

THE ARBITRAGE-FREE EQUILIBRIUM PRICING OF LIABILITIES  
IN AN INCOMPLETE MARKET:  
APPLICATION TO A SOUTH AFRICAN RETIREMENT FUND

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ABSTRACT

In prior work by the author the method of pricing the liabilities of a financial institution by means of dynamic mean–variance hedging is applied to an incomplete market that is nevertheless in equilibrium with homogeneous expectations. In subsequent work a long-term equilibrium model is developed and parameterised for the South African market. The aim of this paper is to apply the latter model to the pricing method with a view to quantifying the effects of incompleteness and the sensitivity of the price of illustrative liabilities to the parameters of the model. The application is to retirement-fund benefits in the South African market.

In an unpublished application of the pricing method it was found that, except for quite short-term liabilities, the computational demands of the pricing algorithm became excessive. The main reason for this was that the algorithm calls for simulations within simulations: for each year of the term of liabilities, a large number of simulations is required, and for each such simulation another large number of simulations is required. In this article consideration is given to the reduction of the computational demands of the algorithm.

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KEYWORDS: Market value of liabilities; Dynamic mean–variance hedging; Equilibrium market models; Incomplete markets