

**Proposal**  
to present a paper called

**The Impact of Stochastic Volatility on Pricing, Hedging, and  
Hedge Efficiency of Variable Annuity Guarantees**

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## Abstract

Variable Annuity sales throughout the world have tremendously increased since insurers have started to include additional guaranteed minimum benefits. These guarantees have been developed in the US, then expanded to the Asian market, in particular Japan, and finally made their way to Europe. Due to the significant financial risk that is inherent within the insurance contracts sold, risk management strategies such as dynamic hedging are commonly applied. While usually delta risk, i.e. the risk of changing stock markets, and occasionally rho risk, i.e. the risk of changing interest rates, are hedged by dynamic hedging, so-called vega risk, i.e. the risk of changing volatility, is – according to industry surveys – commonly not hedged at all. During the recent financial crisis, insurers have suffered from inefficient hedge portfolios within their books.<sup>1</sup> Volatilities have significantly increased leading to a tremendous increase in option values. Thus, the value of the options within variable annuities and thus the cost of the corresponding hedging strategies have also significantly increased.

There already exists some literature on the pricing of different guaranteed minimum benefits and in particular GMWB. However, there is no extensive analysis of the performance of different hedging methods in the case where no perfect hedge exists. Besides that, there is no extensive analysis of the respective guarantees under stochastic volatility. The present paper fills these gaps as described in what follows.

We determine and compare the fair prices for different GMWB for Life products under different model assumptions, first under the Black-Scholes model with deterministic interest rates and volatility, and, secondly, under the Heston model with stochastic volatility. We also present various sensitivity analyses of guarantee prices with respect to different product features and model parameters. Then, we give an overview over different dynamic and semi-static hedging strategies that can be used to manage the risks emerging from the financial market. The main focus will be on dynamic strategies for the hedging of different combinations of so-called “Greeks” like delta, gamma and vega. While a pure valuation of the contract is somehow straightforward, a calculation of the Greeks requires more attention to the numerical methods used. Finally, we analyze and compare the hedging performance of the strategies mentioned above under both asset models. We also examine the effects if the hedging model differs from the data-generating model, i.e. in the case that the insurer calculates the hedging-strategy using a Black-Scholes model whilst market prices of the assets evolve according to the Heston model. This allows a quantification of the model risk the insurer is exposed to if the model does not account for stochastic volatility.

Our results show that, under a model with stochastic volatility the price of the guarantees increases. However, there are certain features such as ratchet features etc. that under certain circumstances may become less valuable under stochastic volatility. We also find that, within the classic Black-Scholes model with deterministic volatility and without jumps in the underlying process, delta-only hedges may lead to tolerable fluctuations in an insurer’s P&L. Under stochastic volatility, however, the same strategies are likely to induce bigger losses and therefore lead to major problems. Overall, volatility risk of such products is significant and should not be neglected. Our results indicate that the lack of volatility hedging could account for a substantial share of the losses several insurers recently suffered from. Therefore some form of vega hedging should be part of an insurer’s risk management strategy if variable annuity guarantees are sold. Thus, our results should be of interest for academics as well as for practitioners and regulators. Our results particularly indicate that regulators who base capital requirements on analyses of the efficiency of an insurer’s hedging strategy should make sure that appropriate models are being used.

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<sup>1</sup> Cf. different articles and papers in “Life and Pensions”, e.g. “A challenging environment“ (June 2008), “Variable Annuities – Flawed product design costs Old Mutual £150m” (September 2008), “Variable annuities – Milliman denies culpability for clients' hedging losses“ (October 2008), “Variable Annuities – Axa injects \$3bn into US arm” (January 2009).