Estimating Portfolio Value-at-Risk with Multivariate Mixture Time Series Models

Wong, Chun Shan
The Chinese University of Hong Kong
E-mail: albertw@baf.msmail.cuhk.edu.hk

In the past decade, our understanding of risk has much improved, and particular advances have been made in the field of value-at-risk. The main purpose of the value-at-risk system is to assess market risks that are due to changes in market prices. The most commonly used approach to estimate portfolio value-at-risk is to assume an independent multivariate lognormal model. In this model, the financial variables, such as the return on interest rates or stock prices, are independent, multivariate, and normally distributed. This model is easy to use and understand, and also has many nice mathematical properties. However, there are a number of drawbacks. (1) The model does not fit the real data well enough, especially the tails of the return distributions; (2) The model does not take into account the serial correlation among the returns; and (3) The constant volatility (variance) assumption in the model is not realistic. In the delta-normal and delta-gamma methods for the estimation of portfolios, the asset returns are assumed to be a function of the independent multivariate lognormal financial variables. Thus, the appropriateness of these methods depends heavily on the validity of the multivariate normal assumption.

Several approaches to overcome the drawbacks of the independent multivariate lognormal model have been proposed in the literature, such as linear multivariate time series models and multivariate time series models with changing conditional variance-covariance matrices. However, it is unclear whether these models fit the tails of the return distributions well.

Recently, the idea of a mixture distribution is generalized to the nonlinear time series context. The mixture Gaussian time series model is actually a mixture of K Gaussian autoregressive models with an ARCH specification. This class of models is capable of overcoming all three deficiencies of the univariate lognormal returns model. It has been shown that for some economic time series, the mixture Gaussian time series models provide better descriptions of return distributions than the linear time series models, especially in the tails of the distribution. In a recent paper, a class of mixture vector autoregressive (MVAR) models consisting of a mixture of K vector autoregressive models has been introduced. It has been shown that the MVAR model is useful in the modeling of several financial time series, such as stock prices and interest rates.

The primary objective of this paper is to develop applications for the multivariate mixture time series models in the estimation of portfolio value-at-risk. When we use value-at-risk as a risk measure, we are concerned with the tails of the return distributions rather the mean returns. Thus, it is important to build a model that can provide a complete description of the return distributions, rather than an accurate estimate of the mean returns. In this sense, the MVAR model is quite promising in providing a reliable estimation of portfolio value-at-risk.

Keywords: Multivariate time series models; mixture distribution; heavy-tailed distribution.