Securitization of longevity risk in reverse mortgages

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Abstract

Reverse mortgages are becoming remarkably popular in the last few years in Australia, and although they have been around a lot longer in the United States, they are receiving renewed interest among the elderly. Increase in life expectancies and decrease in the real income at retirement due to inflation continue to worry the those who are retired or close to retirement. Today, financial products that help alleviate the "risk of living longer" therefore continue to be attractive among the retirees. Reverse mortgages involve various risks from the provider's perspective which may hinder the further development of these financial products. This paper addresses one method of transferring and financing the risks associated with these products through the form of securitization. Securitization is becoming a popular and attractive alternative form of risk transfer of insurance liabilities. Here we demonstrate how to possibly construct a securitization structure for reverse mortgages similar to the one applied in traditional insurance products. Specifically, we investigate the merits of developing survivor bonds and survivor swaps for reverse mortgage products. In the case of survivor bonds, for example, we are able to compute premiums, both analytically and numerically through simulations, and to examine how the longevity risk may be transferred to the financial investors. Our numerical calculations provide an indication of the economic benefits derived from developing survivor bonds to securitize the "longevity risk component" of reverse mortgage products. Moreover, some sensitivity analysis of these economic benefits indicates that these survivor bonds provide for a promising tool for investment diversification.

1 Introduction

Today in many countries around the globe, life expectancy from birth is well over 80 years. While life expectancy has increased in the last few years, the labor market participation rates among the elderly have also dropped. According to Bateman, Kingston, and Piggott (2001), labor participation rates for males aged 60-64, for instance, have fallen from 70-90 percent down to 20-50 percent in recent years. Both living longer and retiring earlier contribute to the risk that the elderly may be unable to fund their retirement in old age. This risk is popularly known as "longevity risk".

Research has found that retired people often have plenty of equity locked in their homes but very few liquid assets to rely on to support their daily needs. For example, in Australia alone, home equity takes a proportion of approximately 50 percent of all household assets. See Australian Bureau of Statistics, Australian Social Trends, 1998.[1] This is a common situation among retired people in many western countries, and we often hear the phrase "house rich and cash poor" to refer to the increasing number of elderly who hold a substantial proportion of their assets in home equity. The financial industry is again starting to tap into this market; today, there is renewed interest to offer reverse mortgage products.

A reverse mortgage, or RM for short, is a form of financial product that allows retirees to convert their substantial home equities into either a lump sum or annuity income and at the same time, to remain in their homes until until they die, sell or vacate their homes to live elsewhere. Loans made through a reverse mortgage accrue with interest and are settled only upon the death of the borrower, sale of the property or the vacation of its residents. There are no repayments made during the course of the loan, and no assets other than the home may be attached to debt repayment. If at the time of settlement, the loan accrued with interest is larger than the sale price of the property, then the provider (or lender), usually a bank or an insurance company, recovers only up to the sale price of the property.

Reverse mortgages have been approved for selling in the United States by the Federal Home Loan Bank Board as early as 1979; see Chinloy and Megbolugbe (1994). Its popularity is only increasing as of late. Today, although the demand for them remains to be scant, there are early indications of a rapid growth of demand for reverse mortgages since the beginning of the millennium. In Australia, these products were introduced only in the early 1990's. Although the market has not yet proven to be as active and viable as the industry had hoped for, the product has a lot of potential as alluded by Reed and Gibler (2003).

The reverse mortgage product involves a variety of risks from the provider's perspective. Most of the loan amount accrues interest at a variable rate, usually adjusted either monthly or yearly based on an index rate, and is repaid only when the borrower dies or sells or permanently leaves the home. The loan balance usually accumulates at a faster rate than the rate of rise of the home equity value, so that in time, it will exceed the value of the home equity. If the outstanding loan balance exceeds the home equity value before the loan is settled, the lender starts to incur a loss. This is often referred to in the literature as the "crossover" risk, and is one of the crucial risk to manage in reverse mortgages. Clearly, this crossover risk is influenced by three underlying factors: mortality, interest rates, and house prices. Improvements in mortality will delay the settlement of the reverse mortgage loan and will therefore increase the chances of hitting the "crossover" mark. A high interest rate will increase the rate at which the loan balance will accrue, and will therefore possibly hit the "crossover" earlier than expected. Finally, a depressed real estate market will worsen the value of the home. Of these three factors, mortality is believed to be the single most important consideration in the pricing and risk management of reverse mortgages. See Chinloy and Megbolugbe (1994) for further discussion of the "crossover" risk.

A traditional method for dealing with the risks associated with reverse mortgages is insurance. The Home Equity Conversion Mortgages (HECM) program in the United States is a clear example of such a scheme. Lenders under this program are protected, to an extent, against losses arising when the

[1] Available at <http://www.abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/4102.01998?OpenDocument>

loan balance exceeds the equity value at time of settlement. In this paper, we suggest securitization as a possible means of managing the longevity risk in reverse mortgages, as mortality securitization is regarded as a flexible approach to transfer the unwanted mortality risk from the insurers to the capital market, allowing the risks to be more efficiently distributed.

Securitization is a financial innovation that emerged in the 1970's in the US financial market. According to Cummins (2004), securitization "involves the isolation of a pool of assets or rights to a set of cash flows and the repackaging of the assets or cash flows into securities that are traded in capital markets". Reinsurance, which has long been a traditional form of insurance risk transfer, is indeed an example of trading insurance risks. However, securitization introduced the bundling of insurance risks into commodities that become available in the larger capital market, allowing therefore investors to tap into the insurance market and to diversify their investment portfolios. Since its introduction, packaging financial instruments through securitization has shown its incredibly fast growth and has expanded its popularity to the insurance industry. An economic justification of securitization of insurance risk is provided by Cox, Fairchild, and Pedersen (2000). Partly a result of increasing natural catastrophes such as the devastation caused by the 1992 Hurricane Andrew, insurance risk-based securities such as catastrophe bonds proved to be popular (Cox and Pedersen (2000)). Later, the idea of securitizing mortality and/or longevity risks was introduced (see Blake and Burrows (2001) and Blake (2003)). There was an increased interest to model these types of mortality-based securities and hence, the ideas of mortality bonds and mortality swaps were proposed in the literature. These proposed concepts were later effectively put into practice. To illustrate, Swiss Re and European Investment Bank issued different types of mortality-based securities respectively in 2003 and 2004. While Swiss Re offered a brevity risk bond to hedge the risk of adverse changes in mortality rates, European Investment Bank issued a survivor bond to address the improvements in mortality rates. See also Creighton, Piggott, Jin, and Valdez (2005).

Following a similar approach used by Lin and Cox (2005b), this paper proposes a securitization method to hedge the longevity risk inherent in reverse mortgage products. Several examples are given, including two types of survivor bonds and a survivor swap. The survivor bonds are priced and a sensitivity analysis is applied to each bond to assess the impact of mortality improvement. Our results demonstrate that securitization can indeed provide an efficient and economical way to hedge the longevity risk in reverse mortgages.

For the rest of the paper, we have it organized as follows. Section 2 reviews the historical development of securitization and discuss the recent developments of mortality securitization. Section 3 examines the history of reverse mortgage loans in the U.S. and discusses the various risks involved in the product. In the end of the section, a pricing model of reverse mortgage is provided. In section 4, we develop a model to securitize the longevity risk in reverse mortgages. Several example schemes are given, including two types of survivor bonds and a survivor swap. In section 5, the survivor bonds are priced both analytically and numerically and the effect of risk hedging is examined at the end of this section. The thesis concludes with a summary and discussion of the findings.

2 The securitization of longevity risk

The first securitized financial transactions can be traced back to 1970 in the United States when the Government National Mortgage Association ("Ginnie Mae") began to sell guaranteed mortgage pass-through certificates. See Hill (1996). In the late 1970s, private securitized residential mortgage transactions began to emerge, in respond to a funding shortfall in the US home mortgage market. At that time, demand of homeowners and potential homeowners for mortgage loans exceeded the lenders capital ability to supply, leading the financial markets to find a more efficient way to transfer funds from investors in the capital markets to the mortgage demanders; Cummins and Lewis (2003).

Since then, the general financial markets have seen securitization grow whip and spur resulting in

the creation of a number of new securities. Two factors have contributed to the blossoming of the securitization market over the last twenty years. The first was changes to the tax code. In 1986, new legislation was introduced in the United States to simplify tax structuring of some complex mortgage-backed securitization; see Hill (1996). The second factor was the rapid development of technology and computing power, since the proper pricing of certain securities required complex and extensive computations (Gorvett (1999)). Since the introduction of mortgage-backed securities in the United States in the 1970s, the volume of securitized transactions has increased dramatically. By year 2002, the volume of newly-issued mortgage-backed securities (MBS) and asset-backed securities (ABS) reached \$1.5 trillion and \$450 billion, respectively; see Cummins and Lewis (2003). By the second quarter of 2003, a total of \$6.6 trillion worth of securities had been issued. Following these explosive developments in the financial securitization market, insurance-based securitization began to be explored.

Mortality-based securities are a very recent subject in the insurance literature. Cummins (2004) analyzed various securitization models and discussed the emerging securitization classes in the insurance industry. He deemed that mortality risk is one of the drivers of demand for insurance securitizations. In a mortality-based securitization, the insurer is protected against possible adverse mortality (or longevity) risks arising from issuing life insurance and annuity products. To illustrate for example, the insurer could issue so-called mortality risk bonds which will provide coverage for losses arising from unfavorable mortality experience, in the case of life insurance, or unfavorable longevity experience, in the case of annuity products. Unfavorable experience would be triggered by a linked mortality experience index. These bonds, which are linked to long duration insurance products, are also usually of long duration and of high capacity. The first known mortality risk bond is believed to be issued in December 2003 by Swiss Re; see Cummins (2004). There are other possible forms of securitization of insurance liabilities many of which are also detailed in Cummins (2004). For securitizing annuity products, the most notable developments are survivor bonds and survivor swaps.

2.1 Survivor bonds

One may think of *survivor bonds* as the annuity-analogue of *mortality risk bonds* issued to securitize the mortality risk component of life insurance products. In a survivor bond, the future coupon payments are linked to the proportion of the cohort at issue who remain to be alive. Clearly because we are hedging the risk that more clients survive than normally expected, we would expect coupon payments made when there is a larger proportion of the cohort surviving.

Blake and Burrows (2001), together with the discussion of Blake (2003), provide for the early papers that tackle the issues associated with the topic of mortality-based securities. The concept of *survivor bonds* was introduced in the Blake and Burrows (2001) paper. The idea behind the survivor bonds is to make the coupon payments linked to the actual number of survivors within the same cohort at issue. Following the same illustration used by Blake and Burrows (2001), the coupon payments to be made 20 years later from issue of survivor bonds linked to a portfolio of annuities issued to 65-year-olds would be proportional to the number of 85-year-old survivors at that time. Furthermore, the annuity bond with coupon payments may be tied to a survivor index published periodically by certain authorities. Blake and Burrows (2001) suggested that survivor bonds have considerable potential as mortality-hedging instruments for insurance companies.

Lin and Cox (2005b) provide for the details of the actuarial computations involved to price a survivor bond to securitize the longevity risk component of annuity products. In particular, to determine the associated risk premium, Lin and Cox (2005b) used the Wang transformation (Wang (1996)) in order to adjust the estimated survivorship probabilities as follows:

$$q^*(x,t) = \Phi\left[\Phi^{-1}(q(x,t)) - \lambda\right], \tag{1}$$

where $\Phi(\cdot)$ denotes the distribution function of a standard Normal, q and q^* are, respectively, the unadjusted and adjusted rates of mortality for a given age and a given cohort. The thresholds of the

survivorship in each period are projected using Renshaw's mortality model based on the Generalized Linear Model (GLM) framework and based on the US mortality experience (1963, 1973, 1983 and 1996 US individual annuity mortality tables). See Renshaw, Haberman, and Hatzopoulos (1996). However in their model, the investors are exposed to "basis risk" or "cohort risk" which refers to the risk that the mortality experience of the linked annuity pool could deteriorate significantly more than that of the mortality tables. On a similar token, the work of Brouhns, Denuit, and Vermunt (2002) on projecting the future mortality rates using the Lee-Carter model can also be applied.

2.2 Survivor swaps

Dowd, Blake, Cairns, and Dawson (2006) recommend another form of mortality-based security: survivor swaps. The authors define a survivor swap as "an agreement to exchange cash flows in the future based on the outcome of at least one survivor index". The authors also discussed several advantages that survivor swaps have over survivor bonds, which include cost effectiveness and flexibility.

Survivor swaps were then discussed briefly by Blake (2003) and Dowd (2003), and in more detail by Dawson (2002) and Lin and Cox (2005b) and Lin and Cox (2005a). In Lin and Cox (2005a), the authors designed a specific mortality swap scheme between a life insurer and an annuity insurer, which they called "natural hedging". The authors show that the mortality swap can avoid basis risk problem since there is no need to project the future mortality thresholds. Dowd, Blake, Cairns, and Dawson (2006) gives two recent examples illustrating how mortality-based securities are applied to hedge adverse mortality risk and longevity risk.

Example 1 *Swiss Re brevity risk bond*

In December 2003, Swiss Re issued a bond linking principal payment to adverse mortality risk scenarios. The bond is designed to hedge the brevity risk in its life book of business, i.e. (the dramatic impact that premature death has on mortality rates) the excessive mortality changes of premature death. To facilitate this transaction, Swiss Re set up a special purpose vehicle (SPV), which raised \$400 million from investors. This issue was the first floating rate bond which links the return of principal solely to a "mortality index". The maturity of the bond is 4 years, and investors receive a floating coupon rate of US LIBOR plus 135 basis points. This coupon rate is higher than other straight bonds, however the principal payment is at risk if "the weighted average of general population mortality across five reference countries (US, UK, France, Italy and Switzerland) exceeds 130% of the 2002 level". Since mortality is generally improving over time, the probability of such high mortality are very low. so investors obtain a high coupon rate in return by tolerating the risk.

Example 2 *EIB survivor bond*

In November 2004, the European Investment Bank (EIB) issued a longevity bond. This bond involves "time t coupon payments that are tied to an initial annuity payment of £50 million indexed to the time t survivor rates of English and Welsh males aged 65 in 2003". Unlike the Swiss Re brevity risk bond dealing with the extreme short-term adverse mortality risk, the EIB bond can be used to hedge against the long-term longevity risk since coupon payments are tied to a survivor index. It was reported that the total value of the issue was \$540 million, and was primarily intended for purchase by UK pension funds.

3 A quick overview of reverse mortgages

In a nutshell, a reverse mortgage provides the elderly to acquire a loan against the value of their property allowing them to receive income for immediate consumption. The loan may be received in the form of either a lump sum, an annuity, a line of credit, or any combination of these. Reverse mortgages differ

from a conventional loan in several respects. First, there are no repayments of principal or interest, but the outstanding balance accrues with interest and possibly administrative fees. Repayment of the outstanding balance is made when the mortgagor dies, or voluntarily leaves the property, and the repayment comes from the proceeds of the sale of the property. Second, although some reverse mortgages may have fixed term at which time loan repayment must be made, the more popular ones contain a tenure guarantee allowing the mortgagor to live in the property until death or voluntary departure. Third, the loan is considered "non-recourse" which means that the lender cannot recover principal or interest from other assets of the borrower. Fourth, unlike conventional loans that can be issued to borrowers of any age, the loan is primarily issued to individuals during their retirement years providing them a source of retirement income. Fifth, reverse mortgages are underwritten on the sole basis of the value of the property, oftentimes, the maximum amount of loan is expressed on a *loan-to-value*, a ratio that depends on the current value of the property. This is in contrast with a conventional loan where underwriting accounts beyond the property value, for example, the borrower's current income. Other details about reverse mortgages may be found in Szymanoski (1994).

The first reverse mortgage loan can be traced back to the one issued by Nelson Haynes of Deering Savings & Loan (Portland, Maine) in the United States in 1961. In 1989, after the Federal Housing Administration (FHA) introduction of the Home Equity Conversion Mortgages (HECM) program, reverse mortgages covered by public insurance became widely available in the United States. And after a decade of slow development, the reverse mortgage market started to boom in the twenty-first century as a result of tight budgets from a record number of older Americans. According to the National Reverse Mortgages Lenders Association, nearly $40,000$ new HECM loans originated in the years between 2000 to 2003. During 2004, a total number of $37,829$ of HECM loans were approved, representing a 109% jump over the previous year, and nearly 500% growth since 2001. The growth continued in 2005 with a total number of $43,131$ HECM reverse mortgages, which increased 14% from the previous year. In response, there was also an increase in the supply side. The number of RM lenders has tripled to 191. As HECMs accounts for about 90% of all reverse mortgages in the United States today, the above statistics well represents the condition of the reverse mortgage market.[2]

3.1 Merits and risks of reverse mortgages

The most notable merit of a reverse mortgage is clearly that the borrower is not required to repay the loan until he or she dies or leaves the property. In comparison with traditional asset-backed loans, reverse mortgage loans provide the elderly with a means of hedging longevity risks by helping to maintain a sustainable level of retirement income. Furthermore, another favorable feature of a reverse mortgage is the "non recourse" clause discussed in the early part of this section. When the loan is terminated, the borrower only needs to repay the loan amount or proceeds from the sale of the house price: whichever is the lesser sum.

While reverse mortgage loans provide many attractive benefits to the borrower, they also involve many risks for the perspective of the lender or loan provider. As earlier alluded, one of the most crucial risk in reverse mortgage loans is usually the "crossover risk". In the case that the loan value exceeds the collateral house value, the lender is limited to recover only the proceeds of sale the house when a reverse mortgage loan is terminated. Any excess is therefore considered a loss to the lender. Since the interest rate is usually higher than the house price appreciation rate, the loan value will certainly exceed the house value at some future point. On the other hand, however, if the loan is terminated before the crossover, any excess of the proceeds from the sale will revert back to the borrower (or his or her heir), rather than becoming the lender's gain. This feature of reverse mortgages makes it very similar to options contracts.

[2]See US Department of Housing and Urban Development, FHA Outlook: 2003-2006, available at <`http://www.hud.gov/offices/hsg/comp/rpts/ooe/olmenu.cfm`>.

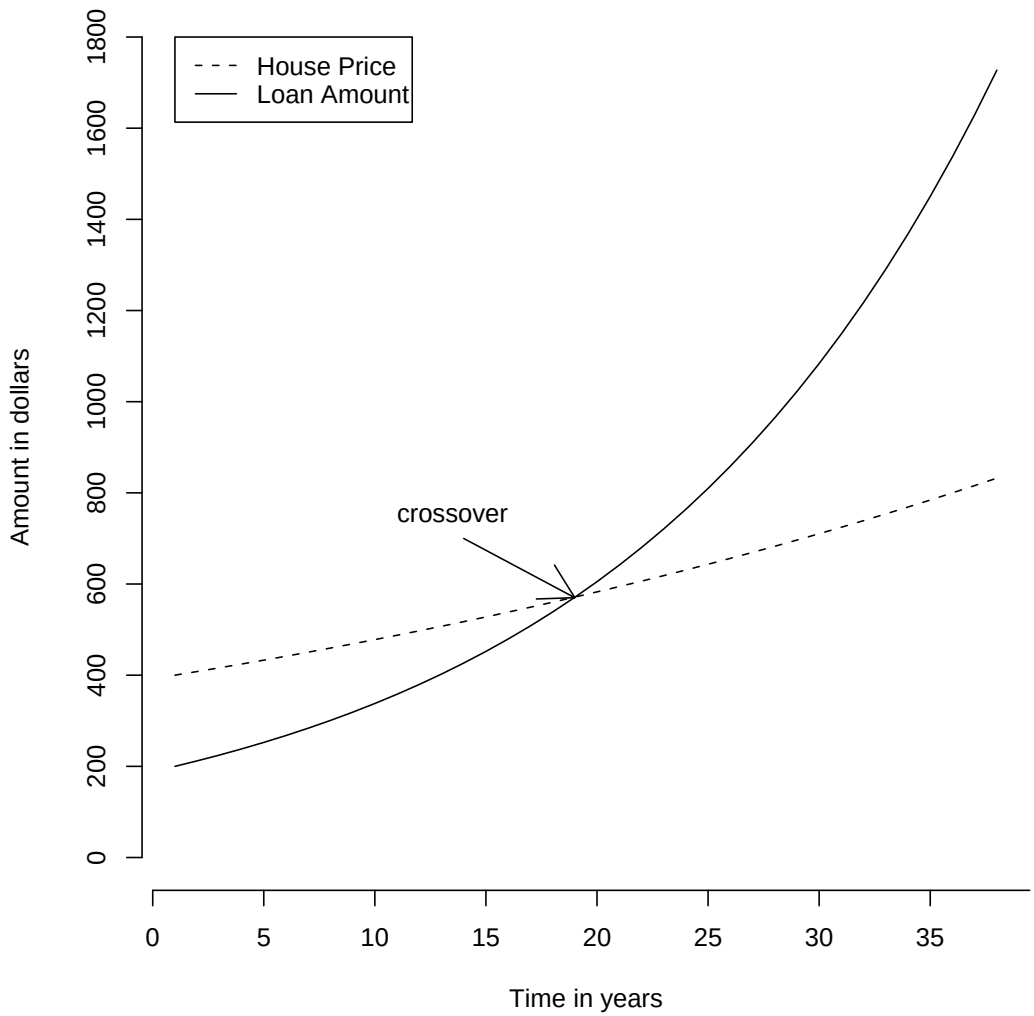

Figure 1: Illustration of the crossover risk

To illustrate these effects on the loss to the lender, a simple example is provided below. In this scenario, it is assumed that the interest rate is at a flat percentage of 6% annually, and the house value appreciation rate is at the constant rate of 3% per year. The loan-to-value ratio is assumed to be 50%, allowing a 62-year-old male to borrow a lump sum amount of $\$200,000$ against his house which is currently valued at $\$400,000$. Figure 1 illustrates the crossover risk. If the loan is repaid prior to the crossover point, there is no loss to the lender. However, if the loan is repaid after this crossover point, the difference between the balance of the loan and the house value is a loss to the lender if settlement has to be made at that point.

Assuming the lender finances the capital at an interest rate of 4% per annum, Figure 2 illustrates the net cash flow to the lender in each year. After the crossover point, the net cash flow in each year typically becomes negative.

The crossover risk is a combination of three major underlying risks — the occupancy risk, interest risk and house price risk.

- **Occupancy risk and longevity risk** The occupancy risk is the risk that the borrower could live in the house too long so that the loan value accumulates to a point where it exceeds the house value. The overall repayment rate is a combination of mortality and mobility rate. Although the

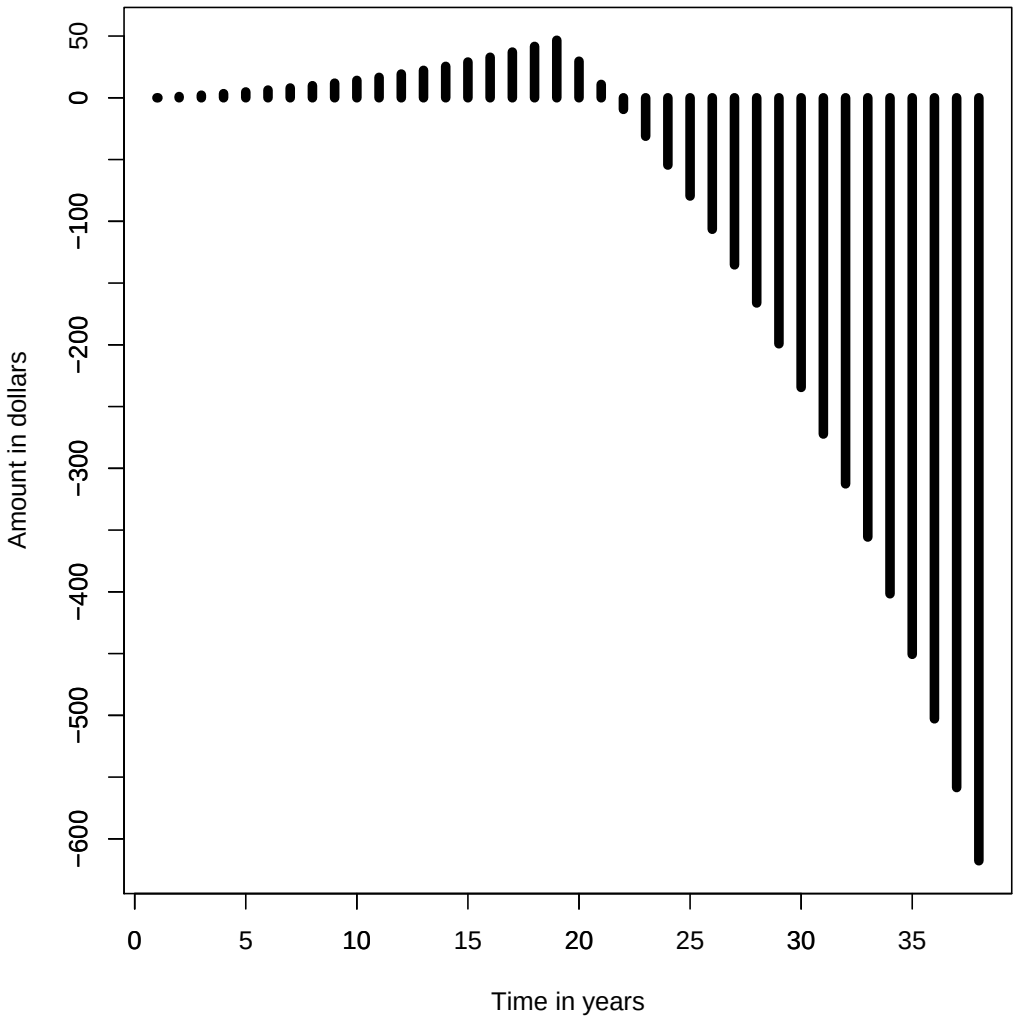

Figure 2: Cash flow analysis for a reverse mortgage lender

decision to move and repay the loan may be affected by the condition of the real estate market and the interest rate, the real attractiveness of the reverse mortgage loan is that the product allows the borrower, who is usually elderly low-income and doesn't want to move, to stay in their home until they die. Thus the duration of the loan is mainly determined by the mortality rate. Due to the dramatic improvement in the mortality rate since the 1970's, the longevity risk has become the most crucial risk in reverse mortgage product.

- **Interest rate and house price risk** Since the loan repayment is capped by the house price, a high interest rate environment and a depressed real estate market can obviously exacerbate the cross-over risk. The difference between the two risk is that the interest risk can not be diversified, while the house price risk can be partially diversified by holding a large portfolio of loans across areas.

- **Other risks** In addition to the three major risks, other important risks include maintenance risk and expenses risk. Maintenance risk is also called "moral hazard", and arises when the reverse mortgage borrowers fail to make the necessary repairs to maintain the value of their homes because they know that the lender bears the risk of the declining home resale value. Discussions on maintenance risk can be found in Miceli and Sirmans (1994) and Shiller and Weiss (1998).

3.2 Pricing lump sum reverse mortgages

This section starts the discussion of how to price for a lump sum reverse mortgage. We also introduce the notation to be used throughout the paper. Consider a retired individual who takes out a reverse mortgage loan for a value of Q_0 dollars, against his house currently valued at H_0 dollars. If at time t the loan amount is Q_t, the house price is H_t, and the cost of the capital is M_t, then by definition, the value of the reverse mortgage loan is

$$V_t = \min\left(Q_t, H_t\right),$$

and the loss to the lender L_t is

$$L_t = M_t - V_t = M_t - \min\left(Q_t, H_t\right).$$

If the loan amount Q_t accumulates at a risk free interest rate of r_t plus a risk premium λ, the house price H_t appreciates at a rate of δ_t and the cost of the capital M_t accumulates at a interest rate of η, then the loan value process can be described as

$$Q_t = Q_0 \exp\left(\int_0^t \left(r_s + \lambda\right) ds\right),$$

the house price process is

$$H_t = H_0 \exp\left(\int_0^t \delta_s ds\right),$$

and the process for the cost of capital is

$$M_t = Q_0 \exp\left(\int_0^t \eta_s ds\right).$$

Since the value of the loan repayment V_t is the smaller of the house price and the accumulated loan amount, we have

$$V_t = \min\left[Q_0 \exp\left(\int_0^t \left(r_s + \lambda\right) ds\right), H_0 \exp\left(\int_0^t \delta_s ds\right)\right].$$

The loss to the lender L_t then at time t becomes

$$L_t = Q_0 \exp\left(\int_0^t \eta_s ds\right) - \min\left[Q_0 \exp\left(\int_0^t (r_s + \lambda)\, ds\right), H_0 \exp\left(\int_0^t \delta_s ds\right)\right].$$

Now, suppose the future life span of the reverse mortgage loan is a random variable T (which could technically be the future remaining lifetime of the borrower), then when the loan is settled, the loss L_T to the lender can be expressed as the random variable

$$L_T = Q_0 \exp\left(\int_0^T \eta_s ds\right) - \min\left[Q_0 \exp\left(\int_0^T (r_s + \lambda)\, ds\right), H_0 \exp\left(\int_0^T \delta_s ds\right)\right]. \tag{2}$$

Applying the actuarial equivalence principle, we have the present value of total expected gain to be equal to the present value of total expected loss so that

$$\mathrm{E}\left(e^{-rT} L_T\right) = 0. \tag{3}$$

Substituting (2) into (3), the pricing equation for the reverse mortgage is therefore as follows:

$$\mathrm{E}\left[Q_0 \exp\left(\int_0^T \eta_s ds\right)\right] = \mathrm{E}\left[\min\left[Q_0 \exp\left(\int_0^T (r_s + \lambda)\, ds\right), H_0 \exp\left(\int_0^T \delta_s ds\right)\right]\right] \tag{4}$$

Using the pricing equation in (4), given a certain level of risk premium λ assessed by the lender, the maximal safe loan amount Q_0 can be determined. On the other hand, given a specific level of initial loan amount Q_0, the actuarially fair risk premium λ that the lender should assess can be determined.

4 Proposed structure of the securitization

Cox, Fairchild, and Pedersen (2000) pointed out that a common structure for asset and liability securitization involves four entities: retail customers, a retail contract issuer, a special purpose company, and investors. These authors then illustrate the process using the examples of several recent catastrophe bonds including the USAA hurricane bonds, Winterthur Windstorm Bonds, and Swiss Re California Earthquake Bonds. In this section, the general structure is applied to a lump sum case of a reverse mortgage product. Here, the process should involve at least five components:

- Borrower (Homeowner)
- Loan originator (Retailer)
- Special Purpose Company (or Special Purpose Vehicle)
- Lender (Investment bank)
- Investors (Capital markets)

Figure 3 illustrates the general structure of reverse mortgage securitization and all the cash flows involved in the process.

The transaction starts from the reverse mortgage retailer. This retailer is the front office that makes contact with the reverse mortgage loan borrower and negotiates the loan. After the retailer initiates the loan, it collects the lump sum from the reverse mortgages lender and then pays the amount to the borrower. To protect itself from the risk of not being able to fully recover the accumulated loan amount, the lender enters into an insurance contract with the Special Purpose Company (SPC). The insurance contract sets up a schedule of fixed trigger levels such that if the loss amount exceeds the pre-specified triggers, the SPC will pay the lender a certain amount of benefit up to an upper limit.

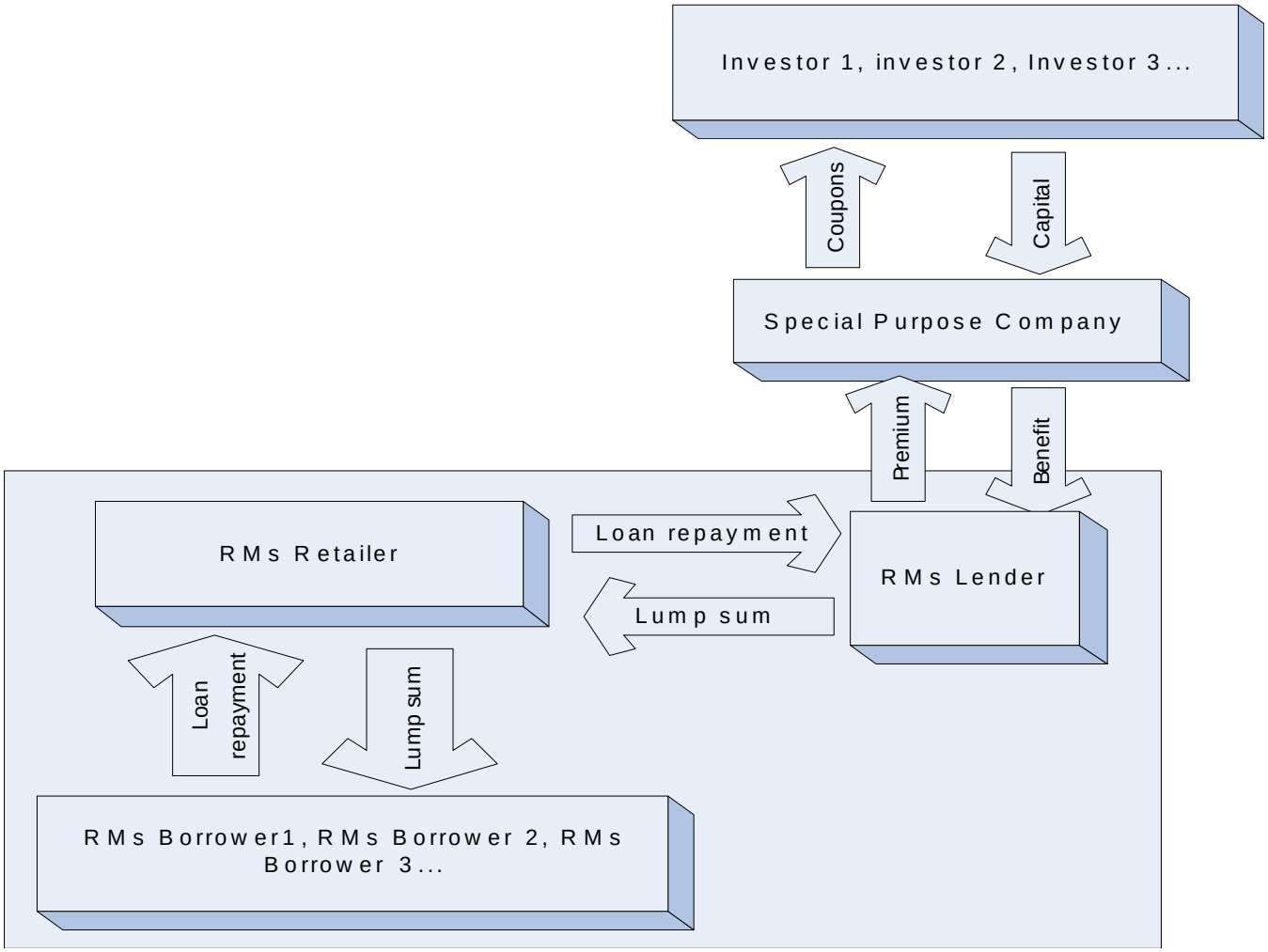

Figure 3: Structure of the reverse mortgage securitization

In exchange, the SPC collects a premium from the lender up front. The SPC issues a survivor bond in the market. The bond is then sold at a price lower than the normal market price, because in the event that the loss of the lender exceeds the trigger, part or all of the coupon could be defaulted to the bondholders and transferred to the insured — the lender.

Readers should notice that the above model only provides a very basic structure for the securitization process. In the real world, the process usually involves many other components which could serve various other purposes. For example, to protect the bond investors from the default risk from the SPC, the process may involve some form of credit enhancements from institutional rating agencies.

4.1 Cash flow analysis for each component

- **For the retailer** As an independent servicing institution, the retailer provides service to the customers, monitors their repayments of the loans, and maintains the integrity of the cash flows and payment process. In each period after the loan starts, the retailer collects the repaid loan amount from the borrowers and then transfers the amount to the lenders. From the perspective of the retailer, the cash inflows from the borrowers are exactly the same as the cash outflows to the lenders and therefore there is no risk of loss at all.

- **For the lender** The lender predicts the number of survivors of the loans and the loss amount in each period by analyzing past mortality improvement, interest rate fluctuation, and the real estate market conditions before the loans start. In each period after the loans commence, the lender's cash inflows are the loan repayments collected from the retailer and the cash outflows are the accumulated cost of capital. If the net of the two is less than the scheduled triggers, no insurance benefit is claimed from the SPC. Otherwise, the lender can collect a benefit from the SPC to cover the loss.

- **For the SPC** The SPC is a passive entity that only exists to securitize the mortality risk and sells the security in the capital market. For this purpose, it collects premiums from the lender and issues a survivor bond. The premium and the capital from selling the bond are assumed to be invested at a risk-free interest rate. In each period after the loans commence, the SPC's only cash inflow is the risk-free investment proceeds. The SPC's cash outflows are the claims from the retailer with high priority and the coupons paid to the bond holders. At the end of the term of the bond, the SPC repays the principal to the investors. The net cash flow should be always zero for the SPC.

- **For the investors** The survivor bond investors purchase the survivor bond at a lower price than a straight bond, but bear the risk of losing some of the future coupons. In each period after the loans commence, the cash inflows for the bond holders are the random coupon payments from the SPC. At the end of the term, the investor collects the full principal. To illustrate the securitization process, several examples of securitization schemes are provided, including two types of mortality bonds and one mortality swap.

4.2 Example 1 - reverse mortgage survivor bond type 1

In this case, to illustrate the effect of longevity securitization, the interest rate and house appreciation rate are both assumed to be constant. Suppose the lender holds a portfolio of l_0 loans. At time 0, all the borrowers are of the same age, say aged 62, and each borrow a lump sum of Q_0 against their home property currently valued at H_0. To hedge the longevity risk, the lender purchases insurance from the SPC at a lump sum premium of P. Under the contract, in each period after the crossover, the SPC will pay the lender a benefit of $A_t\left(l_t - \widehat{l_t}\right)$, up to a ceiling amount of C, if the number of survived loans l_t exceeds the predetermined trigger $\widehat{l_t}$. In period t the loss amount for each loan i is $L_{i,t}$, and since the interest rate and house appreciation rate are constant, $L_{i,t} = L_t$ for all i and all t. The amount A_t is

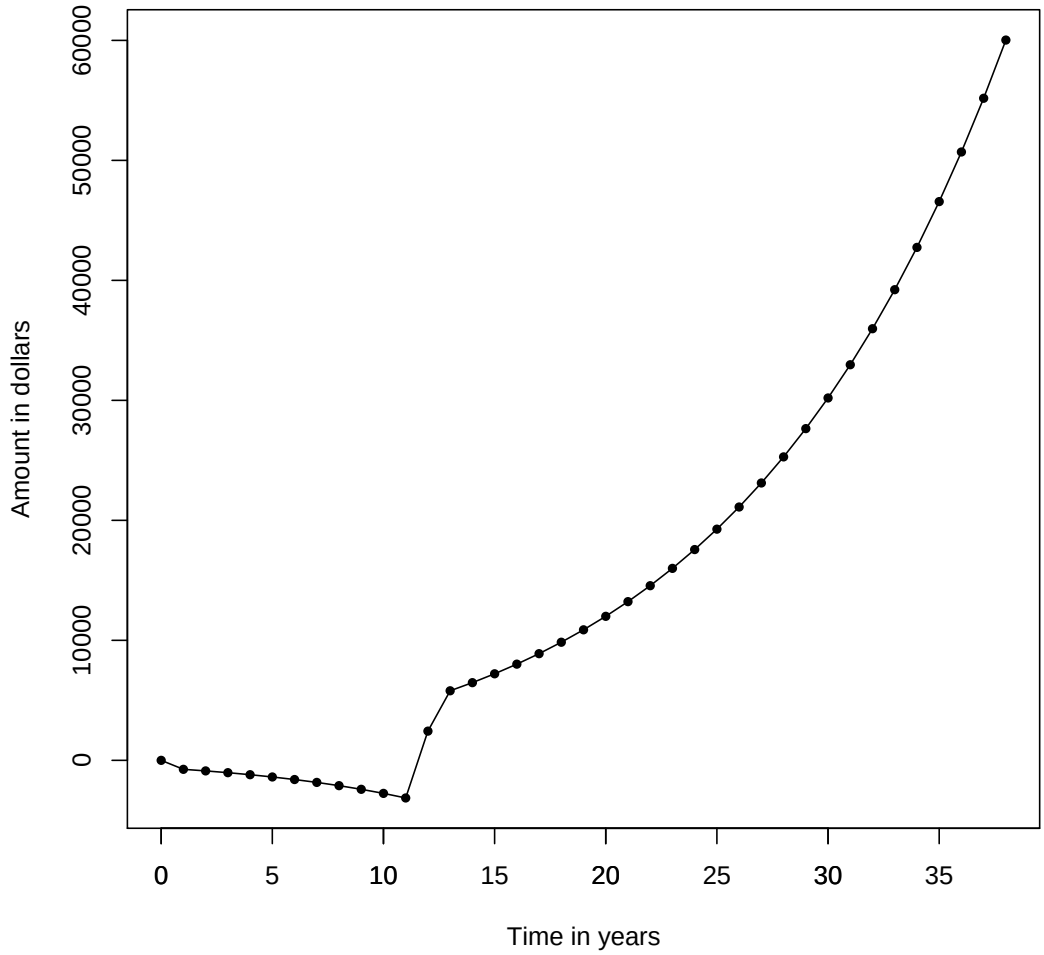

Figure 4: Single loss L_t in each period

determined as the one-period discounted difference between the expected loss between time periods t and $t+1$ so that

$$A_t = \frac{L_{t+1}}{1+r} - L_t.$$

If the risk-free interest rate is r, the house appreciation rate is c, the risk premium the lender charges is λ_1 and the premium that the lender is charged for capital finance is λ_2, then

$$L_t = Q_0 \left(1 + r + \lambda_2\right)^t - \min\left[Q_0 \left(1 + r + \lambda_1\right)^t, H_0 \left(1 + c\right)^t\right].$$

For example, if $r = 6.5\%$, $c = 3\%$, $\lambda_1 = 3\%$, $\lambda_2 = 1.5\%$, $Q_0 = 50,000$ and $H_0 = 100,000$, then L_t and A_t in each period are calculated and graphically displayed in Figures 4 and 5.

In Figure 4, L_t is always increasing with t after the crossover point. This means that after the crossover, the loss amount increases over time. In Figure 5, A_t is always positive and increasing with t after the crossover point. This means the one-period discounted loss L_{t+1} is larger than the current period loss L_t. This finding implies that after the crossover point, the lender is always better off incurring loss at the current time than incurring loss later on. The actual and expected number of

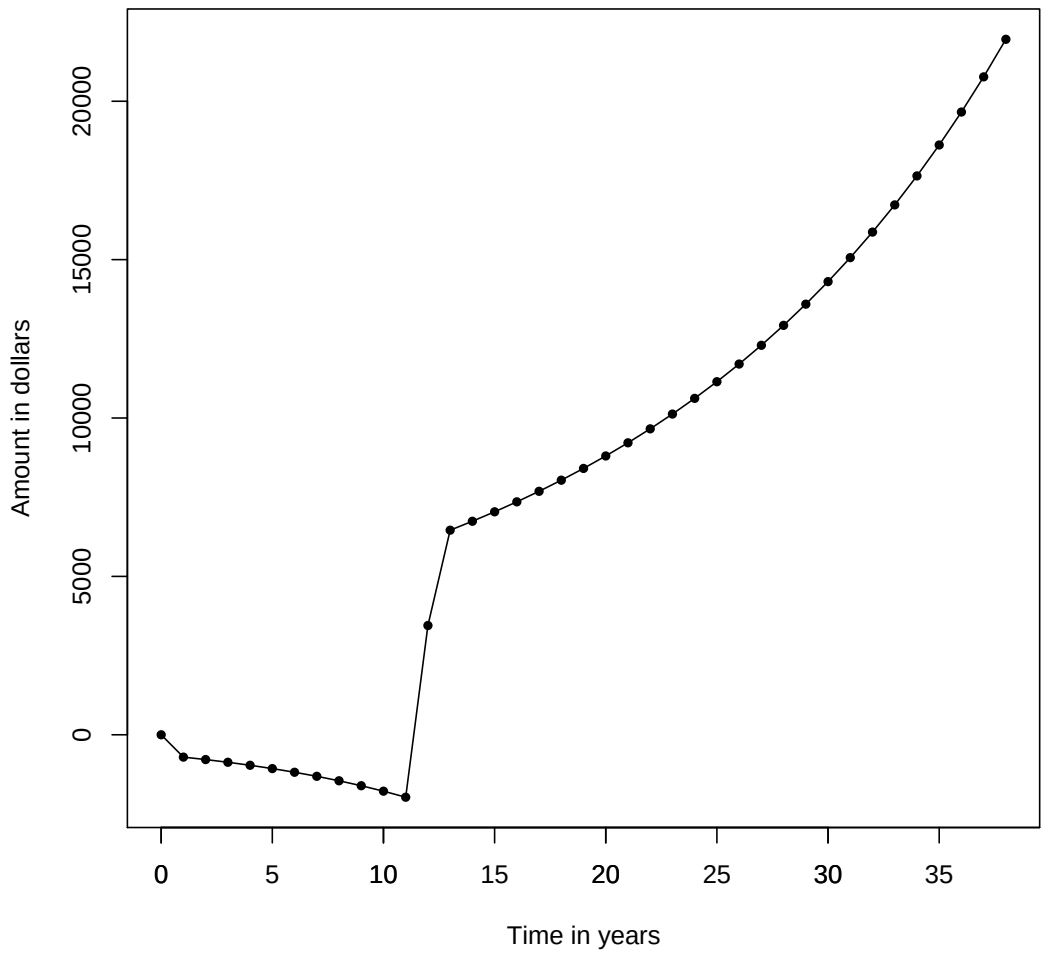

Figure 5: Appreciation A_t of each loss in each period

terminated loans in each period t are, respectively, denoted as d_t and $\hat{d}_t$, and the first period that the lender claims benefit is j. It is straightforward to show

$$\sum_{t=j}^{T}\left(l_t-\widehat{l}_t\right)=\sum_{t=j}^{T}\left(d_t-\widehat{d}_t\right). \tag{5}$$

Equation (5) indicates that less loss incurred in the current period means more loss will be incurred in the future, because the excessive survived loans ultimately will need to be repaid at some time in the future. When these excessive loans are repaid later, the present value of these losses will be greater than if they had been incurred in the present time. The difference between any two losses L_j and L_k, for $(j \leq k, k \leq T)$, in two different periods after the crossover can be expressed as

$$L_k - L_j = \sum_{t=j}^{k} A_t s_{\overline{t-j|}}, \tag{6}$$

where $s_{\overline{t-j|}}$ denotes the actuarial notation for the accumulation of 1 from time t to j at the risk-free rate r.

Protected by the securitization contract after the crossover, the lender can claim payments from the SPC to construct a reserve to cover the unexpected future loss in the event that $l_t > \widehat{l}_t$. The benefit payments B_t of each period are determined in (7). After collection, the benefits are invested at a risk-free interest rate r and accumulate until the excessive survived loans are repaid, and so we have

$$B_t=\begin{cases} 0 & \text{if } \ l_t \leq \widehat{l}_t \\ A_t\left(l_t-\widehat{l}_t\right) & \text{if } \ \widehat{l}_t < l_t < \frac{C}{A_t} \\ C & \text{if } \ l_t > \frac{C}{A_t} \end{cases} \tag{7}$$

Now, let the first period that the lender claims benefit B_t be j, then the reserve R_k the lender accumulates continually with B_t during the loan process to period k is

$$R_k = \sum_{t=j}^{k} B_t s_{\overline{t-j|}}.$$

Denote the actual and expected aggregate loss in each period t as $\underline{L}_t$ and $\hat{\underline{L}}_t$. By definition, the actual aggregate loss is

$$\hat{\underline{L}}_t = \hat{d}_t L_t,$$

and the expected aggregate loss is

$$\underline{L}_t = d_t L_t.$$

In the case of $\underline{L}_t > \hat{\underline{L}}_t$, the lender will have an unexpected loss of

$$\underline{L}_t - \hat{\underline{L}}_t = \left(d_t - \widehat{d}_t\right) L_t.$$

This unexpected loss during the period from j to k could be partly or fully covered by the reserve R_k. Notice that

$$R_k = \sum_{t=j}^{k} B_t s_{\overline{t-j|}} \leq \sum_{t=j}^{k}\left(\underline{L}_t - \hat{\underline{L}}_t\right), \tag{8}$$

which means the accumulated unexpected loss is the upper limit of the reserve R_k. This avoids the problem of being over-insured. The accumulated unexpected loss is fully covered only if

$$R_k = \sum_{t=j}^{k} \left(\underline{L}_t - \hat{\underline{L}}_t \right),$$

or

$$R_k = \sum_{t=j}^{k} A_t s_{\overline{t-j|}},$$

in which case all the benefits collected from the insurer are less than the ceiling C.

The SPC issues a bond with a face value of F and survivorship-contingent coupon payments of C_t at a price of $V \leq F$. The coupon payments are deducted whenever $\widehat{l}_t < l_t$ and the net coupon payments that the bondholders receive in each period are

$$C_t = \begin{cases} C & \text{if } \ l_t \leq \widehat{l}_t \\ C - A_t \left(l_t - \widehat{l}_t \right) & \text{if } \ \widehat{l}_t < l_t < \frac{C}{A_t} \\ 0 & \text{if } \ l_t > \frac{C}{A_t} \end{cases} . \tag{9}$$

In each period, the SPC needs to pay the coupon C_t, and in the final period, the SPC needs to pay back the principal as well. To cover the cash outflow, the SPC invests the premium P and the survivor bond price V in a straight risk-free bond with a face value F sold at a price of W. If $v = \frac{1}{1+r}$ is the one period discount factor, as long as

$$P + V \geq W = Fv^T + \sum_{t=1}^{T} v^t C,$$

the SPC can collect the amount C in each period and will be able to fulfill both his insurance and bond contract. To avoid any arbitrage, we should have $P + V = W$.

4.3 Example 2 - reverse mortgage survivor bond type 2

In this case, the interest rate and house appreciation rate are no longer assumed to be deterministic, but stochastic. Suppose the lender holds the same portfolio of l_0 loans as in survivor bond type 1. At time 0, all the borrowers are aged 62 and each borrow a lump sum of Q_0 against a property currently valued at H_0. If the mortality bond type 1 is applied, there is a chance that the lender is over-insured, that is that the reserve R_k may sometimes exceed what the lender actually needs:

$$R_k > \sum_{t=j}^{k} \left(\underline{L}_t - \hat{\underline{L}}_t \right),$$

where $\underline{L}_t$ and $\hat{\underline{L}}_t$ stand for the aggregate actual loss and the preset trigger amount. This is because L_t may not be necessarily increasing all the time, and thus A_t may not be positive all the time. To avoid this problem, the lender can purchase another type of insurance contract from the SPC at a lump sum premium of P. Under this insurance contract, in each period after the crossover, the SPC will cover the lender's aggregate loss up to a ceiling amount C if the actual total amount of loss $\underline{L}_t$ exceeds the trigger amount $\hat{\underline{L}}_t$, for example 95% percentile of the distribution of $\underline{L}_t$. Under this arrangement, the benefit paid to the lender in period t is

$$B_t = \begin{cases} 0 & \text{if } \ \underline{L}_t \leq \hat{\underline{L}}_t \\ \underline{L}_t - \hat{\underline{L}}_t & \text{if } \ \hat{\underline{L}}_t < \underline{L}_t \leq C \\ C & \text{if } \ \underline{L}_t > C \end{cases} . \tag{10}$$

The lender's net loss after the benefit in period t is

$$\underline{L}_t - B_t = \begin{cases} \underline{L}_t - 0 & \text{if } \underline{L}_t \leq \hat{\underline{L}}_t \\ \underline{L}_t - \left(\underline{L}_t - \hat{\underline{L}}_t\right) & \text{if } \hat{\underline{L}}_t < \underline{L}_t \leq C \\ \underline{L}_t - C & \text{if } \underline{L}_t > C \end{cases}$$

or equivalently, we have

$$\underline{L}_t - B_t = \begin{cases} \underline{L}_t & \text{if } \underline{L}_t \leq \hat{\underline{L}}_t \\ \hat{\underline{L}}_t & \text{if } \hat{\underline{L}}_t < \underline{L}_t \leq C \\ \underline{L}_t - C & \text{if } \underline{L}_t > C \end{cases} . \tag{11}$$

Similar to the above case, the survivor bond has a face value of F and random coupon payments of C_t sold at a price of $V \leq F$. However, in this case, the coupon payments are linked to the lender's aggregate loss, not the number of survived loans. The coupons for the bondholders in period t are

$$C_t = \begin{cases} C & \text{if } \underline{L}_t \leq \hat{\underline{L}}_t \\ C - \left(\underline{L}_t - \hat{\underline{L}}_t\right) & \text{if } \hat{\underline{L}}_t < \underline{L}_t \leq C \\ 0 & \text{if } \underline{L}_t > C \end{cases} . \tag{12}$$

As in type 1, the SPC invests the premium P and the survivor bond price V in a straight risk-free bond with a face value of F sold at a price of W. If v is the one period discount factor, as long as

$$P + V \geq W = Fv^T + \sum_{t=1}^{T} v^t C,$$

the SPC can collect amount C in each period and fulfill both his insurance and bond contract. For the securitization to be actuarially fair, we should have $P + V = W$.

4.4 Example 3 - reverse mortgage survivor swap

Another possible securitization structure is a reverse mortgage swap. In a survivor swap transaction, there is no principal payment at time T. At one side, the SPC pays the same cash flows B_t to the insurer, $t = 1, 2, ..., T$. In exchange for the floating benefit B_t, the lender pays a fixed annual premium x to the SPC instead of paying a lump sum premium P. Eventually, we have

$$P = xa_{\overline{T|}}.$$

On the other side, the SPC pays C_t to the bondholders. The investors pay the SPC a fixed amount y each year in order to receive the same coupons C_t, instead of paying V for the survivor bond. So we have

$$ya_{\overline{T|}} = \sum_{t=1}^{T} v^t \mathrm{E}\left(C_t\right).$$

As in the reverse mortgage bond example, the SPC has cash flows of B_t to the lender and C_t to the investors. Assuming there is no counter-party risk, in each year the SPC gets $x + y$, exactly enough to finance its obligation $B_t + C_t$. One advantage of swaps over issuing mortality bonds is the lower transaction costs, but the trade-off is that swaps could introduce default risk. As part of the solution, the swap might be provided by a broker or investment banker to reduce the default risk.

5 Pricing a reverse mortgage survivor bond

Basically to price a bond is just to discount all the expected future cash flows at the appropriate discount rate. The general bond pricing equation is

$$V = Fv^T + \sum_{k=1}^{T} v^k \mathrm{E}\left(C_t\right) \tag{13}$$

where the notation follows the examples in the previous section. The two types of survivor bonds are priced below.

5.1 Case 1 - survivor bond type 1

In this case, we assume that the interest rate and house appreciation rate are constant. The only difference between survivor bond type 1 and a straight bond is that the former's coupon payments are linked to the survivorship of the loans and is thus uncertain. Suppose a series of $\widehat{l}_t$ are determined as the triggers, for a portfolio of l_0 loans borrowed by persons aged x with the identical house value H_0, the bondholders will receive coupons in each period t

$$C_t = \begin{cases} C & \text{if } \ l_t \leq \widehat{l}_t \\ C - A_t\left(l_t - \widehat{l}_t\right) & \text{if } \ \widehat{l}_t < l_t < \frac{C}{A_t} \\ 0 & \text{if } \ l_t > \frac{C}{A_t} \end{cases}.$$

This is equivalent to

$$C_t = C - \left[A_t\left(l_t - \widehat{l}_t\right), 0\right]_+ + \left[A_t\left(l_t - \widehat{l}_t\right) - C, 0\right]_+.$$

Taking the expectation on both sides,

$$\mathrm{E}\left(C_t\right) = C - \mathrm{E}\left[A_t\left(l_t - \widehat{l}_t\right), 0\right]_+ + \mathrm{E}\left[A_t\left(l_t - \widehat{l}_t\right) - C, 0\right]_+. \tag{14}$$

The pricing equation of the survivor bond type 1 can be obtained by substituting equation (14) into (13):

$$V = Fv^T + \sum_{t=1}^{T} v^t \left\{C - \mathrm{E}\left[A_t\left(l_t - \widehat{l}_t\right), 0\right]_+ + \mathrm{E}\left[A_t\left(l_t - \widehat{l}_t\right) - C, 0\right]_+\right\}. \tag{15}$$

The number of survived loans l_t follows a Binomial distribution at the same termination rate ${}_tq_x^*$. If the loan number is large, for example more than 30, according to the Central Limit Theorem (CLT), l_t is approximately distributed as Normal with mean $\tau_t = l_t\left(1 - {}_tq_x^*\right)$ and variance $\sigma_t^2 = l_t\left(1 - {}_tq_x^*\right){}_tq_x^*$. In equation (14), we can rewrite the expectations term as

$$\begin{aligned} \mathrm{E}\left[A_t\left(l_t - \widehat{l}_t\right), 0\right]_+ &= A_t\mathrm{E}\left[\left(l_t - \tau_t\right) - \left(\widehat{l}_t - \tau_t\right), 0\right]_+ \\ &= A_t\sigma_t\mathrm{E}\left[\frac{\left(l_t - \tau_t\right)}{\sigma_t} - \frac{\left(\widehat{l}_t - \tau_t\right)}{\sigma_t}, 0\right]_+. \end{aligned}$$

Let $\mathrm{E}\left[(Z-k), 0\right]_+ = \Psi\left(k\right)$ for a standard Normal random variable Z, and $k_t = \frac{\left(\widehat{l}_t - \tau_t\right)}{\sigma_t}$. We therefore have

$$\mathrm{E}\left[A_t\left(l_t - \widehat{l}_t\right), 0\right]_+ = A_t\sigma_t\Psi\left(k_t\right).$$

It can be shown that

$$\Psi(k) = \mathrm{E}\left[(Z-k), 0\right]_+ = \phi(k) - k\left[1 - \Phi(k)\right], \tag{16}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are, respectively, the pdf and cdf of a standard Normal random variable. Similarly,

$$\mathrm{E}\left[A_t\left(l_t - \widehat{l}_t\right) - C, 0\right]_+ = A_t\sigma_t\Psi\left(k_t + \frac{C}{A_t\sigma_t}\right).$$

Thus succinctly, equation (14) can be rewritten as

$$\mathrm{E}(C_t) = C - A_t\sigma_t\Psi(k_t) + A_t\sigma_t\Psi\left(k_t + \frac{C}{A_t\sigma_t}\right). \tag{17}$$

Substituting equation (17) in (13), we have the approximated pricing equation of the reverse mortgage survivor bond type 1,

$$V = Fv^T + \sum_{t=1}^{T} v^t\left[C - A_t\sigma_t\Psi(k_t) + A_t\sigma_t\Psi\left(k_t + \frac{C}{A_t\sigma_t}\right)\right]. \tag{18}$$

From equation (18), the survivor bond price can be easily calculated.

Following up on the previous example, let the annual interest rate r be 6.5%, annual house price appreciation c be 3%, the risk premium the lender charges λ_1 and is charged λ_2 be 3% and 1.5%, respectively, the initial loan amount Q_0 be \$50,000 and the house price H_0 be \$100,000,. Then, the L_t and A_t in each period are calculated in Tables (1) and (2).

Table 1: Single loss in each period

Period	Loss L_t	Period	Loss L_t	Period	Loss L_t
1	−588.33	14	−3590.22	27	16738.21
2	−691.29	15	−4059.30	28	18400.75
3	−807.79	16	2500.51	29	20206.01
4	−939.43	17	5935.67	30	22165.68
5	−1087.97	18	6651.23	31	24292.41
6	−1255.36	19	7431.25	32	26599.90
7	−1443.77	20	8281.12	33	29102.90
8	−1655.62	21	9206.63	34	31817.40
9	−1893.57	22	10214.08	32	34760.64
10	−2160.57	23	11310.25	36	37951.27
11	−2459.91	24	12502.48	37	41409.46
12	−2795.21	25	13798.72	38	45156.95
13	−3170.50	26	15207.54		

5.1.1 Projecting the mortality rate

The repayment rate of the contract is the sum of the mortality rate q_{x+t} and the mobility rate m_{x+t}. Following the illustrative example set by DiVenti and Herzog (1991), the mobility rate is assumed to be 30% of the mortality rate. If the overall repayment rate is denoted as q^*_{x+t}, we have

$$q^*_{x+t} = q_{x+t} + m_{x+t}$$

$$q^*_{x+t} = q_{x+t}(1 + 30\%)$$

Table 2: Appreciation of each loss in each period

Period	Appreciation A_t	Period	Appreciation A_t	Period	Appreciation A_t
1	−552.42	14	−2092.50	27	11173.30
2	−613.19	15	−2313.83	28	11712.80
3	−680.39	16	4093.37	29	12284.82
4	−754.69	17	7166.25	30	12891.65
5	−836.83	18	7475.87	31	13535.76
6	−927.60	19	7802.34	32	14219.77
7	−1027.90	20	8146.79	33	14946.54
8	−1138.70	21	8510.39	34	15719.13
9	−1261.07	22	8894.45	32	16540.83
10	−1396.21	23	9300.32	36	17415.19
11	−1545.42	24	9729.49	37	18346.04
12	−1710.12	25	10183.56	38	19337.48
13	−1891.91	26	10664.21		

To capture the time trend of the mortality, we need to treat mortality rate as a function of both age x and time t. Charupat and Milevsky (2002) attached a time-dependent random shock to the widely used Gompertz mortality model in actuarial literature (Frees, Carriere, and Valdez (1996)) and applied the model to determine optimal annuitization policies. Lee and Carter (1992) introduced a regression model for central mortality rates $m(t,x)$ which involves age-dependent and time-dependent terms. Renshaw, Haberman, and Hatzopoulos (1996) also suggested a Generalized Linear Model (GLM) for the force of mortality similar to the Lee and Carter's approach. Cairns, Blake, and Dowd (2004) provided an excellent review of the various mortality rate models and a theoretical framework to pricing mortality derivatives.

In this section, we use the Generalized Linear Model (GLM) suggested by Renshaw, Haberman, and Hatzopoulos (1996) to project the future mortality rates. According to the Renshaw model, the force of mortality $\mu_{x,t}$ is a log-linear function of age x and time t. Based on the Australian life table for 1881-2002, the model is calibrated as:

$$\mu_{x,t} = \exp\left(\beta_0 + \sum_{j=1}^{3} \beta_j L_j(x) + \sum_{i=1}^{2} \alpha_i t^i + \sum_{i=1}^{2}\sum_{j=1}^{3} \gamma_{i,j} L_j(x)\, t^i\right).$$

where $L_j(x)$ is the Legendre polynomial. In S-plus, a GLM regression model is fitted using the mortality rates from the Australian Life Table for 1881-2002. The regression results are listed in Table 3.

Future improvements in the force of mortality are calculated on a 5-year interval basis, as the mortality tables are mostly published every 5 years. In each 5-year interval, the mortality improvement is assumed to be linear. The projected improvements in the force of mortality are listed in Table 4.

With the above improvement factors, the corresponding trigger $\widehat{l}_t$ for each period can be easily obtained. The trigger values are calculated with formula (19).

$$\widehat{l}_t = \begin{cases} l_0\,({}_tp_x)\, e^{0.0014t} & \text{for } 0 < t \le 5 \\ l_0\,({}_tp_x)\, e^{0.007+0.00109(t-5)} & \text{for } 5 < t \le 10 \\ l_0\,({}_tp_x)\, e^{0.0125+0.0008(t-10)} & \text{for } 10 < t \le 15 \\ l_0\,({}_tp_x)\, e^{0.0164-0.00125(t-15)} & \text{for } 15 < t \le 20 \\ l_0\,({}_tp_x)\, e^{0.0102+0.00056(t-20)} & \text{for } 20 < t \le 25 \\ l_0\,({}_tp_x)\, e^{0.0130+0.01147(t-25)} & \text{for } 25 < t \le 30 \\ l_0\,({}_tp_x)\, e^{0.0703+0.026(t-30)} & \text{for } 30 < t \le 35 \\ l_0\,({}_tp_x)\, e^{0.2004+0.0189(t-35)} & \text{for } 35 < t \le 40 \end{cases}. \tag{19}$$

Table 3: Fitted parameters in the GLM model

Parameters	Value	Standard Error	t value
β_0	-2.08911420	0.2313983	-9.02821598
β_1	1.62026486	0.4471106	3.62385658
β_2	-0.06886849	0.5358593	-0.12851972
β_3	-0.01089401	0.4734314	-0.02301074
α_1	-0.31284970	0.2433404	-1.28564656
α_2	-0.31091752	0.4554777	-0.68261856
$\gamma_{1,1}$	0.09956416	0.4690509	0.21226730
$\gamma_{1,2}$	-0.05981924	0.5609072	-0.10664731
$\gamma_{1,3}$	-0.06188307	0.5008452	-0.12355728
$\gamma_{2,1}$	-0.02977038	0.8790215	-0.03386763
$\gamma_{2,2}$	-0.04404868	1.0510683	-0.04190849
$\gamma_{2,3}$	-0.08197422	0.9356919	-0.08760813
Residual Deviance	0.4061248		

Table 4: Projected improvement of the force of mortality

Age range	GLM $\hat{\mu}_{x,t}$	Current table $\hat{\mu}_{x,t}$ (2000-02)	Projected improvement	Improvement rate
$62-66$	0.0096	0.016603	0.007013	0.00140
$67-71$	0.0152	0.027621	0.012461	0.00109
$72-76$	0.0304	0.046858	0.016447	0.00080
$77-81$	0.0656	0.075844	0.010215	-0.00125
$82-86$	0.1171	0.130129	0.012998	0.00056
$87-91$	0.1189	0.189262	0.070341	0.01147
$92-96$	0.0417	0.242077	0.200356	0.02600
$97-101$	0.0027	0.297514	0.294837	0.01890

The results are listed in Table 5. The survivor bond price is calculated with a 1000-run simulation. The details of the calculation of the mortality bond price are listed in Table 6.

Table 5: Projected trigger values in each period

Period	Trigger $\widehat{l}_t$	Period	Trigger $\widehat{l}_t$	Period	Trigger $\widehat{l}_t$
1	986	14	677	27	152
2	973	15	639	28	124
3	958	16	599	29	99
4	942	17	557	30	79
5	924	18	514	31	61
6	904	19	470	32	48
7	883	20	427	33	37
8	859	21	384	34	28
9	834	22	342	32	21
10	807	23	300	36	15
11	778	24	259	37	11
12	746	25	220	38	8
13	712	26	183		

The results show prices for mortality bonds for a group of 1,000 loans with loan amount of $\$50,000$ per person and the trigger levels calculated in Table 5. The annual aggregate cash flow out of the SPC is $\$6,500,000$ and the coupon rate for both the straight bond and the survivor bond is 6.5%. The price

Table 6: Calculation of the mortality bond price (Type 1)

Number of loans	1000
Initial house value	$\$100,000$
Lump sum borrowed	$\$50,000$
Face value of straight bond	$\$100,000,000$
Face value of survivor bond	$\$100,000,000$
Coupon rate for both bonds	6.5% p.a.
Annual aggregate cash flow out of SPC	$\$6,500,000$
Straight bond price	$\$100,000,000$
Survivor bond price	$\$99,902,898$
Premium paid to SPC	$\$97,102$

of the survivor bond is \$999.02 per $\$1,000$ of face value. The total premium is actually quite small relative to the total amount of loans: only $97,102/(1000 \times 50,000)$ which equals 0.2%. For as long as 38 years of protection, this price may be considered inexpensive.

5.1.2 Sensitivity testing

To examine how sensitive the value of a survivor bond is to the mortality change, a random shock is attached to the projected force of mortality $\widehat{\mu}_{x,t}$. Suppose the distribution of mortality shocks ε_t at time t is a Beta distribution with parameters a and b. The mortality improvement shock ε_t is expressed as a percentage of the force of mortality $\widehat{\mu}_{x,t}$, so it ranges from 0 to 1, that is, $0 < \varepsilon_t < 1$ with probability 1. Without the shock, the projected survival probability $\hat{p}_{x,t} = e^{\left(-\widehat{\mu}_{x,t}\right)}$. With the shock, the new survival probability can be expressed as:

$$\hat{p}'_{x,t} = \exp\left(-\widehat{\mu}_{x,t}\right)^{1-\varepsilon_t} = \left(\hat{p}_{x,t}\right)^{1-\varepsilon_t}.$$

It is clear that the following holds

$$\hat{p}'_{x,t} \leq \left(\hat{p}_{x,t}\right)^{1-\varepsilon_t}.$$

After $10,000$ simulation trials, the impact of various mortality shocks is summarized in Table 7. The table lists how many survivors will remain in the portfolio after 20 years, and how much of the total value of the coupons and principal the investors will lose after the shocks. For example, when $a = 1.38$, $b = 26.30$, $\mathrm{E}[\varepsilon_t] = 0.05$, on average the investors will lose only 0.11% of the total value of the coupons and principal. The maximal loss is 0.13%.

The results show that the impact of mortality shocks is very limited in terms of the entire investment. Even in a scenario of a 50% mortality surprise, the investors lose $99,902,898 - 99,663,909$ which equals $\$238,989$ on average, which is less than 3.7% of the total value of expected coupons and 1% of the total value of expected coupons and principal. Therefore, this shows that there is very little chance of investors losing large amounts of coupons.

5.2 Case 2 - survivor bond type 2

In this case, the future coupon payments of the survivor bond are a function of the lender's aggregate loss amount in each period, which is affected by both the number of survivors and the single loss amount of each repaid loan. In this example, the values of trigger $\underline{\hat{L}}_t$ are set to be the 95% percentile of the aggregate loss distribution in each period. To take account of the randomness of mortality improvement, a 1% shock ε_t is attached to the projected force of mortality $\widehat{\mu}_{x,t}$. Following the example in the last

Table 7: Results of sensitivity testing (Type 1)

Shock ε_t	Statistic	l_{20}	PV of coupons and principal	Percentage change
1%	Min	429	99,790,111	−0.11%
	5% percentile	430	99,793,980	−0.11%
	95% percentile	432	99,795,946	−0.11%
	Max	435	99,796,337	−0.11%
	Mean	431	99,795,199	−0.11%
	Stdev	1	635	
5%	Min	435	99,768,435	−0.13%
	5% percentile	440	99,785,248	−0.12%
	95% percentile	453	99,795,809	−0.11%
	Max	465	99,797,493	−0.11%
	Mean	446	99,791,922	−0.11%
	Stdev	4	3,369	
10%	Min	444	99,744,368	−0.16%
	5% percentile	453	99,773,260	−0.13%
	95% percentile	481	99,798,092	−0.11%
	Max	504	99,786,999	−0.10%
	Mean	465	99,786,940	−0.12%
	Stdev	8	6,946	
25%	Min	465	99,663,820	−0.24%
	5% percentile	491	99,714,999	−0.19%
	95% percentile	569	99,783,484	−0.12%
	Max	641	99,793,361	−0.11%
	Mean	528	99,758,254	−0.14%
	Stdev	24	21,573	
50%	Min	411	99,482,588	−0.42%
	5% percentile	483	99,557,089	−0.35%
	95% percentile	699	99,751,549	−0.15%
	Max	825	99,774,291	−0.13%
	Mean	584	99,663,909	−0.24%
	Stdev	65	69,833	

section of a portfolio of l_0 loans borrowed by persons aged x with the identical house value H_0, in each period t, the bondholder will receive coupon

$$C_t = \begin{cases} C & \text{if } \underline{L}_t \leq \hat{\underline{L}}_t \\ C - \left(\underline{L}_t - \hat{\underline{L}}_t\right) & \text{if } \hat{\underline{L}}_t < \underline{L}_t \leq C \\ 0 & \text{if } \underline{L}_t > C \end{cases} .$$

This is equivalent to

$$C_t = C - \left[\left(\underline{L}_t - \hat{\underline{L}}_t\right), 0\right]_+ + \left[\left(\underline{L}_t - \hat{\underline{L}}_t\right) - C, 0\right]_+ .$$

Taking the expectation on both sides,

$$\mathrm{E}\left(C_t\right) = C - \mathrm{E}\left[\left(\underline{L}_t - \hat{\underline{L}}_t\right), 0\right]_+ + E\left[\left(\underline{L}_t - \hat{\underline{L}}_t\right) - C, 0\right]_+ . \tag{20}$$

Substituting Equation (20) into (13), the pricing equation of the survivor bond type 2 is

$$V = Fv^T + \sum_{k=1}^{T} v^k \left\{C - \mathrm{E}\left[\left(\underline{L}_t - \hat{\underline{L}}_t\right), 0\right]_+ + \mathrm{E}\left[\left(\underline{L}_t - \hat{\underline{L}}_t\right) - C, 0\right]_+\right\} \tag{21}$$

Since it is impossible to implicitly express the distribution of $\underline{L}_t$, numerical simulation techniques have to be used to price the bond. To illustrate, a numerical example follows.

5.2.1 A numerical example

Projecting interest rates and house prices Interest rate modeling is extensively covered in the finance and economics literature. In this example, the well-known one-factor stochastic Vasicek model is adopted to project the interest rate. The key feature of Vasicek interest model is mean reversion (Vasicek (1977)). Also in the Vasicek model, interest rate may actually become negative. The Vasicek model describes the short rate's Q dynamics by the following stochastic differential equation (SDE):

$$dr_t = \alpha\left(\beta - r_{t-1}\right)dt + \sigma_r dZ_t, \tag{22}$$

where Z_t is a standard Brownian motion. A discrete approximation can be expressed as

$$r_t - r_{t-\Delta t} = \alpha\left(\beta - r_{t-\Delta t}\right)\Delta t + \sigma_r \varepsilon_{r,t},$$

where $\varepsilon_{r,t} = \left(Z_{r,t} - Z_{r,t-\Delta t}\right)$, is Normally distributed with mean 0 and variance Δt. After fitting the model, the future interest rate can be projected as

$$r_{t_k} = \Delta t\widehat{\alpha}\widehat{\beta} + \left(1 - \Delta t\widehat{\alpha}\right) r_{t_k - \Delta t} + \widehat{\sigma}_r \varepsilon_{r,t},$$

where $\widehat{\alpha}$, $\widehat{\beta}$, and $\widehat{\sigma}_r$ are fitted parameters.

For the house price model, to distinguish between the diversifiable idiosyncratic price risk and the non-diversifiable systematic price risk, a specific and a general house price appreciation rate are introduced. The basic structure of the house price model is a geometric Brownian motion:

$$d\ln\left(H_t\right) = \mu_H dt + \sigma_H dZ_H.$$

where μ_H is the drift and σ_H is the volatility parameter. Using Ito's lemma, this SDE can be solved as

$$H_t = H_{t-1}\exp\left(\mu_H + \sigma_H Z_H\right),$$

where $Z_H \sim N\left(0,1\right)$. A discrete time approximation is

$$H_t = H_{t-\Delta t}\exp\left(\mu_H \Delta t + \sigma_H \varepsilon_{H,t}\sqrt{\Delta t}\right),$$

where $\varepsilon_{H,t}$ are iid (identically and independently distributed) standard Normal variable $N\left(0,1\right)$. With the projected house price, the general house appreciation rate c_t is calculated to represent the systematic part of the price risk:

$$c_t = \frac{H_t}{H_{t-1}} - 1.$$

As previously discussed in section 3, the idiosyncratic part of the house price is greatly affected by the borrower's maintenance behavior and regional economic fluctuation. To capture this characteristic, a random shock is attached to the general appreciation rate to determine the specific house appreciation rate c_t^s for each house:

$$c_t^s = c_t + \sigma_s Z_s,$$

where $Z_s \sim N\left(0,1\right)$. It is obvious that c_t^s is a normal variable with a mean of general house appreciation rate c_t and a variance σ_s^2. Finally each house has its own price process expressed as

$$H_t = H_{t-1}\left(1 + c_t^s\right).$$

Table 8: Fitted parameters for the Vasicek model

Parameters	Values
Mean	4.16%
α	0.5757
β	4.8825
σ_r	4.7891

Calibration of the models In calibrating the Vasicek interest rate model, we use the Australian 10-year government bond yields for the period 1980-2005. Using Ordinary Least Squares (OLS) estimation, the sum of the squares of the difference between the real interest rate and the modeled interest rate is minimized and the fitted results are listed in Table 8.

The house price model is calibrated using the quarterly median house prices from eight capital cities in Australia. Similarly using OLS estimation, the parameters for the house price model were calculated and the results are displayed in Table 9.

Table 9: Fitted parameters for the house price model

Parameters	Values
μ_H	0.034
σ_H	0.1003
σ_s	0.08

The risk premiums The premium for mortality risk can be calculated based on the annuity price data and the mortality table covering the same period. Using Wang transformation (Wang (1996), the transformed distribution of the future lifetime $F^*(t)$ for aged x can be expressed as

$$F^*(t) = g_\lambda [F(t)] = \Phi \left[\Phi^{-1}({}_tq_x) - \lambda\right], \tag{23}$$

where λ is the risk premium. Given the table of mortality rates ${}_tq_x$ (the original distribution) and annuity prices, $F^*(t)$ (the transformed distribution) can be solved, employing Equation (23), numerically to lead us to the risk premium or market price of mortality risk. However, in the absence of actual annuity price data, we simply assumed the risk premium for mortality risk is 2% in the following simulations.

Notice that the total premium for all the risks may not be equal to the sum of the premium for each different risk. Besides, the risk premiums may change over time. For simplicity, we assumed that the total risk premium for the lender is 7% to lend the money and 3.75% to finance the capital. The trigger values $\hat{\underline{L}}_t$ in each year are calculated as the average of $1,000$ simulation trials. The results are listed in Table 10.

The price of the survivor bond is calculated in Table 11. The results show the prices for mortality bonds for a group of $1,000$ loans with loan amount of \$$50,000$ per person and the trigger levels calculated above. The annual aggregate cash outflow of the SPC is \$$6,500,000$ and the coupon rate for both the straight bond and the survivor bond is 6.5% . The price of the survivor bond is \$985.04 per \$$1,000$ of face value. As we can see, the premium here is much larger than that of survivor bond type one, since much greater risks are involved in this case. But the total premium is still very small relative to the total amount of loans: $1,495,125/(1000 \times 50,000)$ which equals 2.99%.

5.2.2 Sensitivity testing

To examine the impact of mortality improvement in some extreme scenario, the value of the shock parameter ε_t is increased. The results of sensitivity testing are summarized in Table 12.

Table 10: The projected triggers with simulation

Period	Trigger $\hat{\underline{L}}_t$	Period	Trigger $\hat{\underline{L}}_t$	Period	Trigger $\hat{\underline{L}}_t$
1	0	14	2,281,434	27	17,132,581
2	0	15	2,907,624	28	17,435,844
3	0	16	3,681,598	29	17,452,028
4	0	17	4,446,271	30	17,168,945
5	0	18	5,419,246	31	15,990,877
6	0	19	6,587,557	32	15,343,864
7	0	20	7,955,241	33	13,715,978
8	116,142	21	9,333,113	34	11,531,379
9	278,462	22	11,068,592	32	10,051,137
10	446,561	23	12,229,021	36	8,538,117
11	837,594	24	13,476,656	37	6,651,498
12	1,182,790	25	15,116,196	38	4,828,433
13	1,641,429	26	16,262,309		

Table 11: Calculation of the survivor bond type 2 price

Number of loans	1000
Initial house value	\$100,000
Lump sum borrowed	\$50,000
Face value of straight bond	\$100,000,000
Face value of survivor bond	\$100,000,000
Coupon rate for both bonds	6.5% p.a.
Annual aggregate cash flow out of SPC	\$6,500,000
Straight bond price	\$100,000,000
Survivor bond price	\$98,504,875
Premium paid to SPC	\$1,495,125

The results show that the impact of mortality shock is (again) very limited in terms of the whole investment. Because in this case other risk variables are not controlled, it is difficult to tell exactly how much the value changes can be attributed to simply the mortality improvement. However, the results show no matter how severe the mortality shock is, the present value of the survivor bond does not change much, which illustrates that the present value is not extremely sensitive to mortality improvements.

6 Concluding remarks and discussion

The reverse mortgage is a promising financial product with many possible economic benefits to both the consumers and the suppliers. The market for reverse mortgages, as indicated in the recent past, has matured and again, triggered by continued longevity, the product is increasingly popular in recent years among retirees. However, due to the various risks involved in reverse mortgages, especially the longevity risk component, the development of the product has to some extent been stunted. In this paper, we suggest using securitization to deal with the risks to the lender, particularly the longevity risk component.

After a brief introduction of securitization and reverse mortgage product, we discussed and proposed a securitization model similarly used by Lin and Cox (2005b) to handle the longevity risk component in a reverse mortgage. In our first example of survivor bond type 1, the interest rate and house appreciation rate are assumed to be constant to emphasize the effect of longevity securitization. We find that through the securitization transaction, the lender can achieve a long time protection with relatively inexpensive

Table 12: Results of sensitivity testing (survivor bond type 2)

Shock ε_t	Statistic	PV of coupons and principal	Percentage change
5%	Min	59,169,754	−39.93%
	5% percentile	88,218,673	−10.44%
	95% percentile	100,000,000	1.52%
	Max	100,000,000	1.52%
	Mean	98,250,787	−0.26%
	Stdev	5,346,482	
10%	Min	59,927,863	−39.16%
	5% percentile	86,700,844	−11.98%
	95% percentile	100,000,000	1.52%
	Max	100,000,000	1.52%
	Mean	98,196,112	−0.31%
	Stdev	1,803,888	
25%	Min	61,486,654	−37.58%
	5% percentile	86,745,006	−11.94%
	95% percentile	100,000,000	1.52%
	Max	100,000,000	1.52%
	Mean	98,070,043	−0.44%
	Stdev	5,708,053	
50%	Min	61,880,827	−37.18%
	5% percentile	87,868,901	−10.80%
	95% percentile	100,000,000	1.52%
	Max	100,000,000	1.52%
	Mean	98,267,554	−0.24%
	Stdev	1,732,446	

premium. The sensitivity analysis also reveals that even a dramatic 50% mortality improvement shock will only result in the investors losing less than 3.7% of the total value of expected coupons. Therefore there is small likelihood that investors can lose large amounts of coupons.

The similar results occur in survivor bond type 2, in which the randomness of interest rates and house prices are taken into account. Because additional risks are involved, the insurance premiums become more expensive than the case of the survivor bond of type 1. However, compared to the total loan amount of the reverse mortgage portfolio, it still only amounts to an extra 2.99%. The investor can still expect little loss of the total expected coupons even when mortality is significantly improved.

Our results indicate that the mortality securitization is a good method to control the longevity risk in reverse mortgages. Given the many benefits of mortality securitization, we believe that it can help further the future development of reverse mortgage products in the capital market.

The results and the methodology employed in this paper are not without limitations. For simplicity, the correlation between the interest rates and the house prices are ignored, although our initial empirical investigation indicated possible significant linear correlation between the two variables. Mortality pattern used was that of the general population which may be different from that of buyers of reverse mortgages. Furthermore, in this paper, only survivor bonds are priced. Some researchers believe that survivor swap provides for certain advantages over survivor bonds and can be applied between the reverse mortgage lender and the life insurer. It would be interesting to see how a survivor swap is priced in reverse mortgages. Dawson (2002), Dowd, Blake, Cairns, and Dawson (2006), and Lin and Cox (2005b) are useful references on this topic. Finally, a copula approach could also be applied to model the correlation between the interest rate and the house price. This would allow the researcher to investigate the effects of the correlation on the pricing and the risk management of reverse mortgages.

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