Economic Values of Contribution Cashflows for a Sponsoring Employer of a DB Pension Plan and Measures to Bring the Economic Values under Control within an Affordable Range[1]

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September 2009

Indication of Topic:

Pensions: Managing accumulations and decumulation

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Key words:

contribution volatility, economic value, stochastic discount factor, marginal utility, funding standards, cashflow matching

[1] Any views or thoughts expressed in this paper are strictly those of the author and have no relation to the organisation to which the author is belonging.

Abstract

In many countries defined-benefit (DB) pension plans are suffering from sudden and significant funding gaps caused by the asset price depreciation and the interest rate decline in the current financial crisis. Many employers are required to pay additional contributions to make up for the shortfalls within a period of time permitted under the country-specific funding standards and/or by the competent supervising authorities. It should be noted that the additional contributions have to be made under such circumstances that the employers face difficulties in raising money.

Even if the plan had accumulated sufficient amounts of assets and its funded status had reached for instance 1.5 times of the minimum liabilities, the accumulated risk buffer would have evaporated in an instant when asset prices fell by 20% on average due to market depreciation and the amounts of liabilities rose simultaneously by 20% due to interest rate decline. This would pose a serious question on the effectiveness of the funding standards on the mark-to-market basis. At the same time, we have to keep in mind that it is inevitable to take into account to some extent the expected rates of return of risky assets in the calculation of contributions.

This paper first evaluates the economic value of the contributions (risk-adjusted costs) for the sponsoring employer using a stochastic discount factor and shows that the economic value of the contributions is much higher than their best-estimated present value. This paper then explores several measures to bring under control the economic values of the contributions from the aspects of benefit designs, funding policies and standards, and investment strategies.

With regard to benefit designs, this paper considers a structure in which a minimum benefits are supplemented by variable components, in order to lessen the burden of accumulating a fairly large amount of risk buffer, which would become indispensable under the mark-to-market accounting and funding standards. On funding policies and standards, this paper proposes the unique payout-year-specific (PYS) funding standard, under which assets and contributions are divided by payout year and loaded respectively on the 'sequentially chained containers.' The PYS funding standards then specify a sequence of minimum permissible funding ratios each of which is assigned to the corresponding container. Each minimum funding ratio would be a function of the period from the measurement date to the payout date when the assets are unloaded from the container. The PYS funding standards may require partial ring-fencing of assets by payout year, but allow taking into account the expected rates of return of risky assets progressively as the investment horizon extends and thus enable us to reduce the volatility of funded status significantly.

Furthermore, this paper considers appropriate investment strategies under this investment sympathetic PYS funding standards. One of the possible strategies would be waiting, separately by each container, for the chance that the amount of assets surpasses the value of corresponding liabilities, seizing the chance and switching the speculative strategies up to that time to a liability-hedging strategy. Since this strategy has countercyclical nature, there is no fear that the issue of "error of synthesis" might occur in the market.

For preserving favourable environments that sponsoring employers are willing to bear some portion of risks with regard to preparing steady post-retirement income streams for their employees, it is essential to retain the costs of maintaining such arrangements within an affordable range even under volatile market conditions and mark-to-market accountings. For this purpose, we have to devise innovative and synthetic measures covering all the aspects of benefit designs, funding policies and standards and investment strategies.

1 Introduction

In many countries, defined-benefit (DB) pension plans are suffering from sudden and significant funding gaps caused by the asset price depreciation and the interest rate decline in the current financial crisis. Many employers are required to pay additional contributions to make up for the shortfalls within a period of time admitted under country-specific funding standards and/or by the competent supervising authorities. It should be noted that the additional contributions have to be made under such circumstances that employers face difficulties in raising money.

Even if a plan had accumulated sufficient amounts of assets and its funded status had reached for instance 1.5 times of the minimum liabilities, the accumulated risk buffer would evaporate in an instant when asset prices fall by 20% on average due to market depreciation and at the same time the amounts of liabilities rise by 20% due to interest rate decline. This would pose a serious question on the effectiveness of the present funding standards on the mark-to-market basis. At the same time, we have to keep in mind that it is inevitable to take the expected rates of return of risky assets into account in the calculation of contributions.

This paper first evaluates the economic values of contributions made by sponsoring employers using a stochastic discount factor and shows that the economic values of employer contributions are much higher than their best-estimated present values. This paper then explores several measures to bring under control the economic values of contributions from the aspects of benefit designs, funding policies and standards, and investment strategies.

With regard to benefit designs, this paper considers a structure in which minimum benefits are supplemented by variable components, in order to lessen the burden of accumulating a fairly large risk buffer, which is normally indispensable under the mark-to-market accounting and funding standards. On funding policies and standards, this paper proposes the unique payout-year-specific (PYS) funding standard, under which assets and contributions are divided by payout year and loaded respectively on the 'sequentially chained containers.' The PYS funding standard then specifies a sequence of minimum funding ratios each of which is assigned to the corresponding container. Each minimum funding ratio would be a function of the period from the measurement date to the payout year when the assets are unloaded from the container. The PYS funding standard may require partial ring-fencing of assets by payout year, but allows taking into account the expected rates of return of risky assets progressively in discount rates as the investment horizon extends and thus enable the pension fund to reduce the volatilities in funded ratios and contributions significantly.

Furthermore, this paper considers investment strategies under this investment sympathetic funding standard. One of the possible strategies would be waiting, separately by each container, for the chance that the amount of assets surpasses the amount of corresponding liabilities, seizing the chance and switching the speculative strategies to a liability-hedging strategy. Since this strategy has countercyclical nature, there is no fear that the issue of "error of synthesis" might be raised in the market.

This paper is composed as follows. Section 2 evaluates the economic values of contribution cashflows using stochastic discount factors and analyses qualitatively the relationship between the economic values of the contribution cashflows and their volatilities, highlighting the difference with the relationship between the economic values of benefit cashflows and their volatilities. Section 3 introduces the idea that variable benefits supplement minimum benefits irrespective of the situation in benefit indexation and shows that this benefit design is a powerful device for pension plans to survive the economically unfavourable environments for pension funds known by the name of 'Japan scenario.' Section 4 explains in detail the idea of the payout-year-specific (PYS) funding standard and shows that there is a rational ground for taking into account the expected return of risky assets into discount rates. Section 5 considers the applicability of the hypothetical investment strategy assumed in the PYS funding standard and its implications. Section 6 concludes.

2 Economic values of contribution cashflows

2.1 Qualitative evaluation based on the covariance pricing formula

It is well known that under the assumption of complete market the economic value (namely, market consistent value) q of a contract that gives a payoff vector $\mathbf{v}$ is given by the following *covariance pricing formula*.

$$q = \frac{\mathrm{E}(\mathbf{v})}{R_F} + \mathrm{cov}(\boldsymbol{\xi}, \mathbf{v})$$

Here, $\mathrm{E}(\mathbf{v})$ denotes the expected value of the payoff vector $\mathbf{v}$ under the *original* probability measure and $\boldsymbol{\xi} = (\xi_j)'$ is the state price density. The risk free rate is denoted by $R_F = 1 + r_F$. On the other hand, we can understand the economic meaning of the state price density as follows. For simplicity let us consider one period model $t = 0, 1$ and the optimal consumption and the portfolio maximizing the following utility under budgetary constraints.

$$u_0(c_0) + \mathrm{E}^P\left[e^{-\beta} u_{1,j}(c_{1,j})\right]$$

Then the state price density ξ_j in the state ω_j is expressed as the ratio of the marginal utility of the optimal consumption $c^*_{1,j}$ in the state ω_j at $t = 1$ to the marginal utility of the optimal consumption c_0 at $t = 0$.

$$\xi_j = \frac{e^{-\beta} u_1'(c^*_{1,j})}{u_0'(c_0)}$$

Normally, the marginal utility of the optimal consumption of sponsoring employers would increase in the state of market downturn since employers face difficulty in raising money in such a state and decrease in a state of market upturn. Then such a cashflow that increases in the state of market downturn and decreases in the state of market upturn correlates positively to the state price density and thus the second term of the *covariance pricing formula* becomes positive. Employer contributions are a typical example of such cashflows.

Thus it is suggested that the economic value of a stream of employer contributions would be evaluated higher than the best-estimated present value of the stream. Besides, it can be said that the greater the volatility of contribution cashflows is the higher the economic value of the stream of the cashflows becomes, while the best-estimate present value of the stream under the *original* probability is largely constant. For instance, introduction of mark-to-market funding standards will increase the economic costs of managing defined benefit (DB) plans if the introduction of the mark-to-market standards would result in greater volatility of employer contributions.

2.2 Difference with the volatility of benefit cashflows

In the case of a stream of pension benefit cashflows, its volatility works toward the opposite direction. Normally, pension benefit cashflows do not increase in the state of market downturn. For instance, when the investment risk is partially or entirely transferred to plan participants, the benefit cashflows will be more or less affected by market downturn. A stream of benefit cashflows thus correlates negatively to the state price density. The economic value of a stream of benefit cashflows would be evaluated less than its best-estimated present value since the second term of the *covariance pricing formula* presented in the previous section becomes negative and the economic value becomes less than the expected value of the stream of benefits discounted by the risk-free rate.

In other words, the risk premium δ , namely the value of δ which makes the following equation hold, with regard to the volatility of the stream of benefits originated from the beta risk of investments, should be positive. On the other hand the risk premium δ of the stream of employer contributions for a DB plan originated from the beta risk of investments is negative, since instruments for hedging such risks are not always available in the market.

$$q = \frac{\mathrm{E}(\mathbf{v})}{R_F + \delta}$$

By the way, with regard to the volatility of benefits originated from the *macro* longevity risk, the risk premium of a life annuity should be negative. In the state that mortality rates improve beyond expectation, beneficiaries have to reduce their consumption due to budgetary constraints, which means that the marginal utility increases in such a state. On the other hand a life annuity provides predetermined amounts of annual benefits irrespective of the length of the remaining life of the annuitant. Thus the payoff of a life annuity increases in such a state that mortality rates improve beyond expectation and the payoff vector correlates positively to the state price density. The value of the second term of the *covariance pricing formula* presented in the previous section should therefore be positive and the risk premium of the benefits originated from the *macro* longevity risk should be negative.

2.3 Suggestions for benefit designs, funding policies and standards and investments

The above intuitive considerations give us a logical basis for incorporating measures for mitigating the volatilities of employer contributions. Namely, it is essential to devise mechanisms of bringing under control the economic values of employer contributions, if we want to preserve desirable risk-sharing features in post-retirement benefit arrangements under the accounting and funding standards founded on ‘mark-to-market’ valuation of assets and liabilities.

Although the minimum level of employer contributions to keep the plan financially sound is determined by the funding standards, more fundamentally the contribution volatility stems from the volatility in the economic values of promised benefits and the market values of assets. For this reason innovative and synthetic measures of mitigating volatilities in employer contributions are desperately needed covering all the aspects of benefit designs, funding policies and standards and investment strategies. In the following sections this paper proposes several measures of mitigating contribution volatility and preserving such favourable environments that sponsoring employers are willing to bear some portion of risks with regard to preparing steady income streams after retirement for their employees.

3 Minimum benefits supplemented by variable benefits

3.1 The rationale of admitting affordable risks in benefit designs

Until now several mechanisms have been devised for transferring part of the risks from sponsoring employers to participants with regard to preparing steady income streams after retirement, while maintaining the favourable characteristics of DB pension plans. It should be reminded that transferring some of the risks to participants is not only for the sake of mitigating volatilities in employer contributions and lessening the burden of employers.

In the present circumstances transferring part of the risks may also be beneficial to the participants. It is because, if the major risks listed in the previous section were entirely borne by a sponsoring employer and participants did not share any of these risks, it would become necessary that the sponsoring employer constructs a fairly large risk buffer in preparation for these risks. However, constructing a risk buffer is nothing but frontloading of employer contributions. And forced frontloading of employer contributions is often accompanied by benefit reduction. Sticking to traditional benefit designs is thus not always beneficial to participants.

Furthermore, it is well known that excessive risk aversion causes a significant fall of expected rates of return. The following example illustrates this relationship between risk aversion and expected rates of return. As an example, let us consider a mechanism that if the cumulative rate of return of an account of a participant is less than the predetermined *minimum* rate at her retirement age, then the ‘shortfall’ will be made up for by a buffer fund. Correspondingly, each participant would be required to leave the ‘surpluses in the buffer fund if her cumulative rate of return is greater than the predetermined *maximum* rate. The latter arrangement is for ensuring an economically fair trade between each participant and the buffer fund.

The essential point in this example is that the *maximum* rate is naturally determined from the given *minimum* rate and the condition of ensuring economically fair trades. When the *minimum* rate increases the *maximum* rate will decrease and eventually the two rates have to coincide at the point of the risk free rate, if there should be no chance of arbitrage. In short, we have to accept certain level of uncertainty on investment rates of return if we anticipate that expected rates of return on investment is greater than the risk free rate.

3.2 Variation of benefit designs and evaluation of their sustainability under the ‘Japan scenario’

There are various benefit designs, some of which transfers part of the risks to participants. Table 1 summarises the outlines of typical benefit designs for reference.

Table 1 Variation on Benefit Designs and Those Bearing Risks

Benefit Design	Outline	Those Bearing the Risk			
		Investment risk	**Interest rate risk at annuity conversion**	***Micro* longevity risk**	***Macro* longevity risk**
Individual DC Plans	Paying contributions by participant, managing balances by personal account and allowing individual choice on investment vehicles and the manners of decumulation	Individual Participants	Individual Participants	Individual Participants	–
Cash Balance (CB) Plans	Accumulation of notional credits and annuitisation of the cumulative credits at retirement by participant	Sponsor	Individual Participants	Pooled among Participants	Sponsor (in Japan)
Sequential Hybrid (Nursery) Plans	Individual DC plan during the accumulation phase and mandatory conversion to annuity at retirement	Employer (Payout Phase only)	Individual Participants	Pooled among Participants	Sponsor (Improvements beyond Expectation)
With Conditional Benefits Plans	Making part of the benefits (such as indexation of benefits) conditional on the financial situation of the plan	Shared between Sponsor and Participants	Sponsor	Pooled among Participants	Shared between Sponsor and Participants
Collective DC (CDC) Plans	Collective management of assets with no possibility of supplemental contributions paid with regard to the accrued benefits	Participants as a Group	Participants as a Group	Pooled among Participants	Participants as a Group
Traditional DB Plans	Final salary plan, carrier average earnings (CAE) plan, carrier average revalued earnings (CARE) plan, etc.	Sponsor	Sponsor	Pooled among Participants	Sponsor

At present, the most prominent design from the aspect of risk-sharing may be the conditional indexation plan that is widespread in the Netherlands. As to this design, annual indexation of benefits and past salaries to inflation or salary escalation is conditional on the cover ratio (funded ratio) of the plan. The following is an example of the conditional indexation of benefits and revaluation of past salaries. Let L_N and L_R denote the nominal and real amounts of the liabilities respectively, and A denote the amount of assets. Then the rate of benefit indexation and past salary revaluation is given by the following formulae (Ponds and Riel [2007]).

$$\text{if } L_R \le A, \quad \text{then} \quad 100\% \times \text{the rate of increase in salary index}$$

$$\text{if } L_N < A \le L_R, \quad \text{then} \quad \left(\frac{A - L_N}{L_R - L_N}\right) \times \text{the rate of increase in salary index}$$

$$\text{if } A < L_N, \quad \text{then} \quad 0\%$$

It should be noted that the surplus with regard to the nominal liabilities virtually functions as a risk buffer for keeping the funded status with regard to the nominal liabilities greater than 100%, on condition that the employer is not required to make supplemental contributions as long as the nominal funded ratio is greater than 100%.

From the aspect of mitigating employer contributions, the essential point of this design lies in the configuration that the normal contributions are determined taking into account the costs of benefit indexation and past salary revaluation, while the indexation component is not taken into account in calculating the minimum liabilities. Besides, the risk buffer is 'self-financed.' Namely, the cost of formulating the virtual risk buffer is evenly distributed among the years. Under this configuration the amount of assets will naturally surpass the minimum liabilities in such an environment that the economic assumptions will be realized progressively. And the 'surplus' thus formulated is expected to function as a virtual risk buffer. It should be noted here that the assumptions on future inflation and salary escalation in determining contributions play the central role in mitigating the volatility in employer contributions.

On the other hand, any assumptions should look realistic. Under the economic environments known by so-called 'Japan Scenario' that very low interest rates, inflation and salary escalation persist for decades while the volatility of stocks remains considerably high, the assumptions on future inflation and salary escalation would become low and as a result of it sufficient surplus with regard to nominal liabilities may not be accumulated. Thus there is a need to devise a more robust mechanism of constructing a virtual risk buffer even under such economically unfavourable environments for pension funds as the situation that the 'Japan Scenario' lasts persistently.

A possible idea may be explicitly introducing the minimum benefit $B^{(0)}$ and the maximum benefit $B^{(1)}$, both of which are indexed to inflation, and making the portion of benefits $B^{(1)} - B^{(0)}$ conditional on the funded status of the plan. Let $L^{(0)}$ denotes the amount of the liabilities corresponding to the minimum benefits $B^{(0)}$ and $L^{(1)}$ denotes the amount of liabilities corresponding to the maximum benefits $B^{(1)}$. The actual amount of benefits B is given by the following formulae.

$$\text{if } L^{(1)} \le A, \quad \text{then} \quad B = B^{(1)}$$
$$\text{if } A = L^{(0)} + \alpha\,(L^{(1)} - L^{(0)}) \quad (0 \le \alpha < 1), \text{ then } B = B^{(0)} + \alpha\,(B^{(1)} - B^{(0)})$$
$$\text{if } A < L^{(0)}, \quad \text{then} \quad B = B^{(0)}$$

Here the question is how to determine the minimum benefit $B^{(0)}$ and the maximum benefit $B^{(1)}$. It may be possible to set the *real* value of the minimum benefit is equivalent to the terminal value of the normal contributions at the payout date applying annually the *actual real* long-term government bond rate. Similarly, the *real* value of the maximum benefit is equivalent to the terminal value of normal contributions applying annually the sum of *actual real* long-term government bond rate and the *targeted real* excess rates of return on investments over the long-term government bond rate.

Then the *real* size of the conditional benefit relative to the minimum benefit is independent from the actual development of inflation. Besides, investment risk is not solely borne by the beneficiaries at the time since funding surpluses with regard to the minimum benefits will be allocated impartially among active participants and beneficiaries. Furthermore, this benefit design is sympathetic to the reality of actual investments.

It should be noted here that market valuation of conditional benefits is problematic since the market-value of the conditional benefits increases along with the funded ratio of the plan (Rooij, Siegman and Vlaar [2007]). Besides, if the contribution rates are determined from the present value of the stream of expected benefit cashflows discounted by market risk free rates, then the volatility of contribution rates increases substantially and the economic value of contributions will become too much expensive for the sponsoring employers. Thus one of the favourable points in introducing conditional benefits is that it enables the pension plan to set the contribution rates with relatively stable discount rates, taking into account the expected returns of actual portfolio (Rooij, Seigmann and Vlaar [2007]). We will give further consideration to this point in section 4.

In any way, there is no possibility that the actual economy develops just as anticipated at the time of setting up the contributions. It is therefore highly desirable to incorporate a *retrospective* adjustment mechanism of benefits (and contributions) *in advance* even in an occupational pension plan, as in the public pensions. In the field of the public pensions, there are several good examples such as the '*automatic balancing mechanism*' of Sweden, the '*sustainability factor*' of Germany and the '*automatic stabilizers*' of Canada. Some of these ideas in the field of public pensions may also be applicable to funded occupational pension plans with slight modifications, although we have to pay attention to the differences with regard to the conditions to be satisfied in funded occupational pensions and those of public pensions financed on a pay-as-you-go basis.

4 Payout-year-specific funding standards

4.1 Partial ring-fencing of plan assets by payout year

4.1.1 A common idea in funding standards and implicit risk-sharing in DB plans

In a typical funding standard, estimated future benefit cashflows are discounted back to the measurement date and summarised into a single value as a total amount of liabilities. Then the total amount of assets at the measurement date is compared to the total amount of liabilities and the ratio thus obtained is used as a single measure on the financial status of the plan. Although there may be several variations about the estimation of future cashflows (which portion should be taken into account, etc.) and discount rates, there is a common idea among various funding standards that the financial status of a pension plan can be measured by simply comparing the two aggregate values, namely the total value of the portfolio and the total value of the liabilities.

This invariant idea might be originated from the common practice in present DB pension plans that no specific correspondence is established between individual liabilities and plan assets. Namely, plan assets are never allocated to specific participants or specific payout years. Here, it should be noted that the meaning of 'individual' is twofold. One meaning is that total liabilities are the composite of the liabilities that the plan (or the sponsoring employer) owes (or is anticipated to owe) against individual participants. The other is that total liabilities are composed of the liabilities which are to be settled in each payout year. As far as funding standards are concerned, we usually pay attention to the former decomposition. For instance, the liabilities against retired participants are evaluated using risk-free rates whereas the liabilities against active participants are often evaluated taking into account the expected investment returns of risky assets. However, in this paper we turn our attention to the second kind of decomposition in order to remedy the shortcoming of present DB plans explained below and devise a more trustworthy and investment-sympathetic funding standard.

We can point out a serious deficiency in present DB plans caused by the practice explained above that no restriction is imposed on how to raise capital for paying out benefits of each year. Namely, as long as the plan is alive, the interests of the beneficiaries are given the most privileged status and the plan has to keep paying out at least their predetermined minimum pensions irrespective of the financial status of the plan. If the plan has a funding shortfall and benefits payable in the year are paid out in full, as a matter of course the funding shortfall will enlarge and as a result the risk of active participants will expand.

A fundamental problem to be reminded here is that in a traditional DB plan there is no effective and timely mechanism of keeping under control the risks of active participants. It may be possible to require immediate supplementation of funding shortfalls. But such a requirement is counterproductive since it would result in substantial increase of the economic value of employer contributions. It is therefore impractical to require *full funding at all times*. Any funding standard thus 'reluctantly' accepts that pension plans or plan sponsors make up for funding deficiencies gradually spending several years or decades. To put it in another way, admitting funding shortfalls should be recognised as an important function of DB plans to enable *intertemporal* risk-sharing. However, at the same time we should remind that under the present practice on raising capital for paying out benefits, cash is draining away from the fund without any hindrance.

On the other hand the assets of an individual DC plan are completely ring-fenced. Even if the account of a participant has a 'surplus' compared to her targeted benefits after retirement, the surplus is never used for making up for the funding deficiencies of other participants. Namely, in an individual DC plan a financial firewall is set up and it protects the interests of individual active participants especially when the retired participants are suffering from large financial shortages. It should be noted that this mechanism gives active participants a certain sense of security, although this complete ring-fencing of assets by participant looks too rigid since it does not allow any risk-sharing among the participants. Introducing a mechanism of partial ring-fencing of plan assets by payout year in a DB plan might be thus desirable from the aspect of remedying the shortcoming in the present DB structure and realising fair treatments of the participants, while providing adequate flexibility in financing of DB plans.

4.1.2 Decomposition of contributions by payout year and partial ring-fencing of assets

The payout-year specific (PYS) funding standard focuses on the characteristic of individual DC plans mentioned in the previous section and introduces partial ring-fencing of assets by payout year rather than by participant. Let us suppose that the contributions are divided by payout year and loaded respectively on the 'sequentially chained containers.' A payout year is assigned to each container and the contributions loaded on the container and their investment income can only be used for the payment of benefits in the year. The funded status of a container thus does not affect those of other containers. A surplus of a container may be used for filling up the shortfalls of other containers as a mechanism of *intertemporal* risk-sharing. However, it should be strictly prohibited to fill up a funding deficiency of a container with aggravating the funding deficiencies of other containers, since we want to keep the risk of active participants under control. Let us define the meaning of the 'partial ring-fencing' of assets in this way.

The PYS funding standard specifies a sequence of minimum permissible funding ratios each of which is assigned to the corresponding container. The essential point of the PYS funding standard is how to determine the minimum funding ratio of each container. A basic idea of determining the minimum funding ratios is explained in the next section. But it will be easily anticipated that the minimum funding ratio shall be a function of the remaining period until the year of maturity, market risk free rates, the expected excess rate of return and the expected volatility, and the degree of risk aversion. Of course any funding deficiency of a container (if any) shall be made up for until the year of maturity assigned to the container. In this way we can naturally determine the period of time during which the funding deficiencies have to be corrected. This is one of the favourable points of the PYS funding standard.

The value of the assets loaded on a container at a measurement date is equivalent to the terminal value of the contributions loaded on the container and their investment income subtracted by the terminal value of benefits already paid out until the measurement date. The question is therefore how to decompose annual contributions into portions with specified payout years. But when the contributions are calculated on the basis of the (projected) unit cost method, this decomposition is naturally given by discounting back the stream of the estimated future cashflows of benefits accrued during the year, using discount rates *appropriate* for each remaining period until the year of maturity. Thus the question of how to decompose the contributions is reduced to the question of how to determine the *appropriate* discount rates for each container depending on the period until the year of maturity.

4.1.3 Discount rates and the possibility of full funding at the payout year

It is inappropriate to taking into account the expected rate of return of the portfolio of the plan entirely in the discount rate of the PYS funding standard since the probability that the value of the assets loaded on a container will surpass the amount of benefits at the year of maturity is no more than 50%. This is of course a trivial matter but it is worth looking at the situation more precisely since it will serve as a background of the main idea of the PYS funding standard. Let us suppose that the value of the portfolio follows the standard geometric Brownian motion:

$$dA_u = r\,A_u\,du + \sigma\,A_u\,dW_u \qquad u \in [t\,,T]$$

Then the portfolio value at time T is expressed as follows.

$$A_T = A_t \exp\{(r - \frac{1}{2}\sigma^2)(T-t) + \sigma(W_T - W_t)\}$$

If the initial portfolio value A_t is equivalent to the discounted value of the benefits L_T payable in the year T using the expected rate of portfolio return r :

$$A_t = L_T \exp\{-r(T-t)\}$$

then the probability P_0 that the portfolio value A_T at time T becomes greater than the amount of benefits equals

$$P_0 = N(-\frac{1}{2}\sigma\sqrt{T-t})$$

where N is the cumulative standard normal distribution function (see Broeders [2006]). It should be noted that the probability that the portfolio value at time T surpasses the amount of benefits in the year is less than 50%. Furthermore, the probability P_0 diminishes as the volatility σ increases or the remaining period $T-t$ extends. Thus it cannot be said prudent to evaluate the funded status and calculate normal contributions using the expected rate of return of the actual portfolio as the discount rate of liabilities.

Of course we can expect supplemental contributions that a sponsoring employer shall make when the plan is in underfunded statuses. However, the probability that the portfolio value will eventually surpass the minimum benefits in the year of maturity should be high enough to give the sponsoring employer and the participants the sense of security under the current investment strategies. On the other hand applying volatile market risk free rates for calculating normal contributions is also not recommended since it increases the absolute amounts and the volatility of contributions significantly and the sponsoring employer might not afford such an expensive economic value of contributions. We have to devise a mechanism of determining appropriate discount rates of liabilities, taking into account the expected rates of return and volatilities of risky assets.

4.2 Determination of investment horizon-specific discount rates

4.2.1 Main ideas of the PYS funding standard

The mechanism of the PYS funding standard is based on the fact that the probability P_1 that the portfolio value will attain the liability value at *some* time until the year of maturity is higher than the probability P_0 that the portfolio value will surpass the liability value at the year of maturity. Therefore, if the portfolio is switched to a liability-hedging portfolio immediately when the portfolio value firstly hits the targeted liability value, then the portfolio value at the year of maturity will be equal to the amount of benefits to be paid in the year. Thus with this hypothetical investment strategy we can expect a greater probability of fully funded status at the year of maturity than the strategy of simply maintaining the 'strategic' portfolio. In this sense, it can be said that this hypothetical strategy is superior to the static strategy of just holding a strategic portfolio. It should be noted here that this hypothetical strategy is a dynamic strategy but has a countercyclical nature.

This hypothetical strategy is made up of the following two components. One is setting up of an upper barrier and the other is automatic switching to a liability-hedging portfolio when the process X_u of the log funded ratio firstly hits the upper barrier. Here we suppose such a benefit structure that minimum benefits are supplemented by conditional benefits as explained in section 3.2. Additionally, we assume that the portfolio value follows the standard geometric Brownian motion as in the previous section. Then the portfolio value at time u is expressed as follows:

$$A_u = A_t \exp\{(r - \frac{1}{2}\sigma^2)(u-t) + \sigma(W_u - W_t)\}$$

where W_u is a Wiener process. We also assume that the liability value B_u at time u, which is corresponding to the targeted benefit $L^{(1)}$ payable in the year of maturity T, follows the differential equation:

$$dB_u = r_F B_u \, du \quad u \in [t, T]$$

Similarly, the liability value C_u, which is corresponding to the minimum benefit $L^{(0)}$ payable in the year of maturity T, follows the differential equation:

$$dC_u = r_F C_u \, du \quad u \in [t, T]$$

Then the liability values at time u are:

$$B_u = B_t \exp\{r_F (u-t)\}, \quad B_t = L^{(1)} \exp\{-r_F (T-t)\}$$

$$C_u = C_t \exp\{r_F (u-t)\}, \quad C_t = L^{(0)} \exp\{-r_F (T-t)\}$$

The log funded ratios of the plan at time u in comparison with B_u and C_u are given by the followings:

$$X_u = \log\frac{A_u}{B_u} = (\mu - \frac{1}{2}\sigma^2)(u-t) + \sigma(W_u - W_t) + \log\frac{A_t}{B_t}$$

$$Y_u = \log\frac{A_u}{C_u} = (\mu - \frac{1}{2}\sigma^2)(u-t) + \sigma(W_u - W_t) + \log\frac{A_t}{C_t}$$

$$\mu = r - r_F$$

We assume here that the funded ratio at time t in comparison to the liability B_t is less than 1, since we want to verify whether including the excess returns of risky assets in the discount rate of liabilities is justifiable. Namely, if we put $\alpha_t = \log(A_t / B_t)$, then

$$\alpha_t = \log\frac{A_t}{B_t} < 0$$

The log funded ratio thus follows a Wiener process with constant drift $\mu - \frac{1}{2}\sigma^2$ and constant diffusion σ , starting at a point α_t .

$$dX_u = (\mu - \frac{1}{2}\sigma^2)du + \sigma dW_u$$
$$X_t = \alpha_t$$

Then the distribution function of the following *running maximum process*:

$$M_X(u) = \sup_{t \le s \le u} X_s$$

is given by the following expression, which holds for $x \ge \alpha_t$ (see for instance Bjork [2004]).

$$F_{M(u)}(x) = N\left(\frac{(x-\alpha_t) - (\mu - \frac{1}{2}\sigma^2)(u-t)}{\sigma\sqrt{u-t}} \right) - \exp\left\{ 2\frac{(\mu - \frac{1}{2}\sigma^2)(x-\alpha_t)}{\sigma^2} \right\} N\left(-\frac{(x-\alpha_t) + (\mu - \frac{1}{2}\sigma^2)(u-t)}{\sigma\sqrt{u-t}} \right)$$

Therefore the probability P_1 that the portfolio value will attain the liability value at *some* time until the year of maturity is given by the following expression:

$$P_1 = 1 - F_{M(T)}(0) = 1 - N\left(\frac{-\alpha_t - (\mu - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \right) + \exp\left\{ 2\frac{(\mu - \frac{1}{2}\sigma^2)(-\alpha_t)}{\sigma^2} \right\} N\left(-\frac{-\alpha_t + (\mu - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \right)$$

$$= N\left(\frac{\alpha_t + (\mu - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \right) + \exp\left\{ -2\frac{\alpha_t(\mu - \frac{1}{2}\sigma^2)}{\sigma^2} \right\} N\left(\frac{\alpha_t - (\mu - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \right)$$

In the case that the expected excess rate of return is fully taken into account in the discount rate, then the initial log funded ratio is equal to

$$\alpha_t = -\mu\,(T-t)\,.$$

Therefore the probability P_1 is expressed as follows:

$$P_1 = N(-\frac{1}{2}\sigma\sqrt{T-t}) + \exp\left\{ 2\frac{\mu(\mu - \frac{1}{2}\sigma^2)(T-t)}{\sigma^2} \right\} N\left(-\frac{2\mu\sqrt{T-t}}{\sigma} + \frac{1}{2}\sigma\sqrt{T-t} \right)$$

The first term of the right hand side of the above equation is equal to the probability P_0 and the second term is always positive. For instance, in the case $T = 10$, $r = 0.08$, $r_F = 0.05$ and $\sigma^2 = 0.02$, the probabilities P_0 and P_1 are 41.2% and 65.2% respectively. In this case, the probability P_1 is 24% points higher than the probability P_0 . We have thus confirmed the basic background on which the PYS funding standard is founded.

If we set up a condition that the probability P_1 should not be smaller than constant p_1 (for instance $p_1 = 0.8$), corresponding to each portfolio on the efficient frontier we can determine the minimum initial funded ratio α_t that satisfies the condition. In other words, we can derive the maximum permissible proportion θ_t of the expected excess return $\mu = r - r_F$ which can be taken into account in the discount rate of liabilities from the condition $P_1 \geq p_1$ and the period $[t, T]$.

Namely, when the proportion θ_t is given, the initial portfolio value A_t is

$$A_t = L_T \exp\{-(\mu\theta_t + r_F)(T-t)\}$$

and the initial log funded ratio α_t equals

$$\alpha_t = -\mu\theta_t\,(T-t)$$

Then the condition $P_1 \geq p_1$ becomes

$$P_1 = N\left(\frac{\{\mu(1-\theta_t) - \frac{1}{2}\sigma^2\}(T-t)}{\sigma\sqrt{T-t}}\right) + \exp\left\{-2\frac{\mu(\mu-\frac{1}{2}\sigma^2)\theta_t(T-t)}{\sigma^2}\right\} N\left(\frac{\{-\mu(1+\theta_t)+\frac{1}{2}\sigma^2\}(T-t)}{\sigma\sqrt{T-t}}\right) \geq p_1$$

We can thus determine the maximum permissible proportion θ_t satisfying the above condition.

4.2.2 Additional conditions restricting the risk of funding shortfalls

The argument in the previous section does not answer the question of which portfolio we should choose from the efficient frontier in order to determine the appropriate discount rates of liabilities as a basis of the PYS funding standard. Especially, it is easily anticipated that the larger the volatility of the portfolio is, the higher the probability P_1 would become when the initial funded ratio is invariant. Therefore, if we want to determine unique discount rates for the PYS funding standard, we have to introduce another condition, especially from the aspect of restricting the risk of underfunding.

Firstly let us consider the case $L^{(0)} = L^{(1)}$. A candidate of such conditions may be restricting the severity of loss when the portfolio value could not have attained the liability value at *any* time before the year of maturity. However, for simplicity we consider here a rough approximation of this condition that the conditional expectation of the portfolio value at the year of maturity is within an affordable range, given that the portfolio value at the year of maturity is less than the value of the liability $L^{(0)} = L^{(1)}$.

$$\frac{E[A_T | A_T < L^{(0)}]}{L^{(0)}} \geq q, \quad 0 < q < 1$$

This conditional expectation of the portfolio value, which is known by the *loss given default*, is given by the following expression (Broeders [2006]).

$$LGD = E[A_T | A_T < L^{(0)}] = A_t \exp\{r(T-t)\} \frac{N\left(-\frac{\log(\frac{A_t}{L^{(0)}}) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right)}{N\left(-\frac{\log(\frac{A_t}{L^{(0)}}) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right)}$$

Therefore, this additional condition in the case $L^{(0)} = L^{(1)}$ is expressed as follows:

$$\exp\{(1-\theta_t)\mu(T-t)\} \frac{N\left(-\frac{\{(1-\theta_t)\mu + \frac{1}{2}\sigma^2\}(T-t)}{\sigma\sqrt{T-t}}\right)}{N\left(-\frac{\{(1-\theta_t)\mu - \frac{1}{2}\sigma^2\}(T-t)}{\sigma\sqrt{T-t}}\right)} \geq q$$

It is well known that any portfolio on the efficient frontier is expressed as a linear combination of a speculative portfolio and a liability hedging portfolio (the *two-fund separation theorem*). Since the above condition gives a restriction on the volatility of the portfolio, we can thus derive the maximum weight w_t of the speculative portfolio from the above condition, when we assume an appropriate combination of assumptions.

Secondly let us consider the case $L^{(0)} < L^{(1)}$. If the gap between the two liabilities is fairly large and $\beta_t = \log\frac{A_t}{C_t} > 0$, then we can introduce a condition that the log funded ratio Y_u in comparison to the minimum liability value approaches from the above and is absorbed into the lower barrier at *some* time until the year of maturity with the probability less than constant p_2. Here, the lower barrier means the minimum liability value:

$$C_u = C_t \exp\{r_F(u-t)\}, \quad C_t = L^{(0)} \exp\{-r_F(T-t)\}$$

As in the case of X_u, the log funded ratio Y_u follows a Wiener process with constant drift $\mu - \frac{1}{2}\sigma^2$ and constant diffusion σ, starting at a point $\beta_t > 0$.

$$dY_u = (\mu - \frac{1}{2}\sigma^2)du + \sigma dW_u$$
$$Y_t = \beta_t$$

Then the distribution function of the following *running minimum process*

$$m_Y(u) = \inf_{t \le s \le u} Y_s$$

is given by the following expression, which hold for $y \le \beta_t$ (see for instance Bjork [2004]).

$$F_{m(u)}(y) = N\left(\frac{(y-\beta_t)-(\mu-\frac{1}{2}\sigma^2)(u-t)}{\sigma\sqrt{u-t}} \right) + \exp\left\{ 2\frac{(\mu-\frac{1}{2}\sigma^2)(y-\beta_t)}{\sigma^2} \right\} N\left(\frac{(y-\beta_t)+(\mu-\frac{1}{2}\sigma^2)(u-t)}{\sigma\sqrt{u-t}} \right)$$

Then the probability P_2 that the portfolio value will be absorbed into the lower barrier of the minimum liability value at *some* time until the year of maturity is expressed as follows:

$$P_2 = F_{m(u)}(0) = N\left(\frac{-\beta_t-(\mu-\frac{1}{2}\sigma^2)(u-t)}{\sigma\sqrt{u-t}} \right) + \exp\left\{ 2\frac{(\mu-\frac{1}{2}\sigma^2)(-\beta_t)}{\sigma^2} \right\} N\left(\frac{-\beta_t+(\mu-\frac{1}{2}\sigma^2)(u-t)}{\sigma\sqrt{u-t}} \right)$$

When the proportion θ_t is given, the initial log funded ratio β_t is:

$$\beta_t = -\mu\theta_t\,(T-t) + \log(\frac{L^{(1)}}{L^{(0)}}) > 0$$

Then the additional condition $P_2 < p_2$ is expressed as follows:

$$P_2 = N\left(-\frac{\{\mu(1-\theta_t)-\frac{1}{2}\sigma^2\}(T-t)+\log(\frac{L^{(1)}}{L^{(0)}})}{\sigma\sqrt{T-t}} \right) + \exp\left\{ 2\frac{(\mu-\frac{1}{2}\sigma^2)\{\mu\theta_t\,(T-t)-\log(\frac{L^{(1)}}{L^{(0)}})\}}{\sigma^2} \right\} N\left(\frac{\{\mu(1+\theta_t)-\frac{1}{2}\sigma^2\}(T-t)-\log(\frac{L^{(1)}}{L^{(0)}})}{\sigma\sqrt{T-t}} \right) < p_2$$

This condition determines the maximum volatility among the set of the portfolios on the efficient frontier and the corresponding maximum permissible proportion θ_t of the excess return that can be taken into account in the discount rates. Thus we can derive a unique discount rate from the two conditions $P_1 > p_1$ and $P_2 < p_2$, on condition that the buffer between the targeted benefit and the minimum benefit is large enough to satisfy the following condition.

$$\log(\frac{L^{(1)}}{L^{(0)}}) > \mu\theta_t\ (T-t)$$

4.3 Numerical examples of discount rates satisfying the two conditions

Table 2 below shows examples of the combination (w_t, θ_t) satisfying the conditions set out in the previous two sections in the case $L^{(0)} = L^{(1)}$ with several combinations of $T-t,\ p_1, q$. Here we assumed $r_F = 0.05,\ r = 0.08,\ \sigma^2 = 0.02$ and used the following approximation of the log return of the portfolio:

$$r_p = w_t\,(r - r_F) + r_F + \frac{1}{2}\sigma^2\, w_t\,(1 - w_t)$$

For simplicity, we also assumed that the return of on the liability-hedging portfolio is constant and the yield curve is flat.

Table 2 Maximum permissible proportions of excess returns in the case $L^{(0)} = L^{(1)}$

$L^{(1)}/L^{(0)}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$T-t$	10.00	10.00	10.00	20.00	20.00	20.00	5.00	5.00	5.00
p_1	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
q	0.80	0.90	0.95	0.80	0.70	0.60	0.80	0.90	0.95
w_t	0.74	0.35	0.17	0.57	0.91	1.26	0.94	0.46	0.23
θ_t	0.49	0.50	0.51	0.44	0.40	0.40	0.69	0.62	0.60
r_P	7.40	6.29	5.66	6.97	7.81	8.45	7.88	6.63	5.85
$\exp\{-(r_P - r_F)(T-t)\}$	0.79	0.88	0.94	0.67	0.57	0.50	0.87	0.92	0.96
$\exp\{\alpha_t\}$	0.89	0.94	0.97	0.84	0.80	0.76	0.90	0.95	0.97

Table 2 justifies the argument in the previous section that a certain proportion of the expected excess return of the speculative portfolio can be taken into account in the discount rate of the PYS funding standard, since there exists a combination (w_t, θ_t) satisfying both the condition on the probability with regard to attaining the targeted liability and the condition on the severity of the loss given default. For instance, when the period until the year to maturity is 10 years and if the pension plan accepts the loss given default (LGD) at the level of 80% of the targeted liability, then we can take into account about 49% of the expected excess return of the actual portfolio in the discount rate and we can still expect that the portfolio will attain the targeted liability with 70% probability. It should be noted that the maximum proportion of the expected excess return of the actual portfolio that can be included in the discount rate diminishes gradually as the acceptable risk on the LGD increases. Besides, the mark-up to the risk free rate (namely the product of w_t and θ_t) also diminishes gradually as the plan accepts greater risks on the LGD.

It should be reminded that the maximum permissible proportion of the expected excess return of the actual portfolio diminishes as the time horizon extends. As a result, the possible mark-up to the risk free rate (namely the product of w_t and θ_t) also diminishes gradually as the time horizon extends, when the acceptable risk on the LGD is kept constant. In other words, if a same mark-up to the risk free rate is assumed, then the pension plan should accept greater risks on the LGD as the investment time horizon extends. For instance, in the case $(T-t,\ p_1, q)=(10, 0.7, 0.2)$ the permissible mark-up is 0.36%, which is roughly same as in the case $(T-t,\ p_1, q)=(20, 0.7, 0.3)$.

These results might look contradictory to the common understanding that the longer the investment time horizon is, the more a pension plan can take risks. However, we should be fully aware that investments with long time horizons do not assure high return with higher probability than investments with short time horizons. The true reason why pension plan can take greater investment risks under long time horizons is merely that funding deficiencies can be made up for with smaller contributions and spending the given long period.

In **Table 2**, we do not consider the relation between the period until the year of maturity and the acceptable risk on the LGD. It is naturally anticipated that the acceptable risk on the LGD diminishes as the period until the year of maturity decreases. **Table 3** below provides some observation on the maximum permissible proportion of the expected excess return under varying investment time horizon.

Table 3 Maximum permissible proportions of excess returns under varying time horizon

$L^{(1)}/L^{(0)}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$T-t$	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
p_1	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80
q	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90
w_t	0.10	0.14	0.17	0.20	0.23	0.27	0.30	0.32	0.35	0.38
θ_t	0.78	0.56	0.46	0.41	0.38	0.35	0.34	0.32	0.31	0.31
$w_t \times \theta_t$	7.51	7.72	7.97	8.31	8.82	9.45	9.97	10.41	10.99	11.50
r_P	5.37	5.54	5.66	5.77	5.88	5.99	6.10	6.18	6.28	6.37
$\exp\{-(r_P - r_F)(T-t)\}$	1.00	0.99	0.98	0.97	0.96	0.94	0.93	0.91	0.89	0.87
$\exp\{\alpha_t\}$	1.00	0.99	0.99	0.99	0.98	0.98	0.97	0.97	0.96	0.96

If the acceptable risk on the LGD increases linearly from 1% of the minimum benefits when the time horizon is 1 year to 10% when the time horizon is 10 years, the maximum permissible proportion of the expected excess return of the speculative portfolio increases gradually from 7.5% to 11.5%. If the acceptable risk on the LGD is 20% of the minimum benefits when the time horizon is 20 years, then the maximum permissible proportion become 25.4%, which can be observed in **Table 2**. Thus it is shown that the expected rate of return of speculative portfolio can be included in the discount rates progressively as the investment horizon extends, when the admissible risk on the LGD increases along with the investment time horizon.

However, it may be natural to consider that there exists a due limit on the acceptable risk on the

LGD irrespective of the time horizon. In such a case it is anticipated that the graph of the maximum permissible proportion of the expected excess return becomes hump shaped. This anticipation coincides with the results by Bodie and Treussard [2007].

For the case $L^{(0)} < L^{(1)}$, **Table 4** below shows examples of the combination (w_t , θ_t) satisfying the conditions set out in the previous sections with several combinations of $T-t$, p_1, p_2 . The assumptions on r_F , r, σ^2 and the approximation of the log return on the portfolio are same as above.

Table 4 Maximum permissible proportions of excess returns in the case $L^{(0)} < L^{(1)}$

$L^{(1)} / L^{(0)}$	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50
$T-t$	10.00	10.00	10.00	20.00	20.00	20.00	5.00	5.00	5.00
p_1	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
p_2	0.20	0.30	0.40	0.20	0.30	0.40	0.20	0.30	0.40
w_t	0.68	0.80	0.92	0.48	0.56	0.64	0.92	1.09	1.27
θ_t	0.49	0.49	0.50	0.46	0.44	0.43	0.69	0.73	0.79
r_P	7.25	7.55	7.84	6.68	6.91	7.14	7.84	8.18	8.47
$\exp\{-(r_P - r_F)(T-t)\}$	0.80	0.77	0.75	0.71	0.68	0.65	0.87	0.85	0.84
$\exp\{\alpha_t\}$	0.90	0.88	0.87	0.86	0.84	0.83	0.91	0.89	0.87
$\exp\{\beta_t\}$	1.34	1.32	1.30	1.29	1.27	1.25	1.36	1.34	1.31

Table 4 also justifies the argument that a certain proportion of the expected excess return of the risky assets can be included in the discount rate. For instance, when the period until the year of maturity is 10 years and the pension plan accepts the risk that the funded ratio hits the minimum liability value at *some* time with 20% probability at maximum, then we can include 49% of the expected excess return of the actual portfolio in the discount rate and we can still expect that the portfolio value will attain the targeted liability value at *some* time with 70% probability. The maximum permissible proportion is almost stable when the probability that the funded ratio is absorbed into the lower barrier is within the range of 20 ~ 30%. The mark-up to the risk free rate (namely the product of w_t and θ_t) increases gradually as the plan accepts greater risks of absorption.

The maximum permissible proportion diminishes as the time horizon extends, and as in the case $L^{(0)} = L^{(1)}$. The permissible mark-up (namely the product of w_t and θ_t) to the risk free rate also diminishes gradually as the time horizon extends, when the acceptable risk of absorption is kept constant. Therefore, **Table 4** also contradicts the common understanding that the longer the investment time horizon is the greater risk a pension plan can take. However, it is quite natural since investments with long time horizon carry greater uncertainty.

4.4 Implications for traditional funding standards and benefit designs

The observations in the previous sections give us important implications for traditional funding standards, although the observations are based on the hypothetical PYS funding standard. As pointed out above, there are two fundamental variables in traditional funding standards, namely the discount rates and the period that a pension plan can spend for correcting funding deficiencies. On discount rates of liabilities, it is shown that there is a rational basis for incorporating the expected excess return to some extent into discount rates. However, the maximum proportion of the expected excess return that can be added to the risk free rates depends on the average period until the payout years. The longer the average period is, the smaller the maximum proportion becomes, when we assume a due limit on the acceptable risk on funding deficiencies. It may look contradictory to the common understanding.

On the periods of correcting funding deficiencies, it is shown that the maximum admissible period should be recognised as a function of the remaining periods until the years of maturity, as far as the funding standards on the ongoing basis are concerned. Namely, we cannot determine the maximum admissible periods arbitrarily even under the general trends toward the ‘mark-to-market’ valuation of liabilities. It may be necessary to allocate the funding shortfalls to each payout year to make progress in the issue of intertemporal risk-sharing with regard to funding deficiencies. For instance, the amount of supplemental contributions in a year should not be less than the sum of the interests on the total deficiencies and the amount of the shortfall allocated to the year.

It is said that putting a mark-up on the risk free rate reduces volatility of contributions (Rooij, Siegmann and Vlaar [2007]). However, the absolute amount of the permissible mark-up can be determined by restrictions on the probabilities that the funded ratio hits the upper and lower boundaries defined by the targeted and minimum liability values respectively. Any funding standard should thus strike an appropriate balance between securing adequate size of the mark-up and ensuring such high probabilities of attaining the targeted liability value and not being absorbed into the solvency trap that give participants and employers a sense of security.

5 Implications for investment strategies

5.1 Pension assets as a composite of target year funds

Irrespective of the funding standards, it is always possible to consider that the portfolio of pension plan assets is a composite of the target year funds of which the year of maturity coincides with the year of disbursement assigned to the fund. Individual target year fund is composed of a speculative portfolio and a liability-hedging portfolio that is made up from zero-coupon TIPS.

The decomposition of the total portfolio by payout year makes the discussion on the time horizon of investments extremely transparent. For instance, it becomes clear that the total portfolio should be rebalanced every year along with the changes in the estimation on the stream of benefit cashflows, even if the prospect on the market is invariant. It is undeniable that traditional funding standards leave opaque the consideration on the appropriate investment time horizon of DB plan assets.

Besides, we can apply lot of knowledge on the target date funds (TDFs). For instance, the degree of risk aversion of each target year fund may vary along with the funded status of the fund. If the individual target year funds should be rebalanced along with its funded status, the total portfolio should also be rebalanced, even if the prospect on the market is invariant. The PYS funding standard is thus very sympathetic to actual investments strategies. Any funding standard cannot continue to exist without paying proper consideration to investments and *vice versa*. It can be said that the PYS funding standard is a bridge connecting the financing issues and investment issues systematically.

5.2 Applicability of automatic switching to a liability-hedging strategy

There is a common practice in the present TDFs strategies that the proportion invested in risky assets is automatically reduced as the year of maturity approaches. However, this practice is not consistent with the dynamic investment strategy assumed in the PYS funding standard that the speculative portfolio is switched to a liability-hedging portfolio immediately when the fund attains the fully funded status.

Here we consider implementation issues of this hypothetical investment strategy. The suggested strategy is waiting, separately by each container, for the chance that the asset value surpasses the value of corresponding liabilities assigned to the container, seizing the chance and switching the speculative portfolio to a liability-hedging portfolio. The question is whether such a dynamic strategy is feasible from the investment viewpoints. For instance, the speculative portfolio has to be sold off when the value of the portfolio appreciates and the participants have to and give up further investment gains which may be highly probable under the market condition at the time. Therefore a firm governance structure is required since this strategy is not always 'desirable,' as pointed out by Bogers [2009]. However, it can also be said that there is no fear that the issue of "error of synthesis" might be raised in the market, since this strategy has countercyclical nature when the market is in upward trends.

On the other hand, when the portfolio value continues staying below the upper barrier and is occasionally absorbed into the lower barrier of the minimum liability, the speculative portfolio has to be sold off to buy the liability-hedging portfolio. Thus this hypothetical strategy is similar to the constant proportion portfolio insurance (CPPI) and thus we have to pay attention to the characteristic that this strategy has procyclical nature when the market is in downward trends. One of the possible measures for remedying this drawback may be not widening excessively the gap between the targeted benefits and minimum benefits.

6 Conclusion

This paper evaluated the economic values of the stream of employer contribution cashflows to a pension plan where the employer bears certain portion of risks using the covariance pricing formula and showed that the greater the volatility of contribution cashflows is, the higher the economic value of the stream becomes. It is thus essential to devise mechanisms of bringing under control the volatility of employer contribution cashflows in order to preserve desirable features of risk-sharing between the employer and the participants in post-retirement benefit arrangements.

As for benefit designs, participants have to accept certain level of downside risks since excessive risk aversion causes a considerable fall of expected rates of return. Under the environment of 'mark-to-market' valuation of assets and liabilities, pension plans are required to construct a fairly large risk buffer for mitigating volatilities in contributions and funded ratios. A benefit structure that minimum benefits are supplemented by conditional benefits is suitable for such environments since the conditional portion functions as a virtual risk buffer. There is another favourable point that the costs of constructing the virtual risk buffer are evenly distributed to future years. This paper proposed a structure that both the minimum and targeted benefits are indexed to inflation in order that the pension plan is able to survive the economically unfavourable environments as in present Japan where very low interest and inflation rates persist for a long time while the volatility of stocks remains high. This paper also proposed incorporating in advance a mechanism of adjusting both the benefits and contributions retrospectively, making reference to the several good examples in the field of public pensions.

This paper pointed out that present DB plans do not establish any specific correspondence between individual liabilities and plan assets. This practice causes a serious deficiency that there is no effective and timely mechanism of keeping under control the risks of active participants. At the same time it should be reminded that requiring immediate supplementation of funding shortfalls is not recommended at all since it would increase the economic values of employer contributions significantly. As a measure of remedying this deficiency in DB plans, this paper proposed decomposing the assets and contributions by payout year and imposing partial ring-fencing of plan assets, while maintaining adequate flexibility that enables intertemporal risk-sharing.

Decomposition of contributions by payout year is naturally given by discounting back the estimated future cashflows corresponding to the accrued benefits using appropriate discount rates. Thus this paper pointed out the question of how to decompose contributions is reduced to the question of how to determine the discount rates. Using the expected rates of return on investments of actual portfolio as the discount rates cannot be said prudent since the probability that the portfolio value will surpass the targeted benefits at the payout year is less than 50%, when the portfolio value follows the standard geometric Brownian motion. On the other hand, using volatile market risk free rates as the discount rates is also not recommended since it increases not only the absolute amounts but also the volatility of contributions and thus raises the economic values of employer contributions.

This paper introduced the idea of the payout-year-specific (PYS) funding standard in order to devise appropriate discount rates including the expected excess return of risky assets and remedy the shortcoming of present DB plans explained in the third paragraph. In the framework of the PYS funding standard, assets and contributions are decomposed by payout year and partially ring-fenced. Then the PYS funding standard assumes a hypothetical dynamic investment strategy that a speculative portfolio is switched to a liability-hedging portfolio immediately when the value of each payout-year-specific fund firstly hits the corresponding targeted liability value. This paper showed that we can expect a greater probability that fully funded status is achieved at the year of maturity by this dynamic strategy than by the static strategy of simply holding a 'strategic' asset allocation.

The PYS funding standard specifies minimum funded ratios to individual payout-year-specific funds. In this paper, the discount rate giving the minimum funded ratios is determined from the following two conditions. First one is that the asset value of the payout-year-specific fund will attain the corresponding targeted liability value at *some* time until the year of maturity with a given probability at the minimum. Second one is a restriction on the severity of the conditional expectation of the fund value (loss given default) when the fund value is less than the targeted benefits at the year of maturity, or a restriction on the probability that the funded ratio of the payout-year-specific fund hits the minimum admissible level at *some* time until the year of maturity. We can thus determine unique discount rates for each payout-year-specific fund, depending on the period until the year of maturity.

This paper showed that the discount rates thus determined include a proportion of the expected excess return of the speculative portfolio. This paper also showed that if we assume that the acceptable risk on the severity of the loss given default increases as the period until the maturity extends, then the maximum permissible proportion of the expected excess return to be included in the discount rates increases progressively. However, if there is a due limit on the severity of the loss given default, then it is anticipated that the graph of the maximum permissible proportion becomes hump-shaped. We should be fully aware that pension funds can take larger investment risks under long time horizons only when the funding deficiencies can be corrected gradually spending the given long period with smaller supplemental contributions. Any funding standard should strike a right balance between assuring stable employer contributions with reasonable prices and ensuring that targeted benefits are paid with reasonably high probabilities.

Irrespective of the funding standards, it is always possible to consider that the portfolio of a pension plan is a composite of the target date funds (TDFs). According to this understanding, it is clear that the portfolio should be rebalanced along with the changes in the stream of the estimated benefit cashflows, even if the long-term prospect on the market is invariant. It is also clear that the portfolio should be rebalanced along with the funded status of the plan since the degree of risk aversion varies along with the funded status. Any funding standard cannot continue to exist without paying proper considerations to investments and *vice versa*. It can be said that the PYS funding standard is a bridge connecting financing issues and investment issues systematically.

7 References

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