

# Liability driven optimization of investment strategies

Petri Hilli

QSA Quantitative Solvency Analysts Ltd,  
Hiilikatu 3, 00180 Helsinki, Finland,  
email: petri.hilli@qsa.fi,  
tel: +358 40 7542090

March 25, 2009

## Abstract

We study the problem of diversifying a given initial capital over a finite number of investment funds that follow different trading strategies. The investment funds operate in a market where a finite number of underlying assets may be traded over a finite discrete time. Our goal is to find a diversification that is optimal in terms of a given convex risk measure. We formulate an optimization problem where a portfolio manager is faced with uncertain asset returns as well as liabilities.

This paper presents a computational procedure for finding an optimal diversification between funds. The procedure combines simulations with large scale convex optimization and it can be efficiently implemented with modern solvers for linear programming.

We illustrate the optimization process on a problem coming from the Finnish pension insurance industry. The liabilities are taken as the claim process associated with current claims portfolio of the private sector occupational pension system and the investment horizon is 82 years. The results reveal a significant improvement over a set of standard investment styles that are often recommended for long term investors.

**Keywords** Liability driven investment, asset liability management, market risk, underwriting risk, large scale optimization

**Topic** Portfolio and risk management