

**Evaluating Post-Retirement Investment Strategies**  
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Abstract:

This paper proposes a framework and ruin probability measure for evaluating investment strategies available to retiring members of a defined-contribution fund (the so called decumulation phase). Within the context of a discrete-time framework we use simulation techniques to compare post-retirement investment strategies available in South Africa.

Keywords:

Defined-contribution; decumulation; ruin probability; life annuity; income drawdown

## Introduction

This paper considers the evaluation of income provision choices available to members of a defined contribution fund who have recently retired. In particular, we consider the case of an individual at the point of retirement who has an accumulation of assets, but no guaranteed retirement income.

The consumption and portfolio optimization problem during retirement has not received as much attention in financial research as investment optimization prior to retirement (Schies, 2008). However, retirement planning is becoming more and more relevant. Retirees continue to live longer and are enjoying a greater concentration of the world's wealth. Individual responsibility for post-retirement income management is increasing as the proportion of retirement provision in social security and defined benefit occupational funds falls, and the corresponding proportion in individual account systems and defined occupational arrangements increases, a trend that is likely to continue for some time. Demand for quality financial planning advice at the end of the accumulation phase is strong and is likely to increase into the future.

A retiree from an individual account system faces many risks in retirement. These risks include inflation risk (the risk of the chosen investment strategy not keeping pace with inflation), longevity risk (the risk that an individual outlives his or her retirement assets), consumption risk (the risk that the chosen investment strategy does not meet the income needs of the individual in retirement) and annuitisation or interest-rate risk (the risk of retiring when interest-rates are low resulting in relatively expensive life annuities). This list is not exhaustive. Risks relating to tax, credit and poor advice exist as well.

The individual member needs to choose an investment strategy that addresses these risks. The optimal investment strategy in retirement is complicated by the irreversibility of the decision.

Lifetime annuitization is not common around the world. There are a number of reasons for this.<sup>1</sup> Some related to crowding out by other forms of provision, commonly social security systems, or tax disincentives. But in those countries in which the individual component of retirement provision is high, lifetime annuitization rates remain puzzlingly low.<sup>2</sup>

In many countries, the most common post-retirement investment of a defined contribution fund is the life annuity (Blake et al, 2003).

In the UK, the fund credit must be used to purchase a life annuity by the time the member reaches age 75.

In South Africa, retirees can choose from a range of different investments. These include level nominal life annuities, escalating nominal life annuities, inflation-linked life annuities, income-drawdown facilities and so called with profit-annuities<sup>3</sup>. A study by the largest employee benefits

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<sup>1</sup> Rusconi (2008) sets out a number of determinants of annuity market size and type and Antolín et al (2008) describes the considerable variance in the regulatory frameworks of different countries that have an impact on annuity market dynamics.

<sup>2</sup> This has been the subject of research by, for example, Yaari (1965), Davidoff et al (2003) and Mackenzie (2006).

<sup>3</sup> A with-profit annuity is a life annuity that guarantees an income for the rest of the annuitant's life. However, the underlying annuity is invested in a wide range of investment assets, and the annual income of the annuitant receives an increase in line with the investment returns of those assets.

firm in South Africa found that the majority of its retirees purchase a level life annuity or invest in an income drawdown facility<sup>4</sup>.

## Literature Review

Whilst research on decumulation strategies has been relatively scarce (compared with the number of papers on accumulation strategies), there are some notable papers.

- Yaari's (1965) seminal paper on annuity choice shows that in the absence of a bequest motive, it is optimal to fully annuitise the fund credit. The reason behind the conclusion is that returns from life annuities are enhanced by the "living" receiving the assets and thus returns of the "dead".
- Milevsky et al (1997) use a ruin probability measure to determine the optimal allocation to risky assets by considering when an individual's assets will be depleted assuming a given rate of consumption. The framework is expressed in real terms for simplicity.
- Albrecht and Maurer (2002) use a ruin probability measure to compare a level annuity with an investment in mutual funds. The ruin probability is evaluated assuming an annual income requirement equivalent to that provided by the level life annuity.
- Blake et al (2003) compare three different distribution programmes for a retiring DC plan member. They determine the value of an annuity with reference to the fixed payment of a level life annuity. They incorporate utility of the member and vary the relative risk aversion parameter.
- Milevsky and Young (2002) propose an approach to determine if and when an individual should purchase an immediate life annuity. They make use of utility theory in their analysis. In particular, the utility function is assumed to be strictly concave.
- Milevsky and Young (2004) find the optimal consumption over time, solving a similar problem, and seek the optimal time of annuitization. However, they use a power utility function and do not take the targeted consumption of the retiree into account.
- Gerrard et al (2004) consider a scenario in which the retiring member specifies the level of income required and manages the investment and consumption in such a way as to minimize the expected discounted loss over the infinite horizon. The loss function in this case is the square of the difference between that achieved and that required. However, even in this context, ruin is defined as the probability of running out of funds. As scope for future research, they state that "it may be worth exploring numerically how the risk attitude, can affect optimal choice."

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<sup>4</sup>Study done by Alexander Forbes Financial Services in 2008.

## **An advice framework**

We submit that an approach that considers the members actual income needs relative to their retirement expenses is appropriate for post retirement investment planning. VanDerhei and Copeland (2003) show that US retirees face an income shortfall of at least \$400 billion between 2020 and 2030 in their ability to cover basic living expenses and those associated with medical care. Fornero (2008) suggests that uncertainty of future medical costs in that country significantly impairs the demand for lifetime annuities.

The research of Milevsky et al (1997) and Albrecht and Maurer (2002) have evaluated the probability of ruin as running out of funds assuming a constant consumption. However, under this measure an individual cannot be “ruined” under a life annuity as the income is guaranteed for the life of the annuitant. We submit that ruin is experienced when an individual is not able to sustain a certain standard of living.

This paper adds to the current body of research by proposing measures that relate to the actual income requirements of a retiree.

## **Funding level**

Actuaries of defined benefit funds evaluate the funding level of the arrangement by considering the value of the assets in relation to the liabilities at a given point in time. We submit that an analogous comparison can be made in respect of the assets and liabilities of a defined contribution (DC) member.

Consider the individual aged 65 who has recently retired. Suppose he has accumulated funds of R1,000,000. With the assistance of a financial advisor, the individual determines that he needs R6,500 a month (in current purchasing power terms) to maintain his current standard of living.

We argue that this individual has a liability value equivalent to the cost of securing this level of income in real terms for the rest of his life. In other words, the market price of the appropriate inflation-linked life annuity can be used to determine the individual’s liability. These annuities are the only financial instrument available that removes both the inflation risk and longevity risk of the individual. By considering the income requirement of the individual, consumption risk is mitigated too.

As at the date of the analysis (1 July 2009), the cost of securing such a life annuity from an insurer was R1,169,847. The individual thus has 85% of the required funds to guarantee his monthly income needs. Analogous to the terminology used in the actuarial valuation of a defined benefit fund, we can state that the individual is 85% funded and is thus in deficit.

An inflation-linked annuity that provides an income of R5,556 can be purchased with a consideration of R1,000,000 from the same insurer. If the individual’s income requirement was R5,556 then we could argue that the individual is 100% funded.

The inflation-linked life annuity has the following important attributes:

- 1) a guaranteed monthly income is paid;
- 2) escalation of this income is in line with consumer price inflation; and

- 3) certainty or receiving this income as long as the retiree is alive (and the life office is solvent).

The consequence of this framework is that whilst the individual is not invested in the matching asset, their funding level will move in line with real interest rates and the performance of their assets.

The funding level at time  $t$  for an individual that requires annual income of  $C$  can thus be stated as follows:

$$\text{Funding Level } (C,t) = \frac{\text{Assets of member at time } t}{\text{Liability of member at time } t}$$

Where:

*Assets of member at time  $t$*  is the accumulated funds of the member at time  $t$

*Liability of member at time  $t$*  is the purchase price of an inflation-linked life annuity that provides an initial annual income of  $C$  at time  $t$

It could be argued that if an individual requires an income equal to or lower than that provided by the inflation-linked annuity (i.e. his funding level is greater than or equal to 100%), then the inflation-linked annuity is the minimum risk investment. This is because the funding level of the individual will be secured at 100% as long as his real income requirement does not increase over his lifetime.

Under this framework, most individuals at retirement in South Africa are in deficit or have funding levels lower than 100% because they have not saved sufficiently to provide for inflation-protected income at the level of their aspiration. There is thus normally no risk-free solution for a retiree and alternatives must be considered within an appropriate framework of risk.

### **Evaluating investment strategies**

In this paper we propose a framework that can assist members with their investment strategy choice.

In particular we evaluate different investment strategies for a male DC plan member retiring at age 65. We assume that the member has no other savings and no bequest motive.

The strategies considered are:

- 1) Purchase a nominal level life annuity

On retirement, the member uses his accumulated retirement savings to purchase a level pension from a life office. The insurer guarantees a series of level income payments for as long as the member is alive.

- 2) Purchase a nominal escalating life annuity

On retirement, the member uses his accumulated retirement savings to purchase a pension from a life office that increases each year. The insurer guarantees a series of income payments that will increase by 3% at the end of every year for as long as the member is alive.

3) Purchase an inflation-linked life annuity

On retirement, the member uses his accumulated retirement savings to purchase a pension from a life office. The insurer guarantees a series of income payments that will increase in line with headline inflation at the end of every year for as long as the member is alive.

4) Commence in an income-drawdown facility

This programme allows the retiree to invest in a managed fund containing both equities and bonds.

The following strategies are considered based on South African insurer rates as at 1 July 2009 ( $t = 0$ ). The 65 year old member under consideration, has retired from a DC fund with accumulated funds of R1,000,000.

The accumulated funds at time  $t$  are denoted by  $F(t)$ . Thus,  $F(0) = R1,000,000$ . The annual income received under strategy  $j$  in year  $t$  is denoted by  $I_j(t)$ .

*Income received under strategies 1, 2 and 3*

In these strategies the fund credit  $F(0)$  is used immediately to purchase a life annuity at the prevailing annuity rates. The cost of an annuity per R1 for strategy  $j$  is denoted by  $P_j$ . Annuity rates and initial income received under each strategy as at 1 July 2009 are shown in table 1 below.

Strategy $j$	$P_j$	Initial level of annual income $I_j(0) = F(0)/P_j$
1 - Immediate/Level Annuity	7.83	R127,720
2 - Escalating Annuity at 3% p.a.	9.62	R103,969
3 - Inflation-linked Annuity	15.00	R66,675

Table 1: Initial income received under life annuities

These annuity rates do not incorporate any guarantee periods or spouse's pension in the event of death of the member.

Annual income under each strategy is then projected for each year using the following formulae set out in Table 2 below.

Strategy	Income Received
Immediate/Level Annuity	$I_1(t) = I_1(0) \forall t$
Escalating Annuity at 3% p.a.	$I_2(t) = I_2(0) * 1.03^t \forall t$

Inflation-linked Annuity	$I_3(t) = I_3(t-1) * (1 + \delta_t) \forall t$ where $\delta_t$ is the annualized inflation rate in year t
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Table 2: Income received under life annuities

Annual income is paid to the member for as long as he is alive. Upon the member's death the remaining funds stay with the insurer and nothing is left to the member's estate.

#### *Income received under strategy 4*

Under this strategy, the member's fund credit  $F(0)$  is invested in a portfolio of assets. For simplicity, we assume there are two assets available for investment: equities and nominal government bonds. The allocation to equities is  $\omega$ , leaving  $1 - \omega$  to be invested in bonds. We consider allocations to equities with  $\omega = 0\%, 25\%, 50\%$  and  $75\%$ .

The member can specify an annual income drawdown based on a proportion of the accumulated assets. Current South African legislation requires the drawdown rate to be between 2.5% and 17.5% per annum of the accumulated assets.

In the case of the income drawdown facility modeling, we assume that the member will preserve the initial income required in real terms i.e. the income drawn each year will increase with inflation until such time that the drawdown rate reaches the maximum of 17.5% per annum. We consider various levels of initial income draw.

The accumulated funds at the end of each year for strategy 4 and income drawn is represented as follows

$$F(t+1) = F(t) + R(t) - I_4(t) \forall t.$$

$$\text{where } R(t) = F(t) * (\alpha_t * \omega + \beta_t * (1 - \omega))$$

$R(t)$  is the total return on the portfolio during year t.  $\alpha_t$  and  $\beta_t$  represent the total return on equities and bonds respectively and inflation is denoted by  $\delta_t$  for year t.

$I_4(t)$  takes on a value of

$$\begin{array}{ll}
 \text{min\_draw} * (F(t) + R(t)) & \text{if } I_4(t) \leq \text{min\_draw} * (F(t) + R(t)) \\
 \max\{\text{max\_draw} * (F(t) + R(t)), I_4(t)\} & \text{otherwise}
 \end{array}$$

where  $\text{max\_draw} = 0.175$  and  $\text{min\_draw} = 0.025$ .  $I_4(t) = I_4(t-1) * (1 + \delta_t) \forall t$  is the annual income requirement of the member.

We have not considered programmes that allow the retiree to defer the purchase of the life annuity. We have also not considered the so called ‘with-profit annuity’. These can quite easily be incorporated in the framework.

### **Individual income requirements**

In the hypothetical scenario discussed above, the individual determines an income level that is required to maintain his standard of living in retirement.

However, it is unlikely that the individual will be able to support this level of consumption in retirement due to insufficient accumulated savings. It is possible for individuals to reduce their income requirements in retirement. Aguiar and Hurst (2005) find that it is possible for retirees to lower their income requirements in retirement by engaging in more “home production” such as shopping for lower prices. This did not necessarily imply a reduction in quality of life.

Eisenberg (2006) finds that some financial planners have evolved into “life planners” who encourage clients to reprioritize their life priorities rather than accept the “status quo of meaningless materialism”. As Skinner (2007) states, “retirees know they can always move to smaller houses or to less expensive regions of the country, or cook at home rather than eating out”. Accordingly we argue that investment strategies should be considered with regard to various levels of income consumption through retirement.

We submit that there are at least three income levels of concern to an individual. The first relates to the level of income required to support a desired current standard of living (an income for comfort level). The second income level is determined by eliminating luxuries and focusing on core monthly expenses (an income for necessities level). The third is the absolute minimum income that needs to be received to support the individual (a survival income level).

### **Individual income requirements – an example**

Suppose the individual determines that his income required for comfort (comfort income level) is R8000 per month and that his income required for necessities (necessity income level) is R5500 per month.

Using the funding level framework, the individual is 69% funded on a Comfort Income basis and 101% funded on a Necessity Income basis as at 1<sup>st</sup> July 2009.

The initial income available from each of the life annuities is presented in the table below.

<b>Strategy and Life Annuity</b>	<b>Initial level of monthly income</b>
1 - Immediate/Level Annuity	R10,643.37
2 - Escalating Annuity at 3% p.a.	R8,664.12
3 - Inflation-linked Annuity	R5,556.28

Table 3: Initial income affordable for member

The inflation-linked annuity guarantees a future funding level in excess of 100% if the individual measures funding level on the necessity income level basis. This is the minimum risk investment assuming the individual is happy to accept the necessity income basis. However, on a comfort income level basis the inflation-linked annuity could be argued to be a high risk investment as it guarantees a deficit for the individual. In this case, risk is not defined as volatility of investment returns but the risk of not meeting a required real income level.

We submit that in South Africa the popularity of level annuities over other life annuities is due to the higher initial income level provided. In this example, the individual is able to secure R2,643.37 more than that initially required on a comfort income level basis. However, the inflation risk of such annuities is often not appreciated.

The income drawdown facility permits the individual to draw income equal to that required by the individual. However, the longevity risk associated with depleting ones funds needs to be considered in this case.

We propose that a modified Ruin Probability can provide a useful measure to evaluate investment strategies.

### Ruin probabilities

Milevsky (2006) defines the ruin probability as follows:

$$\mathcal{O}(w) = \Pr \left[ \inf_{0 \leq t \leq T} W_t \leq 0 \mid W_0 = w \right]$$

This is the probability that the lowest value of the stochastic process  $W_t$  breaches a value of zero at some point prior to the random time of death  $T$ .  $\mathcal{O}(w)$  is an explicit function of the initial level of retirement wealth  $W$  and an implicit function of the mortality dynamics of  $T$ .

We modify the formula as follows:

$$\mathcal{O}(f, C) = \Pr \left[ \inf_{0 \leq t \leq T} C_t \leq C \mid F_0 = f \right] \quad (1)$$

This is the probability that the lowest value of the stochastic process  $C_t$  breaches the real value of a given consumption level  $C$  at some point prior to the random time of death  $T$ . In this instance,  $C$  is the inflation adjusted value of the given income requirement.

In this way, the ruin probability calculates the likelihood of the individual consuming at a real level lower than that required during his lifetime. It thus includes the consumption, mortality and inflation risk that the individual faces.

### Numerical Techniques

We evaluate the available investment strategies in a discrete time framework using simulation techniques.

The probability of ruin (1) can be calculated numerically as follows

$$\mathcal{O}(f, C) = \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^K F_{i,t} S_{i,t}$$

where:

- $N$  is the number of simulations of the strategy;
- $K$  is the maximum age of the mortality table used less the current age of the member;
- $F_{i,t}$  is the financial ruin indicator and takes on the following values:  
 $0$  if  $C_{i,t} \geq C$   
 $1$  otherwise
- $C$  is the required real level of annual income;
- $C_{i,t}$  is the income actually consumed under the investment strategy for the  $i^{\text{th}}$  simulation during year  $t$
- $S_{i,t}$  is the mortality indicator and takes on the following values:  
 $1$  if member has died in year  $t$  or prior to year  $t$   
 $0$  otherwise

#### *Assumptions:*

We model stochastic mortality by simulating random numbers under a Uniform [0,1] distribution.

$q_x$  is the probability that the member will die between age  $x$  and age  $x+1$ .

The mortality indicator  $S_{i,t}$  for simulation  $i$  is given by

$$\begin{aligned} & 1 \text{ if } U_{i,t} \leq q_{x+t} \text{ and } S_{i,t} \neq 0 \forall \text{ years } \leq t \\ & 0 \text{ otherwise} \\ & \text{where } U_{i,t} \sim U(0,1) \end{aligned}$$

In our simulations we make use of the mortality table PA(90)<sup>5</sup>. We make the following adjustments to the mortality table to reflect South African best practice: age adjustment of 3 years downwards and an assumed 1.5% annual mortality improvement.

#### *Stochastic returns*

All simulations have been computed using Matlab. Each strategy was evaluated by simulating 3000 scenarios over 40 years. Stochastic returns for inflation, bonds and equities are based on the Maitland Stochastic Model – a stochastic model used in Employee Benefits companies in South Africa. Market neutral parameters as at 1 July 2009 were used. This is based on academic work by James Maitland (1997, 2001 and 2002). It is assumed that the portfolio composition is rebalanced annually and that the income for each life annuity or drawdown facility is received at the end of each year.

<sup>5</sup> [http://www.actuaries.org.uk/\\_data/assets/excel\\_doc/0019/21970/PA90m.xls](http://www.actuaries.org.uk/_data/assets/excel_doc/0019/21970/PA90m.xls) (accessed 1st July 2009)

### *Analysis and results*

The model generates simulated ruin probabilities for each strategy corresponding to each of the strategies described above. In the case of the income drawdown facility, the annual drawdown is set equal to the income required and is assumed to increase with prevailing inflation each year.

In cases where the individual receives more income than that required, it is not assumed that this is reinvested. This implies that individuals consume income received in excess of their consumption requirements.

The ruin probabilities in respect of the life annuities are set out in Table 4 below.

Income Required	Level Annuity	Escalating Annuity	Inflation-linked Annuity
500	0.00%	0.00%	0.00%
1,000	0.80%	0.00%	0.00%
1,500	6.90%	0.00%	0.00%
2,000	16.70%	0.00%	0.00%
2,500	25.50%	0.80%	0.00%
3,000	34.40%	4.80%	0.00%
3,500	43.80%	10.80%	0.00%
4,000	52.60%	21.20%	0.00%
4,500	59.80%	31.40%	0.00%
5,000	66.10%	42.60%	0.00%
5,500	71.70%	53.50%	0.00%
6,000	76.00%	64.20%	97.20%
6,500	79.70%	73.70%	97.20%
7,000	82.40%	81.10%	97.20%
7,500	85.90%	86.40%	97.20%
8,000	88.30%	91.40%	97.20%
8,500	90.40%	96.20%	97.20%
9,000	92.60%	97.20%	97.20%
9,500	93.70%	97.20%	97.20%
10,000	95.40%	97.20%	97.20%

Table 4: Ruin probabilities for life annuities

The modification to the standard ruin measure means that life annuities can have a non-zero probability of ruin despite their lifetime guarantee of income. Ruin probabilities increase as the income level required increases.

### Inflation-linked annuity

- In the case of the inflation-linked annuity, the ruin probability is either 0% or 97.20%. The value is 97.20% as opposed to 100% as income is drawn at the end of each year. There is a non-zero probability of the member dying in the first year of retirement and hence the ruin probability is not 100%. If income was drawn at the beginning of each year, the inflation-linked annuity would have a ruin probability of 0% or 100%.
- For income requirement levels lower than 5,500, the inflation-linked annuity minimises the ruin probability.

### Level annuity

- The level annuity provides the highest initial income to the retiree.
- The inflation risk present in such annuities is captured in the ruin probability. The ruin probability also incorporates the likelihood of the member dying before receiving a level income payment lower than the real income required.
- Ruin probability measures for level annuities are therefore high for those individuals who require ongoing real income close to the initial income provided by these annuities.
- The ruin probability increases by more than 5% for every R500 required over the income required band of R1000 to R5000.

### Escalating annuity

- For income levels between 6,000 and 7,000, the escalating annuity provides the minimum ruin probability. For income levels in excess of 7,500, the level annuity provides the minimum ruin probability.

Importantly, the results in Table 4 are dependent on the initial income levels payable under each life annuity. The relative difference in initial income payable under these annuities and the absolute level of the initial income is thus important. The annuitisation risk is now incorporated into the framework.

The ruin probabilities in respect of the life annuities are set out in Table 5 below.

Income Required	0% Equity	25% Equity	50% Equity	75% Equity
500	<b>0.00%</b>	<b>0.00%</b>	<b>0.00%</b>	0.00%
1,000	<b>0.00%</b>	<b>0.00%</b>	<b>0.00%</b>	0.40%
1,500	<b>0.00%</b>	<b>0.00%</b>	0.20%	0.50%
2,000	<b>0.00%</b>	<b>0.00%</b>	0.40%	1.50%
2,500	<b>0.00%</b>	0.30%	1.10%	3.30%
3,000	<b>0.50%</b>	1.20%	2.70%	6.20%
3,500	5.10%	<b>2.80%</b>	5.70%	12.90%
4,000	17.60%	<b>10.00%</b>	13.00%	19.90%
4,500	32.10%	<b>21.00%</b>	21.60%	26.50%
5,000	45.70%	33.90%	<b>33.00%</b>	33.70%
5,500	58.20%	46.00%	<b>40.60%</b>	41.30%
6,000	66.20%	57.70%	48.10%	<b>47.40%</b>
6,500	72.20%	65.80%	57.30%	<b>54.10%</b>
7,000	76.20%	72.50%	65.40%	<b>61.10%</b>
7,500	79.00%	75.40%	70.60%	<b>65.70%</b>
8,000	82.30%	79.00%	74.80%	<b>70.30%</b>
8,500	85.50%	81.80%	77.60%	<b>73.80%</b>
9,000	86.90%	84.10%	80.70%	<b>77.70%</b>
9,500	87.80%	86.60%	83.80%	<b>80.40%</b>
10,000	89.10%	88.90%	86.70%	<b>82.40%</b>

Table 5: Ruin probabilities for income drawdown facility

#### Income drawdown facility

- The member is exposed to investment, inflation, longevity and consumption risk.
- In the case of the income-drawdown facility, the individual is assumed to draw their annual income requirement until the drawdown is equivalent to 17.5% of accumulated funds. The individual will thus not be ruined if they die before they consume too much.
- Importantly, ruin is not reached when the accumulated funds are close to zero but rather when the drawdown allowed results in the individual receiving income less than their real requirement.
- Ruin is minimised for higher income requirements (income levels in excess of R6000) when the investment strategy has a 75% allocation to equities. For income levels less than or equal to R3000, ruin is minimised when the asset allocation has no equity exposure.

In the case of the retiring member with comfort and necessity income requirements of R8000 and R5500 respectively, the associated ruin probabilities are set out in the table below.

<b>Investment Strategy</b>	<b>Income requirement of R5,500</b>	<b>Income requirement of R8,000</b>
Level Annuity	71.70%	88.30%
3% p.a. Escalating Annuity	53.50%	91.40%
Inflation-linked annuity	0.00%	97.20%
Income drawdown	40.6%	70.3%

Table 6: Ruin probabilities for an individual member

The level annuity provides an income level that is initially 33% higher than the required comfort income level of R8,000 yet the probability of ruin is as high as 88%. This is due to the high inflation risk that is difficult to quantify when evaluating strategies on the basis of initial income. Even if the strategy is chosen on the basis that the income required is R5,500, the ruin probability of the level annuity is as high as 72%.

A ruin probability of zero is possible if an income requirement of R5500 is used when evaluating an investment strategy which is the case when an inflation-linked annuity is purchased. In the case of an income requirement of R8000, the income-drawdown strategy with 75% equities minimises the probability of ruin. The ruin probability in this case is as high as 70%.

### **Implications for advice**

We believe that the ruin probability measure presented here can be used as part of a consulting framework by financial advisors. The ruin probability measure is often used in academic research to compare income-drawdown strategies against a life annuity. We believe that this is a useful stand alone measure that can be used to evaluate any investment strategy. The risk of financial advisors miss-selling products is reduced in this way.

The framework presented is flexible and allows individuals to understand the extent of risk they face in retirement. The process is interactive and iterative. Members can vary their consumption requirements and evaluate the impact that this has on their ruin probabilities. If the ruin probability is too high, then the individual can attempt to change their household expenditure to produce a ruin probability with which they are more comfortable.

The strategy that minimizes the ruin probability will depend on the income requirements of the member, the initial level of wealth, the market price of life annuities and the longevity expectation of the member concerned. The framework illustrates that there is no unique solution for a retiree. Even a retiree with the same level of accumulated funds at retirement will have a different solution which minimizes their ruin probability.

## **CONCLUSIONS**

This paper proposes a ruin measure that can assist members at retirement with evaluating investment strategies. The approach is an improvement over previous ruin probability approaches by taking into account the actual income needs of retirees. The approach is flexible and can be used to evaluate any investment strategy.

Based on the funding level framework, members who have an income requirement in excess of that affordable under an inflation-linked annuity have a non-zero probability of ruin.

The framework also illustrates the importance of using market-consistent pricing when evaluating investment strategies.

The usefulness of the measure is dependent on the ability to determine the income requirements of the member. The results illustrate how seemingly small increases in real income required can have bigger than expected impact on the ruin probabilities obtained.

## **SCOPE FOR FUTURE RESEARCH**

The funding level framework can allow dynamic investment strategies to be investigated. The extent of market risk taken is dependent on the funding level of the member. For example, individuals in deficit may elect to invest in the income-drawdown facility with the hope of mortality credits, asset performance and a fall in inflation-linked annuity prices resulting in the individual becoming 100% funded at a future point in time. At such a time, an inflation-linked annuity could be purchased. Investigation of deferring the annuity purchase decision with reference to the funding level concept could provide an easier to understand framework for an individual.

The framework can be used to integrate pre- and post- retirement investment strategies. Members can evaluate their funding level prior to retirement and make changes to their pre-retirement investment strategy to target a 100% funding level. This may mean additional retirement saving contributions or investing in an investment strategy that targets 100% funding or preserves funding if the member has assets that exceeds the liability value determined.

The ruin probability measure does not take into account the extent of shortfall relative to the income required. For example, the inflation-linked annuity can produce a ruin probability of 100% if the income required is R1 more than that obtainable under the inflation-linked annuity. The framework thus does not take into account the “extent of ruin”. This should be incorporated as a secondary measure.

The propensity of the retiree to take risk and the utility associated with different income levels is not taken into account in the current framework. Eliciting utility functions that are based on the income requirements of the member should be investigated.

The analysis presented above does not investigate whether different life annuities would have minimised the ruin probability assuming different historic market prices.

Most importantly, the application of such a measure to actual decision making needs to be explored further.

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