] This is just the famous example of Akerlof's "lemons" (Akerlof, 1970) where under asymmetric information without remedies eventually only the worst quality in the market is traded.

[16] In investment-linked annuities as briefly mentioned in Section 2.2, the general idea is to make the reserves related to a specific funds' development. If instead an overall stock index performance or other comprehensive indicator is considered, regardless of how exactly these funds are invested, those contracts would rather be denominated as *equity-indexed annuities*; see for instance Tiong (2000), Lee (2003), or Lin and Tan (2003).

premiums (or benefits, as will be the case here).

For annuities of the above-mentioned type that make benefits contingent on mortality experience the term *mortality-indexed annuity* (MIA) will be used throughout this paper. Surprisingly, such products do not seem to be offered yet by insurers in the most important insurance markets nor visibly discussed in academia.[17]

Acknowledging that it may not be immediately desirable for policyholders not being able to transfer their entire longevity risk exposure, it has to be kept in mind that the aggregate form of longevity risk bears a great threat for insurers. Thus, the "solution" represented by the proposed MIA could be a great step forward for annuity providers to get this highly correlated risk under control. Such a means of risk avoidance (or reduction) might also foster sluggish annuity markets, and eventually it may be better if insurers offer curtailed rather than none.

2.5 Proposed Product Features

As was suggested before, it remains to be decided how specifically an annuity provider can reduce or eliminate aggregate longevity risk exposure from the contracts offered in the insurance market. In the "plain vanilla" case of a single premium immediate annuity (SPIA) considered in this paper there is no recurring premium that could be adjusted like in German health insurance. Consequently the benefits are the remaining parameter, and in fact there is no necessity for benefits to be constant or nominally fixed throughout the contract term nor in any fashion pre-determined.[18]

The innovation proposed here is that at specified intervals the insurance company offering a mortality-indexed annuity will adjust the benefits contingent on survival for remaining periods in a way that reflects the difference of actual mortality experience from anticipated values since the most recent adjustment date – keeping in mind the above mentioned trade-off between basis risk for the insurer versus transparency in terms of the relevant underlying portfolio.[19] Simply speaking, if more than anticipated insureds are still alive, benefits have to be cut in order to guarantee lifelong payments given the higher number of survivors. Alternatively a higher number of deaths will lead to an increase of benefits for the survivors.

More frequent than annual revisions may be meticulous, but are likely to be too time-consuming or complicated due to limited or lagged data availability and thus appear less worthwhile. It is understood that statutory provisions may actually limit the choices for an insurer, but referring again to the role model of German private health insurance, an annual cycle seems both sensible and feasible if the specific portfolio serves as a reference point. As insurers cannot predict future overall mortality tendencies either, it is further assumed that readjustments of benefits should be made on the basis of a best estimate as per actuarial practice and expertise. Again, regulators may determine specific methodologies or models to be applied, and for the subsequent analysis one such provision based on the Lee-Carter model

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[17] A web search on both *www.google.com* and *scholar.google.com* in February 2009 did not yield any results for the keywords `mortality[-| ]indexed annuit[y|ies]`, and for the keywords `indexed annuit[y|ies]` none were related to actual mortality experience.

[18] This is to be understood from a pure actuarial perspective at this point. In reality it is likely that legal and statutory requirements limit the possibility for variable annuity benefits. For instance, to qualify for favorable tax deferral, (non-investment-linked) life annuity contracts in Germany may only offer constant or increasing payments thus excluding decrements; cf. BMF-Schreiben, Decmeber 22, 2005 IVC1-S2252-343/05, Rz 20.

[19] For the sake of simplicity in the following it will be assumed that adjustments in MIA can be based on the respective portfolio's mortality experience without regulatory objection.

that was introduced above will be presented in Section 3.2.

3 Investigating Mortality-Indexed Annuities

3.1 Model Setup

The mortality-indexed annuity with the basic features proposed in Section 2.5 is to be further analyzed here. Especially the question of whether from a policyholder's perspective such an innovation tends to be advantageous or not is not yet answered, while clearly annuity providers should have a special interest in dispelling the aggregate longevity risk inherent to conventional life annuities. By means of a Monte-Carlo simulation different developments of actual, previously unknown mortality are investigated regarding their effect on the mortality-index annuity proposed, i.e. a large number of scenarios is generated and results thus obtained are further analyzed.

Assuming in a first step – as mentioned before – that the experience from the respective insurer's portfolio is drawn upon when making adjustments to the remaining survivors' annuity benefits (or face values) FV_t, a sufficiently large portfolio of identical annuity contracts is considered with an initial size of l_x at time $t = 0$. Insured persons in the considered pool are assumed to be homogeneous in the sense of identical risk exposures and identical monetary amounts stipulated in their contracts.[20] This also includes an identical age x at $t = 0$ as well as the same gender for all insured so that only a single cohort is considered. The contracts are assumed to be sold to policyholders against a single upfront premium in the amount of π_0, which can also be considered the initial per-contract reserve V_0, and this premium is considered exogenously given throughout the entire analysis.

Given a best estimate of future mortality, the annual benefit FV_0 is set at time $t = 0$ for the remaining annuity term of T years using the equivalence formula[21]

$$\pi_0 = V_0 = FV_0 \cdot \ddot{a}^{1}_{x:\overline{T}|} \tag{3.1}$$

$$\text{so that} \quad FV_0 = \frac{V_0}{\ddot{a}^{1}_{x:\overline{T}|}} = \frac{\pi_0}{\ddot{a}^{1}_{x:\overline{T}|}} \tag{3.2}$$

Here, $\ddot{a}^{1}_{x:\overline{T}|}$ is the actuarial present value of a T-year life annuity immediate[22] as determined using the mortality information and projections available at time $t = 0$ and assuming a constant discrete interest rate of i per period (or year). Such a "pure" annuity pays a unit for as long as a person initially aged x is alive, but not beyond his or her $(x + T)$-th birthday, i.e.

$$\ddot{a}^{1}_{x:\overline{T}|} = \sum_{k=0}^{T-1} \{{}_k p_x \cdot v^k\} \qquad \text{where} \qquad v = \frac{1}{1+i} \tag{3.3}$$

[20] For the purpose of profit sharing and attributing investment returns to individual contracts, German insurers are required to split their entire portfolio into smaller groups of similar contracts *(Abrechnungsverbände)* in order to reflect the different characters of e.g. pure life insurance versus life annuities.

[21] The symbols in the formal expressions in this paper follow the International Actuarial Notation standard as introduced in many actuarial text books; see e.g. Bowers et al. (1997).

[22] An *annuity immediate* pays benefits in advance at the beginning of each period whereas an *annuity due* pays in arrears at the end of each period – both contingent on the insured person still being alive at the beginning of the respective year.

For each future point in time t the insurer will determine the respective per-contract reserve V_t for the remaining survivors. Based on this information future annuity benefits FV_t are determined – considered to be valid from time t on for the remainder of the contract term, i.e. time T, unless further deviations from expected mortality developments occur.

Given that l_{x+t-1} insured were alive at the beginning of the previous year $t-1$ the number of survivors at time t is l_{x+t}. The new per-contract reserve V_t for survivors is then determined by considering what could be brought forward from the previous period, i.e. the previous year's reserve less the annuity payment at the beginning of the previous period – accrued at interest, but also accounting for the redistribution of the reserves from those who passed away in the meantime to those still alive. This can also be considered an *inheritance effect* or *survivor bonus*.[23]

More formally, at time t

$$l_{x+t-1} \cdot \frac{V_{t-1} - FV_{t-1}}{v} = l_{x+t} \cdot V_t \tag{3.4}$$

$$\begin{aligned} \text{so that} \qquad V_t &= \frac{l_{x+t-1}}{l_{x+t}} \cdot \frac{V_{t-1} - FV_{t-1}}{v} \\ &= \frac{V_{t-1} - FV_{t-1}}{v \cdot l_{x+t}/l_{x+t-1}} \\ &= \frac{(V_{t-1} - FV_{t-1}) \cdot (1+i)}{l_{x+t}/l_{x+t-1}} \end{aligned} \tag{3.5}$$

Note that all these computations are based entirely on already observable information since at time t it should be known to the annuity provider how many insureds have survived and how many were alive at the beginning of the previous period. At this point projections or forecasts are not yet needed.

Given that the actual per-contract reserve V_t is determined, the new (adjusted) benefit FV_t is set in a similar way as at time $t = 0$; see Equation (3.2):

$$\begin{aligned} V_t &= FV_t \cdot \ddot{a}^{\;1}_{x+t:\overline{T-t|}} \\ \text{so that} \qquad FV_t &= \frac{V_t}{\ddot{a}^{\;1}_{x+t:\overline{T-t|}}} \end{aligned} \tag{3.6}$$

Of course $\ddot{a}^{\;1}_{x+t:\overline{T-t|}}$ is the updated actuarial present value of a $(T-t)$-year annuity immediate for a person now aged $x+t$ made at time t relying on a new best estimate at that time for future mortality.

It is insightful at this point to consider a situation where initial mortality expectations are correct and do not have to be revised, i.e. the longevity trend incorporated at the contract inception exactly corresponds to the actual development of nature. Of course this case is the absence of systematic risk. As mortality is not deterministic either, fluctuations of individuals' mortality around the (correctly forecasted) general average still occur so that in this case unsystematic risk is the only source of uncertainty. However, if we consider a theoretically

[23] Ruß (2000) comments on different mechanisms how to combine death and survivor benefits in investment-linked insurance products recurring to these effects as basic principles of insurance. In fact, the Law of Large Numbers explicitly combines individual differences in mortality to guarantee a stable average value. See also Blake et al. (2003).

infinite number of contracts in a portfolio, the Law of Large Numbers guarantees that *individual* fluctuations balance, and on average the insurer does not experience deviations from the projected trend in the sense that actual relative frequencies of death almost exactly correspond to the probabilities predicted by the model.[24] In this special case, the evolution of reserves as per Equation (3.5) exactly follows the initial calculation made in Equation (3.1), and at every point in time the reserves brought forward from the preceding period V_t exactly match the required funds for future payments in the (then constant) amount of FV_0. Eventually, in year T (or rather at the beginning of that year) no deficits nor excess reserves are due, which effectively means that on an actuarial basis the insurer would not have any risk of losses.

However, as in reality portfolio sizes will always be finite numbers there is some uncertainty with respect to the difference between relative frequencies and projected probabilities, and even in the absence of systematic longevity risk (i.e. if projections are still perfect) the insurer experiences deviations between the actual evolution of the portfolio reserves as per Equation (3.5) and previous expectations so that adjustments of benefits would also be necessary. This deliberation shows that in the proposed product at this stage, both the systematic and the unsystematic longevity risk are shifted back to policyholders, and they are only able to insure the individual longevity risk in the sense that lifelong payments (or rather until age 100 contingent on survival) are guaranteed. However, benefits may be lower than initially expected.

It should also be emphasized here what has been a consistent assumption throughout the above equations: All calculations and subsequent analyses are made on a net actuarial basis, i.e. only the actuarial risk is taken into account. No surcharges or loadings to cover acquisition expenses, no surplus accrual and thus distribution mechanisms, and no statutory or regulatory requirements or restrictions in terms of reserving, investments, or product features are included unless specifically mentioned.

Another major concern that is relevant for the present analysis is model uncertainty, and a vital discussion of quite varying models to predict future mortality has been in the focus of academia and industry for many years now. Yet no satisfactory solution could be identified as the actual randomness of mortality seems to be too complex, and in addition trends and findings thought to be well-fitting have occasionally turned out to be changing over time. However, for the subsequent simulation of mortality a specific choice has to be made. In particular, the PML extension of the Lee-Carter model (cf. Section 1) will be used for the reasons of its widespread acceptance, the apparent robustness, and its relatively easy implementation. It is well understood, though, that there are alternative choices and that certain shortcomings have been discussed. Yet, the drawbacks and restrictions do not seem to outweigh the mentioned advantages.

Naturally, the mentioned restrictions place a caveat on the entire analysis and thus obtained results, as a far more complex reality will ultimately also influence results. This is especially true for e.g. investment risk or potential basis risk that could be due to regulatory provisions regarding admissible choices. However, further insight and a general direction regarding the advantageousness of risk avoidance through offering mortality-indexed annuities is expected.

[24] Again, note that model uncertainty is ignored here, assuming that the general type of stochasticity underlying mortality developments is known.

3.2 Simulation of Mortality

To account for the true insurance character of an MIA where aggregate longevity risk is retransfered to policyholders (or not transfered to the insurer at all) its individual form is explicitly included in the simulation. That way not only does the general average mortality underly random influences but also each individual's lifetime. Put in different words, there is also risk for each insured that his or her individual lifetime deviates from an initial expectation.

What can be considered a "background risk" here is the fluctuation of the overall mortality pattern. More precisely, future mortality is assumed to develop according to the LC/PML model. The vast majority of contributions have fitted the LC/PML model to actual data determining an ARIMA(0,1,0) model as most suitable to describe the $\{\kappa_t\}$ time series. This effectively means that at any point in time t a linear extrapolation from the current level of κ_t is the best estimate (denoted as $\hat{\kappa}_t$),[25] and it will also be met on average under the present assumption.[26]

Of course, this is only true due to the fact that model uncertainty is excluded from the present investigation which, instead, assumes that it is *a priori* known which model underlies the random future mortality paths. Thus, there should not be any objections to basing best estimates for the mortality index $\{\kappa_t\}$ at future times t on the same drift initially determined when calibrating the LC model.

When transferring the proposed product to real annuity markets an additional source of uncertainty may be the question of whether structural changes have occurred or will be occurring to the mortality patterns applicable to the portfolio of contracts under consideration. While especially the latter question could only be answered drawing upon epidemiological, medical, or even philosophical input, the former challenge can be met relatively well with a reestimation (or recalibration) of the LC model to both historical data as well as actual mortality experience between times 0 and t. Thus, forecasts from time t on could be improved compared to just a periodic update. In the present context this is not necessary due to the exclusion of model uncertainty, though, and as it would also drastically increase the computational complexity, a reestimation of model parameters is dismissed here.

However, the disturbance terms from the ARIMA model cause uncertainty about the actual realizations $\tilde{\kappa}_t$.[27] This can be interpreted in a way that at each point in time t nature draws from the spectrum of possible mortality developments which in expected terms meets the previous best estimates $\hat{\kappa}_t$. More formally, it is assumed that at any future time t the uncertainty can be expressed as $\tilde{\kappa}_t = \hat{\kappa}_t + \tilde{\eta}_t$ with $\eta_t \sim N(0, \sigma_\kappa^2)$. Thus, the larger the time horizon the larger the (accumulative) uncertainty about realizations $\tilde{\kappa}_t$ so that also uncertainty accumulates the more distant future central death rates are.

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[25] Note that there is no distinction between a best estimate $\hat{\kappa}_{t+1}$ made at time t for the year after next and $\hat{\kappa}_{t+1}$ as a best estimate made at time $t+1$ for the period just begun. However, from the context it should always be clear at what time the respective estimate has been made so that a further distinction in the notation appears dispensable.

[26] For the remainder of the present contribution a tilde above a random variable symbol $\tilde{\ }$ shall denote actual realizations, whereas a hat symbol $\hat{\ }$ is to indicate projections or forecasts of that random variable.

[27] Hanewald et al. (2009) point out that it may be sufficient to concentrate on uncertainty arising from the ARIMA process adapted to the $\{\kappa_t\}$ time series while the majority of previous contributions have ignored the error term $\varepsilon_{x,t}$ in the original Lee-Carter model equation. They also note that Lee and Carter (1992, Table B2) themselves noted that the former disturbance terms account for roughly 90% of standard errors of age-specific death rate projections.

The individual form of uncertainty about a person's lifetime is also incorporated into the proposed product, and this is further commented below. Given an actually realized level of general mortality as outlined before the result of surviving for another year (from time t to $t+1$) or passing away is the result of a Bernoulli "experiment". The "population size" in this experiment is identical to the number of survivors alive at time t, l_{x+t}, and its "success" probability is $\hat{p}_{x+t}$ as determined by the overall (randomly sampled) mortality implied by $\tilde{\kappa}_t$.

Put in different words, seen from time t the actual number of survivors $\tilde{l}_{x+t+1}$ at time $t+1$ is a random sample from the binomial distribution $\text{Bin}(l_{x+t}, \hat{p}_{x+t})$. However, $\hat{p}_{x+t}$ in turn is only a best estimate for the yet unknown realization of p_{x+t} which itself is subject to the (immediate) draw of nature. The uncertainty about the actual number of survivors is even stronger for more distant points in time $t+k$ where also $l_{x+t+k-1}$ is unknown and has to be replaced by a best estimate $\hat{l}_{x+t+k-1}$.

Note also that the uncertainty about realizations of $\{\kappa_t\}$ accumulates over time in the sense that more future values of κ_t are less predictable. This is due to the fact that at each intermediate point in time $i = 0, \ldots, t$ nature "draws" realizations of the disturbance terms η_i that all accumulate up to time t. Effectively, the distribution of κ_t is thus more dispersed for longer time horizons t. As the adjustment mechanism proposed for the mortality-indexed annuities leads to both higher *and* lower benefits FV_t at future times t since deviations from the expected number of survivors in either direction can occur, the increasing dispersion of the time index $\{\kappa_t\}$ directly translates also into an increasing dispersion of future benefits paths $\{FV_t\}$.

As pointed out above the present model assumes a random nature of mortality that is twofold: neither the underlying general pattern is certain nor whether and to what extent individual lifetimes deviate from the general levels. Obviously, by this choice both aggregate and individual longevity are incorporated in the model. However, with the proposed possibility for adjustments to $\{FV_t\}$ at any time t based on the equivalence principle as per Equation (3.6) and resorting to the best estimates at that time, the insurer offsets the risk that actual mortality deviates from the projected (overall) trend. In this sense, the proposed mortality-indexed annuity is always balanced from the insurer's perspective.

3.3 The Benchmark: Conventional Life Annuities

While annuity providers – as pointed out earlier – might generally be interested in the idea of not having to bear the aggregate longevity risk in mortality-indexed annuities (MIA), it still remains to be discussed how an assessment of the degree of advantageousness of these products can be done. More precisely, the deduction of a suitable measure has not yet been clarified.

Acknowledging that MIA do not yet exist and that the starting point for considering such innovation will necessarily be a conventional life annuity, it seems worthwhile to parallel the key financial figures from MIA with those from conventional products when exposed to the same mortality experience. In brief, a conventional annuity does not provide the possibility for the provider to adjust benefits in any fashion since those will be set in fixed, nominal terms at the contract inception following Equation (3.2). Thus, dependent on how actual mortality within the portfolio under investigation realizes during the contract term the reserve at the beginning of the T-th period (i.e. at time $T-1$), denoted by $V_T^{*,\text{conv}}$ may turn out to be in- or (over)sufficient and will thus result in either positive or negative values.[28]

[28] If *during* the contract term the reserve V_t becomes negative, it is assumed here that such deficit can be financed at the (unique) interest rate i that was applied for discounting, e.g. in Equation (3.3). See also

Conventional Annuities under Aggregate Longevity Risk

The exact distribution of final reserves $V_{T-1}^{*,\text{conv}}$ in conventional annuities is not yet known since it depends on the realized number of survivors in the portfolio of initially l_x contracts, and those numbers fluctuate around the initial best estimates similar to the time series $\{\kappa_t\}$ as discussed above. However, annuity benefits FV_0 are determined as per Equation (3.2) in a way that if actual mortality exactly follows the initial projection no surplus nor deficit in final reserves is possible, i.e. the annuity benefits thus determined can be considered fair.[29] The fact that fluctuations around the best estimates are symmetric implies that losses (or profits) in per-policy reserves at time T can be expected with roughly a 50% chance, and even though it has to be kept in mind that this is seen from a (net) actuarial perspective it is obvious that this observation also represents a major disadvantage for the annuity provider. Given the fairness of the benefits it may be considered "fair" for the insurer to be exposed to the risk of actuarial losses with a high probability, yet it is also quite likely that the insurer would not want to accept such a high level of risk of insufficiently funded contracts and thus might aim at reducing the exposure to a "safe" level. It is self-evident that complete safety in the sense that no actuarial deficits in reserves are possible cannot be attained. However, it is reasonable that the annuity provider in charge of offering the benchmarking conventional life annuities would want to reduce the shortfall (or deficit) risk from roughly 50% substantially, and this can most easily be achieved by incorporating a safety loading into the annuity calculation. More precisely, annual benefits are reduced by a certain amount which in turn accumulates to soothe the high shortfall probabilities to acceptable levels. To facilitate a true comparison of MIA with conventional annuities, a specific exception from a net actuarial calculation must therefore be made to reflect the insurer's perspective as otherwise MIA would be compared to a product which is likely to never exist. Of course, regulatory requirements may also limit the range of actual choices in annuity markets, and this must be kept in mind.

In terms of the distribution of $V_T^{*,\text{conv}}$, a simple way to achieve a reduction of the shortfall risk from roughly 50% to acceptable lower levels consists in a right-shift of the entire distribution, and this can most easily be done by building up contingency funds Δ_T that become available at time T to increase the per-policy reserve that otherwise leads to deficits with almost 50% probability. If the admissible (actuarial) shortfall probability for the annuity provider is denoted by α then the α-quantile is defined as $z_{\alpha,T} = \sup_z \mathbb{P}(V_T^{*,\text{conv}} < z_{\alpha,T}) \geq \alpha$. The effect of subtracting this number $z_{\alpha,T}$ (which for most choices of α will be a negative number) from $V_T^{*,\text{conv}}$ is illustrated by Figure 1. The same effect as subtracting the α-quantile from the per-policy reserve can be obtained when adding the negative of that amount, i.e. use $\Delta_{\alpha,T} = -z_{\alpha,T}$ to increase the reserves.

To build up the additional amount of $\Delta_{\alpha,T}$ required for the distributional right-shift indicated above by the beginning of year T (or at time $T-1$), the insurer could charge a certain premium surcharge or safety loading. As in the present case a single upfront premium is considered, this can also be achieved by making an otherwise identical product more expensive, i.e. adjust the benefits and thus reduce the initially determined face value FV_0 to FV_0^α so that the additional contingency funds $\Delta_{\alpha,T}$ is financed by the policyholder.[30]

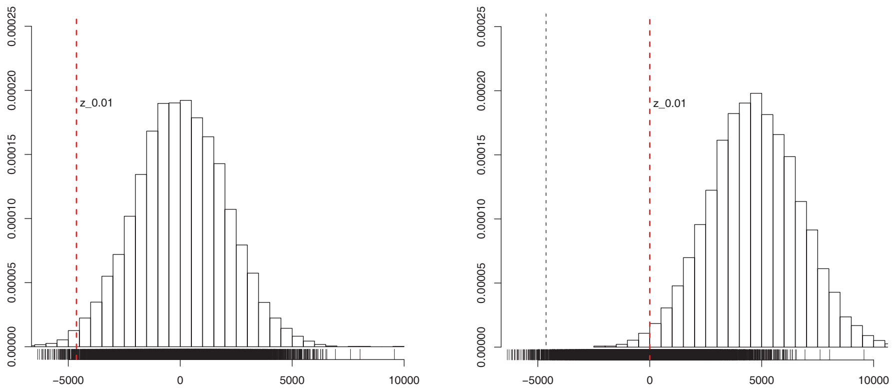
further remarks on this thought below.

[29] Given that the upfront premium π_0 is considered fixed, the usual notion of an *actuarially fair premium* cannot be applied.

[30] As implied by the notation this is of course subject to the admissible likelihood of actuarial losses α chosen to determine $\Delta_{\alpha,T}$.

Figure 1: Right-Shift of Illustrative Density Function by Subtracting Quantile

(a) original (b) after right-shift



Source: authors' calculation.
The plots show the empricial density function for 10,000 random samples from a hypothetical $\mathcal{N}(0, \sigma^2 = 4{,}000{,}000)$ distribution (a) without further changes and (b) after subtracting the 0.01-quantile $z_{0.01}$.

The adjusted benefit $FV_0^\alpha < FV_0$ (not adjusted to actual mortality experience like FV_t in MIA) can be considered basically the original benefit FV_0 determined by Equation (3.2), but from each payment a certain fixed amount is deducted. This deduction in turn can be considered a recurring premium the insurer charges or retains to finance the ultimate contingency funds. Or, put differently, the insurer only sells conventional life annuities if at the same time an insurance savings scheme towards an ultimate financial cushion is embedded in the contract.

The above considerations can be expressed more formally as

$$\begin{aligned}
\pi_0 &= \sum_{k=0}^{T-1} \{FV_0 \cdot {}_k\hat{p}_x \cdot v^k\} \\
&= FV_0 \cdot \ddot{a}^1_{x:\overline{T}|} \qquad \text{see Eq. (3.1)} \\
&= \sum_{k=0}^{T-1} \{FV_0^\alpha \cdot {}_k\hat{p}_x \cdot v^k\} + \Delta_{\alpha,T} \cdot v^{T-1}{}_{T-1}\hat{p}_x \\
&= FV_0^\alpha \cdot \ddot{a}^1_{x:\overline{T}|} + \Delta_{\alpha,T} \cdot {}_{T-1}E_x \qquad (3.7)
\end{aligned}$$

In the last expression of (3.7), ${}_{T-1}E_x$ is a $T-1$-year pure (unit) endowment insurance for a person aged x.[31] By comparing Equations (3.1) and (3.7) the adjusted face value FV_0^α can be determined from the non-adjusted value FV_0 through

$$FV_0 \cdot \ddot{a}^1_{x:\overline{T}|} = FV_0^\alpha \cdot \ddot{a}^1_{x:\overline{T}|} + \Delta_{\alpha,T} \cdot {}_{T-1}E_x \qquad (3.8)$$

$$\begin{aligned}
\text{so that} \qquad FV_0^\alpha &= \frac{FV_0 \cdot \ddot{a}^1_{x:\overline{T}|} - \Delta_{\alpha,T} \cdot {}_{T-1}E_x}{\ddot{a}^1_{x:\overline{T}|}} \\
&= FV_0 - \frac{\Delta_{\alpha,T} \cdot {}_{T-1}E_x}{\ddot{a}^1_{x:\overline{T}|}} \\
&= FV_0 - \Delta_{\alpha,T} \cdot \ddot{s}^1_{x:\overline{T}|} \qquad (3.9)
\end{aligned}$$

[31] Note that the last annuity payments are due at time $T-1$ so that it is sufficient to analyze the provider's financial situation at that time.

where $\ddot{s}^{1}_{x:\overline{T}|}$ is the actuarial *accumulated* value at time T contingent on survival that corresponds to the same unit annuity underlying the *actuarial present value* $\ddot{a}^{1}_{x:\overline{T}|}$.

At this point the question may arise of whether one should also consider the per-policy reserves V_t at any intermediate time t since obviously an insurer offering conventional annuities would neither be willing to accept actuarial net losses at those points in time. For an answer, two important features of the comparable conventional annuities underlying the present investigation have to be kept in mind.

Firstly, if for any actual mortality development the reserve is negative at some intermediate point in time, it was assumed that the annuity provider may finance this shortfall at the same fixed interest rate i that was assumed as the minimum guaranteed interest rate; cf. also Footnote 28. Although external funds raised on capital markets will mostly not be available at such a low rate, this notion appears more realistic when considering the possibility that such shortfall may be financed by borrowing against some other line of business or portfolio of contracts within the insurance company. Also note that due to the inheritance effect[32] negative reserves may become positive thus settling the deficit – although this is not necessarily the case. As a last remark on this note that reserve values analyzed are entirely based on a net actuarial assessment, i.e. except for the safety loading discussed above they do not include any further loadings or surcharges, and the excess investment yields are also not accounted for at this point so that in reality it is likely that actual per-policy reserves are higher in tendency and deficits less likely and less severe.

Secondly, and more importantly, it may be hard (if not impossible) to answer the question whether the safety loading as determined per Equations (3.7) and (3.9) is also sufficient to avoid shortfalls at intermediate points in time $0 < t < T - 1$ or whether the respective shortfall probabilities $\mathbb{P}\left(V_t^{\alpha} < 0\right)$ can thus be reduced to levels well below α. The reason is that, on the one hand, per-policy reserves are reduced over time by the annuity payments due to surviving policyholders whereas on the other hand they are accrued at interest *and* the additional "survivor bonus".

Thus, another way of asserting the adequacy of the safety loading as per the above equations is possible by analyzing the evolution of the α-quantiles of the per-policy reserves $V_t^{*,\mathrm{conv}}$, i.e. $\{z_{\alpha,t}\}_{t=0,\dots,T}$. If those are decreasing in t, i.e. if the lowest value is given by $z_{\alpha,T}$ or, likewise, the highest contingency funds value is represented by $\Delta_{\alpha,T}$, then it is sufficient to use the safety loading as determined above. Simply speaking, if the worst cases (in the relevant are of the quantiles under investigation) are to be expected at the end of the period, it is sufficient to buffer the potential downside risk at this point as thus intermediate shortfalls are also accounted for. For the analysis provided in Chapter 4 the required monotonicity of $\{z_{\alpha,t}\}_{t=0,\dots,T}$ is fulfilled in all treatments (cf. Table 2), justifying to rely on the safety loading established above.

Another issue that has not been further discussed so far is the fact that *on average* there is no actuarial profit or loss with the unadjusted FV_0. By including the safety loading, the additional contingency funds $\Delta_{\alpha,T}$ per surviving policyholder accumulated over the contract period further improves the financial situation with a small effective net deficit risk of only α. The resulting considerable *expected* profits for the conventional annuities that serve as a benchmark for the proposed MIA arise entirely from mortality fluctuations, as it is assumed here that the insurer does not generate higher investment yields than the constant interest rate

[32] See the remarks in Footnote 23 on the "extra" survivor bonus in life annuities.

i, at which the per-policy reserves are carried forward following the equivalence principle; cf. Equation 3.5.[33] For an adaptation of the proposed MIA and the benchmark products into real insurance markets it remains to be further discussed whether and how such expected profits would have to be accounted for and how this might change results presented below. It is also immediately obvious that actual insurers' possibilities and admissible procedures for profit sharing mechanisms that are common to a particular market highly depend on regulatory requirements.[34] In tendency, profit-sharing of "mortality gains" should improve the position of conventional products in comparison to mortality-indexed annuities as those adjust benefits in a way that completely incorporates mortality gains and leaves no surplus due to mortality fluctuations to be redistributed.[35] For the present investigation, however, the details of profit-sharing will not be further investigated.

Comparing Mortality-Indexed Annuities and Conventional Annuities

It is more than obvious that a MIA would be (dis-)advantageous compared to a conventional life annuity exactly when it provides higher (lower) benefits at any given future time. However, due to the fluctuations of mortality (and consequently also of adjusted benefits FV_t) it is not clear how often and to what extent the benefits are higher than FV_0^α in a *comparable* conventional annuity – provided that the latter includes contingency funds as discussed above. Note that with certainty $FV_0 > FV_0^\alpha$ but that no definite statements are possible as to FV_t for future points in time. The standard assumption of risk-averse policyholders is ignored in this context. In a further analysis the fact that risk-averse individuals being reluctant to not transfer the aggregate longevity risk could also be incorporated when determining to what extent MIA are advantageous compared to conventional products. In tendency, the excess of MIA benefits FV_t over FV_0^α is diminished by the increase in risk to be born by the policyholder. Overall effects are thus likely to be highly dependent on the degree of individual risk aversion.

What also needs to be kept in mind is the fact that for most policyholders sooner benefits in excess of "conventional" benefits should have a stronger weight than those in the more distant future. Discounting should reflect such time preferences and also it is less likely for each insured to actually live to see such late excess benefits, requiring "mortality discounting" as incorporated in the concept of actuarial present values; cf. Equation (3.3).

Following the overall principles governing actuarial calculation in life and pension insurance, it thus seems worthwhile to further analyze the actuarial present value of the difference in benefits of MIA compared to conventional annuities as seen from time $t = 0$. One should expect that if the fluctuations of adjusted benefits $\{FV_t\}$ do not turn out to be overly downward drifting below the level of FV_0^α and/or if this only occurs in the more distant future, then the initially higher level of FV_0 in comparison to FV_0^α combined with a higher likelihood for

[33] Only recently German legislation has set off the surplus generated by "mortality (or risk) gains" and "cost gains". In contrast to "investment gains" on policy reserves, of which at least 90% have long been required to be credited to policyholders, of the two last-mentioned items now at least 75% and 50%, respectively, are require to be attributed to individual policies. Cf. also the remarks on regulatory provisions regarding profit sharing in Germany in Footnote 7 on p.4.

[34] While from a pure shareholder value perspective it would be straightforward to retain all surplus generated from deviating mortality, investment yield, or cost development, actual limitations placed on the insurance industry by regulatory specifications are generally intended to protect policyholders' interests. An abundant selection of previous contributions have discussed or analyzed profit-sharing, and for the specific case of Germany cf. e.g. Schwintowski and Ebers (2002), Ebers (2001), or Gruschinske (1989).

[35] This is achieved by the term l_{x+t}/l_{x+t-1} in Equation (3.5).

each insured to actually receive those benefits at all *and* sooner are likely to prevail. As a direct consequence the mentioned "actuarial present value of MIA excess benefits" will become positive, thus being in line with the notion of a measure of advantageousness. Technically, for each scenario in the Monte-Carlo simulation the amount

$$\begin{aligned} ADV_{\mathrm{MIA}}^{\alpha} &= \sum_{k=0}^{T-1} \{(FV_k - FV_0^{\alpha}) \cdot {}_k\tilde{p}_x \cdot v^k\} \\ &= \sum_{k=0}^{T-1} \{(FV_k - FV_0^{\alpha}) \cdot \tilde{l}_{x+k}/\tilde{l}_x \cdot v^k\} \end{aligned} \tag{3.10}$$

is computed where actual survival rates ${}_k\tilde{p}_x = \tilde{l}_{x+k}/\tilde{l}_x$ are used. The thus obtained empirical distribution of $ADV_{\mathrm{MIA}}^{\alpha}$ will be further analyzed next.

4 Simulation and Results

4.1 Data and Parameterization

To perform the simulation of key financial figures (and especially the measure for advantageousness $ADV_{\mathrm{MIA}}^{\alpha}$ developed in Section 3.3) for the mortality-indexed annuities proposed in Section 2.5 several particular choices in terms of data and parameterizations have to be made. Those will be detailed and motivated in the present section.

Firstly, since the simulation intends to investigate variations in the mortality (or longevity) within a stylized homogeneous portfolio of insured and the respective implications for the adjustments of MIA benefits, it is self-evident that the selection effect of (generally lower) annuitant mortality versus the general population mortality should be accounted for. Thus due to the limited availability of such annuitant mortality data the present simulation resorts to the UK data set provided by the CMI.[36] This ultimately led to the LC time index $\{\kappa_t\}$ being modeled as an ARIMA(0,1,0) time series, thus being in line with previous research.

Secondly, since there are more male annuitants than females in the mentioned data set it will be assumed that the homogeneous cohort under investigation is composed of males exclusively. A limited number of years covered by the data also forces to base the calibration (or parameter estimation) of the LC/PML model on calendar years $z = 1983, \ldots, 2003$, and consequently all projections begin in year $z = 2004$ which corresponds to time $t = 0$. Although for calibration purposes only ages $x = 25, \ldots, 110$ could be considered, the cohort's initial age is assumed to be $x = 60$ at time $t = 0$, and the annuity term here is limited to $T = 41$ at the outset. Thus, the last annuity payment is due at the beginning of the year $z = 2044$ (or at time $t = 40$, equivalently) where all surviving cohort members have just completed their 100^{th} birthday with no further payments thereafter. However, ultimate financial data such as per-policy reserves can also be considered in the "second" after the last annuity payments at time $T - 1$ since no further cash flows are to occur for the remainder of the T-th year. This is simply owed to the fact that an annuity immediate was assumed here, which makes payments in advance.

[36] The Continuous Mortality Investigation (CMI) is an institution sponsored by the Faculty of Institute of Actuaries in the UK in charge of fostering research related to mortality modeling issues. Data provided encompasses annuitant mortality data per capita from the British insurance industry.

Lastly, the upfront premium considered to be exogenously given[37] is set as $\pi_0 = 100{,}000$, and the number of generated scenarios in the Monte-Carlo simulation is set to $N = 10{,}000$ throughout the subsequent analyses.[38]

The above outlined parameterization choices are summarized in Table 1.

Table 1: Parameterization for the Simulation of MIA Advantageousness

number of scenarios	initial age	annuity term	upfront single premium
N	x	T	π_0
10,000	60	41	100,000

Two parameterization choices have not yet been further discussed. For the size of the fictitious portfolio, i.e. the initial number of insured, two different choices are considered: On the one hand, to reflect comfortable diversification effects a relatively large (given the assumed homogeneity) portfolio is assumed by setting $l_{60} = 100{,}000$. On the other hand, individual fluctuations of lifetime are modeled as Bernoulli experiments with the actual number of survivors being random samples from suitably parameterized distribution. Given the Law of Large Numbers, which states that sample means will converge to the theoretical "true" mean of the underlying distribution upon increasing sample sizes, it is likely that thus randomly sampled number of survivors will be relatively close to the respective overall survival probabilities "drawn by nature". To challenge individual fluctuations to a larger extent and also accounting for the fact that in reality (primary) insurers may only be able to compose smaller homogeneous annuity portfolios, an alternative portfolio size of $l_{60} = 1{,}000$ is also considered.

For the constant interest rate i incorporated into the calculation of actuarial present values $\ddot{a}^1_{x:\overline{T}|}$ and discounting purposes also two different assumptions were made. Note that the present analysis is made from a net actuarial standpoint where actual (excess) yields on the invested reserves are disregarded. As discussed above, an adequate mechanism for redistribution of surplus must be thoroughly considered. However, a quite conservative yet commonly found value of $i = 3\%$ *p.a.* is assumed as well as the more ambitious choice of $i = 5\%$ *p.a.* In the long run, with the use of long-term governmental bonds insurers in many countries have been able to generate returns on their investments well above the former choice with little difficulties to attain the latter level.[39]

Table 2 provides an overview of the portfolio size and interest rate combinations including the abbreviations for the resulting "treatments" referred to below.

For the admissible shortfall risk α values of 0.01, 0.005, and 0.001 are used to reflect reasonable choices not uncommon in the industry. If necessary for distinction, the specific values

[37] To illustrate the notion of a fixed upfront premium in the amount of π_0 it may be helpful to consider a pre-existing individual savings plan, a term life insurance ending exactly at the annuity inception date, or an inheritance made. In either case proceeds from these financial instruments become available at time $t = 0$ exactly in the amount of π_0. An additional assumption could be that the annuity provider cannot ask the policyholder to pay an increased upfront premium in order to allow the establishment of contingency funds, but accordingly needs to make adjustments to the initial annuity face value, i.e. provide only FV_0^α instead of FV_0.

[38] A precise notation would require to add a symbol j to all formula symbols introduced in this contribution to denote that they differ by scenario. For the sake of clarity, such notation has been ignored so far and will only be used below if explicitly necessary.

[39] See also Kling et al. (2007) for a thorough analysis of the effects on risk exposure that different regimes of surplus distribution as related to investment yields being in excess of interest rate guarantees.

Table 2: Treatments for the Simulation of MIA Advantageousness

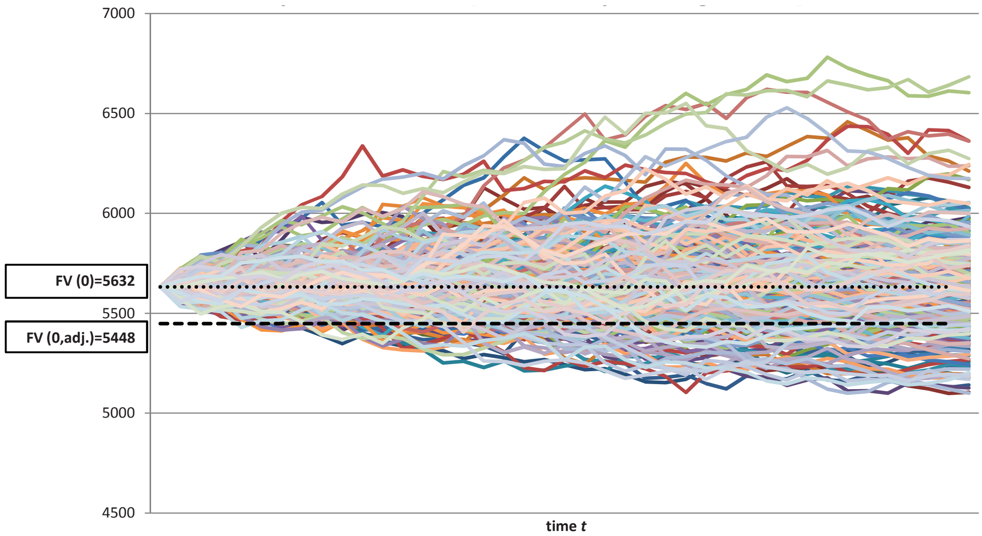
		i	
		3%	5%
l_{60}	100,000	A_I (base case)	A_{II}
	1,000	B_I	B_{II}

will be added as a left superscript, e.g. as $ADV_{\text{MIA}}^{0.001}$.

4.2 Results and Discussion

When applying the parameterization and underlying data outlined in Section 4.1 to the Monte-Carlo simulation model introduced in Section 3.1, the data being computed is extensive: Four treatments with $N = 10{,}000$ scenarios each were run, and in each run j a series of MIA face values $\{FV_t^j\}_{t=0\ldots40}$ was generated,[40] contingent on three different levels of α – among other interim data necessary to actually compute the MIA developments.

Figure 2: Paths of MIA Benefits in Comparison to Conventional Annuity – Treatment A_I, $\alpha = 0.005$



Source: authors' calculation, based on data provided by the CMI.
The dotted line indicates the face value $FV_0 = 5{,}632$ prior to contingency funds inclusion; the dashed line shows $FV_0^{0.005} = 5{,}448$ ("FV(0,adj.)") for a *comparable* conventional annuity.

For instance, for the *base case* treatment using $\alpha = 0.005$ Figure 2 depicts the spectrum of MIA benefit paths as well as those obtained from a *comparable* "conventional" life annuity – both with and without contingency funds.[41] Obviously, it is next to impossible to directly

[40] Note that no payments are due after time $T-1$ so that no value of FV_{41} needs to be determined.
[41] It is obvious and also evident from Figure 2 that for the latter case, benefits from the conventional product coincide with the initial MIA benefit that is determined for the very first period (and later revised), as at time $t = 0$ the respective best estimates coincide as well.

assess this amount of data. While at first glance it may seem that paths tend to fall below the "conventional" benefit level (dashed, lower horizontal line) quickly,[42] it has also to be kept in mind what was mentioned above in the context of deducing the measure for advantageousness: benefits below that threshold in the not-so-near future can be assumed to be less grave due to discounting and mortality in the meantime. In addition, note that an annuity pays (accumulative) benefits at *all* times contingent on survival, i.e. rather the sum of excess or deficit benefits (or the area between each MIA path and the constant of FV_0^{α}) is a relevant quantity.

As the interest and "survival" discounting cannot easily be captured when using visualizations similar to Figure 2, those provide less insight into MIA advantageousness. Instead, MIA face values are boiled down to realizations of $ADV_{\text{MIA}}^{\alpha}$ as an aggregate measure including the twofold discount of interest and mortality, which effectively leaves 120,000 values for further analysis. The remaining complexity of the results can suitably be depicted by using histograms of the empirical distribution of $ADV_{\text{MIA}}^{\alpha}$, such as Figure 3 for different levels of α in the *base case* treatment A_I or Figures 4, 5, and 6 for analogous representations for the other treatments A_{II}, B_I, and B_{II}, respectively. Further analysis of the distributions of $ADV_{\text{MIA}}^{\alpha}$ may work towards an answer to whether the apparent attractiveness of MIA for insurers also applies to potential policyholders.[43]

Treatment A_I – the base case

For the base case treatment A_I, the innovative mortality-indexed annuity turns out to be considerably more advantageous than a conventional annuity in the majority of cases; see Figure 3. In fact, the MIA provides an average level of $ADV_{\text{MIA}}^{\alpha}$ of between roughly 3,100 and 3,700 – depending on α. Given that FV_0 is 5,632, these values correspond to 55% to 65% of an annual benefit payment. For treatment A_I, these values (among a selection of risk measures) are listed in Table 3 separated by the insurer's shortfall risk α. What can also be observed is the fact that the downside or shortfall risk $\mathbb{P}\left[ADV_{\text{MIA}}^{\alpha} < 0\right]$ is in the area of only $2 - 5\%$ so that it is rather unlikely that the innovative product is truly disadvantageous as measured by $ADV_{\text{MIA}}^{\alpha}$. Obviously, the smaller α, i.e. the smaller the insurer's admissible actuarial shortfall risk when including contingency funds, the more advantageous are potential MIA benefits. This is simply due to the fact that a higher α requires FV_0^{α} to be lower, *ceteris paribus*, in order for $\Delta_{\alpha,T}$ to be available by time T.

Another more general measure for the riskiness would be the variability of the (degree of) advantageousness. Figure 3 provides little evidence of whether the increase in downside risk $\mathbb{P}\left[ADV_{\text{MIA}}^{\alpha} < 0\right]$ (when gradually increasing α from 0.001 to 0.005 and 0.010; cf. Figures 3(a), (b) and (c)) is only due to a shift of the respective density functions, or whether also a higher dispersion has its share. The empirical variances are therefore also listed in Table 3, and apart from the fact that the actual differences in relative terms are quite small compared to their high levels, a slight increase of the variability can still be observed. However, the much stronger reduction of empirical means suggests that the former explanation is the dominant cause.

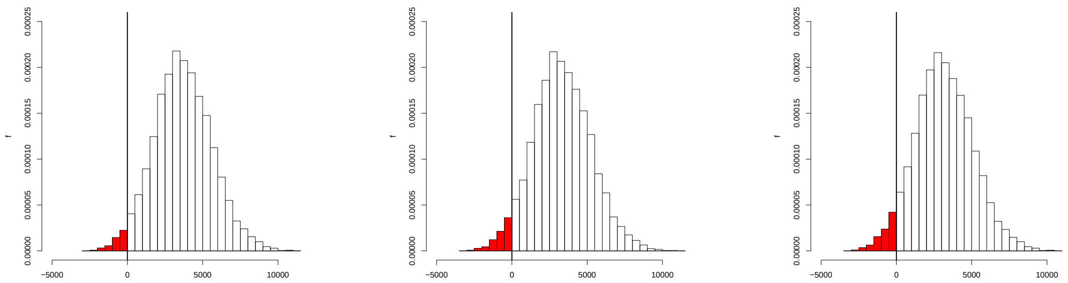
Although it turns out that advantageous scenarios are very likely with the policyholders' downside risk being rather limited, it is interesting to further analyze also *how* negative potential adverse scenarios may turn out to be. For this purpose, also the expected

[42] In fact, roughly half of values FV_{40} are lower than the initial value FV_0 and roughly one third even below the (comparable) level of $FV_{\text{MIA}}^{0.005}$.

[43] Recall that $ADV_{\text{MIA}}^{\alpha}$ was developed as a measure for the advantageousness of MIA in comparison to conventional life annuities if only the insurer needs to include a safety loading or build up contingency funds to reduce the actuarial shortfall risk to α.

Figure 3: Empirical Distribution of $ADV^{\alpha}_{\text{MIA}}$ – Treatment A_I

(a) $\alpha = 0.001$ $FV_0^{\alpha} = 5{,}426.77$ (b) $\alpha = 0.005$ $FV_0^{\alpha} = 5{,}447.78$ (c) $\alpha = 0.010$ $FV_0^{\alpha} = 5{,}456.95$



Source: authors' calculation, based on data provided by the CMI.

[policyholder] shortfall (or rather "expected MIA *dis*advantageousness") is computed, i.e. $\mathbb{E}\left[ADV^{\alpha}_{\text{MIA}} \mid ADV^{\alpha}_{\text{MIA}} < 0\right]$. Those values provide an idea of how severe waiving a transfer of aggregate longevity risk to an annuity provider may be, and the respective quantities are also provided in Table 3. It is not surprising then that with a higher α (and thus also higher values of FV_0^{α}) the general level of $ADV^{\alpha}_{\text{MIA}}$ is lower, and consequently also expected shortfalls take on more negative values.

Table 3: Key Figures from Treatment A_I

Annuity Benefits	$\alpha = 0.001$	$\alpha = 0.005$	$\alpha = 0.010$
FV_0	5,631.67		
FV_0^{α}	5,426.77	5,447.78	5,456.95
Risk Measure			
$\mathbb{E}\left[ADV^{\alpha}_{\text{MIA}}\right]$	3,700.17	3,327.33	3,164.66
$\mathbb{P}\left[ADV^{\alpha}_{\text{MIA}} < 0\right]$	2.34%	3.90%	4.64%
$\text{Var}\left[ADV^{\alpha}_{\text{MIA}}\right]$	3,650,434	3,678,755	3,691,146
$\mathbb{E}\left[ADV^{\alpha}_{\text{MIA}} \mid ADV^{\alpha}_{\text{MIA}} < 0\right]$	−646.16	−692.67	−737.81

Source: authors' calculation, based on data provided by the CMI.

Of course the above mentioned downside risk directly leads to the question of how the insurer's remaining (actuarial) shortfall risk α compares to the extent of aggregate longevity risk that is transferred back to the policyholders in MIA, and from the discussed values it is clear that those two measures for the uncertainty do not coincide at all. Effectively, the downside risk is much larger in MIA, where the policyholders are at risk, compared to conventional annuities where it is the insurer's liability to cover potential deficits in per-policy reserves. However, through the assumed contingency funds this risk reduced to the minimum desired level.[44]

[44] The comparatively large portfolio size of initially $l_{60} = 100{,}000$ in treatments A_I and A_{II} rather plays in favor of the insurer as well, since relative frequencies or proportions (of e.g. survivors) are then not too deviant from theoretical probabilities, effectively also working towards a risk reduction.

A further question that is related to the imbalance of risk bearing is the issue of marketability. More precisely, assuming that the insurer does not intend to (or, for reasons of competitiveness, may not) conceal the downside potential in MIA contracts, it is at least questionable how well it would be received by insurees. Naturally, an immediate attractive counterargument can be brought forward: the (non-reduced) annuity benefit FV_0 that can be provided when policyholders agree to bear aggregate longevity risk is initially higher than the comparable value of FV_0^{α}.

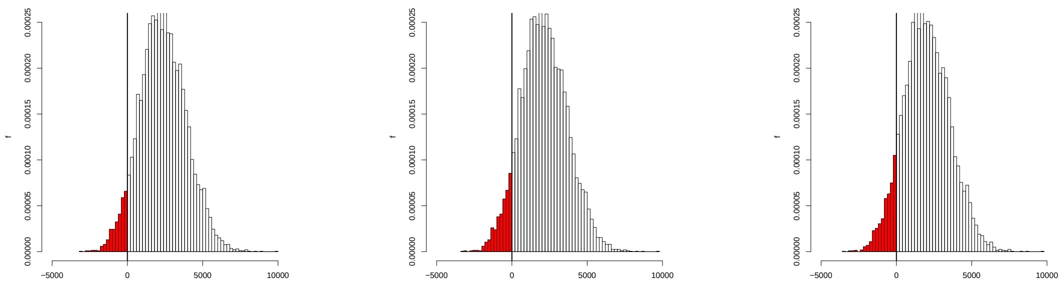
It must thus remain an open question at this point whether a skilled marketing is able to place innovative MIA contracts with interested persons. However, it is beyond the scope of this contribution to further analyze individuals' risk perception, assessment, or willingness to bear longevity risk.

Treatment A_{II}

When transiting to treatment A_{II}, i.e. incorporating a higher interest rate of $i = 5\%$, a very general first observation is that the graphical representations in Figure 4 do not drastically differ from those of treatment A_I. However, on closer inspection it is also evident that the respective distributions of $ADV_{\text{MIA}}^{\alpha}$ are shifted slightly further to the left. This is probably less revealed by Figure 4 but more so by the respective risk measures provided in Table 4.

Figure 4: Empirical Distribution of $ADV_{\text{MIA}}^{\alpha}$ – Treatment A_{II}

(a) $\alpha = 0.001$ $FV_0^{\alpha} = 6{,}814.29$ | (b) $\alpha = 0.005$ $FV_0^{\alpha} = 6{,}829.79$ | (c) $\alpha = 0.010$ $FV_0^{\alpha} = 6{,}838.79$



Source: authors' calculation, based on data provided by the CMI.

In particular, the empirical means are significantly reduced by roughly $1{,}150 - 1{,}350$ compared to treatment A_I to levels of between 2,000 and 2,400 (see Table 4), and the "shortfall" probability, i.e. the frequency of disadvantageous MIA benefit paths, is almost twice of what it was before. Contrarily, the expected shortfalls are smaller (in absolute terms), and these observations demand some further commenting.

Firstly, a higher interest rate primarily has an influence on the actuarial present values $\ddot{a}^{1}_{x:\overline{T}|}$ which are lower compared to the case of $i = 3\%$ due to the stronger discounting; see Equation (3.3). When inspecting Equation (3.2), this obviously results in a higher level of FV_0 given that the upfront premium remains unchanged.[45] Similarly, in Equation (3.6) the higher interest rate results in higher annuity benefits FV_t as the updated actuarial present values $\ddot{a}_{x+t:\overline{T-t}|}^{\;1}$ are also increased. However, the effect on Equation (3.5) is ambiguous. For

[45] Compare the remarks on this assumption in Footnote 37 on p.19.

the case of $t = 1$, a higher value of i obviously yields a higher value of V_1,[46] but for times $t > 1$ both V_{t-1} and FV_{t-1} are higher compared to the base case so that a general statement on the evolution of per-policy reserves V_t is not possible. Naturally, the claimed effect on Equation (3.6) is rendered ambiguous.

For the benchmarking conventional annuity, rearranging Equation (3.7) and taking into account that both $\ddot{a}^1_{x:\overline{T}|}$ and ${}_{T-1}E_x$ are lower under a higher i suggests that FV_0^α must be higher as again the upfront single premium π_0 is considered fixed, but this is also contingent on a constant value of $\Delta_{\alpha,T}$.

The described theoretically motivated effects on both FV_0 and FV_0^α can also clearly be identified in the first lines of Table 4 in comparison to the respective values in Table 3. However, the difference between FV_0 and FV_0^α is lower now; for instance, in the case of $\alpha = 0.001$ it is now only 160 instead of 205 units compared to treatment A_I. Therefore, a higher interest rate i seems to work in favor of the conventional annuity, where only a lower initial benefit adjustment from FV_0 to FV_0^α is necessary to incorporate the safety loading that accumulates to the desired contingency funds and thus reduces the insurer's shortfall risk to a given level of α. Yet the (initial) levels of benefits for *both* the innovative MIA and the conventional product are considerably higher under a higher interest rate i.

Secondly, as the proposed measure of advantageousness, ADV^α_{MIA}, is based on the actuarial present value of the difference between MIA benefits evolution and the comparable conventional benefits FV_0^α, Equation (3.10) reveals that discounting with the fixed interest rate i has an important effect. Obviously, when increasing i from 3% to 5% the stronger discounting through a lower v causes ADV^α_{MIA} to be lower *ceteris paribus*. On the other hand, as pointed out above, the (initially) lower difference between FV_0 and FV_0^α has an adverse effect. However, as is evident from Figure 2, negative (and positive) differences between FV_t and FV_0^α are rather to be expected in the long run, and such later deficit (or excess) benefits for the MIA product tend to be stronger the longer the time horizon is. Even though negative differences are weighed less now, the enormous upside potential for strongly positive realizations of ADV^α_{MIA} is compressed by stronger discounting, and as lower levels of e.g. $\mathbb{E}\left[ADV^\alpha_{\text{MIA}}\right]$ in Table 4 compared to the base case show, the latter effect obviously prevails.

Table 4: Key Figures from Treatment A_{II}

Annuity Benefits	$\alpha = 0.001$	$\alpha = 0.005$	$\alpha = 0.010$
FV_0	6,975.56		
FV_0^α	6,814.29	6,829.79	6,838.79
Risk Measure			
$\mathbb{E}\left[ADV^\alpha_{\text{MIA}}\right]$	2,366.11	2,143.97	2,015.09
$\mathbb{P}\left[ADV^\alpha_{\text{MIA}} < 0\right]$	5.62%	7.50%	8.91%
$\text{Var}\left[ADV^\alpha_{\text{MIA}}\right]$	2,354,463	2,365,189	2,371,423
$\mathbb{E}\left[ADV^\alpha_{\text{MIA}} \mid ADV^\alpha_{\text{MIA}} < 0\right]$	−601.69	−649.60	−667.88

Source: authors' calculation, based on data provided by the CMI.

A stronger discounting in the case of $i = 5\%$ also causes such negative differences to have a lower impact on the advantageousness of MIA. i.e. deficits (and excesses, likewise) are far

[46] Recall that the initial reserve is identical to the upfront single premium, i.e. $V_0 = \pi_0$.

more attenuated aas already mentioned, and the obvious consequence for treatment A_{II} in comparison to the base case can also be observed from Table 4 versus Table 3: the empirical variances are less than two thirds of what they had been before. Interestingly, the expected shortfalls are (in absolute terms) slightly smaller now, which can be interpreted in such a way that *if* the MIA is disadvantageous the extent is less harmful, and in the view of the general left-shift of the distributions of $ADV^{\alpha}_{\text{MIA}}$ compared to the base case, this observation is in perfect accordance with the notably smaller variances.

Treatments B_I and B_{II}

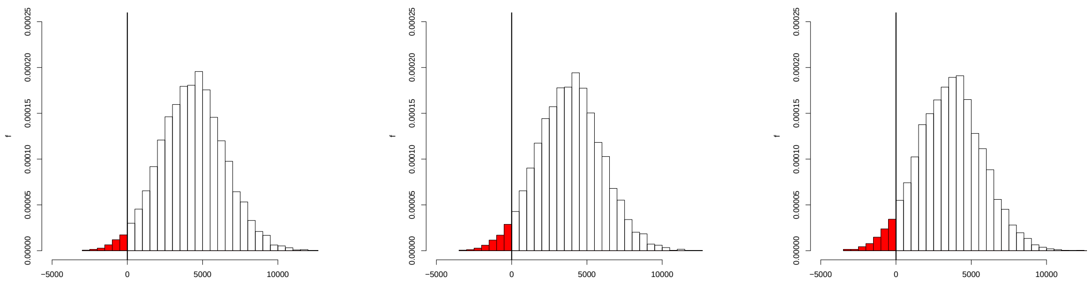
If the much smaller portfolios from treatments B_I and B_{II} with only $l_{60} = 1{,}000$ are considered, the above findings still apply accordingly. More precisely, the higher interest rate of 5% in treatment B_{II} versus a moderate 3% in treatment B_I has an overall effect that is similar to the above comparison of A_I and A_{II}: The initial annuity benefits FV_0 and FV_0^{α} are higher for the higher interest rate (with the reduction in the differences of FV_0 and FV_0^{α} being 200 instead of 240), the shortfall frequency is roughly doubled and the empirical means are significantly lower under higher interest rates, which again is the immediate consequence of a left-shift of the distributions of $ADV^{\alpha}_{\text{MIA}}$, of course contingent on α. This difference can also be identified in the diagrams in Figure 6 versus those in Figure 5, and Tables 5 and 6 give the relevant key figures. Similar to the previous cases of treatments A_I and A_{II}, the distributions under a higher interest rate are also more compressed, which is evident from a reduction of the variances by roughly one third and likewise less negative expected shortfalls (or "expected policyholder disadvantageousness" of MIA). Thus, a higher interest rate works in exactly the same fashion here – despite the lower number of contracts in the underlying portfolios of insured so that one conclusion is that results are much more sensitive to discounting than they are to different portfolio sizes.

Figure 5: Empirical Distribution of $ADV^{\alpha}_{\text{MIA}}$ – Treatment B_I

(a) $\alpha = 0.001$ $FV_0^{\alpha} = 5{,}391.62$

(b) $\alpha = 0.005$ $FV_0^{\alpha} = 5{,}417.04$

(c) $\alpha = 0.010$ $FV_0^{\alpha} = 5{,}431.19$



Source: authors' calculation, based on data provided by the CMI.

However, another observation is the overall difference of the results under treatments B_I and B_{II} compared to those in the case of larger annuity portfolios. Obviously, the fact that the Law of Large Numbers works less well here causes stronger deviations of the actual number of survivors from the initial best estimates. Accumulating over time, these deviations cause a more dispersed distribution of per-policy reserves for the conventional product used as a benchmark. In turn, to reduce deficit reserves here and make the benchmark comparable, the

Table 5: Key Figures from Treatment B_I

Annuity Benefits	$\alpha = 0.001$	$\alpha = 0.005$	$\alpha = 0.010$
FV_0	5,631.67		
FV_0^α	5,391.62	5,417.04	5,431.19
Risk Measure			
$\mathbb{E}\left[ADV_{\text{MIA}}^\alpha\right]$	4,326.52	3,875.51	3,624.41
$\mathbb{P}\left[ADV_{\text{MIA}}^\alpha < 0\right]$	2.02%	3.40%	4.40%
$\text{Var}\left[ADV_{\text{MIA}}^\alpha\right]$	4,550,639	4,593,644	4,617,675
$\mathbb{E}\left[ADV_{\text{MIA}}^\alpha \mid ADV_{\text{MIA}}^\alpha < 0\right]$	−728.69	−805.33	−855.19

Source: authors' calculation, based on data provided by the CMI.

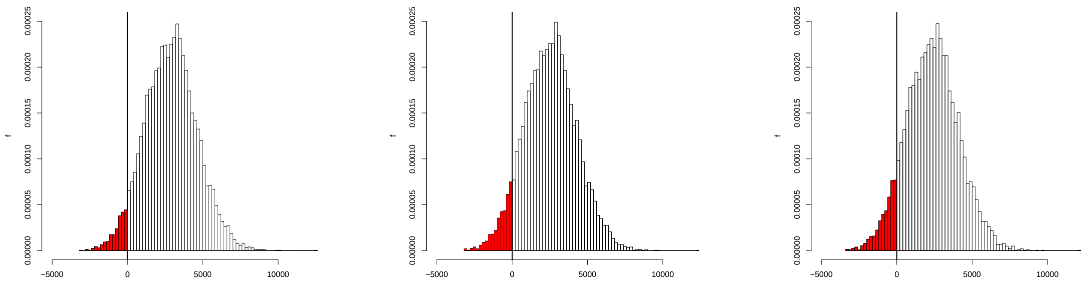
contingency funds need to be increased compared to the case of the larger portfolios discussed before. In turn, conventional annuities deliver benefits that are smaller in the "B" treatments when compared to the respective numbers from the "A" treatments. This is evident from the values for FV_0^α in Tables 5 and 6, respectively. Thus, in Equation (3.10) the differences between FV_t and FV_0^α tend to be larger *ceteris paribus*, and this immediately leads to values of ADV_{MIA}^α that are generally higher. In short: the "B" distributions are shifted to the right, which is also evident e.g. from the higher empirical means or lower shortfall probabilities in Tables 5 and6 when compared to Tables 3 and4, respectively. Interestingly, the dispersions in treatments with smaller portfolios are roughly $1.25 - 1.27$ times what they had been before, and as a direct consequence expected "shortfalls" take on more negative values. In short: disadvantageous scenarios are less likely here, and the overall prospect for benefits under MIA are considerably better for smaller portfolios. However, *if* negative developments occur they are more severe than in the case of larger annuity portfolios.

Figure 6: Empirical Distribution of ADV_{MIA}^α – Treatment B_{II}

(a) $\alpha = 0.001$ $FV_0^\alpha = 6{,}777.00$

(b) $\alpha = 0.005$ $FV_0^\alpha = 6{,}803.44$

(c) $\alpha = 0.010$ $FV_0^\alpha = 6{,}814.68$



Source: authors' calculation, based on data provided by the CMI.

Table 6: Key Figures from Treatment B_{II}

Annuity Benefits	$\alpha = 0.001$	$\alpha = 0.005$	$\alpha = 0.010$
FV_0		6,975.56	
FV_0^α	6,777.00	6,803.44	6,814.68
Risk Measure			
$\mathbb{E}\left[ADV_{\text{MIA}}^\alpha\right]$	2,891.20	2,527.11	2,366.09
$\mathbb{P}\left[ADV_{\text{MIA}}^\alpha < 0\right]$	4.44%	7.04%	8.35%
$\text{Var}\left[ADV_{\text{MIA}}^\alpha\right]$	2,994,518	3,007,693	3,017,638
$\mathbb{E}\left[ADV_{\text{MIA}}^\alpha \mid ADV_{\text{MIA}}^\alpha < 0\right]$	−687.64	−734.58	−771.30

Source: authors' calculation, based on data provided by the CMI.

5 Summary and Conclusion

As discussed in Section 1, the rather welcome longevity trend has also some implications that have to be cautiously accounted for by both individuals and insurance companies. For the latter, when doing annuity and pension business, there is evidence that the aggregate form of longevity risk as distinguished earlier may represent a severe systematic risk. It is obvious that the high correlation of individual risks and thus the potential for accumulative losses represent a major drawback for insurers. More precisely, unforeseen deviations of actual mortality development from previous expectations can barely – if at all – be hedged or diversified so that ultimately the question of whether insurers should decline to assume this particular risk seems legitimate. It has become clear that this form of *risk avoidance* appears to be quite promising for insurers.

Following the example of German private health insurance that was briefly introduced in Section 2.3 as a "role model" for the proposed MIA, a variation to conventional "pure" life annuities is proposed where actual benefits contingent on survival are linked to actual mortality experienced by a certain reference group; see Section 2.5. The fact that such products seem to not exist thus appears rather astonishing given the existence of similar product innovations. These *mortality-indexed annuities (MIA)* are then further analyzed by comparing the variable benefits from such contracts with a comparable conventional product, assuming that the latter would include a safety margin or contingency funds used to avoid actuarial shortfalls for the insurer as discussed in further detail in Section 3.3. Effectively, the difference of benefits in MIA and comparable contracts are aggregated by using the notion of an actuarial present value of these differences, i.e. the sum of annual differences but accounting for the twofold discount by interest *and* mortality.

The results from a Monte-Carlo simulation under various parameterizations presented in Section 4.2 reveal that with a high likelihood policyholders are better off with the innovative MIA, although there is twofold risk: On the one hand, the degree of advantageousness varies greatly, i.e. empirical distributions feature relatively large dispersions, and this is a direct consequence of the fact that aggregate longevity risk is not transfered but born by the insured. The overall tendency though is a pronounced advantageousness since contingency funds need not exist and MIA benefits are generally higher. On the other hand, however, accumulating uncertainty regarding mortality translates into a downside risk in the sense that MIA may actually be disadvantageous for policyholders, i.e. there is a chance that the benefits from MIA are considerably lower than they would have been in a comparable conventional annuity contract.

Results obviously depend on what the insurer's admissible ruin probability and thus the amount of contingency funds is. The smaller the admissible shortfall risk the larger the loading or equivalently the difference in initial benefits. As a higher difference would provide an additional headstart for mortality-indexed annuities it is logically consistent that the degree of advantageousness as measured by $ADV_{\text{MIA}}^{\alpha}$ would also be higher and consequently the downside risk for policyholders lower. The huge upside potential can also be a strong argument in favor of MIA, and it is evident from the histograms where the largest portion of probability mass is not only to the right of zero but also stretches far into highly positive values of $ADV_{\text{MIA}}^{\alpha}$. Similar to the transition from conventional annuities to investment-linked contracts as discussed in Section 2.2 some policyholders may find it appealing to be offered such contracts but a more detailed analysis of this issue is subject to further research. As briefly mentioned before, such analysis also would have to take into account the individual attitude towards risk or, more precisely, the degree of risk aversion.

What needs to be further considered is that for the present investigation it was assumed that both MIA and conventional annuities encompass a fixed guaranteed interest rate (set at only 3% and 5% so that it is not unlikely that the insurer's investment of the collected premiums or policy reserve will yield higher returns), and the latter type also include a sizeable contingency funds (which in the presented model on average delivers high profits to the annuity provider). Therefore, products actually offered in insurance markets provide certain mechanisms to have policyholders participate in the expected profits from investments and excess reserves. For the MIA contracts proposed here, a further analysis aiming at a higher degree of reality could assume stochastic investment yields from a suitable mix of e.g. stocks and bonds. At each point in time t when adjusting annuity benefits, the evolution of the per-policy reserve as per Equation (3.5) would be based on actual investment yields $\tilde{\imath}_{t-1}$ instead of the fixed interest rate i assumed here, and profit *sharing* could be done by taking only e.g. 90% of the simulated investment yield. Yet, the computational complexity of the model presented here would be greatly increased so that such mechanism was not incorporated for the present work. In tendency, however, both the innovative MIA as well as the benchmarking annuities would be even more advantageous for policyholders, and it remains to be seen whether the overall effect would change the results discussed above at all.

As for the first point, one may well argue that it is not adequate to assume the simplest type of an annuity contract – as considered in the present investigation and also referred to as the "plain vanilla" case – to be typical for actual insurance markets. For instance German insurance companies have traditionally almost exclusively offered participating insurance contracts on both the life insurance and the annuities markets.[47] In exchange for a relatively conservative guaranteed minimum interest rate on the policy reserves, at least 90% of returns in excess of that minimum rate have to be accumulated on account of each contract.[48] However, this

[47] For an insightful analysis of annuity types and different designs offered in the German market as well as an evaluation for different types of customers, see von Gaudecker and Weber (2004).

[48] The minimum guaranteed rate that is mandatory for the calculation of policy reserves is set and regularly revised by BaFin, the Federal Regulatory Authority in Germany. The current rate is prescribed by §2 DeckRV, and it is set in accordance with the smoothed development of yields on long term bonds as per §65 VAG. The general decrease of capital markets over the past years led to a current rate of 2.25%, effective for all contracts sold in or after 2007. It had been 2.75% before and was previously as high as 4.00%.

approach leads to an entirely different regime of guarantee(s). Although German insurers have tended to use the argument of historically realized rates of return – including actually attributed surplus – for their marketing, the exact amount is a priori unknown and far from being guaranteed. In fact most part of the assigned surplus participation is not necessarily binding for the insurer until eventually attributed on a per-contract basis.[49]

From a practical point of view the markedly complex mechanisms of determination, individual assignment and actual payout of the respective surplus share make it a special challenge to realistically account for this concept in any model as various such mechanisms have long been in use in the industry, and the actual range of choices available also highly depends on the regulatory and legislative environment. By including profit-sharing mechanisms, several other issues follow that also would need to be discussed: Besides a notable increase of risk exposure for insurance companies,[50] it also raises the question of how to value such mechanisms[51]. Nevertheless, interesting and possibly more realistic results are to be expected from an inclusion in some further investigation.

Another important caveat must be made at this point. As emphasized earlier the problems arising from model uncertainty are not confronted in the present investigation. Rather it was assumed that the LC/PML approach represents the *true* model underlying actual mortality so that the insurer's projections were only subject to the explicit stochasticity incorporated in the LC/PML model. Also, parameter uncertainty might render the model calibration and thus projections erroneous. Reality is likely to originate more sophisticated fluctuations or structural changes, and it remains a challenge for the industry and academia to improve and further the achievements of adequate mortality modeling.

Both uncertainties that are ignored here will actually increase the variations of ultimate reserves V_T for the conventional product, so that contingency funds must be larger to still attain the same low shortfall risk and this would then tend to deliver higher values of the measure of MIA advantageousness. Yet, also the MIA benefits would tend to be more dispersed so that the overall effect remains ambiguous at this point, and the same argument applies when considering alternative mortality models that might be less conservative and produce more deviating projections.[52]

Insofar as the models used for projections by the insurance company (or prescribed by regulators) are considered faulty or insufficient, model uncertainty must be kept in mind when analyzing the above results. Effectively insurers might consider including additional safety margins to account for these risks, and this could further bias the transfer of results obtained in this contribution.

[49] The German Federal Court of Justice recently criticized the prevailing practice of insurers' surplus attribution, which largely favored uncanceled contracts by means of large terminal bonuses, for its lack of transparency; see BGH IV ZR 162/03, IV ZR 177/03, IV ZR 245/03. Providers were thus obliged to "adequately" account for the development of reserves underlying surplus participation. As a consequence §153 VVG (Versicherungsvertragsgesetz) was revised and the *Verordnung über die Mindestbeitragsrückerstattung in der Lebensversicherung (Mindestzuführungsverordnung – MindZV)* as of April 04, 2008 (see BGBl. I S. 690) replaced the previously applicable *Verordnung über die Mindestbeitragsrückerstattung in der Lebensversicherung (ZRQuotenV)* as of July 23, 1996 (see BGBl. I S. 1190) for new business.

[50] See Kling et al. (2007) for an analysis in the case of life insurance companies.

[51] See Bauer et al. (2006).

[52] A relatively simple way to mimic increasing uncertainty about projections over time would consist in multiplying the ARIMA disturbance term used to model the time index in the LC/PML approach with a factor increasing over time, such as e.g. $\left(1+\frac{t}{100}\right)$.

It is also to be noted that for the present contribution the MIA product investigated effectively not only transfers systematic mortality (or longevity) risk back to policyholders but also the unsystematic portion of this risk. By adjusting benefits FV_t according to Equations (3.5) and (3.6) fluctuations in the actual number of survivors $\tilde{l}_{x+t}$ are not further distinguished with respect to their origin, i.e. to what extent they arise from general changes in the underlying mortality pattern (systematic risk) or whether they are due to the deviation of individuals' mortality from the portfolio average.[53]

The chosen implementation, however, causes that more risk than initially discussed is shifted back to policyholders, and it may be worthwhile to reflect on the question whether such a strong cutback of risk born by the annuity provider is conducive to the purpose of risk reduction for the insurer. Recall that the objective of the proposed MIA was to avoid the potential for accumulative losses due to systematic changes in the longevity trend. When also shifting back the unsystematic portion of longevity risk this objective is obviously exceeded. The reason for forgoing such a further distinction is to facilitate the computations. Also, the chosen approach may be far easier for annuity providers to implement as it does not require any further analyses. Instead, at each point in time t calculations may be cased on the current number of survivors l_{x+t}, and this information is likely to be immediately available for the insurer. Eventually the fact that this present case can be considered the most extreme scenario of *risk avoidance* in comparison to the least re-transfer of longevity risk

The possibly most important question here is the effect of not shifting back the unsystematic longevity risk on the results that have been presented and discussed in Section 4.2. Although further analyses are required for a precise assessment, some thoughts on the tendency of changes in results are still possible at this point. Naturally, if opposed to the situation analyzed here the insurer bears the unsystematic longevity risk the resulting mortality-indexed annuity would in tendency be better for the policyholders. As they would not have to bear the unsystematic component of longevity risk the excess or difference of adjusted benefits compared to a comparable conventional product would be larger *ceteris paribus*. In addition, as the annuity provider would have to bear that risk, ultimate per-policy reserves will not always break even for the MIA contracts – opposed to the case analyzed above. Following the argumentation in Section 3.3 to justify the establishing of contingency funds for the benchmark product, this could now also be applied to the (partial) MIA contracts. As this might counteract the improvement of $ADV^{\alpha}_{\text{MIA}}$ pointed out before, the overall effect is ambiguous. Therefore, further analyses are necessary.

An extension for a further investigation of the proposed mortality-indexed annuities could be an issue that is inseparably linked to an actual implementation into real annuity markets: costs and expenses. As pointed out earlier, the present assessment was entirely made on a net actuarial basis, i.e. without costs, expenses, or profit-sharing etc. – with the exception being the safety loading for conventional annuities when serving as a benchmark for MIA. Actual products offered in insurance markets do almost never provide a fair premium without any surcharges or loadings, and costs that are typically included in gross actuarial calculation are those pertaining to the policy acquisition (i.e. commissions payable to insurance agents and

[53] Of course, the latter cause would be the effect of an imperfect diversification across individual risks in the portfolio of contracts. More formally: the effect asserted by the Law of Large Numbers only holds for a theoretically infinite number of contracts, and as this never coincides with reality there is the risk of such deviations.

the costs of checking and processing applications) as well as recurring costs for running the insurance company, annual statements on policy reserves etc.).[54]

In the specific case of MIA it remains to be discussed whether and how exactly expenses and loadings have an influence and the results obtained above. For a true comparison, not only MIA but also the benchmark product would have to include expenses and loadings, and it is not obvious at this point in which case these are lower. Further research in this direction is required to provide a satisfactory appraisal.

Not to be overlooked is the observation that the notion of benefits being linked to demographic development in general, or mortality experience in particular, may actually be not so uncommon in some countries. For instance, the German public pension system, which is organized on a pay-as-you-go basis, delivers benefits that are set by statutory ordinance including a so-called “demographic factor” for some time. Thus no ultimate guarantee as to the exact amount of retirement provision can be made during the working life, and also thereafter benefits may be changed by legislators as appropriate. Naturally, the demographic transition has also had an impact on population age structures in Germany, and as a consequence social security benefits have rather not been increased or barely compensated for inflation. Despite an entirely different range of problems and challenges associated with public pay-as-you-go pension systems, this thought raises hope that the notion of benefits being contingent on mortality experience is not considered absurd or generally dismissed.

Further discussion and deliberations regarding this innovative idea, which at a closer investigation does not appear so much surprising, may be necessary to ultimately convince both insurers and insurees of the overall benefits that mortality-indexed annuities (MIA) can provide.

Eventually individuals who in general seem to be rather reluctant to buy life annuities[55] could be provided additional incentives to do so given the potentially advantageous features of mortality-linked annuities.

[54] Many actuarial textbooks provide an introduction of how to include initial and recurring costs into pricing and reserving of insurance contracts as a transition from a net perspective presented in the first place; see e.g. Bowers et al. (1997, Ch.15).

[55] This empirical observation in contrast to the theoretical optimality of complete annuitization of wealth is generally discussed in the literature as the “annuity puzzle”; see Yaari (1965); Davidoff et al. (2005).

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