Mean-Variance efficiency in proportional reinsurance under group correlation: friendly optimization without Kuhn-Tucker theorems

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Abstract

The paper concerns optimal mean variance proportional reinsurance under group correlation.

In order to solve the corresponding constrained quadratic optimization problem, we exploit the smart friendly technique originally proposed by B. de Finetti in his pioneering paper [2].

Such a procedure is based on a set of so called "key functions" describing the local advantage, measured by (one half) the decrease of variance over the decrease of expected gain, coming from an additional (or initial) reinsurance of any risk. Then, the set of mean variance efficient retentions could be simply seen as a path in the $n-$ dimensional space of retentions. This path connects the natural starting point, the one with the largest maximum expected gain (full retention), to the final point of minimum variance (zero retention). At any point of the path, the simple idea is to provide additional (or initial) reinsurance exclusively to those risks sharing the maximal advantage.

Besides being able to solve with closed form formulas the case of no correlated risks, de Finetti gave also some insights about the handling of a particular type of correlation. That is just the case of group correlation, namely with risks divided in groups with constant "group specific" correlation *within* each group and no correlation *between* them. Moreover, insurance as well as reinsurance premiums come from the application of the standard deviation premium principle with a constant, group specific, loading coefficient.

In our paper, exploiting the mentioned friendly procedure, we are able:

- to show that the efficient path is continuous piecewise linear in the space of retentions;
- to verify that the corner points of the path correspond to matching points. In those points the advantage to begin reinsuring a new risk (previously fully retained) matches the current advantage shared by going on with reinsurance of the set of best performers (already partially reinsured risks). Among other things, this implies that (as already noted by de Finetti) the order of entrance in reinsurance is dictated, within any group, by the standard deviation degree;
- to express the values of the efficient retention vector corresponding to any value of the advantage parameter;
- to give formula to compute mean and variance of any efficient retention as a function of the advantage parameter;
- to provide the algebraic expressions of the set of parabolas which is the geometric representation of the efficient set in the mean-variance space;

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- to prove that such a set is continuous and without kinks.

Even if all these results could be formally obtained by a mechanical application of the (Karush) Kuhn-Tucker conditions, the application of de Finetti's friendly procedure has the advantage (as we shall see in detail in the paper) to put in clear evidence the intrinsic characteristics of the solution as well as an immediate key of interpretation of its reinsurance meaning.

Moreover, this approach is a natural way to treat other interesting related issues linked to proportional reinsurance problems, such as ruin probability, risk measures and so on.

Keywords. Mean-variance efficiency; constrained quadratic optimization; proportional reinsurance; group correlation.

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